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## Modeling Deformable Surfaces from Single Videos

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## Talk Outline

- 2D Deformable Surfaces
- Problem Formulation.
- Fast Matching.
- Robust Optimization Scheme.
- Illumination Correction.
- 3D Deformable Surfaces
- Linear Formulation.
- Inextensible surfaces.
- Sharply folding surfaces.
- Eliminating the reference image.


## 2D Deformable Surfaces



Estimating:

- Deformations
- Lighting parameters
- Occlusions


## 2D Deformable Surfaces

- Problem Formulation
- Fast Matching
- Robust Optimization
- Lighting Correction


## Problem Formulation



- Input:
- Correspondences between a reference and input image.
- No a priori pose information.
- Output:
- A mapping $F$ from model to input image.


## Challenges

Non-rigid deformation without a priori pose:

- High dimensionality (200+ DOF)
- Large search space
- Wide baseline matching

Real-time requirements:

- Fast optimization scheme
- Fast matching


## Deformable Model

## Wide Baseline Matching



## Regularization Term



$$
\varepsilon(S)=\overrightarrow{\varepsilon_{C}}(S)+\lambda_{D} \varepsilon_{D}(S)
$$

$$
S=(X, Y)
$$



Reference Image
Input Image

## $\varepsilon_{D}$ Regularization Term

Quadratic function vertex coordinates:

$$
\varepsilon_{D}(S)=\frac{1}{2}\left(X^{T} K X+Y^{T} K Y\right)
$$

- penalizes non uniform scaling;
- penalizes excessive bending;

- allows perspective distortion;
- allows smooth surface deformation.



## $\varepsilon_{c}$ Correspondence Term



$$
\varepsilon_{C}=-\sum_{c \in C}\left\|c_{1}-T_{\mathbf{S}}\left(c_{0}\right)\right\|^{2}
$$

## Real-Time Augmentation



Once upon a time animated pictures at EPFL

## Key Ingredients

- Classification-based approach to matching.
- Robust minimization scheme.
- Intensity ratios for illumination correction.


## 2D Deformable Surfaces

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## $\varepsilon_{C}$ Correspondence Term

Standard approach:


Classification-based approach:

- One class per keypoint.
- Shifts computational burden to offline training.



## Binarized Tests for Keypoint Matching



## Randomized Tree

Generic tree: The nodes contain simple tests of the form "Is $\mathrm{I}\left(\mathrm{m}_{1}\right)>\mathrm{I}\left(\mathrm{m}_{2}\right)$ ?"


Posterior probabilities can be learnt from:

- synthetically warped images

- video sequences



## Multiple Trees



Where should the tests be performed?
-Choose locations to maximize information gain.
-Choose locations randomly.

## Random vs Optimized Locations

Recognition rate


Recognition rates for 200 keypoints.

## FERNS: Flattening the Tree



The distributions can be expressed simply, as:


## Bayesian Interpretation

We are looking for:

$$
P\left(C=c_{i} \mid f_{1}, f_{2}, \cdots f_{n}, f_{n+1}, \cdots \cdots f_{N}\right)
$$

proportional to

$$
P\left(f_{1}, f_{2}, \cdots f_{n}, f_{n+1}, \cdots \cdots f_{N} \mid C=c_{i}\right)
$$

but complete representation of joint distribution infeasible.
Naive Bavesian:

$$
\approx \prod_{j} P\left(f_{j} \mid C=c_{i}\right)
$$

Compromise:

$$
\approx P\left(f_{1}, f_{2}, \cdots f_{n} \mid C=c_{i}\right) \otimes P\left(f_{n+1}, \cdots f_{2 n} \mid C=c_{i}\right) \otimes \cdots
$$

--> probabilities stored in the leaves.

## Sum vs Product



Number of structures (Depth / Size 10)


500 classes, no orientation or perspective correction.

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## Scale and Orientation Invariance



## Planar or Not



Reference image vs Input Images


Reference video


Input Images

## $B P E$

Very simple computation that can be seen as computing gradients:
patch

smooth

binary tests


- Most smooth kernels work, even simple box filters.
- 128,256 , or 512 binary tests usually suffice.
- Random arrangment of tests effective iff evenly sampled.
- Not rotationally invariant.

