



#### Modeling Deformable Surfaces from Single Videos

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# **Talk Outline**



#### 2D Deformable Surfaces

- Problem Formulation.
- Fast Matching.
- Robust Optimization Scheme.
- Illumination Correction.

#### • 3D Deformable Surfaces

- Linear Formulation.
- Inextensible surfaces.
- Sharply folding surfaces.
- Eliminating the reference image.





#### **2D Deformable Surfaces**

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#### Estimating:

- Deformations
- Lighting parameters
- Occlusions

Pilet et al., IJCV 2008





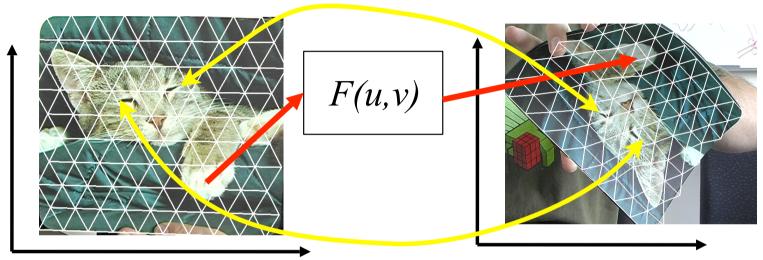
# **2D Deformable Surfaces**

- Problem Formulation
- Fast Matching
- Robust Optimization
- Lighting Correction





### **Problem Formulation**



- Input:
  - Correspondences between a **reference** and **input image**.
  - No a priori pose information.
- Output:
  - A mapping **F** from model to input image.





# Challenges

Non-rigid deformation without a priori pose:

- High dimensionality (200+ DOF)
- Large search space
- Wide baseline matching

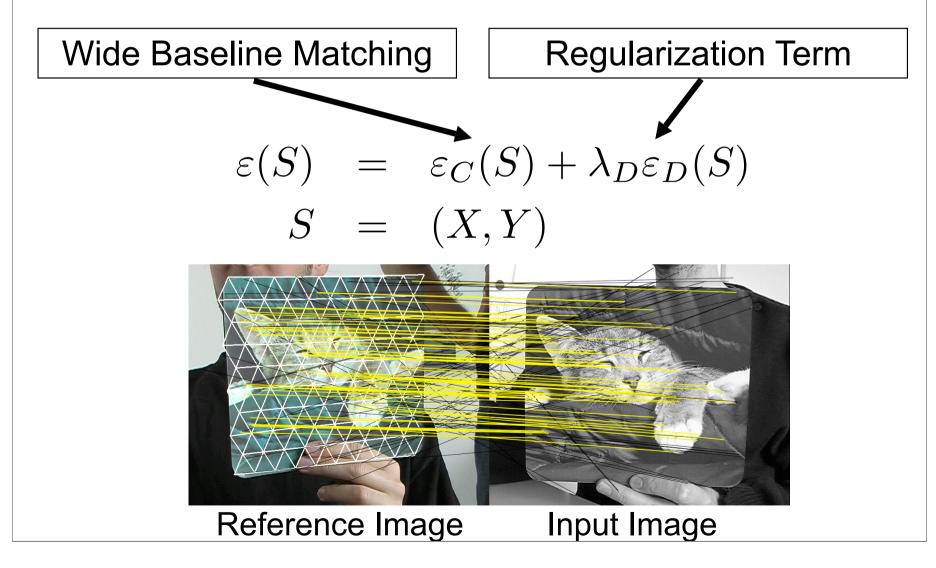
Real-time requirements:

- Fast optimization scheme
- Fast matching





# **Deformable Model**







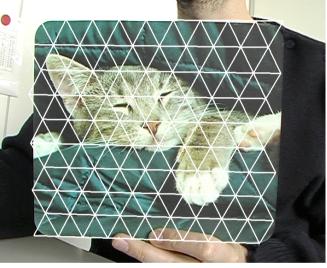
# $\varepsilon_D$ Regularization Term

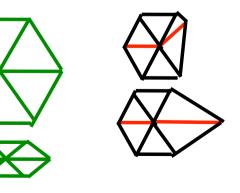
Quadratic function vertex coordinates

$$\varepsilon_D(S) = \frac{1}{2} \left( X^T K X + Y^T K Y \right)$$

- penalizes non uniform scaling;
- penalizes excessive bending;
- allows perspective distortion;
- allows smooth surface deformation.



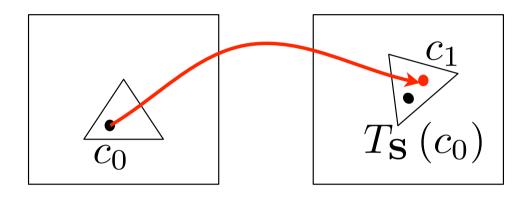


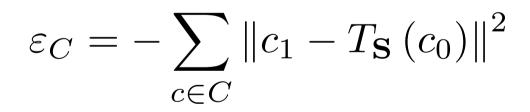






#### $\epsilon_c$ Correspondence Term

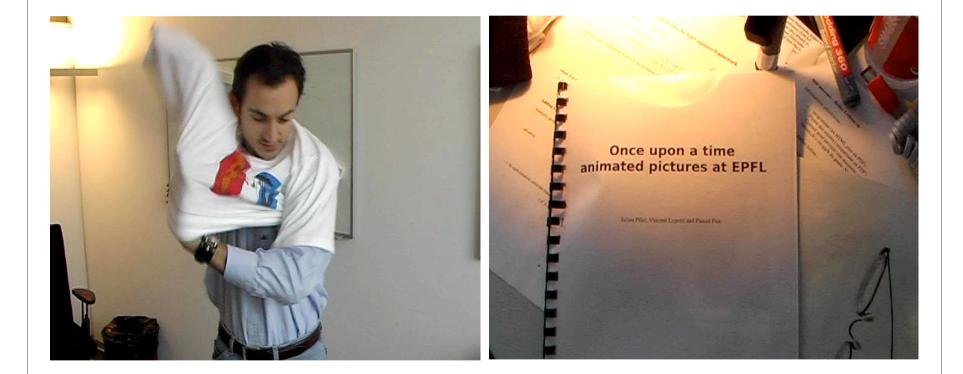








#### **Real-Time Augmentation**







# **Key Ingredients**

- Classification-based approach to matching.
- Robust minimization scheme.
- Intensity ratios for illumination correction.





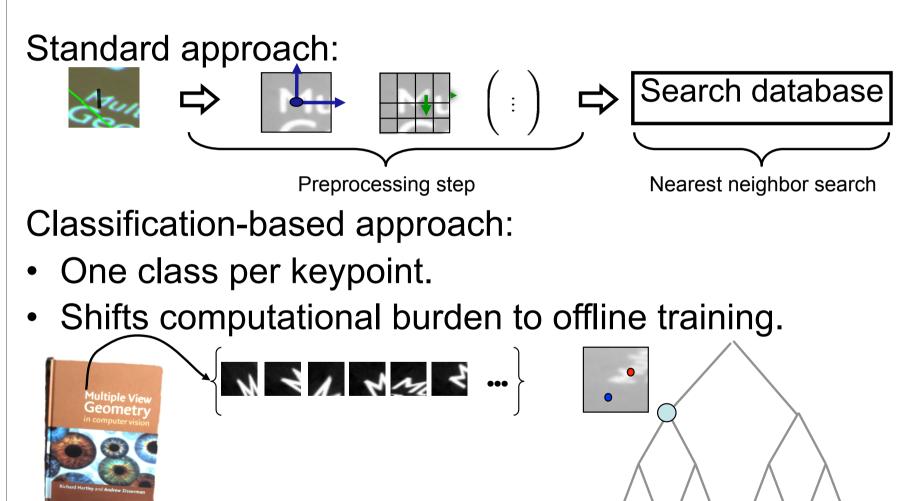
# **2D Deformable Surfaces**

- Problem Formulation
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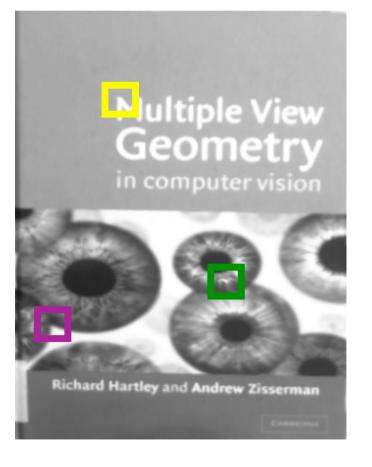
# ε<sub>C</sub> Correspondence Term







# **Binarized Tests for Keypoint Matching**



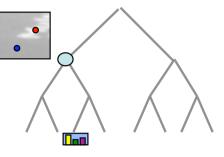






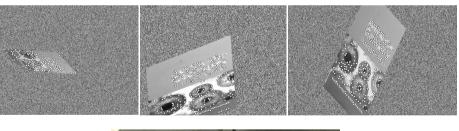
# **Randomized Tree**

Generic tree: The nodes contain simple tests of the form "Is  $I(m_1) > I(m_2)$ ?"



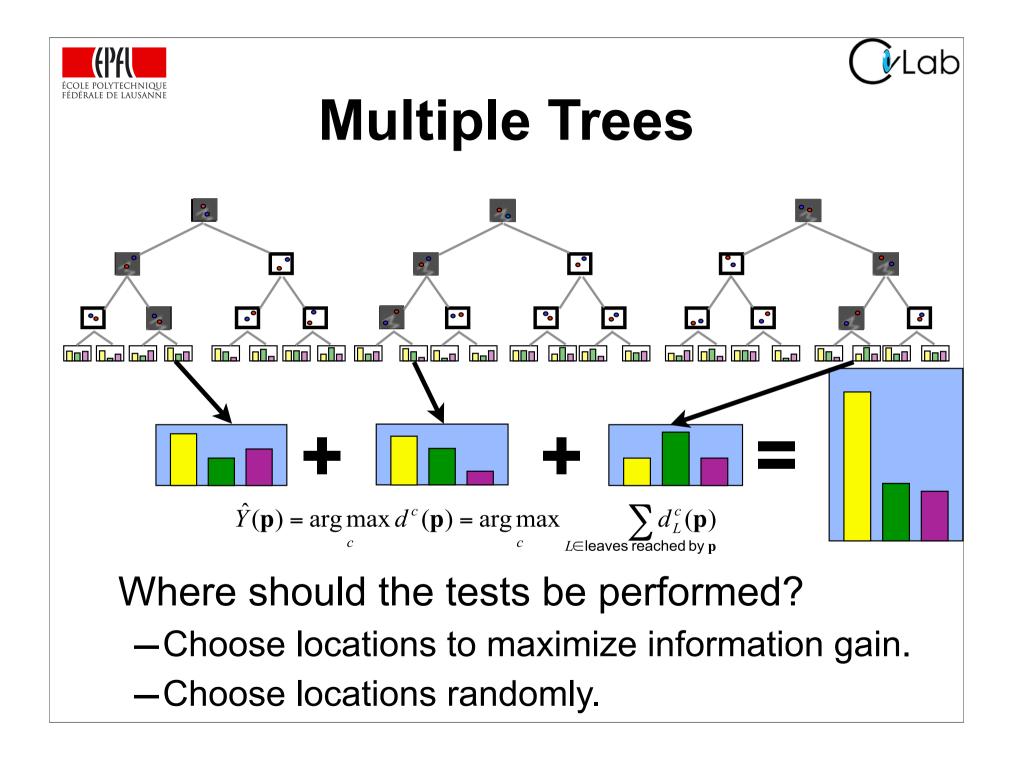
Posterior probabilities can be learnt from:

