Realizable Group Diversification Effects

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Abstract

The impact of capital mobility restrictions on the diversification benefit for risk at the group level of a financial conglomerate is an important aspect in risk management. In this paper we propose a new bottom-up approach for realizing diversification benefits using some predetermined capital and risk transfer instruments, taking counter-party default risk into account.

1 Introduction

The impact of capital mobility restrictions on the acknowledged diversification benefit of the internal model of an insurance group, or any financial conglomerate, is an important topic in the Solvency II discussion, see e.g. Section 6 in the recent CEIOPS document [3]. It is generally accepted that for diversification to work at a group level, capital needs to flow freely between business units. Regulators, rating agencies and local companies management may constrain this fungibility unless there are some predetermined and legally binding capital and risk transfer (CRT) instruments in place.

It is difficult—if not impossible—to quantify the diversification effect of CRTs using a standard aggregation approach, such as the covariance method. This leads us to propose a simple but sufficiently realistic model for a bottom-up risk assessment of an insurance group. We assume that available capital above a sufficient tied capital level for the subsidiaries is transferable to the parent company, which in turn provides guarantees to the subsidiaries. Default of the parent company on its guarantees is explicitly taken into account. The resulting realizable diversification effect in terms of tied capital levels and possible guarantees can be evaluated and compared to the diversification effect assuming full

fungibility of capital. A simulation study illustrates these effects for stop loss and quota share guarantees.

The proposed model can easily be extended and applies equally well to banking groups and financial conglomerates. It has been developed in a joint project of Munich Re Group and Filipović to examine how much of the diversification benefit showing up in Munich Re's Internal Risk Model can be realized in a Solvency II consistent framework based on legally binding CRT instruments. Munich Re subsequently applied this model to determine a haircut on acknowledged diversification due to restrictions on fungibility of capital[1]. This framework has also been suggested by the CRO Forum response to rating agency requests for comments on internal model reviews, see [4]. A more thorough study of optimal CRTs can be found in Filipović and Kupper [5, 6].

2 Basic setup

We consider an insurance group which is assumed to consist of three[2] entities, labeled by $i = 0, 1, 2$. Entity $i = 0$ denotes the parent company that owns the subsidiary entities $i = 1, 2$. The effects of ownership will be explicitly described below.

A one year solvency horizon is considered. That is, random discounted terminal values at a one year time horizon are compared with the corresponding deterministic current values of today.

The current available capital (i.e. value of asset-liability portfolio) of entity i is denoted

$$c_i = a_i - \ell_i$$

where a_i is the market value of assets (net of subsidiaries' available capital[3]), and ℓ_i equals the best estimate value of liabilities[4].

The terminal value of the asset-liability portfolio of entity i on a stand alone basis is denoted

$$V_i = A_i - L_i$$

where A_i is the terminal value of assets (net of subsidiaries' available capital), and L_i equals the terminal value of best estimate liabilities, including claims payments made during current year.

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[1]See disclosure of the risk capital figures according to Munich Re's Internal Risk Model in the Analyst's Conference at May 2007.

[2]The following analysis can easily be extended to comprise arbitrarily many subsidiaries.

[3]In principle, the subsidiary's available capital represents an asset of the parent company (we assume that the parent company is unique shareholder and owns 100% of subsidiary's equity which equals available capital). But to avoid double counting of available capital, we write parent company's current asset value a_0 net of c_i. Capital can only be used once in case of a need. The ownership structure will be taken into account in Section 3.3 below.

[4]cash flows of expected outflows discounted at actual risk free rate (no market value margin applies in this setup)

3 Required capital

The group required capital strongly depends on the way the entities' risks are aggregated. In what follows, we consider and compare three different views on risk aggregation: stand alone (no diversification), consolidated (full diversification), and via legally binding CRT instruments (realizable diversification). The respective diversification effect is measured as relative difference between the stand alone and the aggregated group required capital.

As for risk measurement, two risk measures will be applied: Value-at-Risk (VaR) at a 99.5% and Expected Shortfall (ES) at a corresponding level α. We recall that $\mathrm{VaR}[X] = Q_{99.5\%}[-X]$ for any position X, where $Q_\lambda[-X]$ denotes the λ-quantile of the losses $-X$. If X has a continuous distribution function then ES equals the expected loss that is incurred in the event that VaR at level α is exceeded[5]:

$$\mathrm{ES}[X] = \mathbb{E}[-X \mid -X \geq Q_\alpha[-X]].$$

From this equation we can infer α such that, for normal random variables, ES and VaR coincide:

$$\mathrm{VaR}[Z] = \mathrm{ES}[Z] = \frac{1}{(1-\alpha)\sqrt{2\pi}} \int_{Q_\alpha[-Z]}^\infty x\mathrm{e}^{-x^2/2} dx = \frac{e^{-Q_\alpha^2[-Z]/2}}{(1-\alpha)\sqrt{2\pi}}$$

where Z denotes a standard normal distributed random variable. We thus obtain $\alpha = 98.7\%$.