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## Benchmarks Datasets



## BRIEF vs SURF






## BRIEF vs SIFT





## SIFT > BRIEF > SURF.

Be careful about interpreting benchmarks!

## Computational Issues

Computing $N=512$ descriptors.


Matching $N=512$ descriptors against $N$ others.


- Integral images can further decrease BRIEF's description time by making smoothing faster.
- Intel Core i7 CPU's POPCOUNT instruction will drastically speed-up the matching of binary vectors.
- Scale and rotational invariance need to be added in some cases.


## 2D Deformable Surfaces

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- Lighting Correction


## $\varepsilon_{C}$ Correspondence Term



$$
\varepsilon_{C}=-\sum_{c \in C}\left\|c_{1}-T_{\mathbf{S}}\left(c_{0}\right)\right\|^{2}
$$

Not robust to outliers!

## Robustness to Mismatches

$$
\varepsilon_{C}=-\sum_{c \in C} w_{c} \rho\left(\left\|c_{1}-T_{\mathbf{S}}\left(c_{0}\right)\right\|, r\right)
$$

where $\rho$ is a robust estimator whose radius of confidence is $r$ and $w_{c} \in[0,1]$ a weight associated to each correspondence.

$$
\begin{aligned}
& \rho(\delta, r)=\left\{\begin{array}{cc}
\frac{3\left(r^{2}-\delta^{2}\right)}{4 r^{3}} & \delta<r \\
0 & \text { otherwise }
\end{array}\right. \\
& \int_{-\infty}^{\infty} \rho(x, r) d x=1 \quad \forall r>0
\end{aligned}
$$

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## Iterative Reweighting



## Gauging Robustness

Probability of having $90 \%$ mesh vertices within 2 plxels of the solution

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## Visualizing the Deformations



## Semi-Implicit Optimization Scheme

Minimize:

$$
\begin{aligned}
\varepsilon(S) & =\lambda_{D} \varepsilon_{D}(S)+\varepsilon_{C}(S) \\
\varepsilon_{D}(S) & =\frac{1}{2}\left(X^{T} K X+Y^{T} K Y\right)
\end{aligned}
$$

Satisfied when:

$$
\begin{aligned}
0 & =\frac{\partial \varepsilon}{\partial X}=\frac{\partial \varepsilon_{C}}{\partial X}+K X \\
0 & =\frac{\partial \varepsilon}{\partial Y}=\frac{\partial \varepsilon_{C}}{\partial Y}+K Y
\end{aligned}
$$

## Semi-Implicit Optimization Scheme

Introduce viscosity term:

$$
\begin{aligned}
K X+\alpha \dot{X} & =-\frac{\partial \varepsilon_{C}}{\partial X} \\
K Y+\alpha \dot{Y} & =-\frac{\partial \varepsilon_{C}}{\partial Y}
\end{aligned}
$$

Time discretization:

$$
\begin{aligned}
0 & =K X_{t}+\alpha\left(X_{t}-X_{t-1}\right)+\left.\frac{\partial \varepsilon_{C}}{\partial X}\right|_{X=X_{t-1} Y=Y_{t-1}} \\
0 & =K Y_{t}+\alpha\left(Y_{t}-Y_{t-1}\right)+\left.\frac{\partial \varepsilon_{C}}{\partial Y}\right|_{X=X_{t-1}, Y=Y_{t-1}}
\end{aligned}
$$

## Semi-Implicit Optimization Scheme

Solve at each iteration:

$$
\begin{aligned}
(K+\alpha I) X_{t} & =\alpha X_{t-1}-\left.\frac{\partial \varepsilon_{C}}{\partial X}\right|_{X=X_{t-1}, Y=Y_{t-1}} \\
(K+\alpha I) Y_{t} & =\alpha Y_{t-1}-\left.\frac{\partial \varepsilon_{C}}{\partial Y}\right|_{X=X_{t-1}, Y=Y_{t-1}}
\end{aligned}
$$

--> Fast because K has only a few non zero diagonals.


