Mortgage Supply and Capital Regulation in a Low Interest Rate Environment

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Preliminary

Abstract

Low-for-long nominal interest rates have resulted in strong growth in mortgage lending and real house prices in Switzerland. Domestically-oriented, small banks have mostly contributed to this expansion in mortgage lending while the two big, global systemic banks (UBS and Credit Suisse) have lost market share. We develop a model with two types of banks and monopolistic competition in the deposit and mortgage market, which we calibrate to the Swiss banking sector. In this model, a contemporaneous expansion in the housing market and change in market shares as in the data emerges only if the monetary policy rate is reduced and capital requirements on the big banks are tightened. Any of the two policies in isolation fails to match the empirical evidence.

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1 Introduction

The global financial crisis has led to a reduction in nominal interest rates in many countries around the world. In Europe, the debt crisis of 2010-12 has put further downward pressure on interest rates, which remain at or below zero. Switzerland has been no exception. The Swiss National Bank (SNB) reduced its policy rate to near zero in 2008 and in negative territory in 2015 to stem pressure on the Swiss Franc to appreciate.[1] This low-for-long interest rate environment and the emergence of negative nominal interest rates have brought attention to the banking lending channel. Eggertsson et al. (2019) and Brunnermeier and Koby (2016) argue that the standard transmission mechanism of monetary policy breaks down in such environment so that low interest rates may in fact be associated with a reduction in output due to lower lending. Yet, there is little debate on the effect of low rates on the supply of mortgages and on housing market conditions.

The empirical evidence for Switzerland suggests that low nominal interest rates are fueling an expansion in the housing market. The left panel of Figure 1 reports evidence that the reduction in the monetary policy rate has transmitted to lending and deposit rates, although the pass through has been sluggish and partial. The right panel of Figure 1 displays housing market conditions. Since the monetary policy rate was reduced to almost zero in 2008Q4 until 2018Q4, the ratio of domestic mortgages to GDP has increased by 35 percentage points and the real house price index has increased by 80 percentage points.[2]

The Swiss banking system consists of two large, global systemic financial institutions, UBS and Credit Suisse, and several smaller domestically-oriented banks. Henceforth we refer to UBS and Credit Suisse as the two big banks while we label the domestically-oriented financial institutions as small banks. The small banks have been mostly responsible for the expansion in mortgages over the period under consideration. The left panel of Figure 2 shows that the share of mortgages issued by the big banks as fraction of GDP has remained stable at 40 percent while the share of small banks has increased by almost 40 percentage points. The increase in mortgage lending by small banks was funded primarily by an increase in deposits, as shown in the right panel of Figure 2.

The global financial crisis negatively affected UBS and Credit Suisse because of their global nature. In the aftermath of the financial crisis, bank capital regulation

[1]The SNB exempts most commercial banks sight deposits from the negative rate, which is levied only on deposits in excess of 20 times the minimum reserve requirement.

[2]On an annualized basis, nominal GDP growth rate was negative in 2009Q1 to 2009Q3 and positive in the rest of the sample period.

(a) Nominal money market rates

(b) Mortgage lending and real house price

Figure 1: Switzerland

has been significantly tightened in Switzerland. Banks in Switzerland are subject to regulatory capital requirements, which are based on international standards (Basel I at the beginning of the period to Basel III at the end of the period) supplemented by Swiss specificities. Additional capital requirements by bank category have been gradually imposed since 2008 and have resulted in much higher requirements for the big banks (Category 1) relative to the small ones (Category 2 to 5), as shown in Table 1.[3]

In this paper we show that both the reduction in the monetary policy rate and the asymmetric tightening of capital requirements are necessary to generate a mortgage expansion and an increase in house prices as experienced in Switzerland since 2008. We develop a model with two types of banks, big and small, which collect deposits from savers, issue mortgages to borrowers and invest in safe assets. Capital and liquidity requirements aim to limit leverage in the banking sector and maintain a minimum ratio of safe assets to deposits; these requirements constrain the behavior of financial institutions. The banking sector is imperfectly competitive; we assume that the semi-elasticity of mortgages and deposit demand is a function of the market share of the bank so that markups and markdowns also respond to market shares. We calibrate our model to match observed interest rates and regulatory requirements before entering the low interest rate environment. We then reduce the monetary policy rate and tighten capital requirements for the big banks. The reduction in the monetary policy rate from its initial positive level to zero generates a housing boom, namely an increase

[3]Bank category is determined by the size of its balance sheet, assets under management, and privileged deposits.

in mortgages and in real house prices; however, bringing the monetary policy rate below zero is not sufficient to reverse the housing boom. An asymmetric tightening in capital requirements is able to generate a reduction of mortgages and house prices and is consistent with an increase in the mortgage and deposit share of small banks.

It is the combination of the two policies that is consistent with actual developments. A reduction in the monetary policy rate with either unchanged or slack capital constraints results in the two types of bank behaving identically, market share not changing, an expansion followed by a flattening out of mortgages and deposits. On the other hand, if the monetary policy rate is held constant at a positive level while capital requirements are tightened for the big but not for the small banks leads, the market share of big and small banks move (as in the data), but aggregate mortgages contract (unlike in the data).

In our model, savers demand liquidity services provided by money and/or deposits and/or bonds. When the policy rate falls into negative territory, bonds are substituted away but deposits are not eliminated because banks keep deposit rates from becoming negative. At this point, banks stop expanding mortgages and deposit if capital constraints are unchanged; if capital constraints are tightened for big banks, mortgages go down for this type of banks and the housing boom is partly reversed. We allow for money and deposits to become perfect substitutes when the policy rate becomes negative and our results are unaffected.

We extend our model to study the effects of sectoral counter-cyclical capital buffer targeted to mortgages and find it helps to avoid excessive credit growth but does not alleviate concentration of market share and default risk in small banks.

We draw several policy implications from our analysis. First, tightening capital requirements as the policy rate is reduced is successful in limiting and/or undoing a boom in the housing market. Second, if requirements are raised only for big banks, small banks become more exposed to the housing market. If an increase in lending is associated with an increase in risk taking, as it is typically the case, then small banks become more risky. Asymmetric capital regulation may have the unintended consequence of shifting risk from big to small banks. Third, the sectoral counter-cyclical capital buffer as well as the exemption from negative rates on reserves help in either reducing credit growth or raising bank net worth; quantitatively, however, these effects are small.

The rest of the paper is organized as follows. Section 2 reviews the literature. Section 3 presents the model and section 4 discusses the calibration. In section 5 we present our main results and provide intuition; section 6 studies the robustness of our results to alternative modeling assumptions. Section 7 analyzes two policy

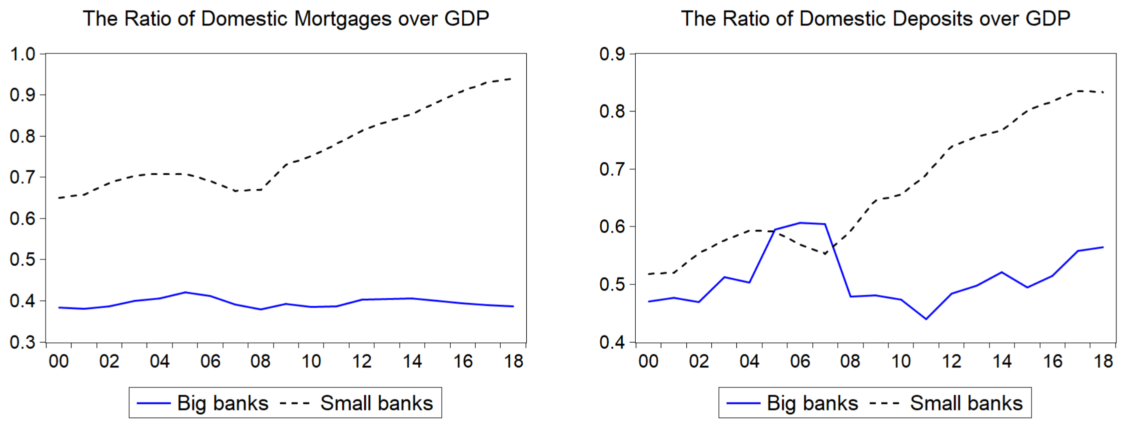


Figure 2: Mortgage supply and deposit across banks

Bank Category	Total capital requirement
Category 1	26.8
Category 2	18.8
Category 3	12
Category 4	11.2
Category 5	10.5

Table 1: 2018 Banking regulation in Switzerland

experiments: the counter-cyclical capital buffer and the exemption on reserves. Section 8 concludes.

2 Literature

Transmission of monetary policy

Our work builds on four main strands of literature. First, it relates to a rich literature that studies monetary policy transmission through the banking sector (Gertler and Karadi (2015), Brunnermeier and Sannikov (2016)). Drechsler et al. (2017) and Polo (2018) document the slow adjustment of deposit rates, and show that it amplifies the response of output to monetary policy changes (”deposit funding channel”). While Wang (2018) argues in a banking model with perfect competition that lower equilibrium real rates dampens the transmission of monetary policy.

There is a growing literature focusing on the negative interest rate policy(NIRP). This includes substantial empirical work: Bech and Malkhozov (2016) and Claessens et al. (2018) show that lower interest rate implies lower net interest margin, which has an adverse effect on banks’ profitability. Basten and Mariathasan (2018) study the case of Switzerland, it turns out that more affected banks by NIRP reduce costly reserves and bond financing, and get higher fee and interest income as a compensation. From a theoretical view, Brunnermeier and Koby (2016) provide a New Keynesian banking model and demonstrate when the policy rate below a reversal rate interest rate cuts are contractionary for capital constrained banks. Eggertsson et al. (2019) show that as the policy rate turns negative the usual transmission mechanism of monetary policy breaks down because of the importance of deposit funding and the zero lower bound on deposit rate. Relative to these literature, our work embeds incomplete pass-through to deposit and mortgage rates, and analyzes the difference in banks' rate setting in terms of monetary policy transmission and how capital regulation and deposit funding matters for driving these differences.

Macroprudential regulation

Within our framework an important driver of the differences in rate setting across banks is capital regulation, so our paper also fits into the literature that discusses the influence of macroprudential regulations on the behavior of the banking sector. Kashyap et al. (2010), Baker and Wurgler (2015), and Kisin and Manela (2016) empirically document that an increase in capital ratios would lead to a modest increase in banks' cost of capital. Consistent with their findings, Bichsel et al. (2019) estimate an increase in corporate lending spreads between 0 and 5 bps for one percentage point increase in capital requirements using the bank- and loan-level data for Switzerland, and document the institution-specific difference. Auer and Ongena (2019) also study the case of Switzerland, they identify that the introduction of countercyclical capital buffer, an additional requirement of capital holdings targeted on mortgages, may cause banks to increase lending to firms that involved in housing activities.