For the sake of exposure, we will write ρ as a placeholder for either VaR or ES in what follows.

3.1 Stand alone view (no diversification)

Here, no diversification effect between the legal entities of the group is taken into account. The stand alone required capital for entity i is

$$k_i^{stal} = \rho[V_i - c_i].$$

The stand alone solvency requirement for entity i is $k_i^{stal} \leq c_i$, which, due to cash-invariance[6] of ρ, is equivalent to $\rho[V_i] \leq 0$. In other words: the risk of insolvency ($V_i < 0$) is acceptably low.

The resulting group required capital k^{stal} under stand alone view is the sum of the required capital of the separate entities, i.e.

$$k^{stal} = \sum_{i=0}^2 k_i^{stal}.$$

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[5]See e.g. Section 2 in McNeil et al. [7].
[6]Cash-invariance of ρ implies $\rho[V_i - c_i] = \rho[V_i] + c_i$, see e.g. Section 6 in McNeil et al. [7].

3.2 Consolidated view (full diversification)

The consolidated view assumes one group balance sheet under full fungibility of capital. Group required capital k^{cons} is determined by applying the risk measure on the consolidated profit and loss $\sum_{i=0}^{2}(V_i - c_i)$, i.e.

$$k^{cons} = \rho\left[\sum_{i=0}^{2}(V_i - c_i)\right].$$

The resulting relative diversification effect becomes

$$RDE^{cons} = 1 - \frac{k^{cons}}{k^{stal}}.$$

3.3 CRT view (realizable diversification)

Here, group diversification effects are realized via use of legally binding CRT instruments. This approach is most promising for being approved by the regulators. It is similar to the group level Swiss Solvency Test, see [2] and [6].

We follow a two-step scheme and assume first that each subsidiary i faces a tied capital level m_i above which capital is fungible on a going concern basis. The tied capital level is a deterministic parameter, exogenously determined by local accounting rules and/or regulatory capital requirements.

According to the ownership structure, the surplus $(V_i - m_i)^+$ of subsidiary i exceeding the tied capital level m_i can be transferred to the parent company. This leads to the following a priori distribution of total surplus across the entities:

$$C_0 = V_0 + \sum_{i=1}^{2}(V_i - m_i)^+$$
$$C_i = \min\{V_i, m_i\}, \quad i = 1, 2.$$

The parent company, in turn, provides a guarantee to subsidiary i with cash flow G_i if available. We shall consider the following two types of guarantees:

(i) Stop loss guarantees with cash flow $G_i = (m_i - V_i)^+ = (m_i - C_i)$. Hence these are parental guarantees on the economic balance sheet of its subsidiaries. This asserts that the available capital of the subsidiary does not fall below the tied capital level m_i.

(ii) Quota share guarantees with cash flow $G_i = qL_i$, where q denotes the quota. The parent company takes a proportional share in the liabilities of its subsidiaries (under an economic balance sheet view), backed by the available capital of the parent company after the excess capital transfer from its subsidiaries. We will assume a standard quota of $q = 40\%$.

An important feature of our approach is that parental default is taken into account: if the sum of the guarantees exceeds the actual surplus capital of the parent company, that is,

$$(C_0 - m_0)^+ < \sum_{i=1}^{2} G_i,$$

then the surplus capital is distributed to the subsidiaries on a pro rata approach with fractions

$$F_i = \frac{G_i}{\sum_{j=1}^{2} G_j}, \quad i = 1, 2.$$

That is, the actual cash flow for the guarantee is

$$Y_i = \begin{cases} G_i & \text{if } (C_0 - m_0)^+ \geq \sum_{i=1}^{2} G_i, \\ F_i \times (C_0 - m_0)^+ & \text{otherwise.} \end{cases}$$