Taylor expansion of data term:

$$
\begin{aligned}
\varepsilon(X, Y) & =\lambda_{D} \varepsilon_{D}(X, Y)+\varepsilon_{C}(X, Y) \\
\varepsilon_{D}(X, Y) & =\frac{1}{2}\left(X^{T} K X+Y^{T} K Y\right) \\
\varepsilon_{C}(X+d X, Y+d Y) & =A+B d X+C d Y+\frac{1}{2} d X^{t} D d X+\frac{1}{2} d Y^{t} E d Y
\end{aligned} \quad \text { Zhu and Lyu, ECCV'07 } \quad \text {. }
$$

## Newton

## Optimization Scheme

At the minimum:

$$
\begin{aligned}
& 0=\frac{\partial \varepsilon}{\partial X}=B+D d X+K(X+d X) \\
& 0=\frac{\partial \varepsilon}{\partial Y}=C+E d Y+K(Y+d Y)
\end{aligned}
$$

Solve at each iteration:

$$
\begin{aligned}
(K+D) d X & =-B-K X \\
(K+E) d Y & =-C-K Y
\end{aligned}
$$

## Semi-Implicit vs Newton

Residuals as a function of the number of iterations: Semi-Implicit in green and Newton in blue.





## 2D Deformable Surfaces

- Problem Formulation
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## Lighting

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## Intensity Ratios

Reference image: $I_{r, p}=L_{r} A_{p}$
Input image: $I_{i, p}=L_{i, p} A_{p}$

White image: $I_{r, w}=L_{r} A_{w}$
Synthetic image: $I_{x, p}=L_{i, p} A_{w}$
$=A_{w} L_{r} \frac{I_{i, p}}{I_{r, p}}$
$=I_{r, w} \frac{I_{i, p}}{I_{r, p}}$


## (f)fl <br> ÉCOLE POLYTECHNIQU <br> fédérale de lausann <br> Background Subtraction



Standard approach:

- Pixel-wise statistical background model.

Modified approach:

- Account for the fact that illuminations changes tend to be correlated.
- Model variations of intensity ratios as GMMs.
--> Effective for occlustion detection.


## Realistic Augmentation




## Problem Formulation

Input:

- Reference image.
- Corresponding 3D surface.
- Projection matrix P.
- 3D-to-2D correspondences between reference configuration and input image.


Unknowns:

- Mesh vertex coordinates corresponding to input image

$$
\mathbf{X}=\left[x_{1}, y_{1}, z_{1}, \cdots, x_{n_{v}}, y_{n_{v}}, z_{n_{v}}\right]^{T}
$$

## Ambiguity



Scale ambiguity


Bas-Relief Ambiguity

- 3D Shape or deformation models are needed.

How can we design models that do not make unwarranted assumptions?

## 3D Deformable Surfaces

- Linear Formulation.
- Inextensible surfaces.
- Sharply folding surfaces.
- Eliminating the reference image.
- Applications.


## Linear Formulation

- Calibrated camera, $\mathbf{A}$ intrinsic parameters matrix.
- Coordinates expressed in the camera referential.
- Unknown mesh vertex coordinates: $\mathbf{X}=\left(\mathbf{v}_{1}^{\mathrm{T}}, \ldots, \mathbf{v}_{\mathrm{nv}}^{\mathrm{T}}\right), \mathbf{v}_{i}=\left(x_{i}, y_{i}, z_{i}\right)^{\mathrm{T}}$
- Correspondences
- Barycentric coordinates from reference configuration: $\left(a_{i}, b_{i}, c_{i}\right)$
- Current image location: $\left(u_{i}, v_{i}\right)^{T}$

$$
\left(\begin{array}{c}
u_{i} \\
v_{i} \\
1
\end{array}\right)=\frac{1}{k_{i}} \mathbf{A}\left(a_{i} \mathrm{v}_{1}+b_{i} \mathrm{v}_{2}+c_{i} \mathrm{v}_{3}\right)
$$



Salzmann et al., CVPR'07

## Linear Formulation

$$
\mathbf{A}\left(b_{1} \mathbf{v}_{1}+b_{2} \mathbf{v}_{2}+b_{3} \mathbf{v}_{3}\right)=k\left[\begin{array}{c}
u \\
v \\
1
\end{array}\right]
$$

$\rightarrow k$ can be expressed in terms of the vertex coordinates using the last row.