• synthetically warped images





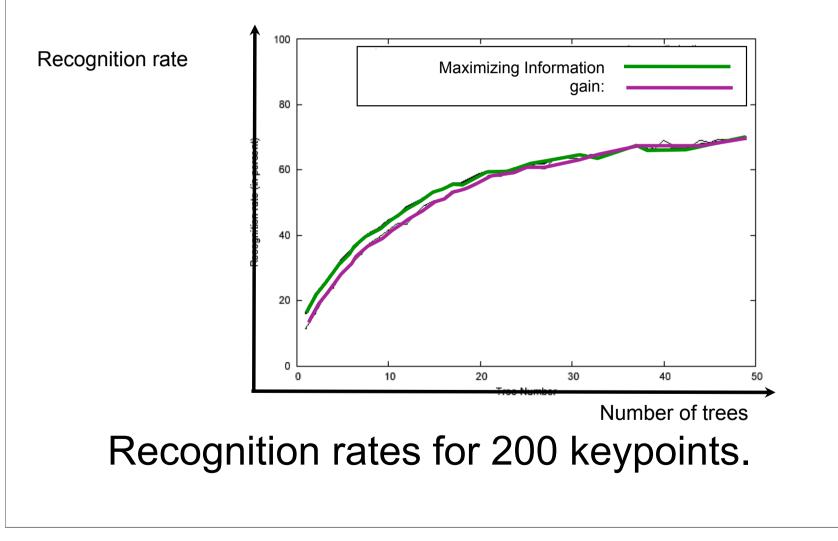
• video sequences

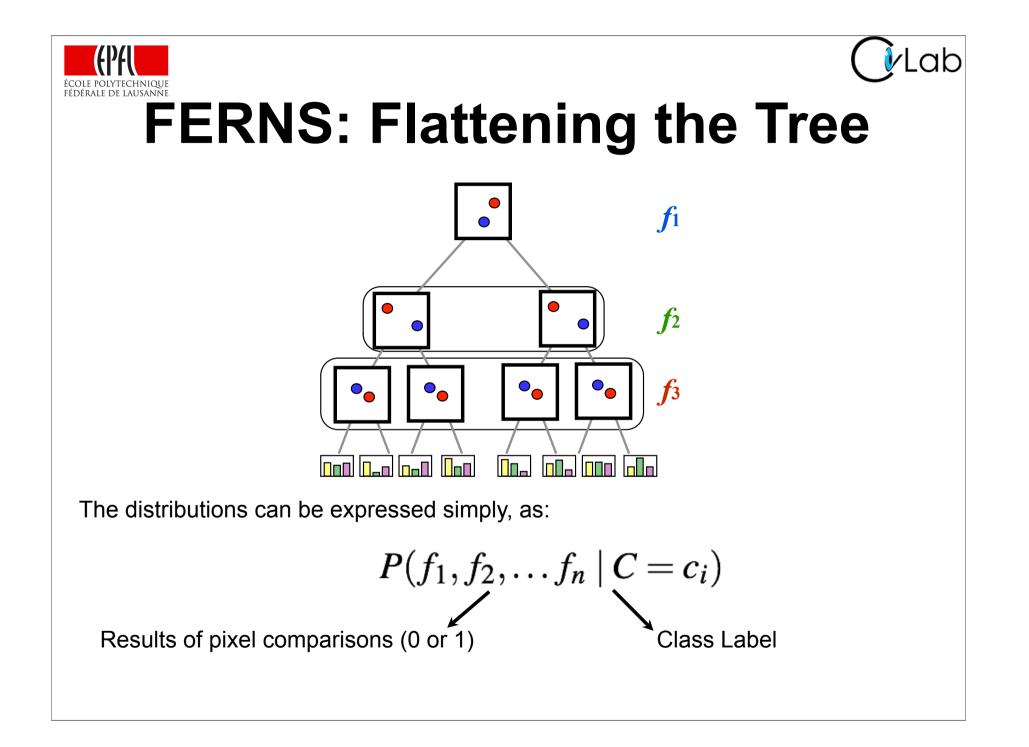






# **Random vs Optimized Locations**







# **Bayesian Interpretation**

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We are looking for:

$$P(C = c_i \mid f_1, f_2, \cdots f_n, f_{n+1}, \cdots \cdots f_N)$$

proportional to

$$P(f_1, f_2, \cdots f_n, f_{n+1}, \cdots \cdots f_N \mid C = c_i)$$

but complete representation of joint distribution infeasible.

Naive Bavesian:

$$\approx \prod_{j} P(f_j \mid C = c_i)$$

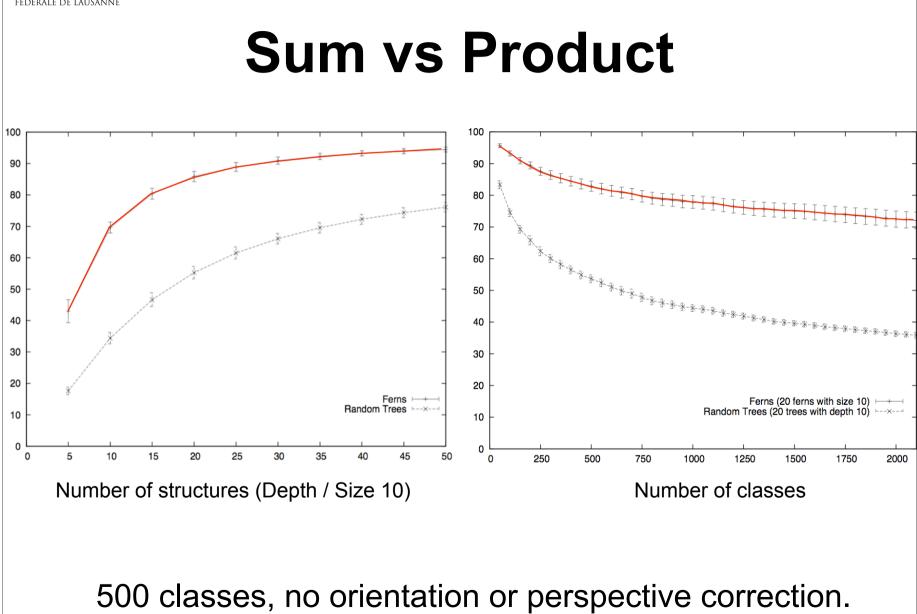
Compromise:

$$\approx P(f_1, f_2, \cdots f_n \mid C = c_i) \bigotimes P(f_{n+1}, \cdots f_{2n} \mid C = c_i) \bigotimes \cdot$$

--> probabilities stored in the leaves.



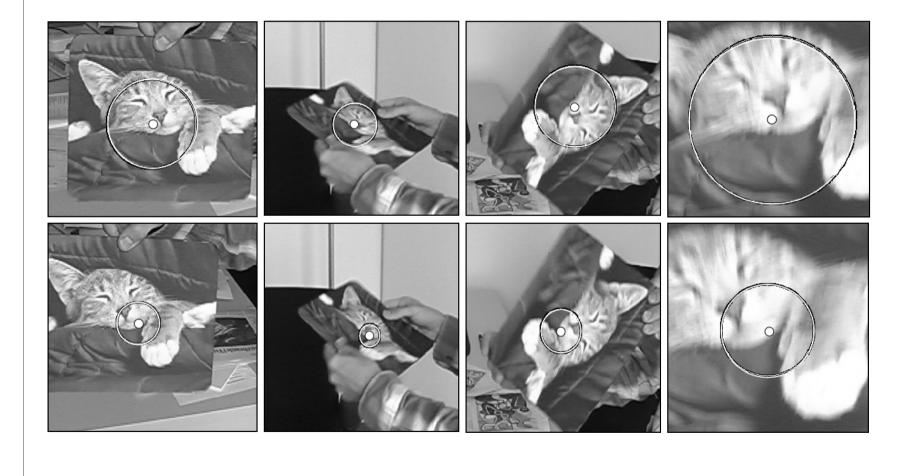








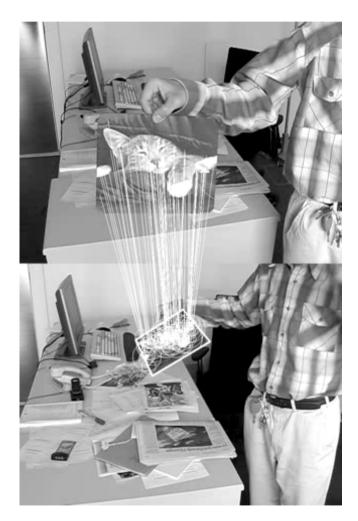
### **Scale and Orientation Invariance**







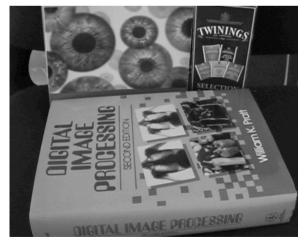
#### **Planar or Not**



#### Reference image vs Input Images



Reference video



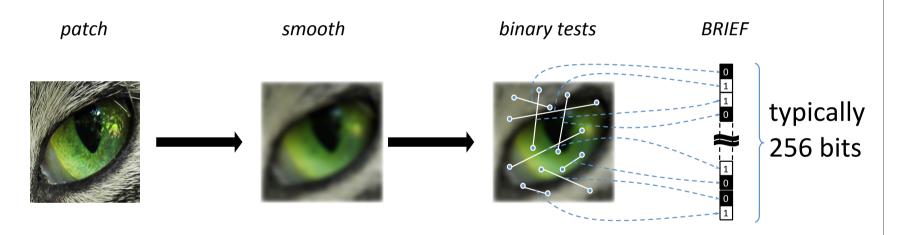
Input Images





# BRIEF

Very simple computation that can be seen as computing gradients:



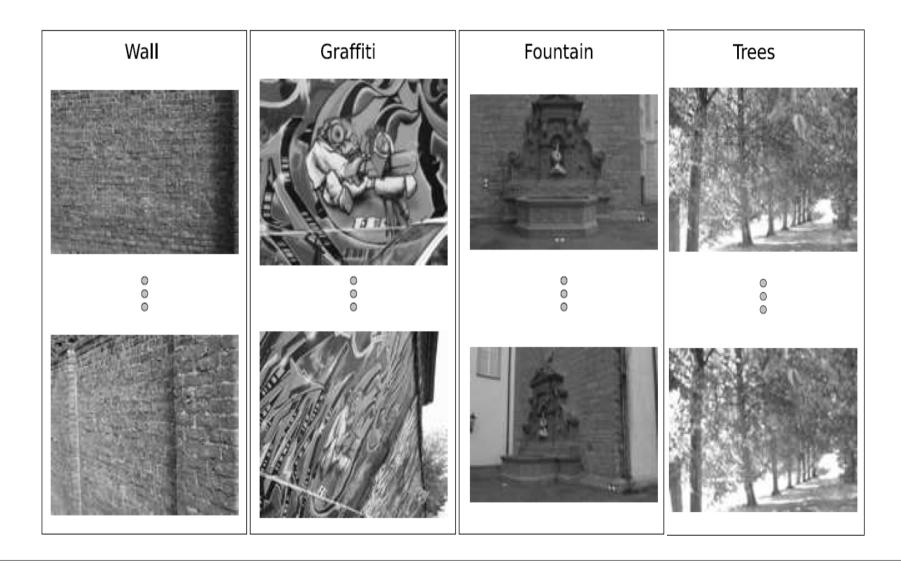
- Most smooth kernels work, even simple box filters.
- 128, 256, or 512 binary tests usually suffice.
- Random arrangment of tests effective iff evenly sampled.
- Not rotationally invariant.