Note that $\sum_{i=1}^{2} Y_i = \min\{(C_0 - m_0)^+, \sum_{i=1}^{2} G_i\}$. The corresponding probability that the parent company defaults on its guarantees is

$$p^{df} = \mathbb{P}\left[(C_0 - m_0)^+ < \sum_{i=1}^{2} G_i \right].$$

This two-stage CRT yields the following realizable distribution of available capital across the entities:

$$C_0^{CRT} = C_0 - \sum_{i=1}^{2} Y_i,$$

$$C_i^{CRT} = C_i + Y_i, \quad i = 1, 2.$$

The resulting group capital requirement is the sum

$$k^{CRT} = \sum_{i=0}^{2} k_i^{CRT}$$

of the stand alone requirements after CRT

$$k_i^{CRT} = \rho[C_i^{CRT} - c_i].$$

The resulting relative diversification effect becomes

$$RDE^{CRT} = 1 - \frac{k^{CRT}}{k^{stal}}.$$

Benchmark is the consolidated diversification effect RDE^{cons}. From an economic point of view, we expect that the consolidated diversification effect with

full fungibility of capital dominates the realizable diversification benefit using CRT:

$$RDE^{CRT} \leq RDE^{cons}.$$

Theory asserts that this is true for ES. Indeed, subadditivity[7] of ES implies

$$\mathrm{ES}\left[\sum_{i=0}^{2} V_i\right] = \mathrm{ES}\left[\sum_{i=0}^{2} C_i^{CRT}\right] \leq \sum_{i=0}^{2} \mathrm{ES}\left[C_i^{CRT}\right].$$

However, this is not true for VaR, as we shall see below!

In this CRT framework, restrictions on fungibility of capital due to capital requirements imposed by local regulation could be captured as follows: the tied capital levels m_i need to be chosen in such a way that the regulatory capital requirements are respected for each entity. This means that the contingent capital transfers of repatriation excess capital on subsidiary level can be made legally enforceable by an appropriate choice of m_i.

It is interesting to note that the group level Swiss Solvency Test (SST, see [2]) assumes that subsidiaries can be sold by the parent company at their economic value. The latter is defined as available capital minus some market value margin mvm_i. The SST field tests [1] have shown that mvm_i ranges between 10% and 60% of the stand alone required capital k_i^{stal}. On the other hand, it is shown in a study [6] that the tied capital level m_i should typically be greater than mvm_i (since otherwise the sale of the subsidiaries becomes a "standard scenario" within the model, which is not reasonable).

It seems reasonable that the major part of fungible capital is held at the parent company, whereas not much fungible capital is held at the subsidiary entities. This means, that standard tied capital ratios m_i/c_i should be around 80% for $i = 0$ and 90% to 100% for $i = 1, 2$, respectively.

4   Stochastic model

We assume that the current asset and liability values are distributed as follows:

initial values	parent company	subsidiary 1	subsidiary 2	group
a_i	92	11	24	127
l_i	48	2	18	68
c_i	44	9	6	59

For terminal asset and liability values we assume a multi-dimensional normal distribution:

$$A_i = a_i(1 + \sigma_i^A W_i^A), \qquad L_i = l_i(1 + \sigma_i^L W_i^L), \qquad i = 0, 1, 2,$$

where the relative asset and liability standard deviations σ_i^A, σ_i^L are given by

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[7]ES is subadditive means $\mathrm{ES}[X + Y] \leq \mathrm{ES}[X] + \mathrm{ES}[Y]$. This property does not hold for VaR in general. See e.g. Section 6 in McNeil et al. [7].

rel. std. dev	parent company	subsidiary 1	subsidiary 2
σ_i^A	0.03	0.03	0.03
σ_i^L	0.07	0.50	0.07

and $(W_0^A, W_1^A, W_2^A, W_0^L, W_1^L, W_2^L)$ is a 6-dimensional normal distributed random vector, which is normalized,

$$\mathbb{E}[W_i^A] = \mathbb{E}[W_i^L] = 0\,, \qquad \mathbb{E}[(W_i^A)^2] = \mathbb{E}[(W_i^L)^2] = 1 \qquad i = 0, 1, 2$$

and has correlation matrix, for $i \neq j$,

$$\left(\begin{array}{c|cc|cc} & W_i^A & W_i^L & W_j^A & W_j^L \\ \hline W_i^A & 1 & 0 & 0.8 & 0 \\ W_i^L & & 1 & & 0.5 \end{array} \right).$$

This means that subsidiary 1 holds more volatile (e.g. P&C) liabilities than the parent company and subsidiary 2. Moreover, assets A_i in entity i are uncorrelated with liabilities L_j in any other entity $j \neq i$. Meanwhile, assets A_i are positively correlated with assets A_j, and so are liabilities L_i and L_j, respectively.

5 Results

The following results are obtained from a simulation with sample size 10^6.

Due to the normal distribution assumption, ES and VaR based required capitals coincide for the stand alone and consolidated view. They are as follows:

view	parent company	subs. 1	subs. 2	group	RDE
stand alone	11.2	2.7	3.7	17.6	
consolidated				15.4	12.4 %

We next present the results for the CRT view, both for stop loss and quota share guarantees, for varying tied capital levels.

