APPENDIX A
PROPERTIES OF THE REGULARIZATION TERM

Here we prove the claim made in Section 4.1.1 that $\mathbf{A}\mathbf{x} = \mathbf{0}$ when $\mathbf{x}$ represents an affine transformed version of the reference mesh and that $\|\mathbf{A}\mathbf{x}\|^2$ is invariant to rotations and translations.

The first equation in Eq. 13 applies for each of three spatial coordinates x, y, z and can be rewritten in a matrix form as

$$[w_1 \quad w_2 \quad w_3 \quad w_4] \begin{bmatrix} \mathbf{v}_{1\text{ref}x} & \mathbf{v}_{1\text{ref}y} & \mathbf{v}_{1\text{ref}z} \\ \mathbf{v}_{2\text{ref}x} & \mathbf{v}_{2\text{ref}y} & \mathbf{v}_{2\text{ref}z} \\ \mathbf{v}_{3\text{ref}x} & \mathbf{v}_{3\text{ref}y} & \mathbf{v}_{3\text{ref}z} \\ \mathbf{v}_{4\text{ref}x} & \mathbf{v}_{4\text{ref}y} & \mathbf{v}_{4\text{ref}z} \end{bmatrix} = \mathbf{0}^T \ . \tag{25}$$

It can be seen from Eq. 25 and the second equation in Eq. 13 that the three vectors of the x, y, z components of the reference mesh and the vector of all 1s lie in the kernel of the matrix $\mathbf{A}'$. It means our regularization term $\|\mathbf{A}\mathbf{x}\|^2$ does not penalize affine transformation of the reference mesh. Hence, $\mathbf{A}\mathbf{x} = \mathbf{0}$ as long as $\mathbf{x}$ represents an affine transformed version of the reference mesh.

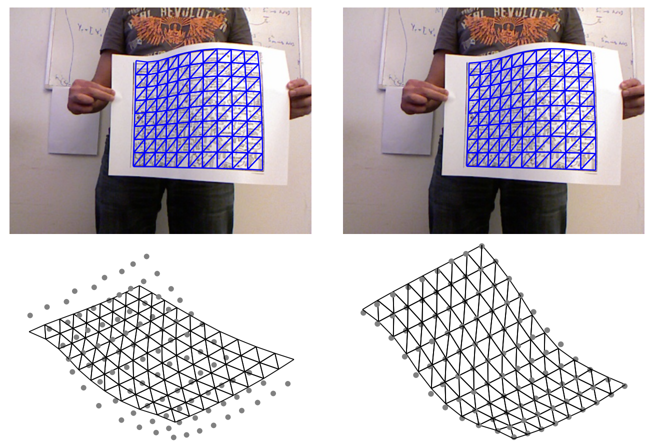
Let $\mathbf{v}'_i = \mathbf{R}\mathbf{v}_i + \mathbf{t}$ be the new location of vertex $\mathbf{v}_i$ under a rigid transformation of the mesh, where $\mathbf{R}$ is the rotation matrix and $\mathbf{t}$ is the translation vector. Since $\mathbf{R}^T\mathbf{R} = \mathbf{I}_3$ and $w_1 + w_2 + w_3 + w_4 = 0$, we have

$$\begin{aligned} & \left\|w_1\mathbf{v}'_{i_1} + w_2\mathbf{v}'_{i_2} + w_3\mathbf{v}'_{i_3} + w_4\mathbf{v}'_{i_4}\right\|^2 \\ = \ & \big\|\mathbf{R}(w_1\mathbf{v}_{i_1} + w_2\mathbf{v}_{i_2} + w_3\mathbf{v}_{i_3} + w_4\mathbf{v}_{i_4}) \\ & \qquad + \mathbf{t}(w_1 + w_2 + w_3 + w_4)\big\|^2 \\ = \ & \left\|w_1\mathbf{v}_{i_1} + w_2\mathbf{v}_{i_2} + w_3\mathbf{v}_{i_3} + w_4\mathbf{v}_{i_4}\right\|^2 \ . \end{aligned} \tag{26}$$

Hence, $\|\mathbf{A}\mathbf{x}'\|^2 = \|\mathbf{A}\mathbf{x}\|^2$ when $\mathbf{x}'$ is a rigidly transformed version of $\mathbf{x}$. In other words, $\|\mathbf{A}\mathbf{x}\|^2$ is invariant to rotations and translations.