Calonder et al. ECCV'10





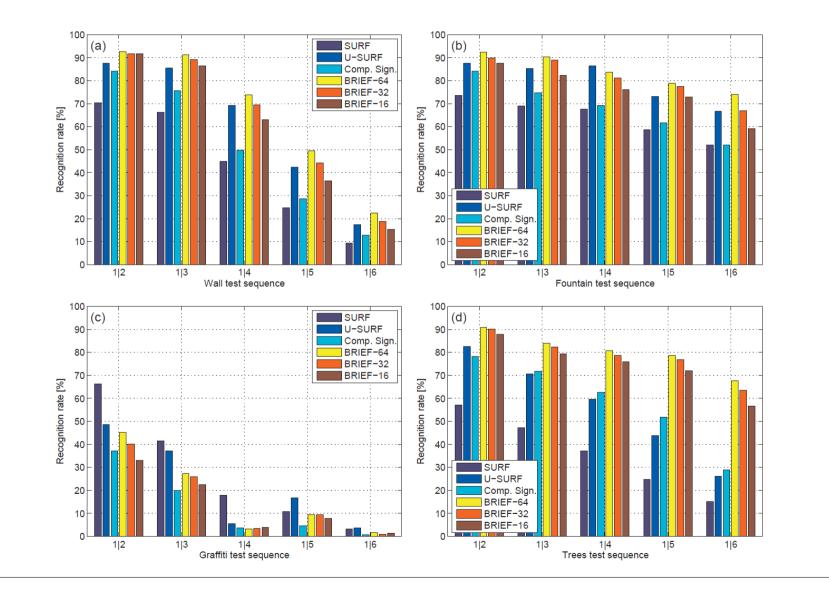
#### **Benchmarks Datasets**







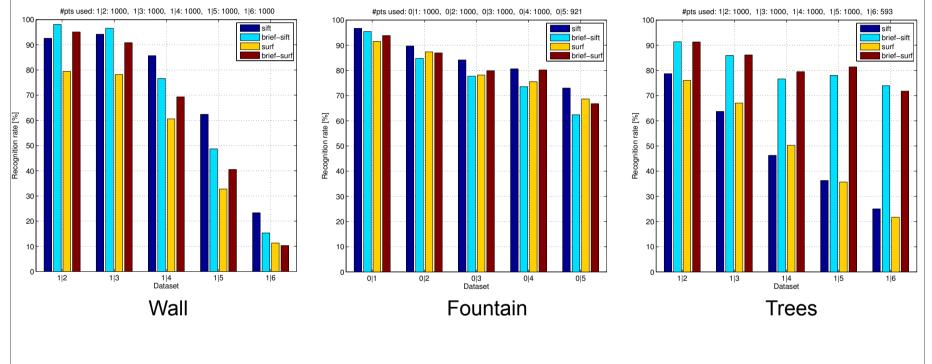
#### **BRIEF vs SURF**





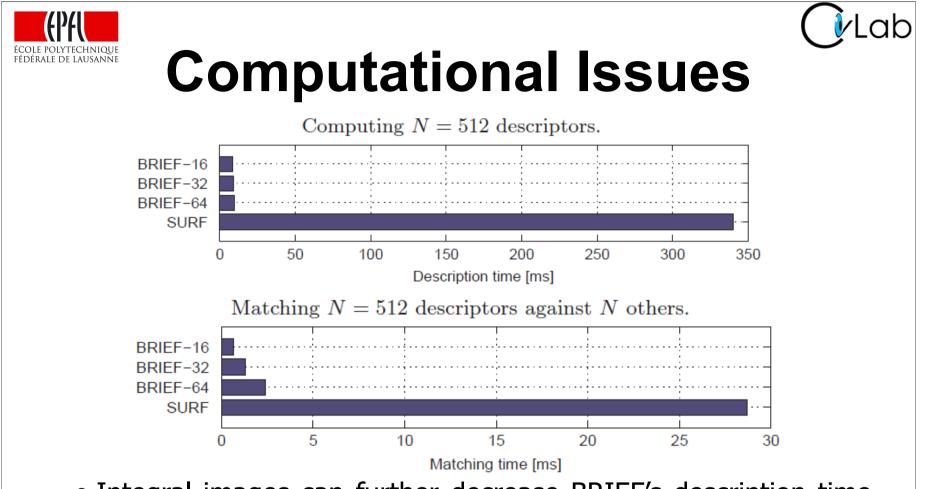


#### **BRIEF vs SIFT**



#### SIFT > BRIEF > SURF.

Be careful about interpreting benchmarks!



- Integral images can further decrease BRIEF's description time by making smoothing faster.
- Intel Core i7 CPU's POPCOUNT instruction will drastically speed-up the matching of binary vectors.
- Scale and rotational invariance need to be added in some cases.





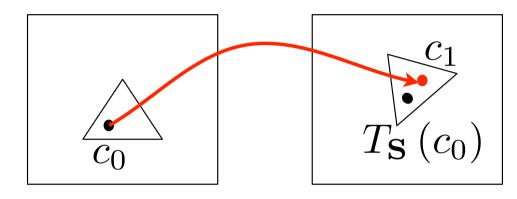
# **2D Deformable Surfaces**

- Problem Formulation
- Fast Matching
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- Lighting Correction





#### $\epsilon_c$ Correspondence Term



$$\varepsilon_C = -\sum_{c \in C} \left\| c_1 - T_{\mathbf{S}} \left( c_0 \right) \right\|^2$$

Not robust to outliers!





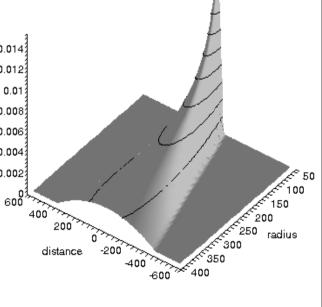
# **Robustness to Mismatches**

$$\varepsilon_{C} = -\sum_{c \in C} w_{c} \rho \left( \left\| c_{1} - T_{\mathbf{S}} \left( c_{0} \right) \right\|, r \right)$$

where  $\rho$  is a robust estimator whose radius of confidence is r and  $w_c \in [0, 1]$  a weight associated to each correspondence.

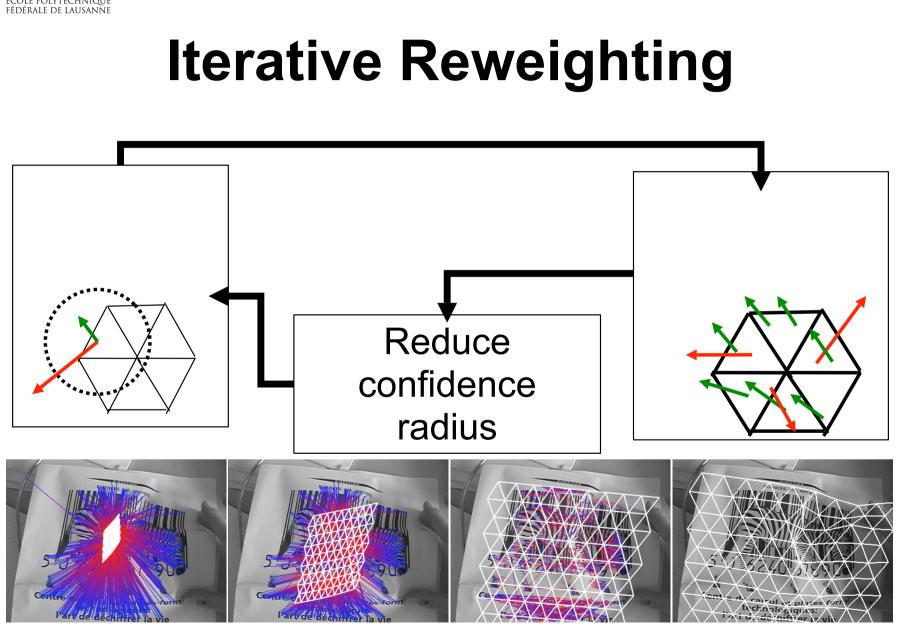
$$\rho\left(\delta,r\right) = \begin{cases} \frac{3\left(r^2 - \delta^2\right)}{4r^3} & \delta < r\\ 0 & otherwise \end{cases}$$

$$\int_{-\infty}^{\infty} \rho(x,r) dx = 1 \quad \forall r > 0$$







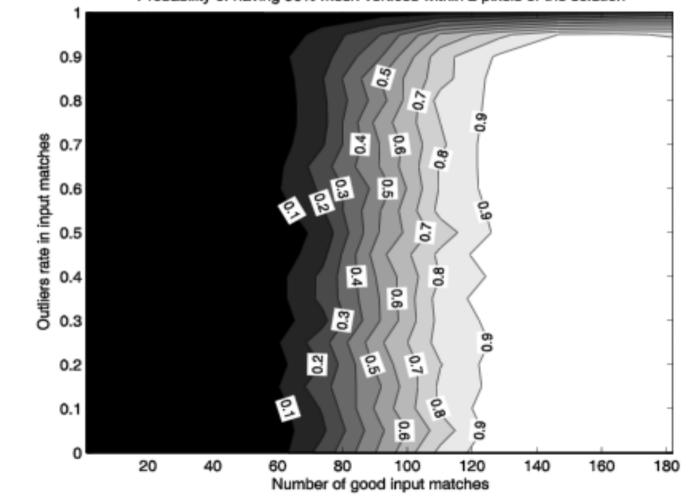






### **Gauging Robustness**

Probability of having 90% mesh vertices within 2 pixels of the solution







# **Visualizing the Deformations**









Minimize:

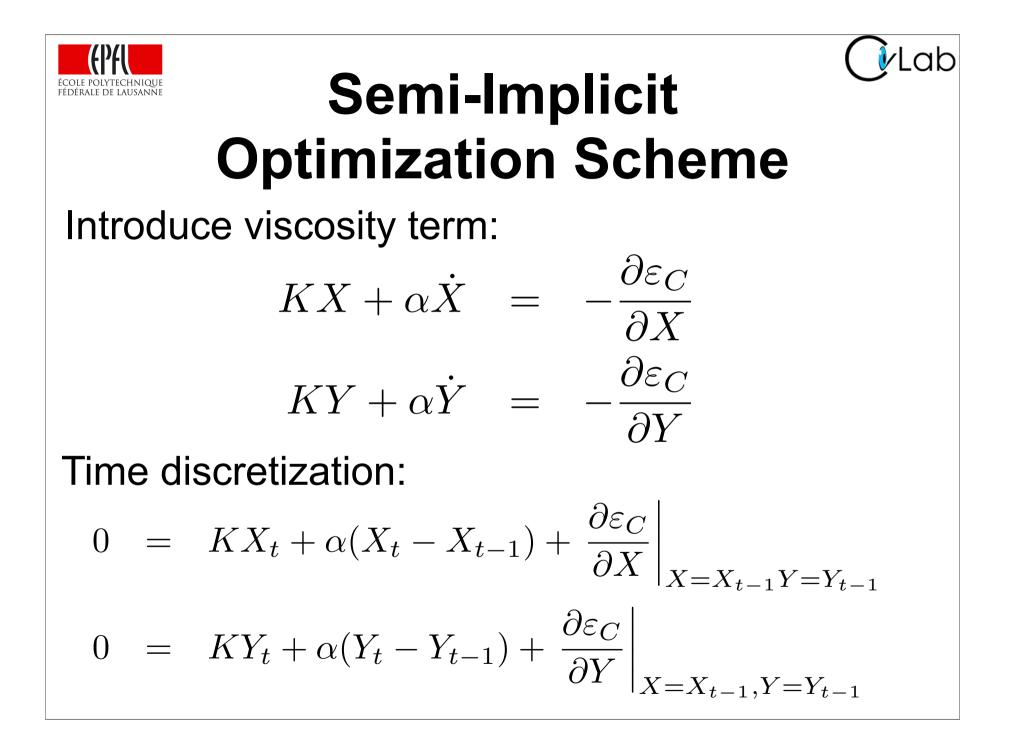
$$\varepsilon(S) = \lambda_D \varepsilon_D(S) + \varepsilon_C(S)$$
  
$$\varepsilon_D(S) = \frac{1}{2} \left( X^T K X + Y^T K Y \right)$$

Satisfied when:

$$0 = \frac{\partial \varepsilon}{\partial X} = \frac{\partial \varepsilon_C}{\partial X} + KX$$
$$0 = \frac{\partial \varepsilon}{\partial Y} = \frac{\partial \varepsilon_C}{\partial Y} + KY$$

M. Kass, A. Witkin, and D. Terzopoulos. Snakes: Active Contour Models. IJCV, 1988.

Lab





## Semi-Implicit Optimization Scheme

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Solve at each iteration:

$$(K + \alpha I)X_t = \alpha X_{t-1} - \frac{\partial \varepsilon_C}{\partial X}\Big|_{X = X_{t-1}, Y = Y_{t-1}}$$
$$(K + \alpha I)Y_t = \alpha Y_{t-1} - \frac{\partial \varepsilon_C}{\partial Y}\Big|_{X = X_{t-1}, Y = Y_{t-1}}$$

--> Fast because K has only a few non zero diagonals.



#### Taylor expansion of data term:

$$\begin{split} \varepsilon(X,Y) &= \lambda_D \varepsilon_D(X,Y) + \varepsilon_C(X,Y) \\ \varepsilon_D(X,Y) &= \frac{1}{2} \left( X^T K X + Y^T K Y \right) \\ \varepsilon_C(X+dX,Y+dY) &= A + B d X + C d Y + \frac{1}{2} d X^t D d X + \frac{1}{2} d Y^t E d Y \\ \text{Zhu and Lyu, ECCV'07} \end{split}$$

#### Newton **Optimization Scheme** At the minimum:

$$0 = \frac{\partial \varepsilon}{\partial X} = B + DdX + K(X + dX)$$
$$0 = \frac{\partial \varepsilon}{\partial Y} = C + EdY + K(Y + dY)$$

Solve at each iteration:

$$(K+D)dX = -B - KX$$
$$(K+E)dY = -C - KY$$



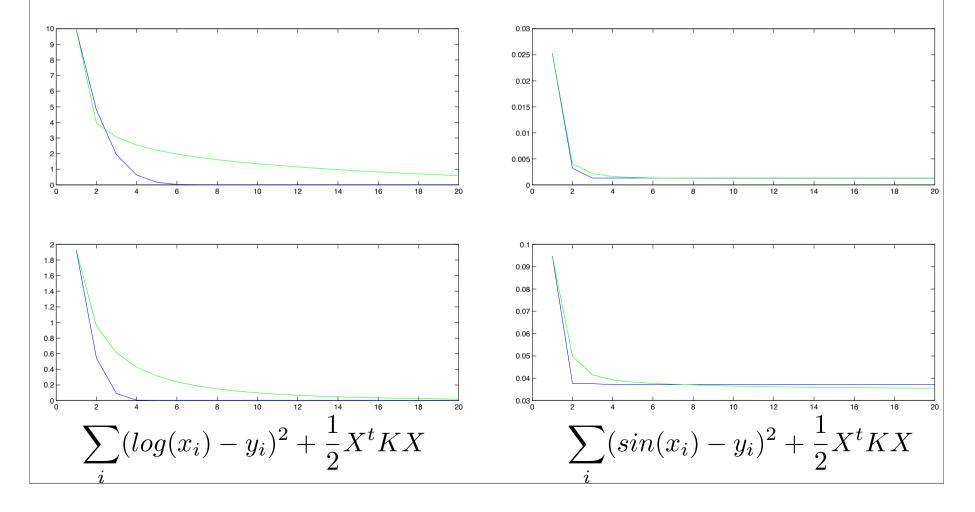






#### **Semi-Implicit vs Newton**

Residuals as a function of the number of iterations: Semi-Implicit in green and Newton in blue.