5.1 Stop loss guarantees

ES based results are sensitive but robust with respect to varying tied capital levels for parent company and subsidiaries, respectively, see Figures 1 and 4.

The realizable diversification effect RDE^{CRT} is monotone decreasing for increasing tied capital levels. This effect is intuitive, since an increase of the tied capital level is related to a decrease of fungibility of capital and hence of diversification. For tied capital ratios m_i/c_i less than 75% the consolidated diversification effect under full fungibility assumption can be almost completely realized.

In fact, we obtain the following particular values for the ES and VaR based realizable diversification effects in percentage of the consolidated one:

tied capital ratios m_i/c_i	ES based	VaR based	default probability
(80%, 95%, 95%)	58%	55%	5%
(80%, 80%, 80%)	85%	82%	2%

The default probabilities of the parent company are small. This seems counter intuitive, as the primary stop loss guarantees and parent company's surplus value are considerably negatively correlated, i.e. $\text{corr}(\sum_{i=1}^{2} G_i, C_0) = -57\%$. The reason is that in this setting, the mean and the standard deviation of the primary stop loss guarantees ($\mathbb{E}[\sum_{i=1}^{2} G_i] = 0.67$, $\text{std}[\sum_{i=1}^{2} G_i] = 1.03$) are relative small compared to the mean of parent company's excess capital $\mathbb{E}[(C_0 - m_0)^+] = 10.26$.

5.2 Quota share guarantees

Results are very sensitive with unpredictable behavior with respect to varying tied capital levels for parent company and subsidiaries, see Figures 5 and 8.

The realizable diversification effect RDE^{CRT} as a function of the tied capital level for the subsidiaries is hump shaped with a sharp maximum. Away from this optimal tied capital level, the realizable diversification effect decreases significantly. The sensitivity with respect to the parental tied capital level is similar. The maximal realizable diversification effect of about 80% of the consolidated one is achieved for combinations of parental and subsidiary tied capital ratios m_i/c_i which lie on a diagonal ranging from $(90\%, 40\%)$ to $(70\%, 100\%)$, see Figure 5.

We obtain the following particular values for the ES and VaR based realizable diversification effects in percentage of the fully consolidated one:

tied capital ratios m_i/c_i	ES based	VaR based	default probability
$(80\%, 95\%, 95\%)$	51%	48%	37%
$(80\%, 80\%, 80\%)$	75%	73%	27%

The default probabilities of the parent company are large. Though similar negative correlation of primary quota share guarantees and parent company's surplus value $(\text{corr}(\sum_{i=1}^{2} G_i, C_0) = -59\%)$ as in the stop loss case, the mean and the standard deviation of the primary quota share guarantees ($\mathbb{E}[\sum_{i=1}^{2} G_i] = 8.0$, $\text{std}[\sum_{i=1}^{2} G_i] = 0.79$) are of a similar size as parent company's expected excess capital $\mathbb{E}[(C_0 - m_0)^+] = 10.26$.

5.3 Parental default

The implicit parental default on its guarantees towards the subsidiaries causes unpredictable (i.e. non-monotone) dependence of the diversification effects on the tied capital levels.

Generally speaking, for a small tied capital level m_0 of the parent company, the guarantees of the parent company towards its subsidiaries are likely to be met. Hence these guarantees are a material liability for the parent company, resulting in a high capital requirement k_0^{CRT} and, in turn, low capital requirements k_1^{CRT} and k_2^{CRT} for the subsidiaries. Conversely, for a large tied capital level m_0 of the parent company, the implicit default option for the parent company is likely to be in the money, implying a low capital requirement k_0^{CRT}

and, since the parental default option represents a material counter-party default risk for the subsidiaries, high capital requirements k_1^{CRT} and k_2^{CRT} for the subsidiaries. The trade off between these two opposite effects results in different curves for the diversification effect, depending on the type of guarantees. The small default probabilities of the parent company indicate that the implicit default option has little effect for stop loss type guarantees. While for the quota share guarantees this effect is much more accentuated.

In both examples, the default probability of the parent company is monotonically increasing for increasing tied capital levels, see Figures 3 and 7.

5.4   ES vs VaR

Figures 4 and 8 show that ES based realizable relative diversification effects are essentially the same as for VaR for parental and subsidiaries' tied capital ratios m_i/c_i of 80% and above 75%, respectively.

However, below these tied capital levels, ES and VaR based relative diversification effects differ. Indeed, ES based realizable relative diversification effects are more stable than for VaR. However, the most striking negative feature of VaR is that the realizable relative diversification effect may become larger (up to almost 25%) than the consolidated diversification effect! This applies for stop loss and quota share guarantees, see Figures 2 and 6.