Other recent work stand on the theoretical side. Kashyap et al. (2017) exteneds the original Diamond-Dybvig model and find that capital regulations result in more lending, while liquidity regulations reduce lending. Mendicino et al. (2018), Nguyen (2015), and Begenau (2019) focus on the optimal level of capital requirement. Begenau and Landvoigt (2018) propose a tractable quantitative general equilibrium model with regulated and unregulated banks, it shows that tightening the capital requirement leads to a safer banking system despite riskier shadow banking activity. Some papers study other unintended consequences caused by the regulatory heterogeneity across different lenders and asset classes, such as potential regulatory arbitrage and reduced competition in the market (Acharya et al. 2013; Greenwood et al. 2017). Our paper's analysis contributes to this line of research, we build up a partial equilibrium model to look at the effect of differentiated capital regulations on banks' competition in the mortgage market, especially when the policy rate falls, and examine how this matters for aggregate default risk.

Market structure of the banking sector

Since we assumed imperfect competition in the banking sector, our work also connects to another line of literature studying the banking market structures. Egan et al. (2017) provide empirical analysis on imperfect competition in the market for deposits and quantitatively study how the competition affects the feedback between probabilities of bank default and deposits. Cuciniello and Signoretti (2014) develop a New Keynesian dynamic stochastic general equilibrium (NKDSGE) model with imperfect competition in the banking sector and collateral-constrained borrowers to address how much the banking industry market structure amplify business cycles. Drechsler et al. (2017) document that pass-through of the policy rate to deposit rates depends on market power by banks in local deposit markets in a static model. Benetton (2018) develops a structural model of mortgage demand and lender competition to study how leverage regulation affects the equilibrium in the UK mortgage market. Other recent banking theories suggest a positive relation between capital and market share(Allen et al. 2011; Mehran and Thakor 2011). Our work builds up on that and combines the method used in Atkeson and Burstein (2008) to examine the interaction between banking regulations and bank competition in the mortgage market.

Banks' risk taking

Finally, this paper studies the risk choice of banks. Jiménez et al. (2014) and Dell'Ariccia et al. (2017) show that banks' loan portfolio tend to be riskier when interest rates are low. Ravn (2016) builds a macroeconomic model in which countercyclical lending standards emerge as an equilibrium outcome. Michelangeli and Sette (2016) state that banks' risk choice depends on their capital, higher bank capital is associated with a higher likelihood of application acceptance of riskier borrowers and lower offered interest rates. Nucera et al. (2017) also document the different responses across banks in the Euro Area, large banks with more diversified income become less systemically risky under negative rates, while riskiness increases for smaller banks. Coimbra and Rey (2017) model the difference in risk taking behaviour by incorporating heterogeneous VaR constraints and Ferrante (2018) do it by pooling different loans. Relatedly, our paper document different degree of risk exposure between big and small banks by measuring the proportion of substandard borrowers accepted when the policy rate falls.

3 The Model

We now present a two-period equilibrium model that captures the relationships between monetary policy, capital requirement and bank lending. Later this model will be used to illustrate how a reduction in the policy rate, even into negative territory, pass through to the rates on mortgages and deposits, and how it interacts with a tightening of capital requirement and influences mortgage supply.

In the model, we fix growth rate of house prices and set the policy rate exogenously. All other prices and quantities are chosen optimally by the agents in the economy. The model features savers, borrowers and two types of banks. Savers consume final goods and invest on liquid assets from the options: bonds, deposits and cash. Borrowers consume and finance their house purchases with mortgages. Banks work as the financial intermediates that collect deposits from savers and then lend to borrowers, and they operate under imperfect competition in the deposit and mortgage market.

The timing is as follows:

- In period 1, banks set the rates they charge on mortgages and pay for deposits, borrowers choose how much to borrow for their housing and consumption demand, savers also decide the amount of consumption and how much liquid asset to hold.

- In period 2, there is an idiosyncratic shock to house values, which leads some borrowers to default on their mortgages. Banks get payment from repaying borrowers and then pay out savers. Thus savers and borrowers' final consumption, and banks net return are determined.

3.1 Savers

In the case of Switzerland, big and small banks have branches at each canton which offer differentiated services to savers and borrowers. Their compete on interest rates and take different level of riskiness. Individual savers and borrowers establish their own perception of which bank offers most favorable depository and lending services, and they are reluctant to switch banks. This gives banks a degree of market power in rate setting. If we take bank rates as given, the optimal problem of savers/borrowers can be split into two parts: first, deciding the amount of consumption/housing service and savings/debt, and second, choosing whichever bank to save in/borrow from. We will address the first issue at the family level for savers and borrowers in Section 3.1 and 3.2 respectively, and disclose their individual preferences on banks later in Section 3.3. Hereafter variables indexed by s refer to savers, and by b refer to borrowers.

The economy is populated by a saver family which consists of many individual savers. Each of them has his own investment choice on liquid assets. We further assume that a family leader will aggregate returns on all liquid assets and assign the amount of consumption evenly across members.

The goal of this household is to maximize the present discounted value of utility, given by:

$$V_s(C, Liq) = lnC_{s,1} + \frac{j_m}{1-\eta}(Liq)^{1-\eta} + \beta_s lnC_{s,2}, \tag{1}$$

where β_s is the discount factor, j_m governs the weight of utility from holding liquid assets Liq relative to consumption, η is the intertemporal elasticity of substitution. Savers have preferences over consumption in period 1 and period 2 ($C_{s,1}$ and $C_{s,2}$). Besides, our paper relates to debate on the importance of deposit supply by savers in a low rate environment. To this end, we also need to model savers' preference for liquidity, either by imposing a cash-in-advance constraint or by using a money-in-the-utility function specification. Following Drechsler et al. (2017) we assume savers' utility from liquidity services takes form of the second term on the right-hand side of (1). Liquidity assets are only held by savers and the amount is determined in period 1, hereafter, we drop the relevant index s and time subscript for convenience.

Liquidity services are produced from bonds B and deposits D, according to a CES aggregator:

$$Liq(B, D, M) = [\alpha_B B^{1-\frac{1}{\xi_m}} + D^{1-\frac{1}{\xi_m}}]^{\frac{\xi_m}{\xi_m - 1}},$$

where ξ_m is the elasticity of substitution between bonds and deposits, and α_b measures the liquidity quality of bonds relative to deposits and cash. First, bonds, deposits and cash all provide liquidity, they are substitutes. But, as seen in the data, there is no significant drop in the propensity to save in deposits even though deposit rate is heading towards zero, they are not perfectly substitutable. Hence we have $\xi_m > 1$. Second, nowadays electronic payments are becoming the norm and deposits can be used for transactions very conveniently, leading the economy into a cashless state, therefore we can assume that savers only have access to deposits and bonds to transfer their resources to period 2. To assess the role of cash, in Section 6 we will introduce cash into the aggregator for liquidity assets, and assume that cash share the same value in terms of liquidity quality as deposits. However, bonds still pay traditional coupon and must be paid back at maturity date, so $\alpha_B < 1$ means that bonds are less liquid than deposits and cash.

In period 1, savers finance their consumption demand and investment on liquid assets by initial wealth W_s and sales of house endowment h. In period 2, they receive returns on liquid assets, lump-sum transfers and taxes, and then all are spent on buying consumption goods. Their budget constraints in the two periods are:

$$C_{s,1} + D + B = W_s + q_{h,1}h, \tag{2}$$

$$C_{s,2} + T = RB + R^d D + \pi^b. \tag{3}$$

The policy interest rate R is the return rate that applies to government bonds, R^d is the average deposit rate offered by banks and cash has no interest. T is lump-sum tax paid to the government for issuing bonds. Banks' profits π^b are also rebated in a lump–sum fashion to savers.[4]

Maximizing the utility (1) subject to their budget constraints (2) and (3) we derive the first-order conditions relative to $C_{s,1}$, $C_{s,2}$, B and D:[5]

$$\frac{1}{C_{s,1}} = \mu_{s,1}, \tag{4}$$

$$\beta_s \frac{1}{C_{s,2}} = \mu_{s,2}, \tag{5}$$

$$\mu_{s,1} = j_m (Liq)^{-\eta - \frac{1}{\xi_m}} \alpha_b B^{-\frac{1}{\xi_m}} + \mu_{s,2} R, \tag{6}$$

$$\mu_{s,1} = j_m (Liq)^{-\eta - \frac{1}{\xi_m}} D^{-\frac{1}{\xi_m}} + \mu_{s,2} R^d, \tag{7}$$

$\mu_{s,i}$ denotes the Lagrangian multiplier on the budget constraint in period i. Equations (6) to (7) show savers' demand for bonds and deposits. Abstracting (7) from (6) we obtain

$$j_m (Liq)^{-\eta - \frac{1}{\xi_m}} (D^{-\frac{1}{\xi_m}} - B^{-\frac{1}{\xi_m}}) = \mu_{s,2}(R - R^d),$$

These equations show saver family's asset allocation based on the relevant rates. When the policy rate is positive and bigger than deposit rate, saver family invests more money in bonds rather than deposits. Then as the policy rate falls accompanied by a decrease or even a reverse in the spread between policy rate and deposit rate, it then likes to cut bond holdings and deposit more money in banks. When we introduce cash, it is initially a comparatively expensive source of liquidity, any substitution out of bonds is almost entirely into deposits. Note, however, as deposit rate is heading to zero, the spread $R^d - 1$ also decreases. Cash becomes a less expensive source of liquidity, thus saver household become more likely to put any substitution out of bond holdings into cash, even transfer deposits into cash. This suggests that deposit supply elasticity increases as rates fall, banks have to shrink the degree of rate cuts to pass onto deposit rate to avoid large outflows from deposits into cash.

[4] $\pi^b = N_G + N_D$ is interest income earned by big and small banks

[5] More details on how to solve the maximize problem are given in Section A of the Appendix.

3.2 Borrowers

As for borrowers, we also assume that their consumption is fully insured against the idiosyncratic risk in housing investment. The borrower family aggregates the amount of housing and mortgages, and then divides the respective returns and consumption equally among household members.