$$
\left[\begin{array}{lll}
b_{1} \mathbf{H} & b_{2} \mathbf{H} & b_{3} \mathbf{H}
\end{array}\right]\left[\begin{array}{l}
\mathbf{v}_{1} \\
\mathbf{v}_{2} \\
\mathbf{v}_{3}
\end{array}\right]=\mathbf{0}
$$

with

$$
\mathbf{H}=\mathbf{A}_{\mathbf{2} \times \mathbf{3}}-\left[\begin{array}{l}
u \\
v
\end{array}\right] \mathbf{A}_{\mathbf{3}},
$$

where $\mathbf{A}_{\mathbf{2} \times \mathbf{3}}$ contains the first two rows of $\mathbf{A}$, and $\mathbf{A}_{\mathbf{3}}$ is the third one.
--> Each correspondence gives rise to two linear equations.

## Linear System and Singular Values

$\mathbf{X}$ must be solution of $\mathbf{M X}=\mathbf{0}$


Singular values of M

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## Inextensible Meshes

A solution of the linear system belongs to the kernel of $\mathbf{M}$ :

$$
\mathbf{M X}=0 \Rightarrow \mathbf{X}=\sum_{i} \beta_{i} \mathbf{p}_{i}
$$

where the $\mathbf{p}_{i}$ are the eigenvectors corresponding to small eigenvalues.

Inextensible mesh :

$$
\left\|\sum_{i} \beta_{i} \mathbf{p}_{i}^{\mathrm{j}}-\sum_{i} \beta_{i} \mathbf{i}_{i}^{\mathrm{k}}\right\|^{2}=\text { cte }
$$

for all neighboring vertices $j$ and $k$.
--> A system a quadratic equations that could be solved in closed form using extended linearization, but with too many variables for existing solvers.

## Dimensionality Reduction



$$
\begin{aligned}
\mathbf{X} & =\mathbf{X}_{0}+\sum_{i} \alpha_{i} \mathbf{S}_{i} \\
& =\mathbf{X}_{0}+\mathbf{S A}
\end{aligned}
$$

$$
\text { with } \mathbf{A}=\left[\begin{array}{lll}
\alpha_{1} & \ldots & \alpha_{N}
\end{array}\right]^{T}
$$

## Degrees of Freedom

For an inextensible triangulation with V vertices, $E=E i+E b$ edges, and $F$ facets with no holes:
-Euler formula

$$
V+F-E=1
$$

- Interior edges shared by two facets $3 F=2 E i+E b$.
-Degrees of freedom

$$
3 \mathrm{~V}-\mathrm{E}=6+\mathrm{Eb}
$$



## Spinnaker Modes



## Reduced System


where the $\mathbf{W}$ is a diagonal matrix of modal penalty terms that depends on the eigenvalues of the training data covariance matrix.

- A can also be written as a weighted sum of eigenvectors of the extended matrix.
- The inextensibility constraints give rise to a smaller set of quadratic equations than can now be solved.


## Independent Detection



## Limitation



In the presence of sharp folds:

- The Euclidean distance between discrete points decreases.
- Inextensibility constraints are not appropriate anymore.


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## Handling Creases



Replace inextensibility constraints by distance inequalities that:

- Let us reconstruct surfaces with sharp folds.
- Yield a convex formulation of the reconstruction problem.


## Inequality Constraints

In the presence of sharp folds, geodesic distances remain constant, but Euclidean ones may decrease.


Naive formulation :

$$
\mathbf{X}_{\text {opt }}=\arg \min \|\mathbf{M} \mathbf{X}\|,
$$

subject to

$$
\left\|\mathbf{v}_{j}-\mathbf{v}_{k}\right\| \leq d_{j k}
$$

for all neighboring vertices $j$ and $k$.

## Pushing the Mesh Away <br> Lab



- Inequality constraints do not prevent the mesh from shrinking.
- To this end, we push the points along their lines-of-sight as far as the constraints allow.


## Convex Formulation



This is an SOCP problem, which can be solved using standard numerical routines.

## Shape Regularization

- Regularization is needed to enforce smoothness on poorly textured parts.
- To handle sharp folds, the global models must be replaced by local ones.

$\rightarrow$ Introduce a linear model for individual surfaces patches

$$
\mathrm{X}^{i}=\mathrm{X}_{0}^{i}+\Lambda \mathrm{c}^{i}
$$

## Local Deformation Model

- To avoid having to explicitly force the coefficients of overlapping patches to be consistent, we express them as

$$
\mathrm{c}^{i}=\Lambda^{\mathrm{T}}\left(\mathrm{X}^{i}-\mathrm{X}_{0}^{i}\right),
$$

which arises from the orthonormality of the modes.