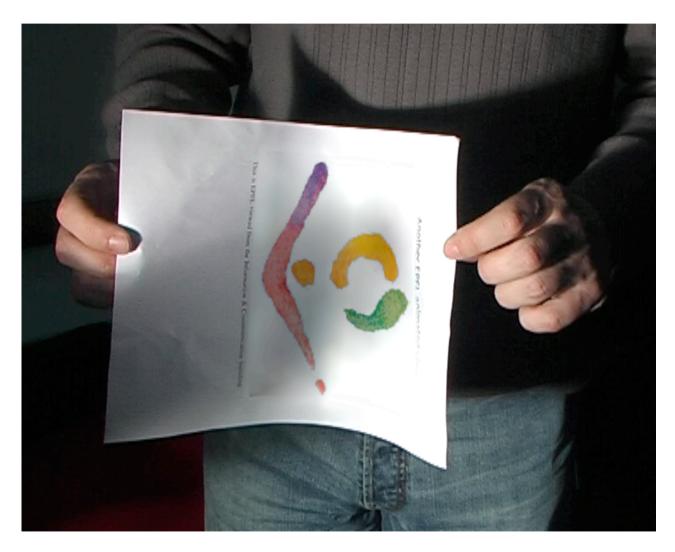
## **2D Deformable Surfaces**

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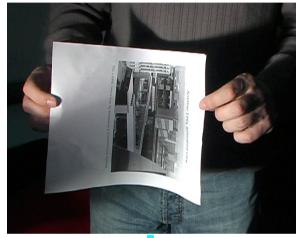


### **Intensity Ratios**

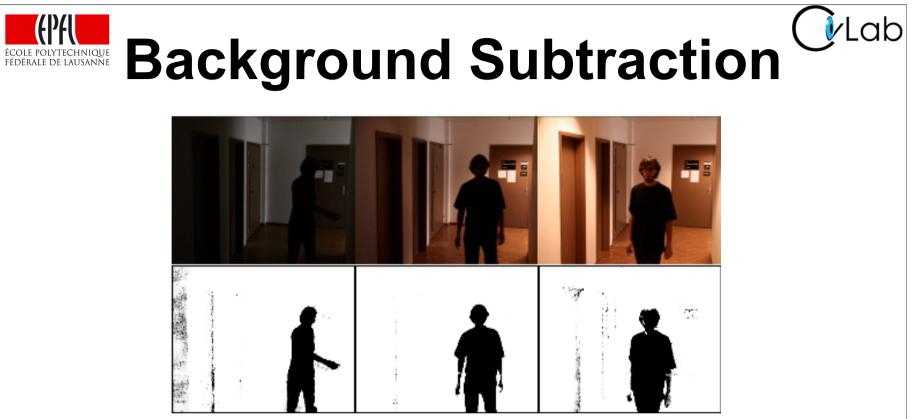
Reference image:  $I_{r,p} = L_r A_p$ Input image:  $I_{i,p} = L_{i,p} A_p$ 

White image:  $I_{r,w} = L_r A_w$ Synthetic image:  $I_{x,p} = L_{i,p} A_w$ 

$$= A_w L_r \frac{I_{i,p}}{I_{r,p}}$$
$$= I_{r,w} \frac{I_{i,p}}{I_{r,p}}$$







Standard approach:

• Pixel-wise statistical background model.

Modified approach:

- Account for the fact that illuminations changes tend to be correlated.
- Model variations of intensity ratios as GMMs.

#### --> Effective for occlustion detection.





#### **Realistic Augmentation**







#### **3D Deformable Surfaces**

Reference









# **Problem Formulation**

#### Input:

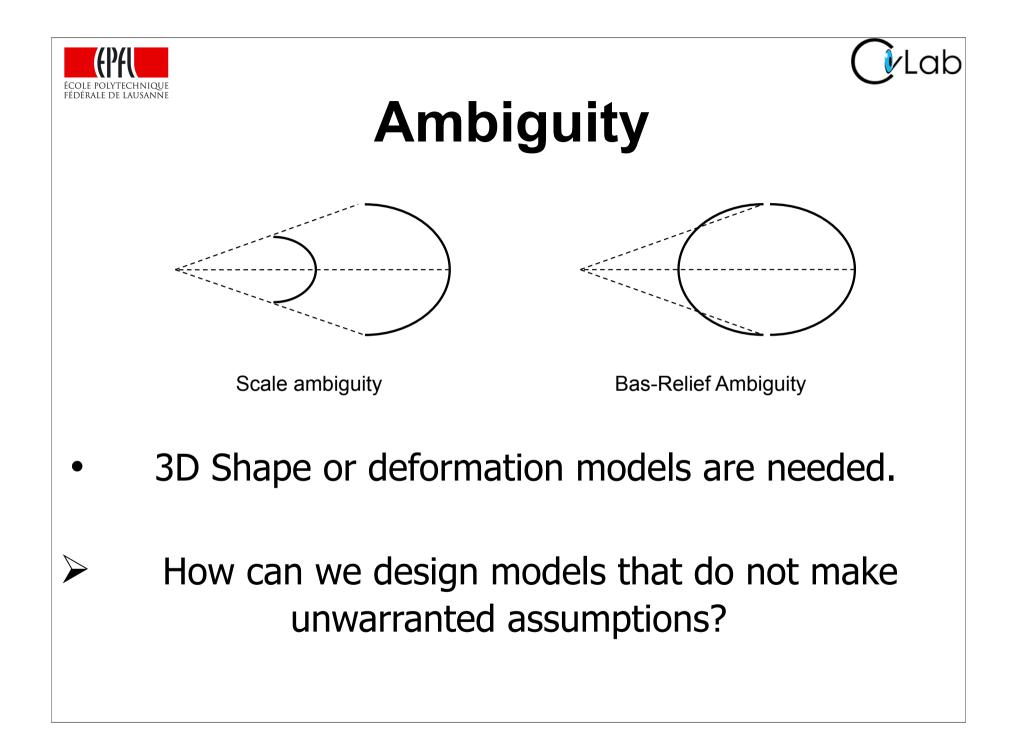
- Reference image.
- Corresponding 3D surface.
- Projection matrix P.
- 3D-to-2D correspondences between reference configuration and input image.



#### Unknowns:

Mesh vertex coordinates corresponding to input image

 $\mathbf{X} = [x_1, y_1, z_1, \cdots, x_{n_v}, y_{n_v}, z_{n_v}]^T$ 







#### **3D Deformable Surfaces**

- Linear Formulation.
- Inextensible surfaces.
- Sharply folding surfaces.
- Eliminating the reference image.
- Applications.





#### **Linear Formulation**

- Calibrated camera, A intrinsic parameters matrix.
- Coordinates expressed in the camera referential.
- Unknown mesh vertex coordinates:  $\mathbf{X} = (\mathbf{v}_1^T, \dots, \mathbf{v}_{n_v}^T), \ \mathbf{v}_i = (x_i, y_i, z_i)^T$
- Correspondences
  - Barycentric coordinates from reference configuration:  $(a_i, b_i, c_i)$
  - Current image location:  $(u_i, v_i)^T$

 $\begin{pmatrix} u_i \\ v_i \\ 1 \end{pmatrix} = \frac{1}{k_i} \mathbf{A} (a_i v_1 + b_i v_2 + c_i v_3)$ Salzmann et al., CVPR'07





# **Linear Formulation**

$$\mathbf{A} \left( b_1 \mathbf{v}_1 + b_2 \mathbf{v}_2 + b_3 \mathbf{v}_3 \right) = k \begin{bmatrix} u \\ v \\ 1 \end{bmatrix}$$

 $\rightarrow k$  can be expressed in terms of the vertex coordinates using the last row.

$$\begin{bmatrix} b_1\mathbf{H} & b_2\mathbf{H} & b_3\mathbf{H} \end{bmatrix} \begin{bmatrix} \mathbf{v}_1 \\ \mathbf{v}_2 \\ \mathbf{v}_3 \end{bmatrix} = \mathbf{0} ,$$

with

$$\mathbf{H} = \mathbf{A_{2\times 3}} - \left[\begin{array}{c} u \\ v \end{array}\right] \mathbf{A_3} ,$$

where  $A_{2\times 3}$  contains the first two rows of A, and  $A_3$  is the third one.

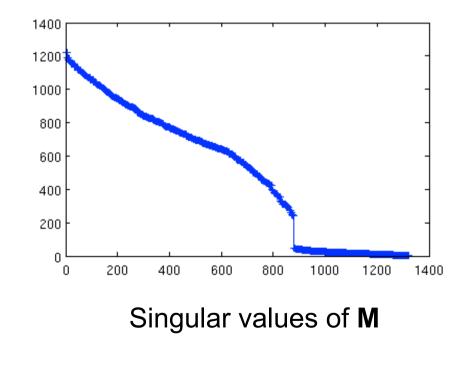
--> Each correspondence gives rise to two linear equations.





## Linear System and Singular Values

**X** must be solution of **MX** =0







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# **Inextensible Meshes**

A solution of the linear system belongs to the kernel of  $\boldsymbol{M}$  :

$$\mathbf{M}\mathbf{X} = 0 \implies \mathbf{X} = \sum_{i} \beta_{i} \mathbf{p}_{i} ,$$

where the  $\mathbf{p}_i$  are the eigenvectors corresponding to small eigenvalues.

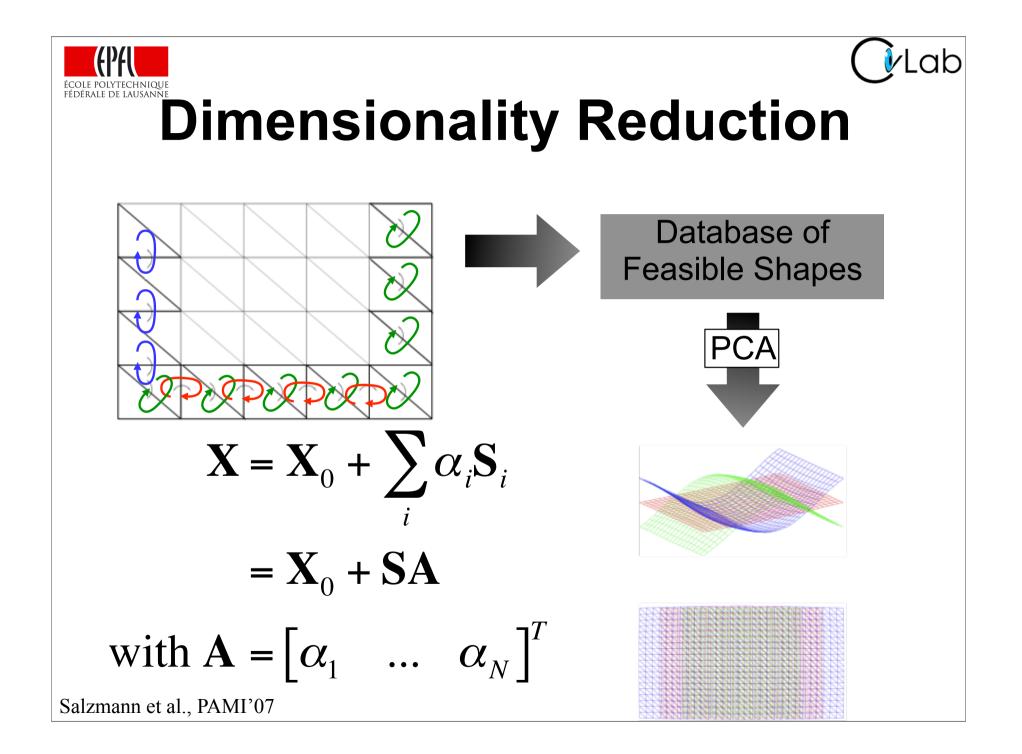
Inextensible mesh:

$$\left\|\sum_{i}\beta_{i}\mathbf{p}_{i}^{j}-\sum_{i}\beta_{i}\mathbf{p}_{i}^{k}\right\|^{2}=\text{cte}$$

for all neighboring vertices j and k.