Borrowers differ from savers in their initial endowment, discount factor, and preferences. Their initial wealth is lower ($W_b < W_s$), and they have no houses, so that they need to borrow in order to finance their housing demand. This is consistent with the evidence that wealthier households hold less debt, see the discussion in Iacoviello and Neri (2010).[6] The discount factor is smaller ($\beta_b < \beta_s$), which implies that borrowers are less patient than savers and therefore willing to borrow from banks for current consumption demand. Like savers, borrowers gain utility from consumption $C_{b,i}$ in period i, but they also enjoy housing services h, thus borrower family's lifetime utility is:

$$V_b(C,h) = lnC_{b,1} + j_h ln(h) + \beta_b lnC_{b,2}, \tag{8}$$

where j_h measures the weight of housing service in the utility function.

In period 1, besides using initial wealth, borrowers take out mortgages L at an average rate R^l to purchase housing at a relative price $q_{h,1}$ and consumption goods. In period 2, an idiosyncratic shock occurs which affects the value of their houses. Following Bernanke et al. (1998) we assume that this shock is independently and identically distributed across borrower members within the family and follows the log-normal distribution:

$$ln(\omega) \sim N(-\frac{\sigma_\omega^2}{2}, \sigma_\omega^2) \ .$$

After the shock, borrowers decide whether to default or not by comparing the housing value against mortgage payment. This leads to a threshold of this idiosyncratic shock denoted by:

$$\bar{\omega} = \frac{R^l L}{q_{h,2} h}, \tag{9}$$

at which borrowers are indifferent between default and repaying. More specifically, when the idiosyncratic shock ω is smaller than $\bar{\omega}$, the aftershock housing value ($\omega q_{h,2} h$) is below mortgage payment and thus borrower will default. If borrowers default on their mortgages banks will repossess their houses; if not borrowers keep their houses and sell them in order to repay their mortgages and consume. $1 - F(\bar{\omega})$ denotes the

[6] Iacoviello and Neri (2010) find that credit-constrained agents have a lower labor income share than the unconstrained.

share of borrowers who repays, where

$$F(\bar{\omega}) = \int_0^{\bar{\omega}} f(\omega) d\omega,$$

and $1 - G(\bar{\omega})$ is the fraction of housing stock owned by repaying borrowers, where

$$G(\bar{\omega}) = \int_0^{\bar{\omega}} \omega f(\omega) d\omega.$$

Summarizing, we can aggregate the budget constraints across the borrower family:

$$C_{b,1} + q_{h,1}h = W_b + L, \tag{10}$$

$$C_{b,2} = (1 - G(\bar{\omega}))q_{h,2}h - (1 - F(\bar{\omega}))R^l L. \tag{11}$$

Maximizing the utility (9) subject to above constraints (11) and (12), and substituting $\bar{\omega}$ by (10), we obtain first-order conditions relative to $C_{b,1}$, $C_{b,2}$, h and L:

$$\frac{1}{C_{b,1}} = \mu_{b,1}, \tag{12}$$

$$\beta_b \frac{1}{C_{b,2}} = \mu_{b,2}, \tag{13}$$

$$\mu_{b,1}q_{h,1} = \frac{j_h}{h} + \mu_{b,2}[(1 - G(\bar{\omega}))q_{h,2} + G^{'}(\bar{\omega})\frac{R^l L}{h} - F^{'}(\bar{\omega})\frac{(R^l L)^2}{q_{h,2}h^2}], \tag{14}$$

$$\frac{\mu_{b,1}}{\mu_{b,2}} = (1 - F(\bar{\omega}))R^l - F^{'}(\bar{\omega})\frac{(R^l)^2 L_t}{q_{h,2}h} + G^{'}(\bar{\omega})R^l, \tag{15}$$

where $\mu_{b,i}$ denotes the Lagrangian multipliers on the budget constraint in period i. Notice that R^l is predetermined for borrower family at the beginning of period 1, therefore there is no incentive compatibility constraint for them to satisfy as in Bernanke et al. (1998) and the threshold value depends on the leverage chosen by the family according to (10).

Plugging (13) and (14) into (16) gives the consumption Euler equation of borrowers. It equalizes the marginal rate of substitution between consumption in period 1 and period 2, to the actual cost of an addition unit of mortgage, which involves the repayment rate in case of no default (the first term on the RHS) and marginal impact on $\bar{\omega}$ (the last two terms on the RHS). An increase in mortgage leads to a higher default threshold value $\bar{\omega}$, this translates into a smaller share of borrowers repaying and therefore total mortgage repayment cost shrinks. Equation (15) shows that marginal utility of forgone consumption for buying one additional unit of housing must equal marginal utility gain from housing service in period 1 (the first term on the RHS), plus the utility from consumption in period 2 stemming from the sale of the non-defaulted housing stock. The latter internalizes the impact of an increase in housing on the threshold value as shown by the last two terms on the RHS.

3.3 Aggregation of Mortgages and Deposits

We now turn to describe the composition of family demand for mortgage and supply of deposit taking bank rates as given, and the rates setting will be explained in section 3.4. Inspired by Allen and Gale (2004), we assume that members in the saver and borrower family are dispersed in a continuum of locations indexed by $i \in [0, 1]$. At each location, there are branches of both big and small banks offering differentiated depository and lending services, and we index big banks by G and small banks by D. As in previous work by Gerali et al. (2010), we assume that mortgages taken by borrowers are CES aggregate of big and small banks' loans and similarly for deposits held by savers. Agents' demand for a particular bank's service depends on its rates. Local branches act atomistically, this is to say that when a branch sets its rates., it affects local mortgage granting and deposit collecting but it does not have effect on the aggregate demand and rates.

Mortgage market: Aggregate mortgage is a CES aggregator of loans taken out by members at all locations:

$$L = [\int_0^1 L_i^{1-1/\rho_l} di]^{\rho_l/(\rho_l-1)}, \tag{16}$$

where ρ_l is the elasticity of substitution across locations. The aggregate mortgage rate R_t^l is defined as

$$R^l = [\int_0^1 (R_i^l)^{1-\rho_l} di]^{1/(1-\rho_l)}. \tag{17}$$

Minimizing total mortgage repayment over all locations $\int_0^1 R_i^l L_i$ subject to (17), we obtain the demand for mortgages at location i:

$$L_i = (\frac{R_i^l}{R^l})^{-\rho_l} L. \tag{18}$$

At location i, a mortgage is a CES composite of differentiated loans supplied by local big and small banks, according to the following function

$$L_i = [(1-\alpha_{l,D})^{\frac{1}{\xi_l}} L_{i,G}^{1-\frac{1}{\xi_l}} + \alpha_{l,D}^{\frac{1}{\xi_l}} L_{i,D}^{1-\frac{1}{\xi_l}}]^{\frac{\xi_l}{\xi_l-1}}, \tag{19}$$

where ξ_l is the elasticity of substitution across banks. Later we calibrate $\alpha_{l,D}$ to match initial share of small banks in the mortgage market. The average mortgage rate at location i is defined as

$$R_i^l = [(1-\alpha_{l,D})(R_{i,G}^l)^{1-\xi_l} + \alpha_{l,D}(R_{i,D}^l)^{1-\xi_l}]^{\frac{1}{1-\xi_l}}.$$

Again, maximizing mortgage repayment to both big and small banks at location i subject to local CES mortgage constraint, yields demand for mortgages granted by bank j at location i as:

$$L_{i,j} = \alpha_{l,j}(\frac{R^l_{i,j}}{R^l_i})^{-\xi_l} L_i, \quad j = G, D.$$

Plugging (17) in, we can write it as

$$L_{i,j} = \alpha_{l,j}(\frac{R^l_{i,j}}{R^l_i})^{-\xi_l}(\frac{R^l_i}{R^l})^{-\rho_l} L, \quad j = G, D \ . \tag{20}$$

With $\xi_l > 1$, the higher mortgage rate $R^l_{i,j}$ a bank sets, the less mortgage demand $L_{i,j}$ it faces.

Deposit market: This nested CES setting is also applicable to the deposit market. Aggregate deposit supply and rate are respectively given by:

$$D = [\int_0^1 D_i^{1-1/\rho_d} di]^{\rho_d/(\rho_d-1)}, \quad R^d = [\int_0^1 (R^d_i)^{1-\rho_d} di]^{1/(1-\rho_d)} \ .$$

Maximizing the revenue of deposits over all locations $\int_0^1 R^d_i D_i$ subject to the above CES supply constraint gives the supply of deposits at location i as:

$$D_i = (\frac{R^d_i}{R^d})^{-\rho_d} D. \tag{21}$$

At each location, deposit supply is a composite of deposits supplied to local branches of big and small banks, that takes the form of

$$D_i = [(1-\alpha_{d,D})^{\frac{1}{\xi_d}} D_{i,G}^{1-\frac{1}{\xi_d}} + \alpha_{d,D}^{\frac{1}{\xi_d}} D_{i,D}^{1-\frac{1}{\xi_d}}]^{\frac{\xi_d}{\xi_d - 1}}, \tag{22}$$

the local deposit rate is denoted by

$$R^d_i = [(1-\alpha_{d,D})(R^d_{i,G})^{1-\xi_d} + \alpha_{d,D}(R^d_{i,D})^{1-\xi_d}]^{\frac{1}{1-\xi_d}}.$$

Maximizing returns on deposits from local banks subject to (23) generate the deposit supply to bank j, and then by plugging in (22) we obtain

$$D_{i,j} = \alpha_{d,j}(\frac{R^d_{i,j}}{R^d_i})^{-\xi_d}(\frac{R^d_i}{R^d})^{-\rho_d} D, \quad j = G, D. \tag{23}$$

With $\xi_d < -1$, the higher deposit rate $R^d_{i,j}$ a bank sets, the more deposits $D_{i,j}$ it takes.

Due to the assumption that savers and borrowers are equally distributed across locations, branches of each type of bank at different locations will behave symmetrically: $L_{i,j} = L_j$ and $D_{i,j} = D_j$. We can drop the index i going forward to focus on bank behavior of each type.

3.4 Banks

Banks raise deposits from savers, invest in safe assets (government bonds) and grant mortgages to borrowers. As mentioned before, there are two types of banks competing in both deposit and mortgage markets. Savers and borrowers treat their services as imperfectly substituable products, thereby giving banks a degree of market power. Banks can internalize mortgage demand (21) and deposit supply (24) of agents when choosing rates so as to maximize discounted value of final net worth. Since banks are owned by savers, they share same discount factor β_s and transfer their profits to savers in the second period.