This is caused by high impact low probability events of a large claim for a subsidiary and simultaneous default of the parental guarantee. If this probability lies below the 0.5% risk tolerance, VaR cannot account for the loss. In contrast, ES measures the loss extent in such events.

It is not acceptable and may indicate wrong decisions, both from a management and regulatory point of view, if a risk measurement method charges less capital for stand alone risks (albeit including CRTs) than for aggregated risks under full fungibility of capital.

6   Asymptotic analysis

We now provide an asymptotic result that formally explains the observed difference in the behavior of the realizable diversification effect for stop loss and quota share guarantees with respect to varying tied capital levels.

For both type of guarantees, it is obvious that for ever greater tied capital levels $m_0, m_1, m_2 \to +\infty$ we obtain the asymptotic limits $C_i^{CRT} \to V_i$, for $i = 0, 1, 2$. That is, in the limit the realizable diversification effect becomes zero. This is in line with Figures 1, 2, 5 and 6.

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6.1 Stop loss guarantees

For ever smaller parental tied capital level $m_0 \to -\infty$ we obtain the following asymptotic limits

$$C_0^{CRT} + \sum_{i=1}^{2} m_i \to \sum_{i=0}^{2} V_i$$
$$C_i^{CRT} - m_i \to 0, \quad i = 1, 2.$$

The same holds true for $m_1, m_2 \to -\infty$. Cash-invariance of ρ thus implies

$$k^{CRT} \to k^{cons}$$

for either $m_0 \to -\infty$ or $m_1, m_2 \to -\infty$. That is, in the limit the realizable equals the consolidated diversification effect. This becomes apparent in Figures 1 and 2 for small tied capital levels.

We conclude that—asymptotically—the observed behavior of the realizable diversification effect for stop loss guarantees does not change if we apply more skewed and heavy tailed distributions in the stochastic model.

6.2 Quota share guarantees

For ever smaller tied capital levels $m_0, m_1, m_2 \to -\infty$ we obtain the following asymptotic limits

$$C_0^{CRT} + \sum_{i=1}^{2} m_i \to \sum_{i=0}^{2} V_i - q \sum_{i=1}^{2} L_i$$
$$C_i^{CRT} - m_i \to q L_i, \quad i = 1, 2.$$

Again, by cash-invariance of ρ, we obtain the asymptotic group capital requirement

$$k^{CRT} \to \rho \left[\sum_{i=0}^{2} V_i - q \sum_{i=1}^{2} L_i \right] + \sum_{i=1}^{2} \rho[q L_i] - \sum_{i=0}^{2} c_i.$$

As it becomes apparent in the upper left corner of Figures 5 and 6, this asymptotic group capital requirement can become even greater than the stand alone group required capital k^{stal}—thus yielding a negative diversification effect.

We conclude that it is vital to optimize the design and allocation of the intra-group CRTs.

7 Conclusion

We have presented a simple aggregation method which allows an objective quantification of the fungibility of capital haircut that could reasonably be applied to consolidated diversification benefits (i.e. those assuming full fungibility). The main findings were

- A substantial part (about 80%) of the consolidated diversification effect (assuming full fungibility) can be realized by intra-group CRT instruments even for tied capital levels of about 80% of initial surplus.

- Realizable diversification effects are sensitive with respect to the specification of the CRTs. Hence, in view of realizing diversification benefits, it is vital to optimize the design and allocation of intra-group CRTs.

- The implicit default of the parent company on its guarantees towards the subsidiaries is an important factor which may cause complex dependence of the realizable diversification effects on the tied capital levels. E.g., the default probability is much lower for stop loss (less than 5%) than for quota share guarantees (more than 30%) for standard tied capital levels.

- ES is preferable to VaR when it comes to group diversification. Results under ES are more stable with respect to varying CRT model parameters. Moreover, VaR is not sub-additive, which may result in larger diversification benefits for stand alone risks than for consolidated risks. Note that our example builds on multi-normal distributions. And yet, the simple use of plain vanilla stop loss and quota share guarantees yields these fallacies for VaR.

We note that the above multi-normal model including two subsidiaries may be too simple to capture realistic risk characteristics of a financial conglomerate. This simple setup was chosen for the sake of illustration of the proposed method. Extensions in all possible directions are straightforward to implement. For instance, the assumption that the tied capital levels are exogenous deterministic parameters can be relaxed. E.g., they can be made dependent on the terminal asset-liability portfolio value; just as the guarantee payments in the above model, which were contingent on non-default of the parent. Finally, it is possible—and only limited by computer power—to enlarge the group and the number and structure of intra-group CRTs.