--> A system a quadratic equations that could be solved in closed form using extended linearization, but with too many variables for existing solvers.

Salzmann et al., ECCV'08







# **Degrees of Freedom**

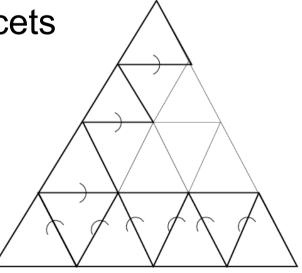
For an inextensible triangulation with V vertices, E=Ei+Eb edges, and F facets with no holes:

-Euler formula

V+F-E=1 .

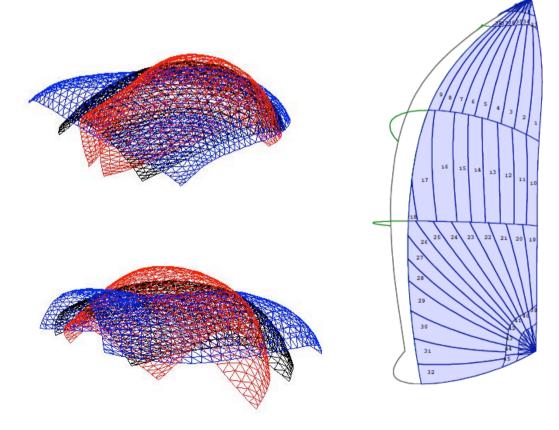
- Interior edges shared by two facets
   3F=2Ei+Eb.
- -Degrees of freedom

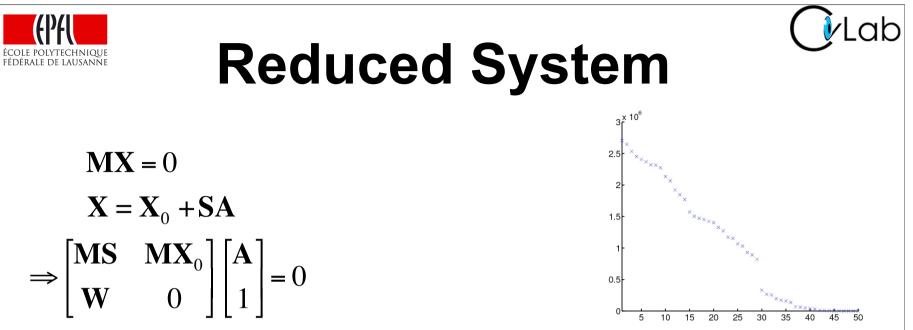
3V-E=6+Eb



#### **Spinnaker Modes**







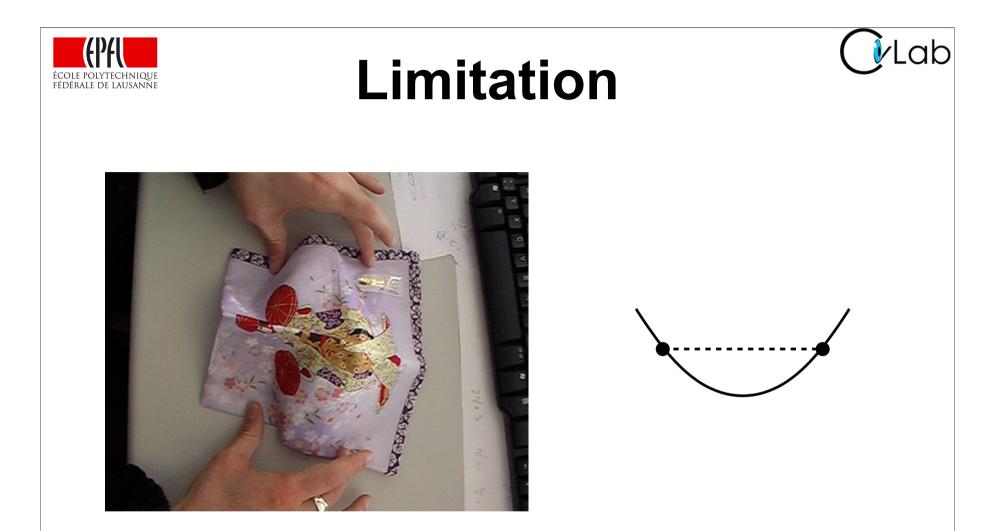
where the W is a diagonal matrix of modal penalty terms that depends on the eigenvalues of the training data covariance matrix.

- A can also be written as a weighted sum of eigenvectors of the extended matrix.
- The inextensibility constraints give rise to a smaller set of quadratic equations than can now be solved.









In the presence of sharp folds:

- The Euclidean distance between discrete points decreases.
- Inextensibility constraints are not appropriate anymore.





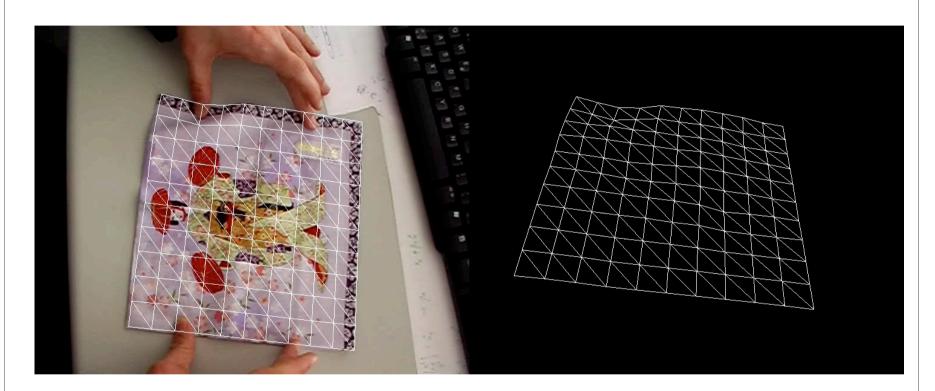
#### **3D Deformable Surfaces**

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#### **Handling Creases**





Replace inextensibility constraints by distance inequalities that:

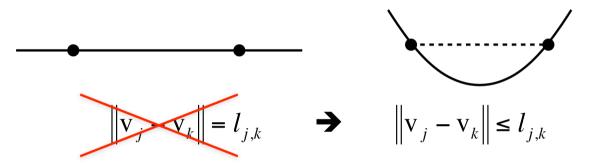
- Let us reconstruct surfaces with sharp folds.
- Yield a convex formulation of the reconstruction problem.

Salzmann et al., CVPR'09



# **Inequality Constraints**

In the presence of sharp folds, geodesic distances remain constant, but Euclidean ones may decrease.



Naive formulation:

$$\mathbf{X}_{opt} = \arg\min \|\mathbf{M}\mathbf{X}\| ,$$

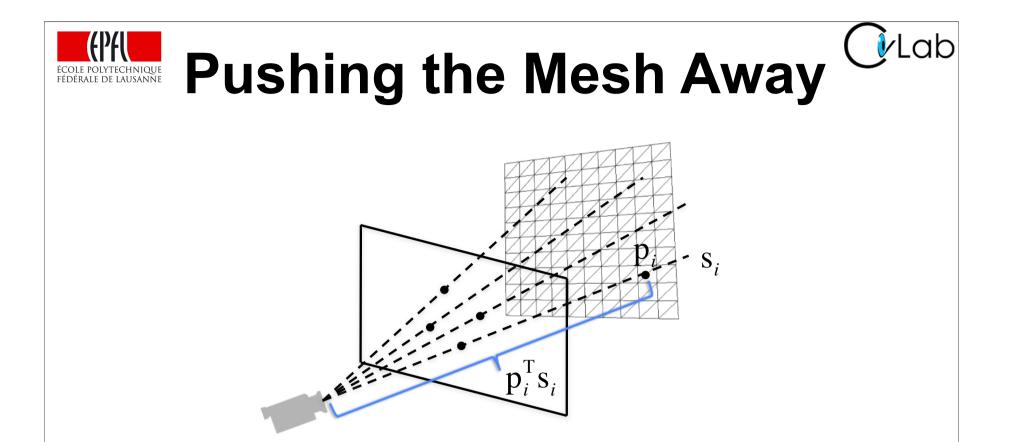
subject to

$$\left\|\mathbf{V}_{j} - \mathbf{V}_{k}\right\| \leq d_{jk}$$

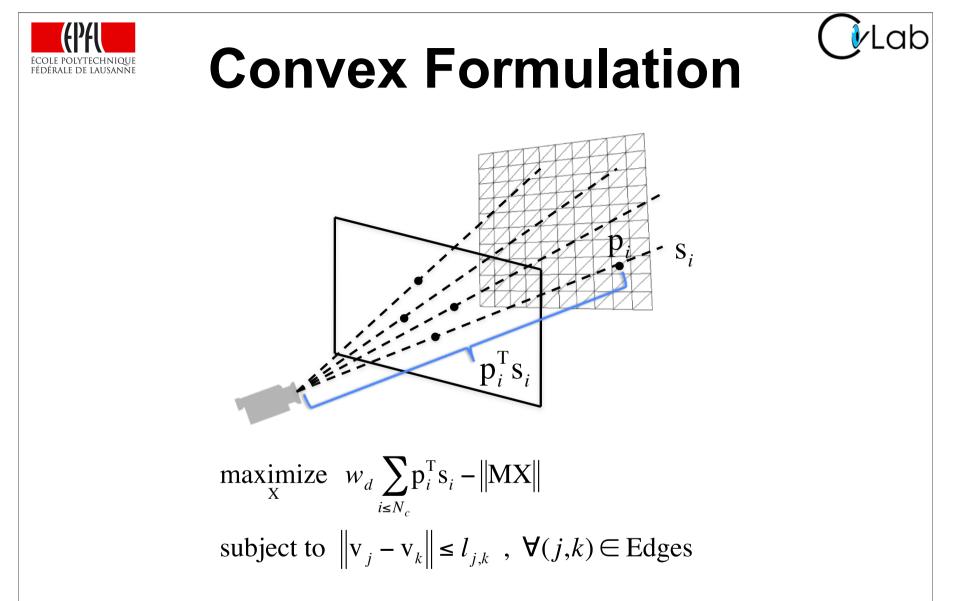
for all neighboring vertices j and k.

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- Inequality constraints do not prevent the mesh from shrinking.
- To this end, we push the points along their lines-of-sight as far as the constraints allow.



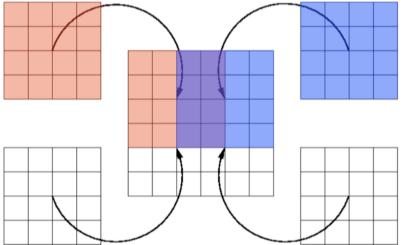
This is an SOCP problem, which can be solved using standard numerical routines.





# **Shape Regularization**

- Regularization is needed to enforce smoothness on poorly textured parts.
- To handle sharp folds, the global models must be replaced by local ones.



 $\rightarrow$  Introduce a linear model for individual surfaces patches

$$\mathbf{X}^i = \mathbf{X}_0^i + \mathbf{\Lambda} \mathbf{c}^i$$





# **Local Deformation Model**

• To avoid having to explicitly force the coefficients of overlapping patches to be consistent, we express them as

 $\mathbf{c}^{i} = \boldsymbol{\Lambda}^{\mathrm{T}} (\mathbf{X}^{i} - \mathbf{X}_{0}^{i}) ,$ 

which arises from the orthonormality of the modes.

• Regularization is achieved by penalizing the coefficients associated to high energy modes, which is done by minimizing

$$\sum_{i\leq N_{\tau}} w_i \left\| \Sigma^{-1/2} \Lambda^{\mathrm{T}} (\mathbf{X}^i - \mathbf{X}_0^i) \right\|,$$

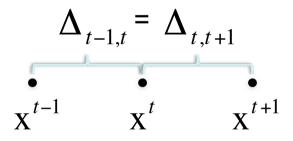
where  $\Sigma$  contains the eigenvalues of the training data covariance matrix.