The balance sheet identity of bank j is given by:

$$L_j + S_j = D_j + N_{j,0}, \quad j = G, D.$$

It is funded by initial net worth $N_{j,0}$ and deposits D_j taken from savers, and invest in mortgage lending L_j and safe assets S_j. Its net worth in period 2 N_j is the returns on these investments net of the funding cost

$$N_j = R_j^{l,2} L_j + RS_j - R_j^d D_j, \quad j = G, D.$$

where R_j^d is deposit rate chosen by the bank. Safe assets receive a riskless interest rate R, and the gross return on mortgage is given by:

$$R_j^{l,2} L_j \equiv (\frac{L_j}{L}) \int_0^{\bar{\omega}} (1-\mu)\omega q_{h,2} h \ f(\omega) d\omega + \int_{\bar{\omega}}^{\infty} R_j^l L_j f(\omega) d\omega,$$

which comprises repayments from non-defaulting borrowers (the second term on the RHS), plus the proceeds from seizing the housing stock of defaulting borrowers, net of monitoring cost as a proportion μ of the housing value (the first term on the RHS). Since the borrower family holds a composite mortgage, in case of default each bank seizes the fraction $\frac{L_j}{L}$ of the defaulted housing stock.[7] Banks sustain losses when borrowers default. We assume that banks anticipate that a fraction of borrowers will default, and incorporate the expectation in their optimal choice of deposit and mortgage rates.

Macroprudential Regulations: Banks are subject to two constraints, capital constraint and liquidity constraint, as follows

$$\varphi_j^c(R) L_j \leq N_j, \quad j = G, D. \tag{24}$$

[7] The housing stock seized by each type of bank is dependent on the bank's share in the mortgage market.

$$\varphi^l D_j \leq S_j, \quad j = G, D. \tag{25}$$

The banks' risk-weighted assets are mortgages weighted by a risk measure φ_j^c plus safe assets weighted by zero, as consistent with reality,[8] and capital constraint (20) requires a proportion of these risk-weighted assets to be covered by banks net worth. Liquidity constraint (21) requires that a fraction φ^l of deposits should be invested in safe assets to maintain sufficient liquidity.

After the financial crisis, these requirements have been tightened significantly based on the revised Basel III international standards and the Swiss-specific "too big to fail" regulation. In particular, the required ratio of total capital to risk-weighted assets has increased from around 9.6 percent in 2008 to 26 percent in 2017 for two big banks, and this modification has also been done for small banks, although to a lesser extent. Since the policy rate fell during this period, for simplicity we model the risk-weighted capital ratio φ_j^c as an increasing function of the policy rate. Hence it increases throughout this period. More importantly, we assume that the initial level of risk-weighted capital ratio is lower for small banks, and capital constraint on small banks is also less tightened when the policy rate falls. That is, $\varphi_D^c < \varphi_G^c$ all along.

Banks choose their rates to maximize discounted value of net worth in period 2 taking mortgage demand and deposit supply as given. Thus their optimization problem can be written as

$$\begin{aligned} max\ \beta_s\{R_j^{l,2} L_j + RS_j - R_j^d D_j\} + \lambda_j^b\{D_j + N_{j,0} - L_j - S_j\} \\ + \lambda_j^c\{R_j^{l,2} L_j + RS_j - R_j^d D_j - \varphi_G^c L_j\} + \lambda_j^l\{S_j - \varphi^l D_j\} \\ + \lambda_j^m\{L_j - \alpha_{l,j}(\frac{R_{i,j}^l}{R_i^l})^{-\xi_l}(\frac{R_i^l}{R^l})^{-\rho_l} L\} + \lambda_j^d\{D_j - \alpha_{l,j}(\frac{R_{i,j}^d}{R_i^d})^{-\xi_d}(\frac{R_i^d}{R^d})^{-\rho_d} D\}, \quad j = G, D. \end{aligned}$$

where λ_j^b, λ_j^c, λ_j^l, λ_j^m and λ_j^d are Lagrangian multipliers on balance sheet constraint, capital constraint, liquidity constraint, mortgage demand and deposit supply functions respectively.

The first-order conditions relative to R_j^L and R_j^d are:

$$R_j^l = \underbrace{\frac{1}{1 - \frac{1}{\epsilon_j^l}}}_{\text{markup}} \frac{1}{1 - F()}[R - (1-\mu)\frac{G(\bar{\omega})}{\bar{\omega}} R^l + \underbrace{\frac{1}{\beta_s + \lambda_j^c}(\varphi_j^c \lambda_j^c + \lambda_j^l)}_{\text{regulation cost}}], \quad j = G, D. \tag{26}$$

$$R_j^d = \underbrace{\frac{1}{1 - \frac{1}{\epsilon_j^d}}}_{\text{markdown}} [R + \underbrace{\frac{1}{\beta_s + \lambda_j^c}(1 - \varphi^l)\lambda_j^l}_{\text{regulation cost}}], \quad j = G, D. \tag{27}$$

[8] Safe assets, as government bonds, are secured by the government and carry no risk, while residential mortgages without guarantee from the government are weighted anywhere from 35% to 100% depending on the related LTV ratios in Switzerland.

where ϵ_j^l and ϵ_j^d denotes the semi-elasticity of demand for mortgages and deposits respectively. Following Atkeson and Burstein (2008) these are functions of bank's market share:

$$\epsilon_j^l = \xi_l(1 - s_j^l) + \rho_l s_j^l,$$
$$\epsilon_j^d = \xi_d(1 - s_j^d) + \rho_d s_j^d,$$

where s_j^l and s_j^d denotes the share of bank j in the mortgage and deposit market, which is the ratio of mortgages/deposits granted by it relative to the total mortgages/deposits granted by all banks. With $\xi_l > \rho_l$ the elasticity ϵ_j^l is a decreasing function of bank's market share and thus markup is an increasing function of that. With $\xi_d < \rho_d$ the elasticity ϵ_j^d is an increasing function of bank's market share and thus markdown is a decreasing function of that. Accordingly, banks with bigger market share have larger market power, and are able to set a relatively higher markup and a lower markdown.

The Kuhn-Tucker conditions are

$$\lambda_j^c[N_j - \varphi_j^c L_j] = 0, \quad j = G, D.$$

$$\lambda_j^l[S_j - \varphi^l D_j] = 0, \quad j = G, D.$$

$$\lambda_j^c \geq 0, \quad j = G, D.$$

$$\lambda_j^l \geq 0, \quad j = G, D.$$

All these equations characterize the optimal interest rates chosen by banks under the supervision of macroprudential regulations. First, because of imperfect competition across banks in the deposit and mortgage market, mortgage rate is a markup on the policy rate while deposit rates is a markdown. Second, when capital constraint binds banks have to charge higher margin between mortgage rate and policy rate, to depress loan demand and push down their leverage. When liquidity constraint binds, banks have to increase both deposit rate and mortgage rate, to increase their liquidity ratio. This effect could be so strong as to reverse the markdown relationship between deposit rate and policy rate. And when both constraints are slack, the respective Lagrangian multipliers are simply zero, that means there is no effect of regulations on bank rates setting.

There is one point worth noting that the optimization problem is formed similarly for big and small banks with only difference in the capital requirement, and this is a key aspect driving the shift in market share between them. As the police rate falls, banks increase their mortgage supply by lowering mortgage rates. An expansion in assets makes capital constraint easy to bind, and stricter requirement on big banks means that it is more likely to bind first for them. As shown in equation (27), lending spreads are affected by binding regulatory requirements, thus big banks would set a relatively higher lending spreads than small banks do, implying a limited lending capacity. Taking advantage of big banks' weaknesses in loan supply, small banks gain share in the mortgage market. Additionally, small banks can derive a competitive advantage from higher market share in increasing markup and lowering markdown, which contributes to raise their net worth.

3.5 Markets Clearing

The quantity of houses is fixed at

$$h = 1, \tag{28}$$

which implies that borrowers' demand for housing determines entirely its price. There is no mechanism in the model to price housing in the second and final period of the economy. As typical in this class of models, we fix house prices in the final period; more precisely, we fix the rate of growth of house prices from period 1 to 2 to be g_q. In section B of the appendix, we study the robustness of our results to different values of g_q.

$$q_{h,2} = g_q * q_{h,1}. \tag{29}$$

Government elastically supplies bonds to savers and banks. We assume that government can keep the money attracted from the bond market in period 1 with a costless technology. In period 2 government pays interest on bonds with lump-sum tax transfers T.

$$T = (R - 1)(B + S). \tag{30}$$

Similarly, we assume that cash M is elastically supplied by the government with the costless technology; the demand for cash pins down its supply.

Equilibrium: Given the policy rate exogenously chosen by the central bank, an equilibrium is a set of prices (mortgage rates, deposits rates and house prices) and quantities (consumption, cash, deposits, bonds and mortgages) such that savers, borrowers and banks optimize, and all markets clear.

4 Parameterization

All parameter values are specified in Table 2. Most parameter values are calibrated to match the average value of relevant variables over the period 2000-2007; during this period, market share of big and small banks remained roughly constant.

Targets: We have four sources of data. The first is nominal interest rates which we then deflate by the CPI inflation. The SNB publishes monthly interest rates for new transactions, but we are limited to use the variable mortgage rate and term deposit rate because other interest rate data starts in 2008. The average real mortgage rate we calculate for the period 2000-07 is 2.5% and deposit rate is 0.25%. We use the three-month Swiss Franc Libor as the policy rate, and the average real rate over 2000-07 is 0.7%. Then markdown and markup can be derived from dividing the average deposit rate and mortgage rate by this benchmark rate, which is 0.995 and 1.018 respectively.

The second set of data is the monetary aggregates. We take the Swiss banknotes in circulation and the total value of demand, time and saving deposits from the SNB, and then calculate the average money-to-deposit ratio for the period 2000-07, which is 0.05. We find an average general government debt to GDP ratio for the period 2000-07 as 40% using data from the Federal Statistics office, and calculate the average bond-to-deposit ratio, which is 0.38.[9]

The next source of data is statistics on banks balance sheet, including mortgages, deposits and reserves, which is published by the SNB. We use these data to calculate the market share of the small banks in deposit and mortgage markets, as seen below in Panel (b) of Figure 4.