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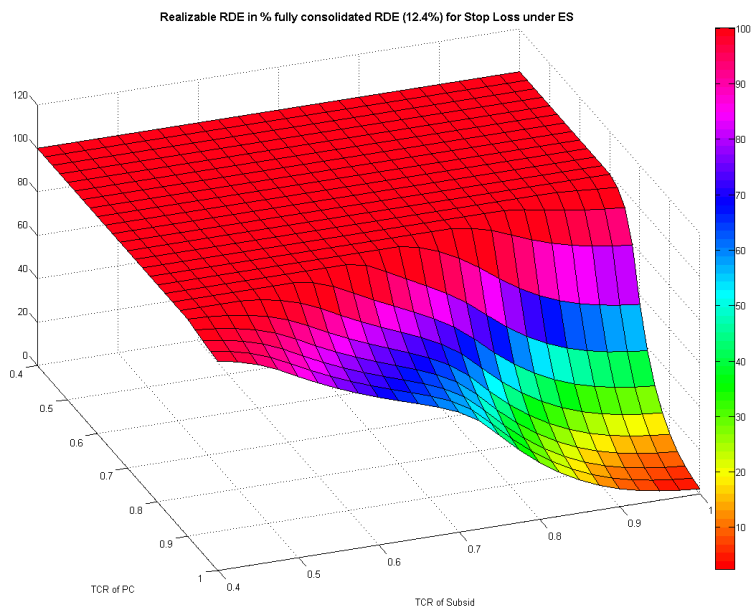[7] McNeil, A., Frey, R. and Embrechts, P. (2005), *Quantitative Risk Management*, Princeton University Press