APPENDIX B
IMPORTANCE OF NON-LINEAR MINIMIZATION

Fig. 16 illustrates the importance of the non-linear constrained minimization step of Eq. 12 that refines the result obtained by solving the linear least-squares problemof Eq. 10. In the left column we show the solution of the linear least-squares problem of Eq. 10. It projects correctly but, as evidenced by its distance to the ground-truth gray dots, its 3D shape is incorrect. By contrast, the surface obtained by solving the constrained optimization problem of Eq. 12 still reprojects correctly while also being metrically accurate. Fig. 17 depicts similar situations in other frames of the sequence.



(a) (b)

Fig. 16: Unconstrained vs constrained optimization results. (a) The surface obtained by solving the unconstrained minimization problem of Eq. 10 and rescaling the result. It is projected on the original image at the top and shown from a different angle at the bottom. (b) The surface obtained by solving the constrained minimization problem of Eq. 12. The projections of both surfaces are almost identical but only the second has the correct 3D shape.

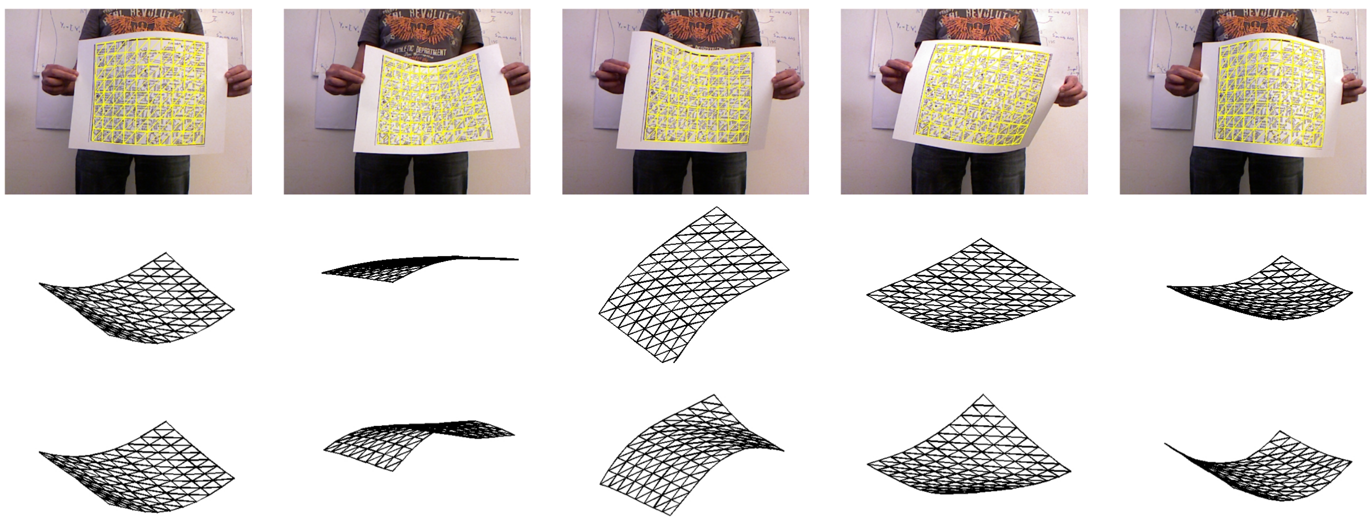


Fig. 17: Additional unconstrained and constrained results for the paper sequence. Top row: Reprojection on the input images of the final reconstructions obtained by solving the constrained minimization problem of Eq. 12. **Middle row:** Intermediate reconstructions obtained by solving the unconstrained minimization problem of Eq. 10 and seen from a different view-point. **Bottom row:** Final reconstructions seen from the same view-point as the intermediate ones. Note that the 3D shapes can be quite different, even though their image projections are very similar.