Finally, we use regulatory data that includes the Basel I to III capital requirements plus Swiss specific requirements. More precisely, we take the required risk-weighted capital ratios for big and small banks from Bichsel et al. (2019) and express them as a function of the policy rate.[10]

Calibrated parameters: Savers' discount factor β_s and initial wealth W_s are calibrated jointly to pin down the average real deposit rate of 0.3% along with a policy rate at 0.75%. Borrowers' discount factor and initial wealth are calibrated to be $\beta_b = 0.95$ and $W_b = 5$ to obtain an average real mortgage rate of 2.5%. To match initial bond-to-deposit ratio, we set the measurement of bonds' relative liquidity quality α_b to be 0.6, the weight on liquid assets in the Savers' utility function to be $j_m = 0.006$, and Borrower's housing preference to be $j_h = 0.075$. The curvature parameter of Savers' utility with respect to liquid assets η affects the pass-through of changes in the policy rate to deposit rates and we set it equal to 1, which implies log utility from liquidity.

Initial net worth for big and small banks is chosen to be the same and equal to 0.4;

[9] Government Debt accounts for 40% of GDP for the period 2000-2007, and total domestic deposits accounts for 105% of GDP. So we get the bond-to-deposit ratio to be 0.38.

[10] See section D in the Appendix for the data about total required capital ratio. We set $\varphi^c_j = 0.1 + (R <= 1.005) * \alpha^c_j * (1.0075 - R)$ to match the increasing capital ratio when the policy rate falls, where $\alpha^c_G = 6$ and $\alpha^c_G = 3$.

Parameter	Value	Description	Targets
β_s	0.99	discount factor for savers	deposit rate 0.3%
β_b	0.95	discount factor for borrowers	mortgage rate 2.5%
j_m	0.006	weight on liquidity in savers' utility	
ξ_m	2.5	elasticity of substitution between deposits and cash	money-to-deposit ratio 0.05
η	1	utility curvature in liquid assets	
α_b	0.4	liquidity quality of bonds	bond-to-deposit ratio 0.38
j_h	0.075	weight on housing in borrowers' utility	
φ_G^c	0.1	capital constraint for big banks	required capital ratio 0.096
φ_D^c	0.08	capital constraint for small banks	
φ^l	0.1	liquidity constraint	
$N_{G,0}, N_{D,0}$	0.4	initial net worth of banks	
W_s, W_b	8,5	initial wealth of savers and borrowers	
ξ_d	-160	elasticity of substitution of deposits across banks	markdown 0.996
ξ_l	140	elasticity of substitution of mortgages across banks	markup 1.018
ρ_d	-10	elasticity of substitution of deposits across locations	
ρ_l	10	elasticity of substitution of mortgages across locations	
g_q	0.0	growth rate of housing price	
μ	0.12	liquidation cost for banks	Bernanke et al. (1998)
σ_ω	0.16	std of idiosyncratic shock	Bernanke et al. (1998)

Table 2: Model parameters

before the financial crisis, the joint capital of the two big banks and the domestically-oriented banks was roughly of the same magnitude. In the CES aggregators, the elasticity of substitution of deposits across banks ξ_d is set to -160 to match a markdown for the deposit rate $\frac{R^d}{R}$ which is equal to 0.996, and the elasticity of substitution of mortgages across banks ξ_l is set to 140 to match a markup for the mortgage rate $\frac{R^l}{R}$ equal to 1.018.

The initial required risk-weighted capital ratios for big banks and small banks are 0.1 and 0.08 respectively, which broadly match required capital ratios before 2008. Since the financial crisis capital requirements have been gradually tightened to reach 0.24 and 0.14 by the end of our sample in 2018, therefore, as explained earlier, we model risk-weighted capital requirements that increase as the policy rate falls so as to match their actual temporal profile, see **??**. We set the parameter φ^l of the liquidity requirement equal to 0.1 for both big and small banks. This implies that banks must hold 10 percent of their deposits in the form of liquid assets.

Finally, we set the growth rate of house price to 0.[11] μ is set to 0.12 as in Bernanke et al. (1998), so that the monitoring cost after default is 12 percent of housing value.

[11] We perform a sensitivity analysis in Section B of the Appendix by choosing a positive growth rate of house prices, and find that our qualitative results remain unchanged.

The standard deviation of idiosyncratic shock σ_ω is at 0.16 to reach an average default rate around 3%.

5 Model Evaluation

We simulate our model for levels of the policy rate R between 1.0075 and 0.985, which match the real interest rate at the beginning and the end of our sample (2008-2018). We present the period-2 equilibrium level of mortgages, deposits, the relevant rates and banks net worth below, see Figure 3. We remind the reader that a reduction in the policy rate is accompanied by a tightening of capital constraint for the two sets of banks as shown in.[12]

Figure 3: The impact of a decreasing policy rate and a tightening of capital requirement

For all plots, the horizontal axis measures the level of the policy rate, 0.75% (R=1.0075) of our baseline calibration to -1.5% (R=0.985) as going from left to right. The lower left panel shows that, starting from the value of 1.0075, the reduction in R is fully transmitted to both deposit and mortgage rates. Cheaper mortgage drives up housing demand, leading to an increase in mortgage lending. Both big and small banks respond in the same way to the initial reduction in the policy rate, collecting more

[12]To isolate the effect of a declining policy rate and a tighter capital requirement, we proceed by eliminating one at a time and then repeat our analysis. Section C in the Appendix presents our results.

deposits and then issuing more mortgages. This process continues until the policy rate reaches 0.25% when capital constraint becomes binding for big banks.

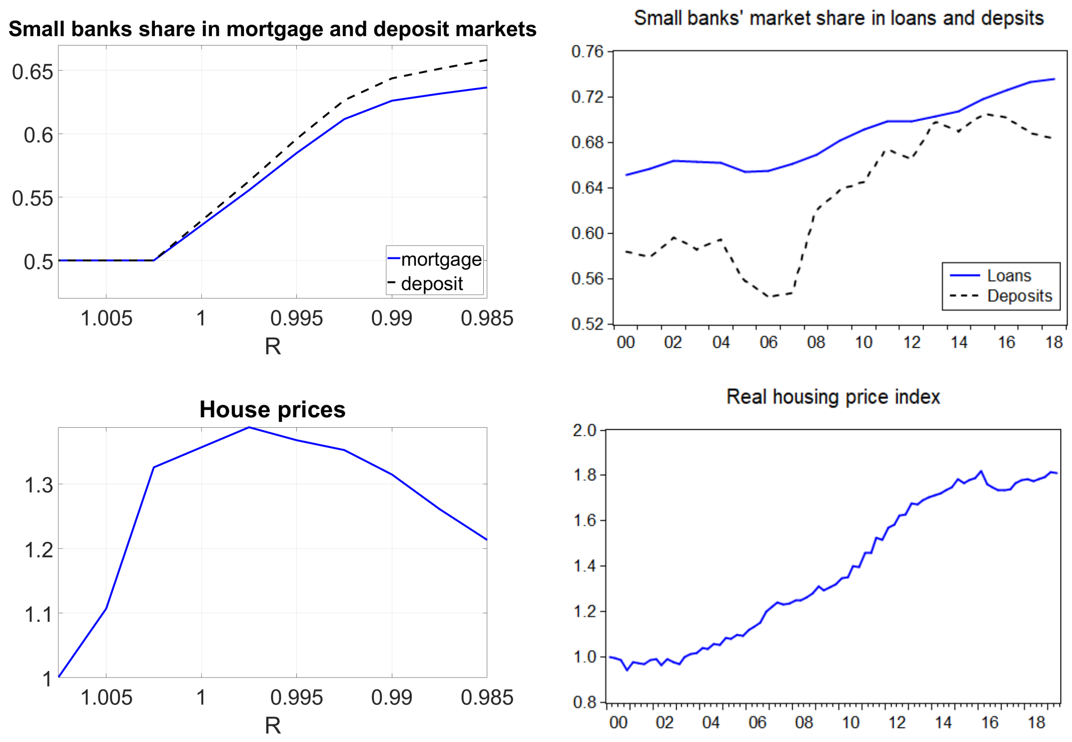
Further reductions in the policy rate are accompanied by higher capital requirements, which the big banks meet by cutting their supply of mortgages and their demand of deposits. The reduction in mortgages by big banks is achieved by raising the mortgage rate above the level offered by the small banks; similarly, the reduction in deposits is achieved by bringing the rate on deposits below the small banks' counterpart. As a result, a fraction of mortgages and deposits shifts from big to small banks, raising the market share of the latter. Small banks expand their mortgages and deposits as the policy rate falls below 1.0025 because their capital requirement is lower relative to the big banks and not yet binding. In fact, small banks absorb borrowers and depositors that leave the big banks because of their widened interest margins. The lower left panel confirms that big banks widen their interest rate margin by raising R^l and reducing R^d relative to the small banks; the red and green line are the average mortgage and deposit rate.

When the policy rate falls, savers switch out of bonds into deposits because they need liquidity services and deposits are better remunerated than bonds. It is thanks to this mechanism that small banks can expand their deposits and their lending even if R is below one. However, when the policy rate reaches -0.75%, capital requirement becomes binding for small banks, which constrains their lending capacity as well. Afterwards small banks follow big banks to reduce their supply of mortgage and demand of deposits. The asymmetric response of banks to further rate reductions ends.

The lower right panel of Figure 3 displays bank net worth in period 2. When the policy rate is reduced from its initial level of 1.0075, banks issue more mortgages, which improves bank profits. Once the capital constraint becomes binding, the big banks increase mortgage rate, partly offsetting the negative effect of limited lending capacity on their profits. However, their net worth starts to grow at a lower pace than in small banks. The difference in capital constraint results in differing lending capacity, and thereby net worth disparities between big and small banks.

The upper left panel of Figure 4 plots the mortgage and deposit market share of small banks implied by the model. The market share of small banks is constant until the capital requirement becomes binding for big banks; at this point, the big banks curtail lending while small banks continue expanding in response to further reduction in the policy rate. As a result, the market share of small banks increases steadily. Once the capital constraint also binds for small banks, the market share of small banks continues to increase albeit at a reduced rate, because small banks still offer better rates than the big banks, as documented in Figure 3. We find that our model rationalizes fairly well the actual change in market share that occurred between 2008 and 2018, which is plotted in the right panel of Figure 4.

Our model predicts that a decreasing policy rate fuels a growth in housing price, but only up to a point where capital constraint becomes binding for both banks. Our model predicts a 39 percentage point increase in the real house price, which is broadly consistent with the 46 percentage point increase in the real house price index in Switzerland since 2008, when our data sample starts from. However, our result overestimates the decrease in house prices relative to the data once the real policy rate falls below 0.9925. In our model, deposits and therefore mortgages fall because capital constraint becomes binding for both banks; in the data, deposits and mortgages continued to grow after 2015. It is also important to note is that our model-predicted deposit and mortgage rates are able to track the evolution of actual data that we see in Figure 1.