Figure 1: Stop loss guarantees: dependence of ES based relative diversification effect RDE on tied capital ratios (TCR) m_i/c_i of parent company and subsidiaries, respectively.
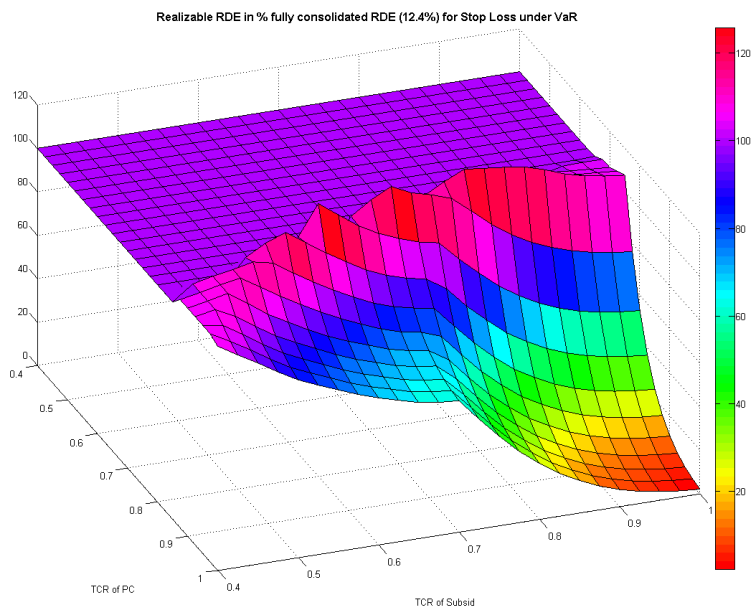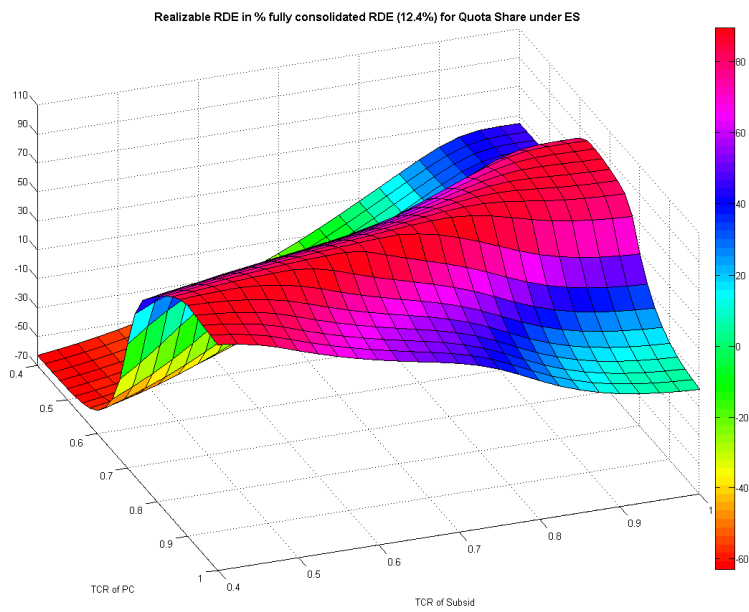
Figure 2: Stop loss guarantees: dependence of VaR based relative diversification effect RDE on tied capital ratios (TCR) m_i/c_i of parent company and subsidiaries, respectively.
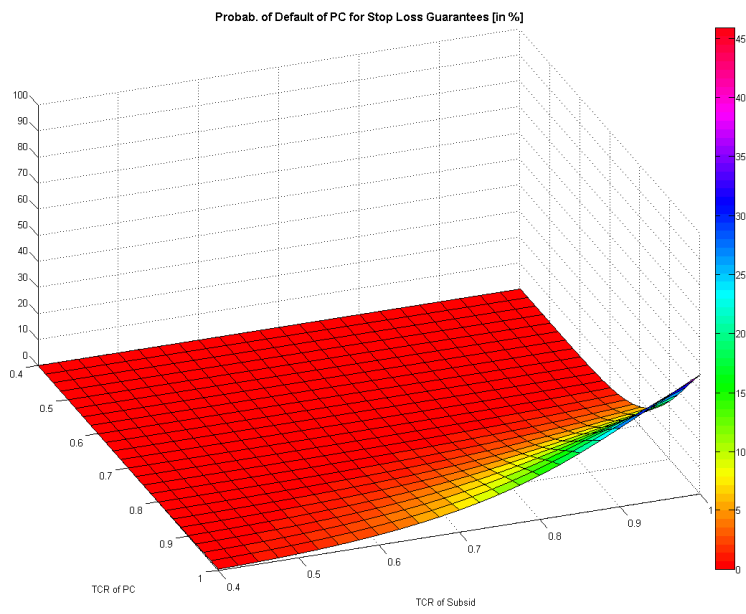
Figure 3: Stop loss guarantees: dependence of parent company's default probability p^{df} on tied capital ratios (TCR) m_i/c_i of parent company and subsidiaries, respectively.
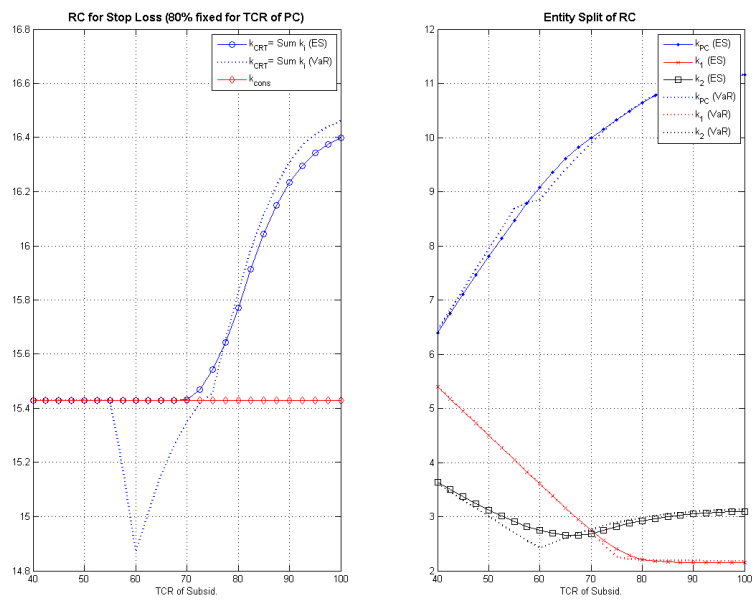
Figure 4: Stop loss guarantees: dependence of group and solo capital requirements on tied capital ratios (TCR) m_i/c_i of the subsidiaries. TCR of parent company is set to standard value $m_0/c_0 = 80\%$. Solid lines indicate the figures based on ES, the dotted lines represent the corresponding VaR figures.