## **Temporal Regularization**

For short video-sequences, we can enforce temporal consistency by introducing a second order---constant speed---motion model:



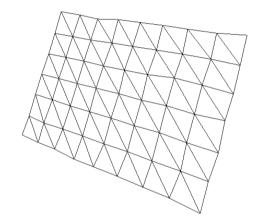
 $\rightarrow$  We solve our optimization problem for 3 frames simultaneously, and regularize the motion between frames by minimizing

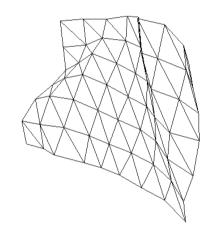
$$\left\| \mathbf{X}^{t-1} - 2\mathbf{X}^{t} + \mathbf{X}^{t+1} \right\|$$





• Optical motion capture:







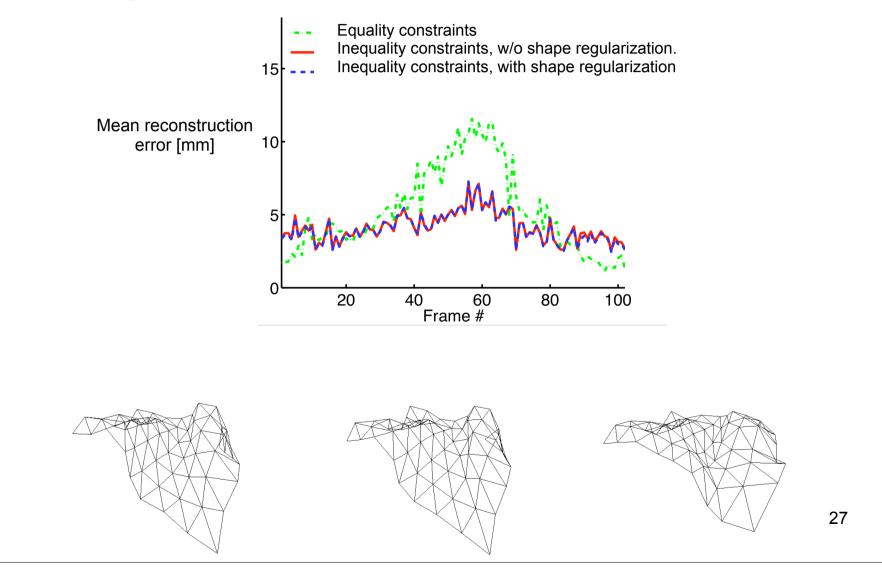
- Correspondences:
  - Sample the barycentric coordinates, project the 3D points, add Gaussian noise with variance 5 to the image locations.
  - Compute SIFT matches between the input images and the reference.
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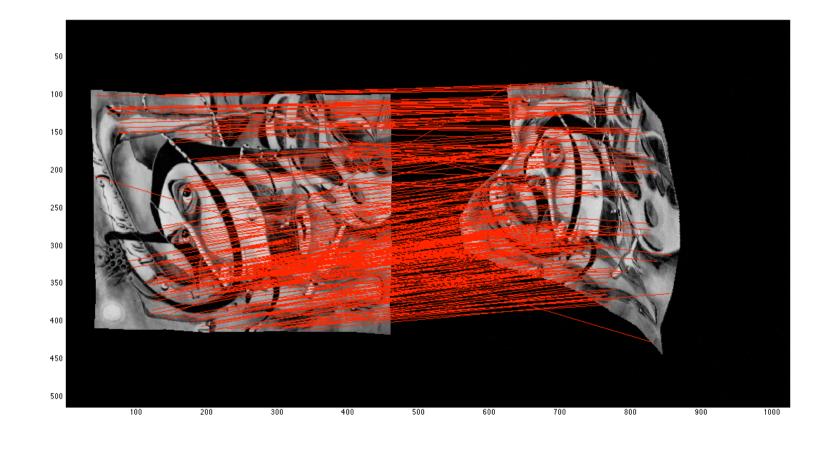
#### **Synthetic Correspondences**







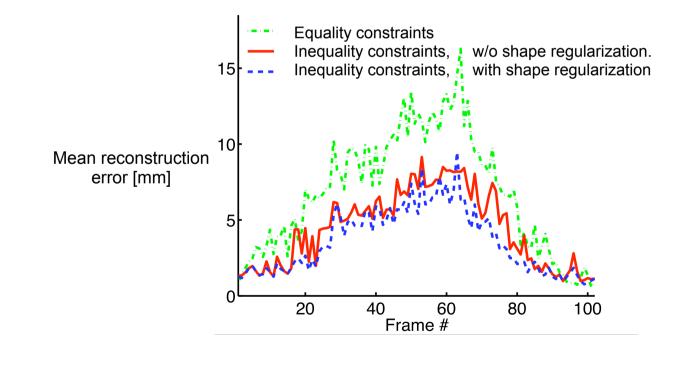
#### **SIFT Correspondences**







# **SIFT Correspondences**

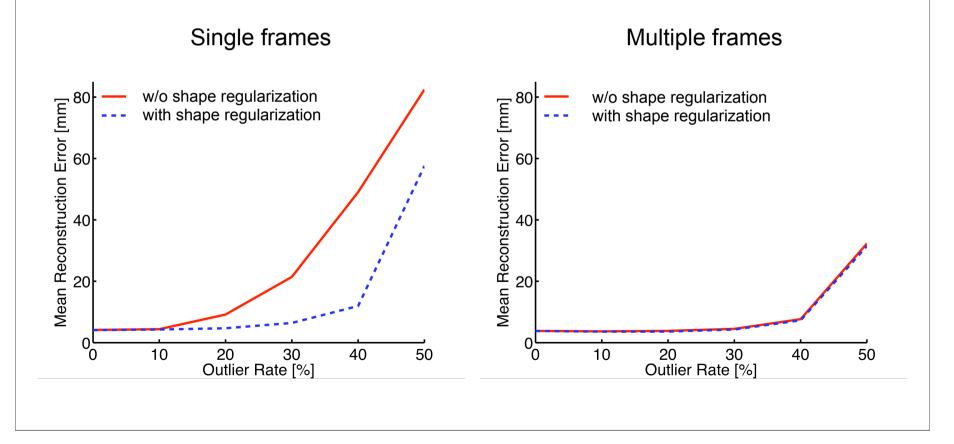


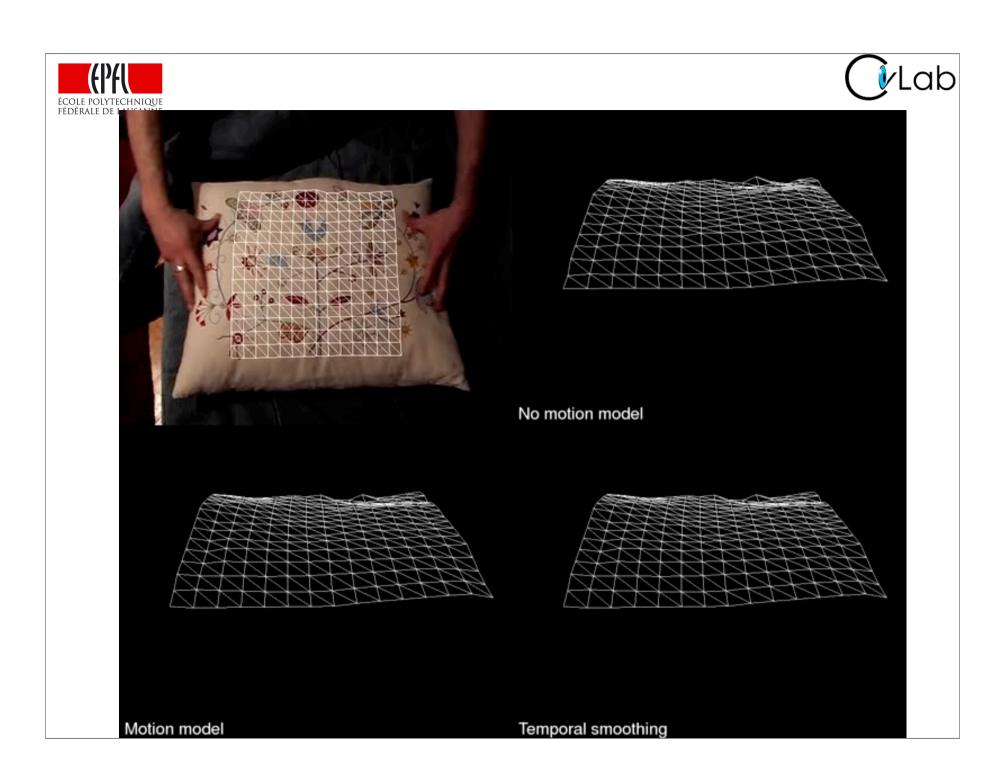


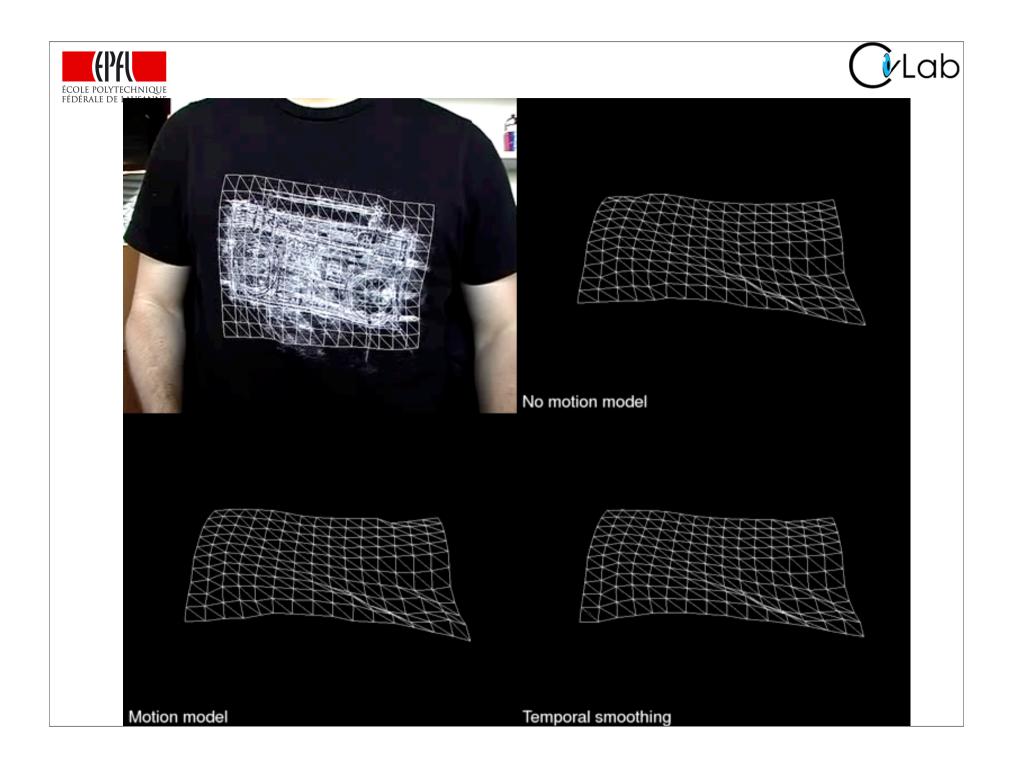


### **Introducing Outliers**

- Synthetic correspondences.
- Varying outlier rate.



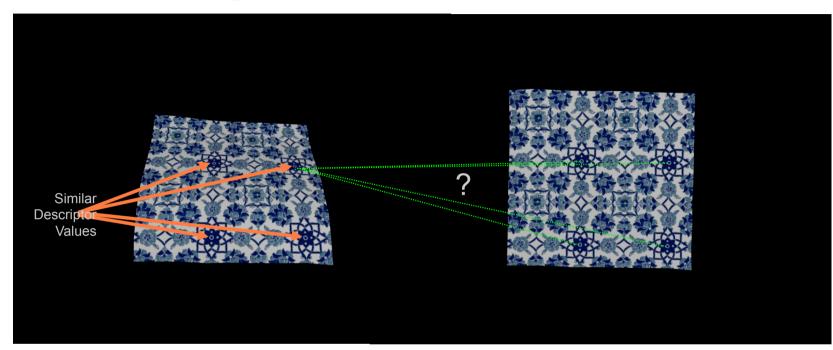








### **Repetitive Patterns**



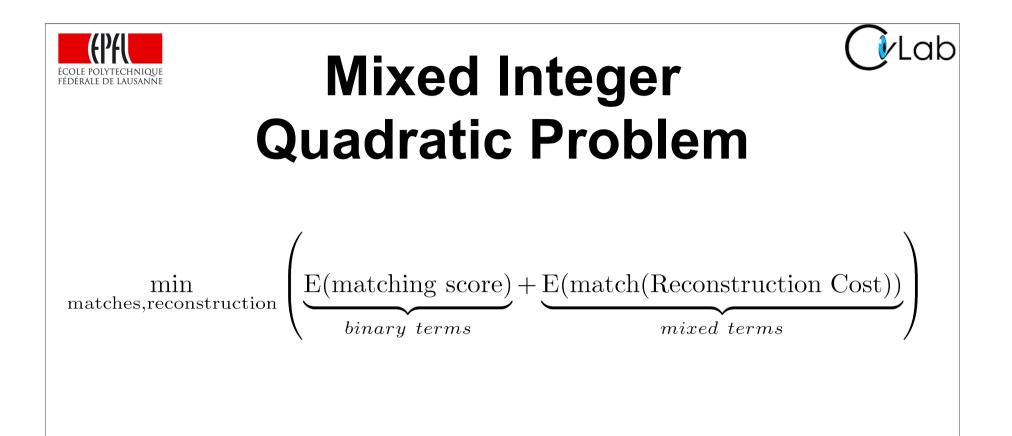
### **Problem:**

• Correspondences are difficult to establish.