(a) model (b) data

Figure 4: Small banks' market share and house prices, model v.s. data

Note that the initial level of interest rate at 0.75% in the simulated results is set to match the average value during the period 2000-2008, thus we calculate the increase in small banks' market share and house prices since the period 2000-2008. To make it easy to compare the results, we normalize initial level of house prices to 1 on both sides.

Figure 5 plots the evolution of saver bond and deposit holdings, which helps better understand the dynamics of deposits. When the policy rate is positive, savers enjoy

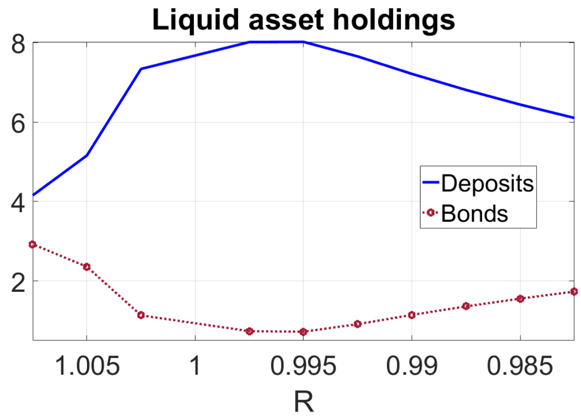


Figure 5: Movement of saver family's liquidity holdings

positive returns on bonds. As the policy rate falls, savers reduce bond holdings and transfer them into deposits since the deposit rate is not reduced one-to-one with the policy rate. When capital constraints bind, banks need to reduce deposits, savers substitute part of deposits back into bonds since banks lower deposit rate nearly one-for-one along with further rate cuts.

6 Understanding the Mechanism

This section analyzes the role played by our modeling assumptions on the results presented in Section 5. We also augment our benchmark model with substandard mortgages, to capture the fact that an expansion of credit typically goes hand-in-hand with an increase in risk, especially for small banks.

6.1 With Propensity to Substitute into Cash

In the baseline framework, savers only have access to deposits and bonds to smooth consumption across periods. As the policy rate falls, savers switch out of bonds into deposits because they need liquidity services and deposits are better remunerated than bonds. Even when the policy rate becomes negative, banks still can expand their deposit funding, and therefore their lending increases.

It is possible that cash provides liquidity services and may dominate liquid assets with negative nominal returns. In this section we introduce cash into our model. Savers can transfer resources from period 1 to 2 by holding cash, which pays zero nominal interest rate; cash and deposits provide identical liquidity services and are substitutable, although not perfectly. For capturing this, the aggregator of liquidity assets now becomes

$$Liq(B, D, M) = [\alpha_b B^{1-\frac{1}{\xi_m}} + [(D^{1-\xi_c} + M^{1-\xi_c})^{\frac{1}{1-\xi_c}}]^{1-\frac{1}{\xi_m}}]^{\frac{\xi_m}{\xi_m - 1}}, \tag{31}$$

with $0 < \xi_c < 1$. We consider the case of perfect substitutability where ξ_c goes close to zero in the next subsection. The main effect of having cash in our model is that deposits and mortgages start falling for a higher policy rate; qualitative dynamics of the model is otherwise identical, as seen in Figure 6.

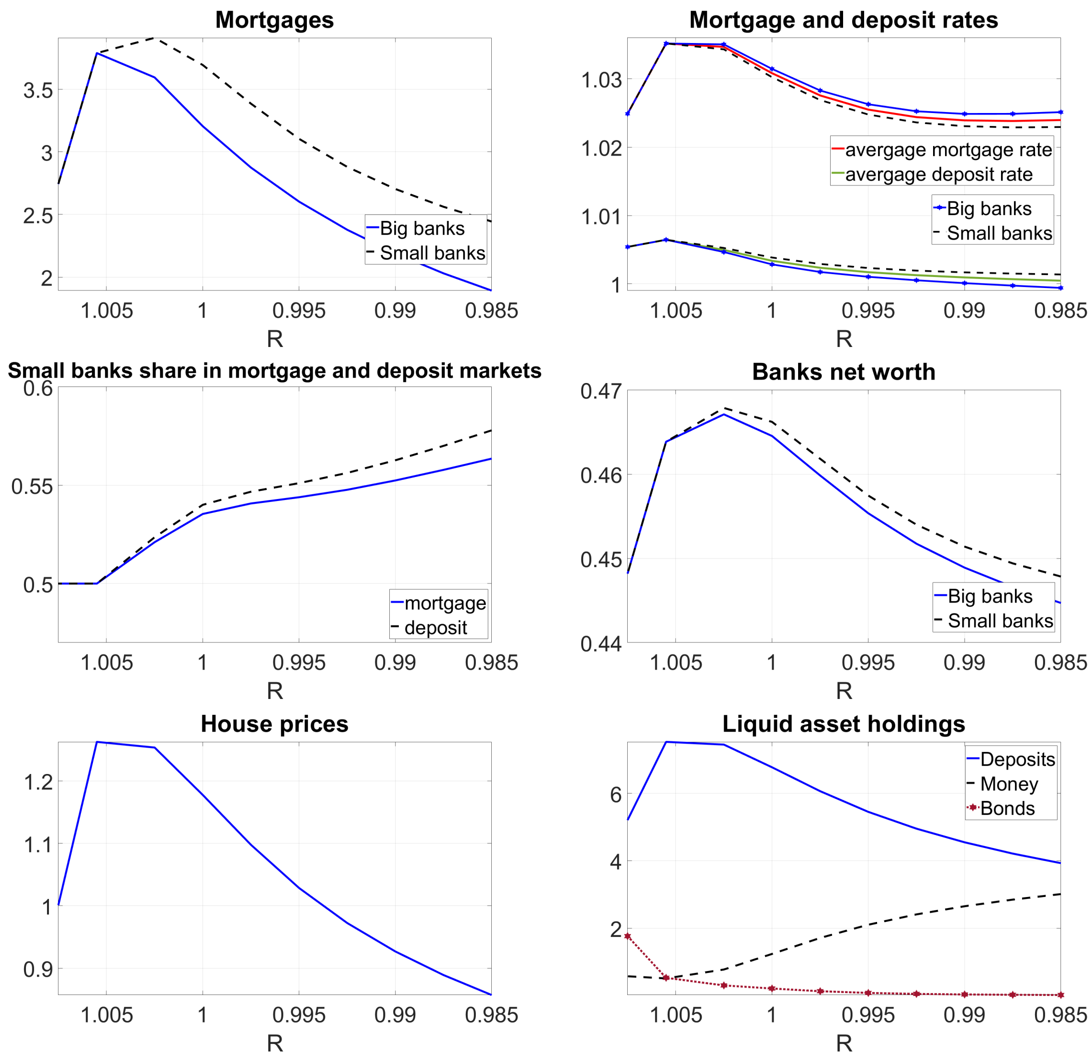


Figure 6: With cash holdings

When the real policy rate gets close to negative, savers switch out of deposits into cash so that banks start to lose deposits. This funding squeeze negatively influences their mortgage supply, and consequently total mortgage lending decreases. An accomplished result is that default risk becomes smaller, leading average mortgage rate to fall. To avoid large outflows from deposits into cash, banks cannot push the deposit rate too much into negative territory; the markdown is first eliminated and then becomes a markup, eventually flipping the sign of conventional markdown. Once the policy rate falls below one, we find a decaying pass-through of the policy rate cuts onto mortgage and deposit rate across banks.

The second right panel shows that banks net worth falls significantly when the policy rate becomes negative. There are a number of reasons. First, the reduction in the policy rate cannot be passed-through to the deposit rate because savers have the option to hold cash, whose net return is zero. Banks therefore make a loss from holding deposits. Second, banks must hold safe assets due to the liquidity constraint; when the policy rate turns negative, banks make a loss from holding safe assets. Third, decreased deposits make banks cut lending; once lending falls, the net worth of the bank falls.

The differences in capital requirements still make banks respond differently to the rate cuts. Similar to our baseline result, the market share of small banks increases, although at a reduced rate. It is because the funding constraint restrict banks to expand their lending as before.

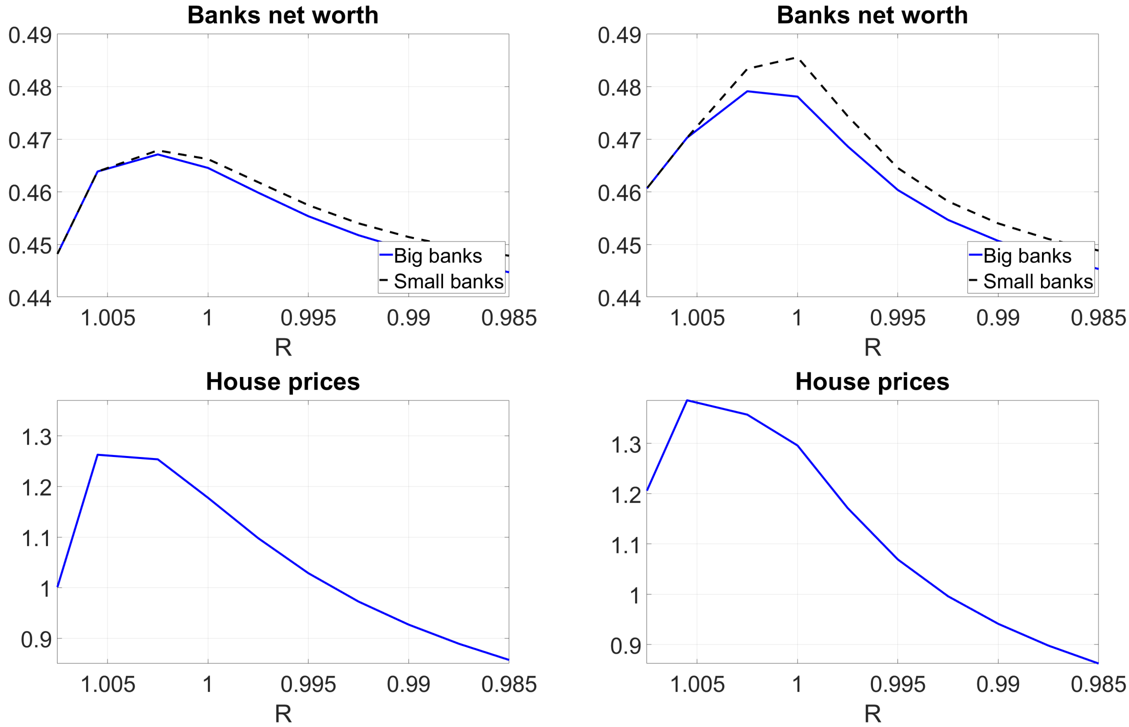
6.2 Cash and Deposits as Perfect Substitutes

Now we assume that cash and deposits are perfect substitutes for the provision of liquidity services by setting ξ_c equal to 0.1. We choose this positive value rather than zero for ξ_c to avoid a corner solution where savers only hold deposits as long as $R > 1$ and fully switch to cash when $R < 1$. Panel (b) of Figure 7 shows that banks net worth and houses prices drop more sharply than in our baseline result when the policy rate turns negative. Intuitively, when cash competition becomes stronger banks lose deposits and and therefore curtail mortgage lending faster. As a result, the fall in both banks net worth and house prices from the peak is around 1.5 times larger than under the baseline setting.