Figure 5: Quota share guarantees: dependence of ES based relative diversification effect RDE on tied capital ratios (TCR) m_i/c_i of parent company and subsidiaries, respectively.
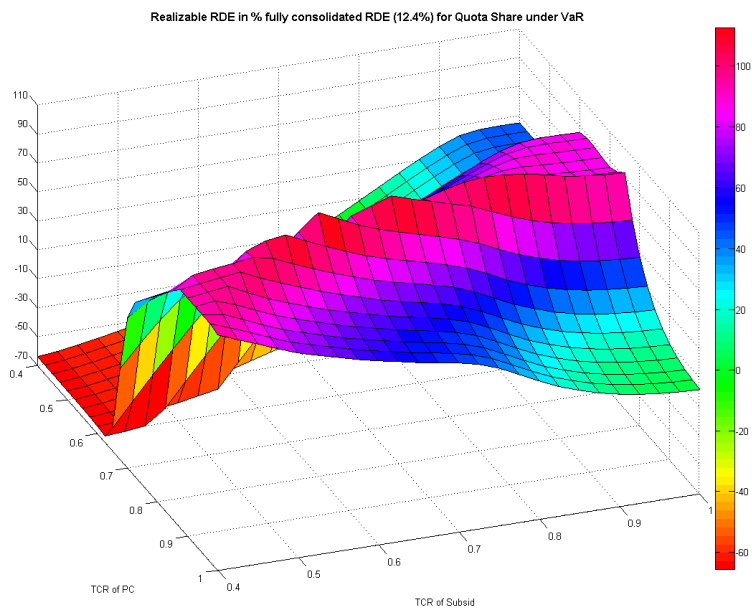
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Figure 6: Quota share guarantees: dependence of VaR based relative diversification effect RDE on tied capital ratios (TCR) m_i/c_i of parent company and subsidiaries, respectively.
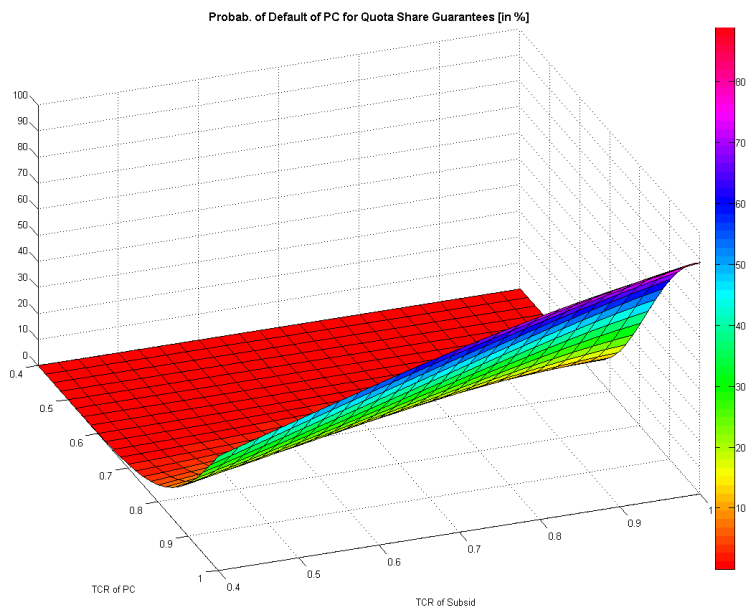
Figure 7: Quota share guarantees: dependence of parent company's default probability p^{df} on tied capital ratios (TCR) m_i/c_i of parent company and subsidiaries, respectively.
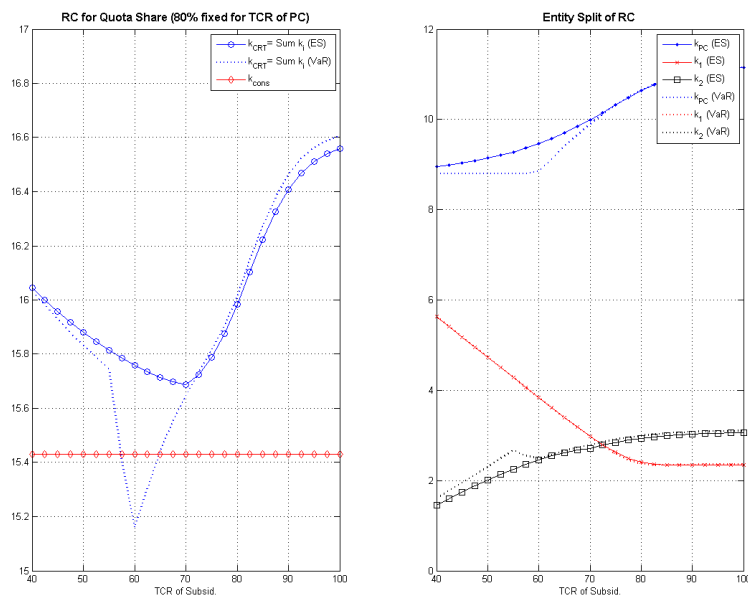
Figure 8: Quota share guarantees: dependence of group and solo capital requirements on tied capital ratios (TCR) m_i/c_i of the subsidiaries. TCR of parent company is set to standard value $m_0/c_0 = 80\%$. Solid lines indicate the figures based on ES, the dotted lines represent the corresponding VaR figures.