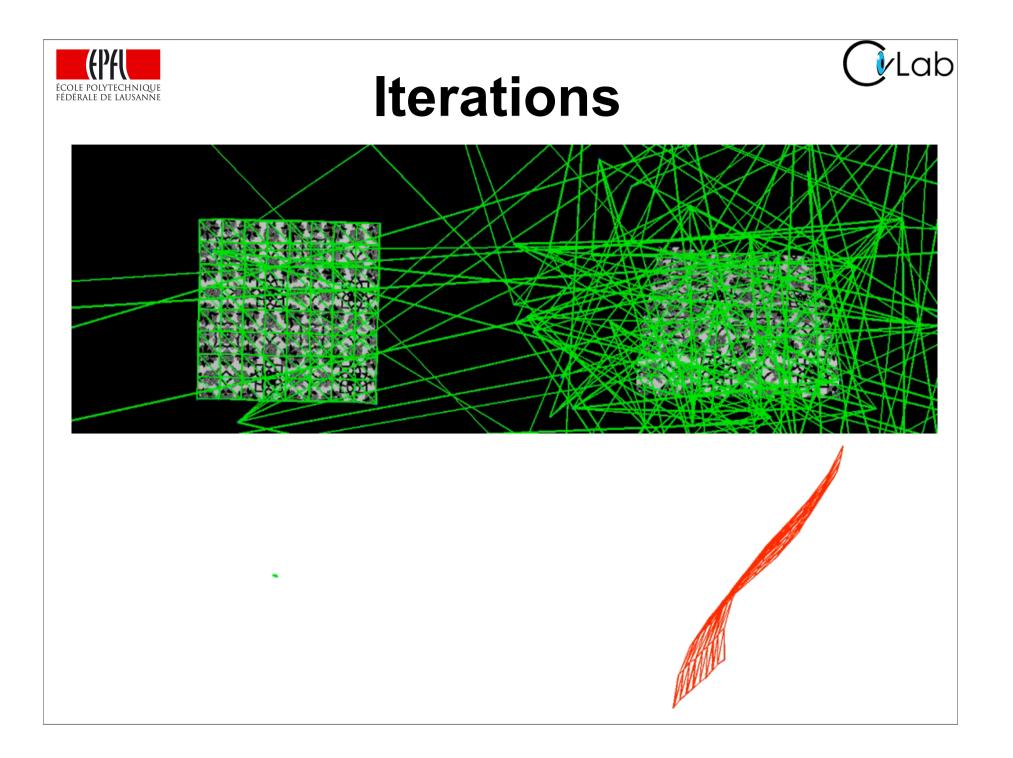
### Solution:

• Simultaneously solve for correspondences and 3D shape.

Shaji et al., CVPR'10



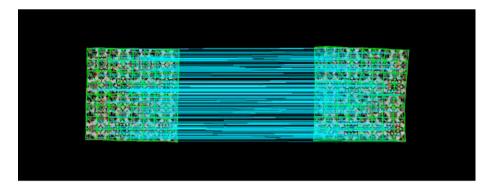
- Instance of a NP Hard Problem.
- Branch-and-bound methods that works well for this particular problem.





## Comparison





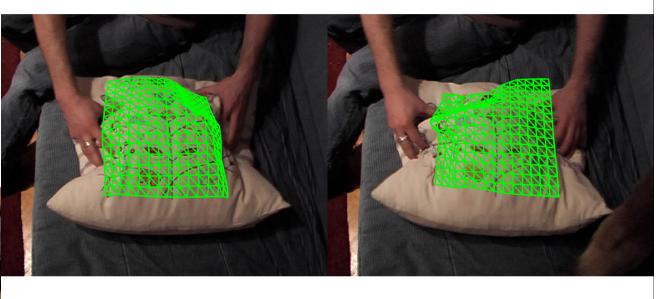
Ground Truth Mesh Reconstruction by our method Reconstruction by method of Salzmann et.al



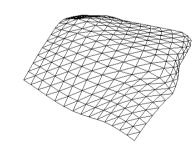


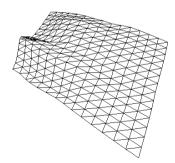


### Cushion













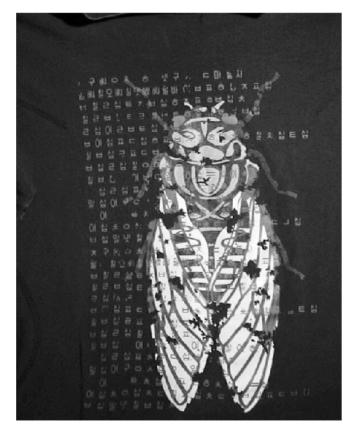
# **Talk Outline**

- Linear Formulation.
- Inextensible surfaces.
- Sharply folding surfaces.
- Eliminating the reference image.
- Applications.

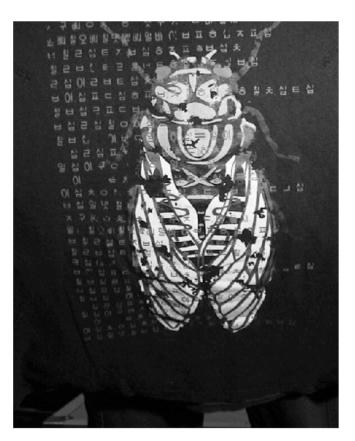




### **Problem Formulation**



**Input Frame** 



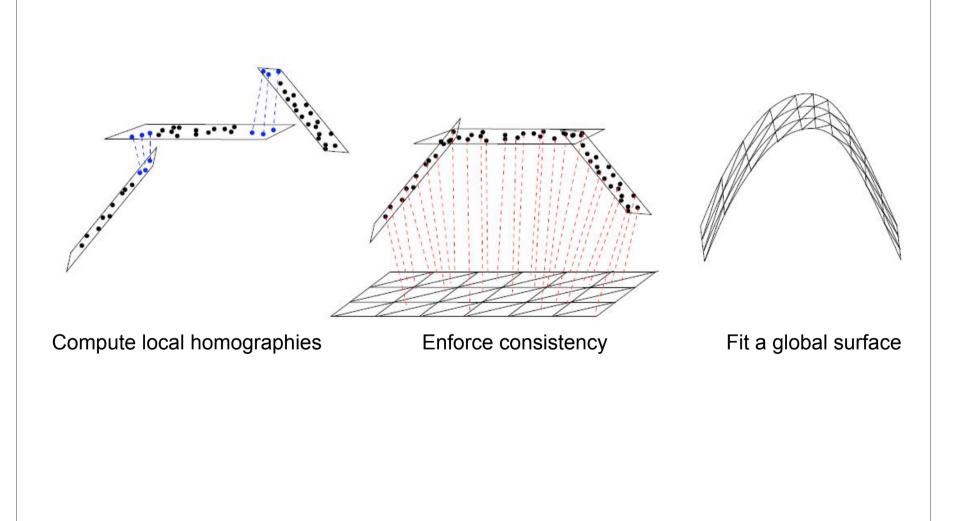
### Support Frame

Varol et al., ICCV'09





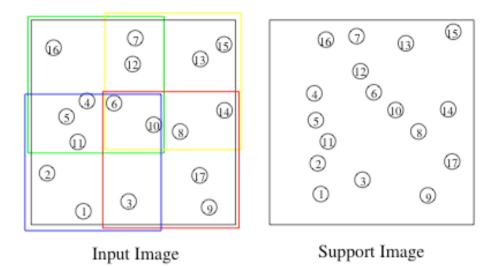
# From Local to Global







# **Local Homographies**



Assuming that the patch is fixed and that the support camera moves

$$\mathbf{P}_{i} = \mathbf{K}[\mathbf{R}_{i}|\mathbf{t}_{i}]$$
$$\mathbf{H}_{i} = \mathbf{R}_{i} - \frac{\mathbf{t}_{i}\mathbf{n}_{i}^{T}}{d^{i}} = \mathbf{R}_{i} - \mathbf{t}'_{i}\mathbf{n}_{i}^{T}$$

 $\rightarrow \mathbf{R}_i, t_i, \text{ and } n_i \text{ can be recovered up to a scale factor.}$ 





# **Enforcing Consistency**

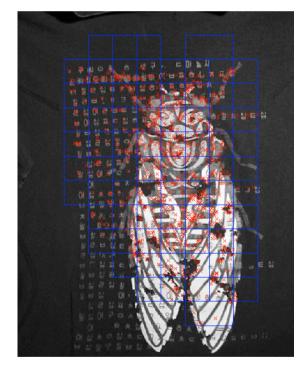
- Turn pairs of matching points in each patch into 3D points that lie on a plane by solving a linear system per patch. The reconstruction is performed up to a local scale factor.
- Compute the scale factors so that points shared by several patches have the same 3D coordinates by solving a global linear system.

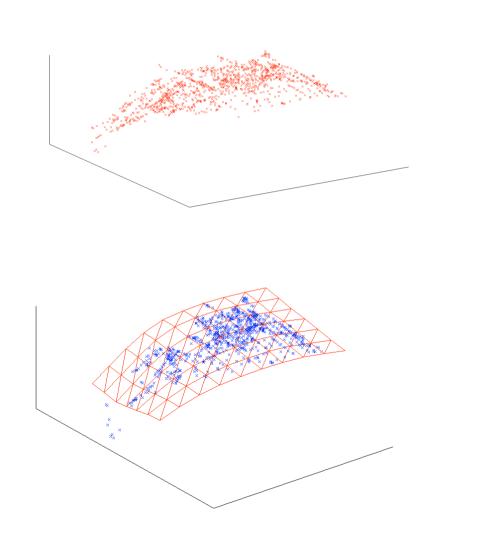
--> 3D cloud up to a global scale factor.