We draw an important implication from our analysis. In cashless economies (as Denmark and Sweden), there is no flight from deposits in response to negative policy rates; however, credit expansion may also continue well into negative territory for the policy rate.

6.3 Constant Market Power

Bank market power plays an important role in banks' responses to the policy rate cuts because they can adjust markups and markdowns, thereby influencing their net worth and the amount of mortgages issued. The benchmark model allows for the market power to increase with the market share of the bank; as a result, differences in size across banks imply different lending and funding decisions. In this subsection,



(a) $\xi_c = 0.3$ (b) $\xi_c = 0.1$

Figure 7: The impact of cash and deposits becoming perfect substitutes

we compare our baseline result to that emerges from the setting where banks have constant market power. In this case, changes in the relative size and market share of banks do not cause a change in markups and markdowns.

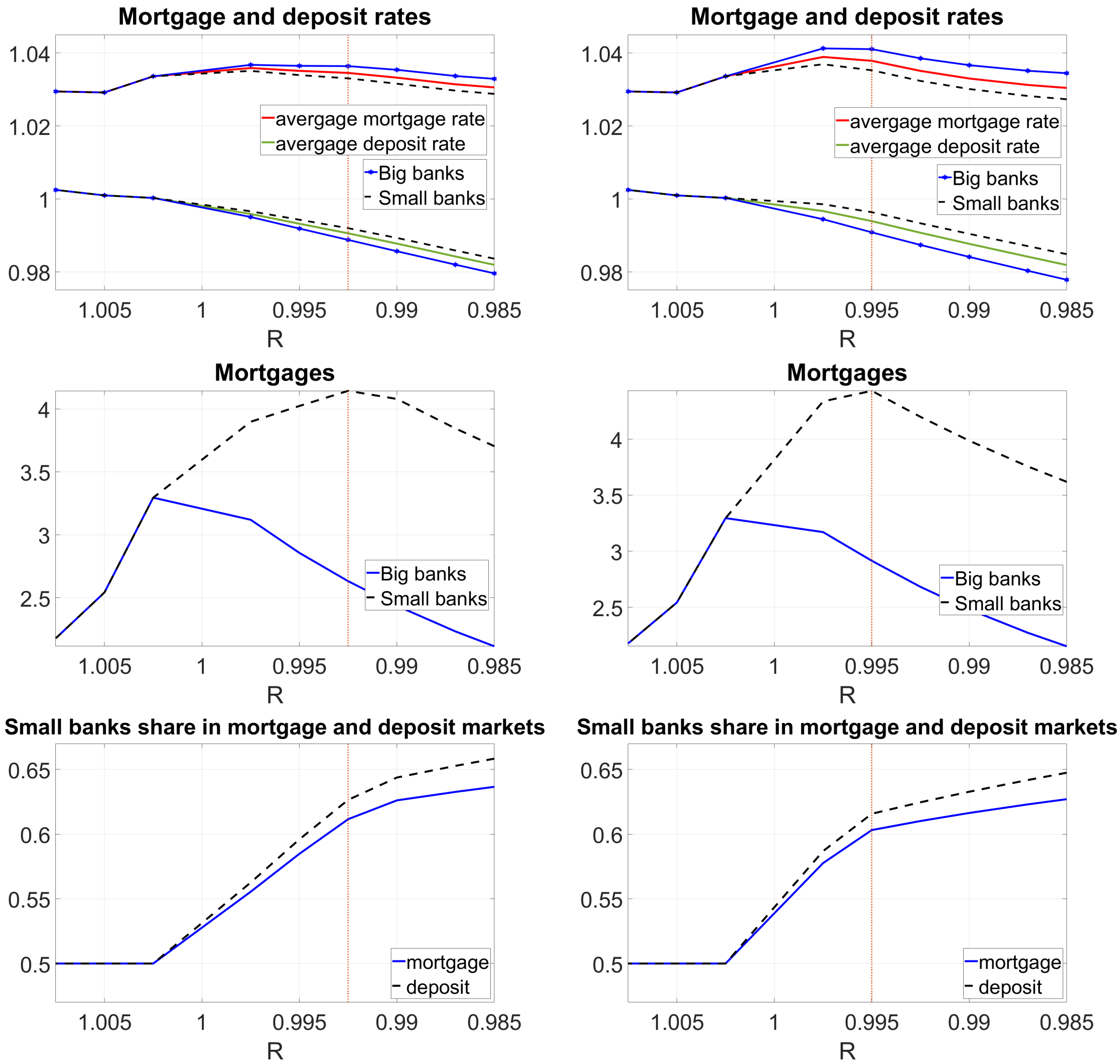
We solve our model under the assumption that bank j, when choosing the mortgage and deposit rate in location i, does not internalize the influence on the CES composite rate in the location, R_i. The first-order conditions now become:

$$R_j^l = \underbrace{\frac{1}{1-\frac{1}{\xi_l}}}_{\text{markup}} \frac{1}{1-F()}[R-(1-\mu)\frac{G(\bar{\omega})}{\bar{\omega}}R^l + \underbrace{\frac{1}{\beta_s+\lambda_j^c}(\varphi_j^c\lambda_j^c+\lambda_j^l)}_{\text{regulation cost}}], \quad j=G,D. \tag{32}$$

$$R_j^d = \underbrace{\frac{1}{1-\frac{1}{\xi_d}}}_{\text{markdown}} [R+\underbrace{\frac{1}{\beta^s+\lambda_j^c}(1-\varphi^l)\lambda_j^l}_{\text{regulation cost}}], \quad j=G,D. \tag{33}$$

The difference relative to equations (26) and (27) is that the markup (for mortgages) and markdown (for deposits) are only a function of the elasticity of substitution across banks, ξ, which does not depend on market size.

Figure 8 compares the results with variable and constant market power. With constant market power, small banks increase market share in both mortgage and deposit markets as the policy rate falls, but less than in the benchmark model. Intuitively, as small banks gain market share, variable market power enables them to raise markup and lower markdown (top panel), which affects their lending behavior in two ways. On the one hand, since small banks charge a relatively higher mortgage rate and raise their mortgage supply at a slower pace, this helps to make capital constraint bind at a lower level of policy rate (second panel). On the other hand, a higher interest margin raise the net worth of small banks, which also contributes to making the capital requirement bind at a lower policy rate. And when capital constraint becomes binding at the level 0.985 of the policy rate, mortgage supply would be constrained by the net worth; accordingly a larger net worth strengthens small banks' lending capacity and brings in higher market share (bottom panel).



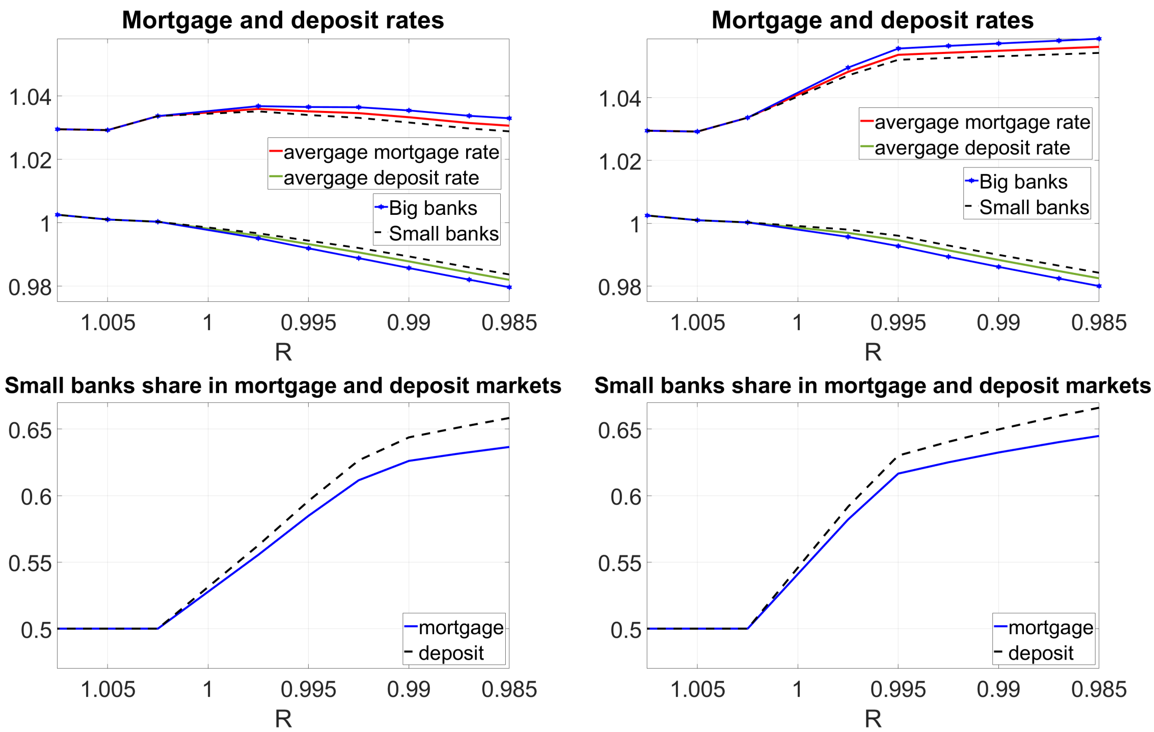
(a) Variable (b) Constant

Figure 8: Variable and constant market power

6.4 An Increase in Idiosyncratic Risk

The banking literature (Jiménez et al. 2014; Dell'Ariccia et al. 2017) has found that a decrease in the policy rate comes with a reduction in the lending rate and an increase in the leverage, which is associated with a shift to riskier assets, as a credit expansion is typically achieved by a reduction in lending standards. To study how an increase in mortgage risk affects our analysis, we assume that the standard deviation of idiosyncratic shock is a linear decreasing function of the policy rate. As a result, mortgages become more risky and the rate of default increases as the policy rate falls.