- Regularization is achieved by penalizing the coefficients associated to high energy modes, which is done by minimizing

$$
\sum_{i \leq N_{p}} w_{i}\left\|\Sigma^{-1 / 2} \Lambda^{\mathrm{T}}\left(\mathrm{X}^{i}-\mathrm{X}_{0}^{i}\right)\right\|
$$

where $\Sigma$ contains the eigenvalues of the training data covariance matrix.

## Temporal Regularization

For short video-sequences, we can enforce temporal consistency by introducing a second order---constant speed---motion model:

$$
\Delta_{t-1, t}=\Delta_{t, t+1}
$$

$\rightarrow$ We solve our optimization problem for 3 frames simultaneously, and regularize the motion between frames by minimizing

$$
\left\|X^{t-1}-2 X^{t}+X^{t+1}\right\|
$$

## Synthetic Data

- Optical motion capture:

- Correspondences:
- Sample the barycentric coordinates, project the 3D points, add Gaussian noise with variance 5 to the image locations.
- Compute SIFT matches between the input images and the reference.


## Synthetic Correspondences




## SIFT Correspondences



## SIFT Correspondences



## Introducing Outliers

- Synthetic correspondences.
- Varying outlier rate.

Single frames


Multiple frames




## Repetitive Patterns



## Problem:

- Correspondences are difficult to establish.

Solution:

- Simultaneously solve for correspondences and 3D shape.


## Mixed Integer Quadratic Problem



- Instance of a NP Hard Problem.
- Branch-and-bound methods that works well for this particular problem.

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## Iterations



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## Comparison



Ground Truth Mesh
Reconstruction by our method
Reconstruction by method of Salzmann et.al

## Cushion



## Talk Outline

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## Problem Formulation



Input Frame


## Support Frame

Varol et al., ICCV'09

## From Local to Global



Compute local homographies
Enforce consistency


Fit a global surface

## Local Homographies



Input Image


Support Image

Assuming that the patch is fixed and that the support camera moves

$$
\begin{aligned}
\mathbf{P}_{i} & =\mathbf{K}\left[\mathbf{R}_{i} \mid \mathbf{t}_{i}\right] \\
\mathbf{H}_{i} & =\mathbf{R}_{i}-\frac{\mathbf{t}_{i} \mathbf{n}_{i}^{T}}{d^{i}}=\mathbf{R}_{i}-\mathbf{t}_{i}^{\prime} \mathbf{n}_{i}^{T}
\end{aligned}
$$

$\rightarrow \mathbf{R}_{i}, t_{i}$, and $n_{i}$ can be recovered up to a scale factor.

## Enforcing Consistency

- Turn pairs of matching points in each patch into 3D points that lie on a plane by solving a linear system per patch. The reconstruction is performed up to a local scale factor.
- Compute the scale factors so that points shared by several patches have the same 3D coordinates by solving a global linear system.
--> 3D cloud up to a global scale factor.

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## Consistent Point Cloud




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Fitted mesh using multiple-frames

## 

Reconstructed point cloud
Fitted mesh using single-frame

## Triangle Soup



1. Track triangle vertices over 4 or more frames.
2. Assume edge-length is preserved and reconstruct in 3D.
3. Enforce consistency of the resulting triangle soup.

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## Augmented Reality



Magic Book


Magic Cushion

## Hydroptère



Runs at 8 Hz on an ordinary PC

## Ele Yiew Iools Yindow Help <br> $\square B$ П (6)

Matching Workspace Detection Workspace 4 d


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## Wing Deformation



- Compare predicted and observed values.
- Improve simulation software until the two match.
--> Virtual wind tunnel.


## Wing Deformation



## Intelligence Gathering



- Automated reading of those banners requires unwarping the surfaces.


## Laparoscopic Surgery



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## A Generic Paradigm

Automated 3D deformable surface detection:

- Reconstruct textured parts of a surface.
- Learn a deformation model from those.
- Apply it to reconstruct the rest of the surface.
$\rightarrow$ A robust method that is easy to deploy.


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