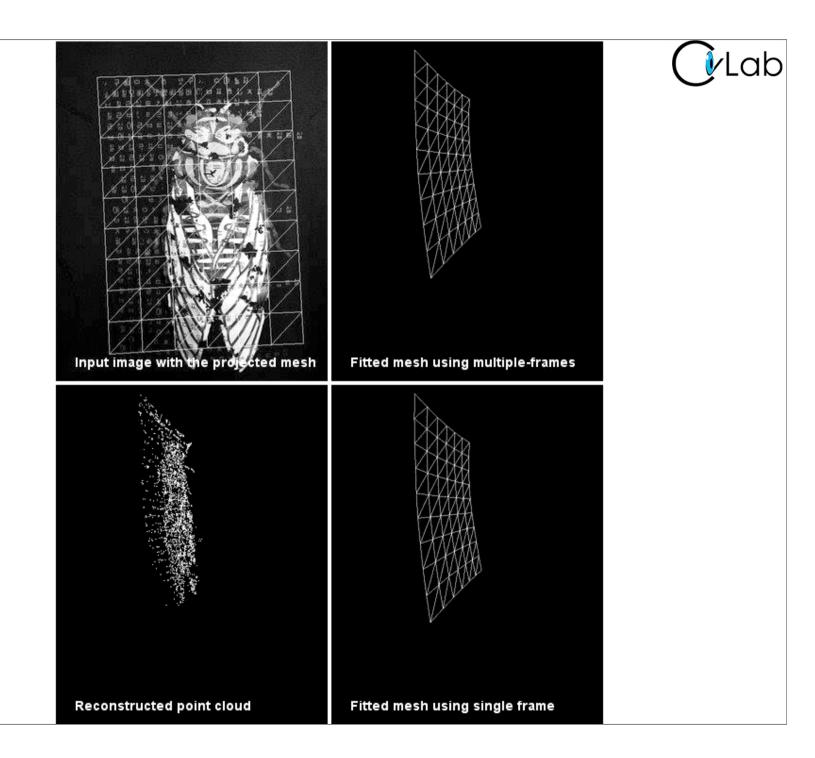


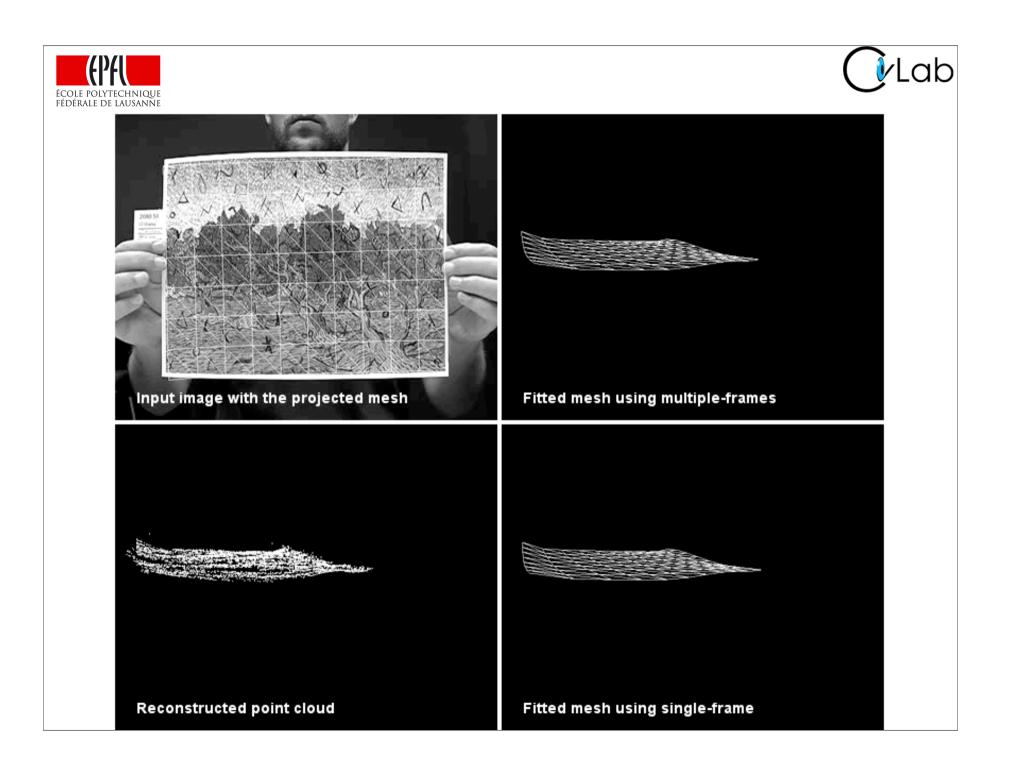
## **Consistent Point Cloud**

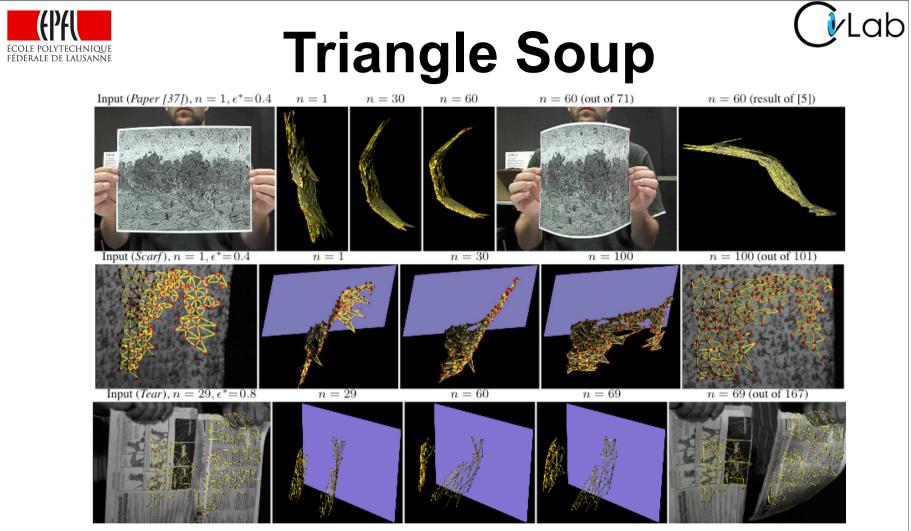












- 1. Track triangle vertices over 4 or more frames.
- 2. Assume edge-length is preserved and reconstruct in 3D.
- 3. Enforce consistency of the resulting *triangle soup*.

Taylor, Jepson, and Kutulakos, CVPR'10





# **3D Deformable Surfaces**

- Linear Formulation.
- Inextensible surfaces.
- Sharply folding surfaces.
- Eliminating the reference image.
- Applications.





### **Augmented Reality**



### Magic Book

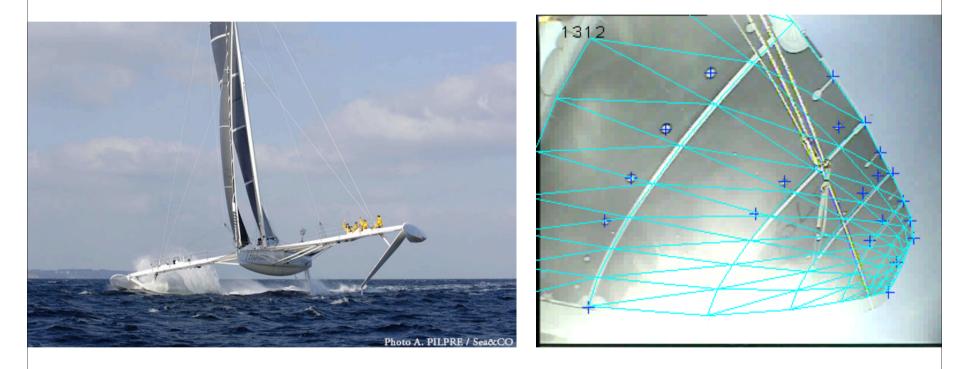
### **Magic Cushion**

Scherrer et al., Leonardo, special issue SIGGRAPH'09

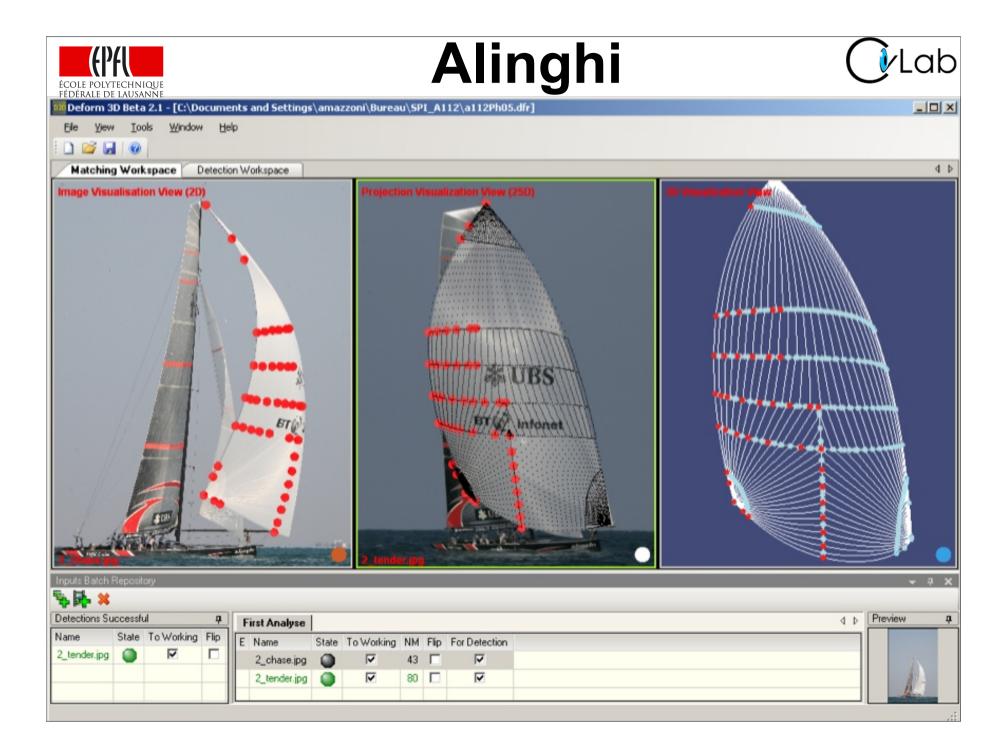




## Hydroptère



Runs at 8 Hz on an ordinary PC







# Wing Deformation

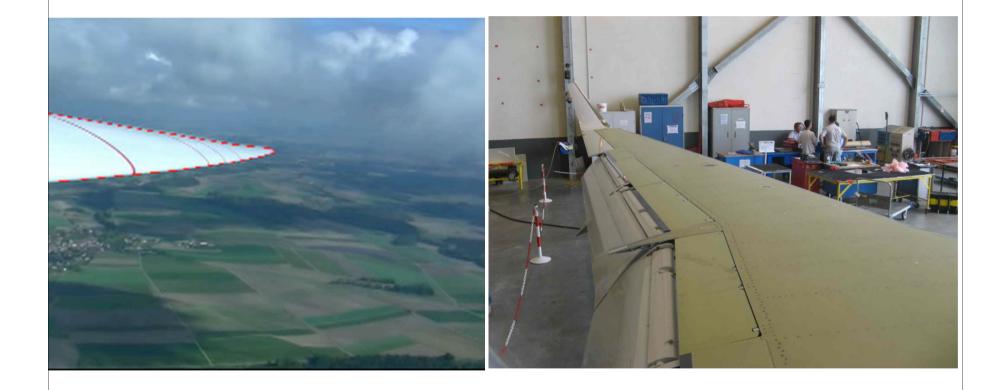


- Compare predicted and observed values.
- Improve simulation software until the two match.
   --> Virtual wind tunnel.





# Wing Deformation







## **Intelligence Gathering**

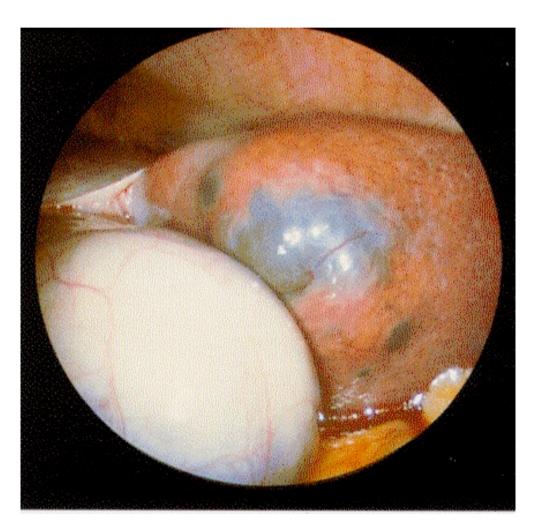


• Automated reading of those banners requires unwarping the surfaces.





### Laparoscopic Surgery







# A Generic Paradigm

Automated 3D deformable surface detection:

- Reconstruct textured parts of a surface.
- Learn a deformation model from those.
- Apply it to reconstruct the rest of the surface.
- $\rightarrow$  A robust method that is easy to deploy.





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- M. Calonder
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- J. Pilet
- M. Salzmann
- A. Shaji
- E. Tola
- R. Urtasun
- A. Varol



### References



#### 2D Deformable Surfaces

• J. Pilet, V. Lepetit, and P. Fua, **Fast Non-Rigid Surface Detection**, **Registration and Realistic Augmentation**, International Journal of Computer Vision, Vol. 76, Nr. 2, February 2008.

#### 3D Deformable Surfaces

• M. Salzmann, J.Pilet, S.Ilic, P.Fua, <u>Surface Deformation Models for Non-Rigid 3--D Shape Recovery</u>, IEEE Transactions on Pattern Analysis and Machine Intelligence, Vol. 29, Nr. 8, pp. 1481 - 1487, August 2007.

• M. Salzmann and P. Fua, Linear Local Models for Monocular Reconstruction of Deformable Surfaces, IEEE Transactions on Pattern Analysis and Machine Intelligence, 2011, In Press.

#### Point Matching:

• M. Ozuysal, M. Calonder, V. Lepetit and P. Fua, <u>Fast Keypoint Recognition using Random Ferns</u>, IEEE Transactions on Pattern Analysis and Machine Intelligence, Vol. 32, Nr. 3, pp. 448 - 461, March 2010

• M. Calonder, V. Lepetit and P. Fua, **BRIEF: Binary Robust Independent Elementary Features**, European Conference on Computer Vision, Heraklion, Greece, 2010.