The impact of an increase in idiosyncratic risk is shown in Figure 9. Relative to our baseline result, higher default risk leads to higher mortgage rate to compensate for higher expected default losses. Notably, small banks gain more market share relative to the baseline case. This is because a higher lending rate has a disproportionate impact on big banks, whose mortgage rate is already high because the capital constraint is higher and already binding. A proportional increase in mortgage rate driven by larger rate of default further widens the interest differential between big and small banks,so that big banks lose even more borrowers. The bottom panel shows that when the policy rate falls to 0.985, small banks' market share is about 2 percentage points larger than for the baseline case.



(a) Baseline

(b) σ_ω increases when R falls

Figure 9: An increase in idiosyncratic risk

6.5 Risk Taking

Previously we assumed an overall shift in default risk and analyzed its effect. It is not yet clear which type of bank may actually take more risk. To figure this out, we augment our baseline framework to include bad borrowers. The scale of bad borrowers accepted by a bank can be used as a measure of its risk-taking. That is, the larger fraction of mortgages a bank lends to bad borrowers, the higher default risk it is exposed to.

We now refer to the borrowers in our baseline model as good borrowers. There also exists two families of bad borrowers getting loans from big and small banks separately,[13] and we index them by the bank j they borrow from. Like in the good borrower family, members' consumption is also fully insured against idiosyncratic risk in housing investment. The bad borrower family uses initial wealth $w_b^{'}$ and mortgages Lb_j taken from banks j at rate of R_j^* to purchase houses h_j^* and buy consumption goods. In period 2, there is substandard idiosyncratic shock across bad borrowers to the value of their house holding, it is different from that to good borrowers with a larger variance. Following Nuño and Thomas (2017) we assume that this substandard idiosyncratic shock is identically and independently distributed, and follows a log-normal distribution

$$ln(\omega_j^*) \sim N(\frac{-\nu\sigma_\omega^2 - \Phi}{2}, \sqrt{\nu}\sigma_\omega^2), \ j = G, D.$$

Hence the budget constraints on bad borrower family in two periods are given by:

$$C_{j,1}^{bb} + q_{h,1}h_j^* = w_b^{'} + Lb_j, \ j = G, D. \tag{34}$$

$$C_{j,2}^{bb} = (1 - \tilde{G}(\bar{\omega}_j^*))q_{h,2}h^* - (1 - \tilde{F}(\bar{\omega}_j^*))R_j^*Lb_j, \ j = G, D. \tag{35}$$

where $C_{j,i}^{bb}$ denotes consumption demand of bad borrower family j in period i, and $\bar{\omega}_j^*$ is the default threshold for bad borrowers.

Housing supply to good borrowers and two bunches of bad borrowers in total is fixed as

$$h + h_G^* + h_D^* = 1.$$

We study the robustness of our results to different levels of housing supply in section B of the appendix.

[13] It is also possible to assume that there is only one family of bad borrowers, big and small banks compete for these bad borrowers as they do for good borrowers. The main difference from the result we have here is that the competition cost would make both banks lend relatively less to bad borrowers and thereby resulting in a relatively smaller default rate than in Figure 10.

Aggregate mortgage for a bad borrower family is a CES aggregator of loans taken out by members at all locations:

$$Lb_j = [\int_0^1 Lb_{i,j}^{1-1/\rho^*} di]^{\rho^*/(\rho^*-1)},\ j = G, D. \tag{36}$$

where ρ^* is the elasticity of substitution across locations.

At each location i bank j's branch offers lending service to both good and bad borrowers. The balance sheet and net worth in period 2 become

$$L_{i,j} + Lb_{i,j} + S_{i,j} = D_{i,j} + N_0,\ j = G, D.$$

$$N_{i,j} = R^{l,2}_{i,j} L_{i,j} + [(1-\mu)\tilde{G}(\bar{\omega}^*_j) q_{h,2} h^*_j + (1 - \tilde{F}(\bar{\omega}^*_j)) R^*_{i,j} Lb_{i,j}] + RS_{i,j} - R^d_{i,j} D_{i,j},\ j = G, D.$$

Mortgages granted to bad borrowers are associated with a higher level of risk-based capital ratio than to good borrowers,[14] therefore capital constraint becomes

$$\varphi^c_j L_{i,j} + 1.3 * \varphi^c_j Lb_{i,j} \leq N_j,\ j = G, D.$$

Note that big and small banks lend to these two groups of bad borrowers separately, there is no price competition for these substandard mortgages. Thus markups are no longer dependent on banks' market share. The implied first-order condition for $Lb_{i,j}$ is:

$$R^*_{i,j} = \frac{1}{1-\frac{1}{\rho^*}} [\frac{\bar{\omega}^*_j}{\bar{\omega}^*_j (1 - \tilde{F}(\bar{\omega}^*_j)) + (1-\mu)\tilde{G}(\bar{\omega}^*_j)}][R + \frac{1}{\beta^s + \lambda^c_j}(1.3\varphi^c_j \lambda^c_j + \lambda^l_j)],\ j = G, D.$$

The average probability of default for a bank is denoted by

$$P(\omega)_j = \frac{L_j}{L_j + Lb_j} F(\bar{\omega}_j) + \frac{Lb_j}{L_j + Lb_j} \tilde{F}(\bar{\omega}^*_j),\ j = G, D.$$

The fraction of loans granted to bad borrowers by two banks and the average default rate are shown in Figure 10. We find that small banks lend out more to bad borrowers and thereby facing a higher default rate than do big banks.

Less competition cost on bad borrowers relative to good ones encourages banks to grant more substandard mortgages to bad borrowers and load up on aggregate risk, in particular, before capital constraint binds. When the policy rate falls to 0.25%, capital constraint binds for big banks, since then they have to cut these substandard mortgages which bear a higher risk-based capital ratio. However, less tightened capital regulation has not yet become binding for small banks so that they are still able to

[14] We assume that risk-based capital ratio of bad mortgages is 1.3 times that of good mortgages, it is driven by the fact that mortgages with LTV ratio above/below 0.8 are given a risk weight 100%/75% on average.

increase mortgage lending to bad borrowers. Thus the share of substandard mortgages in total loans for small banks rises up until capital constraint also binds for them at the level -0.1% of the policy rate. This indicates that small banks hold a larger fraction of riskier mortgages and consequently they are exposed to larger default risk than big banks. Even as the policy rate keeps falling, small banks also have to cut substandard mortgages sharply, this result still holds for a while. Small banks have a higher percentage of substandard mortgages on their balance sheet than big banks do until the policy rate falls to -1.5%. Overall, the competitive advantage from less tightened capital regulation brings small banks higher market share, as well as larger exposure to default risk.

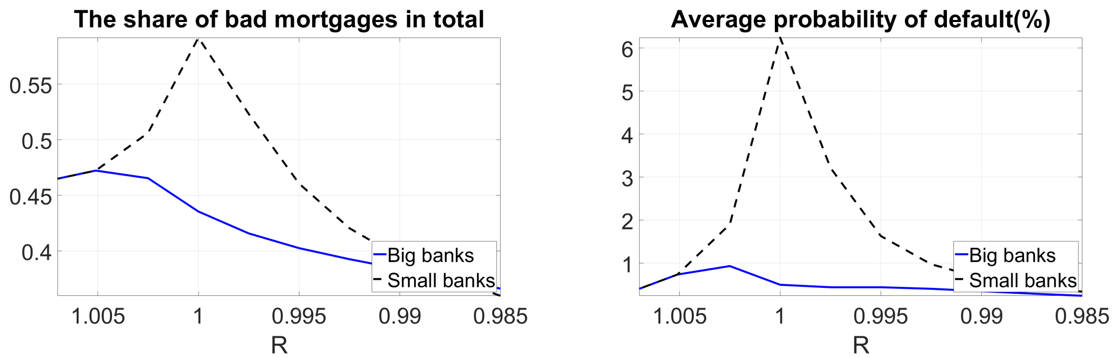


Figure 10: Risk-taking by banks

7 Policy Analysis

Last we analyze the effects of two policies: countercyclical capital buffer and negative rate exemption on reserves. Both have been implemented in Switzerland.

7.1 Countercyclical Capital Buffer

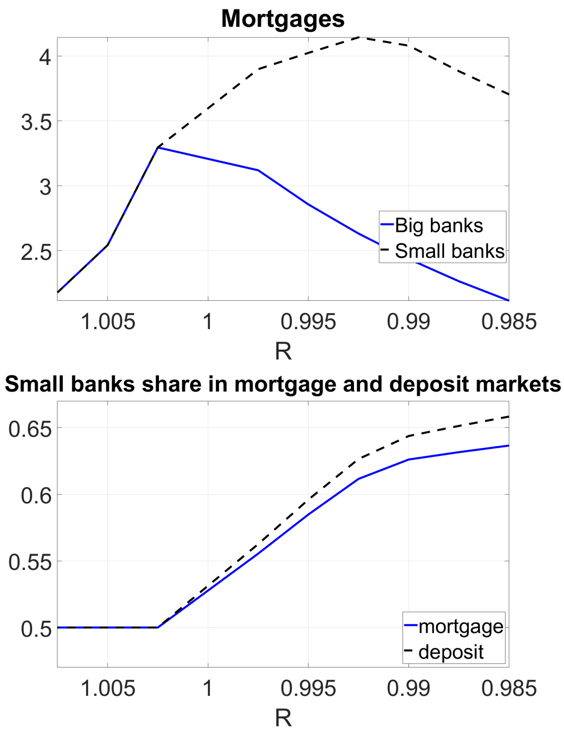
Switzerland activated the sectoral Countercyclical capital buffer (CCyB) requirement targeting mortgage loans at 1% in 2013, and further raised to 2% in 2015. This measure aims to mitigate housing boom and to protect the banking system from periods of excessive credit growth.

The CCyB can be easily introduced in our model by modifying the capital requirement to

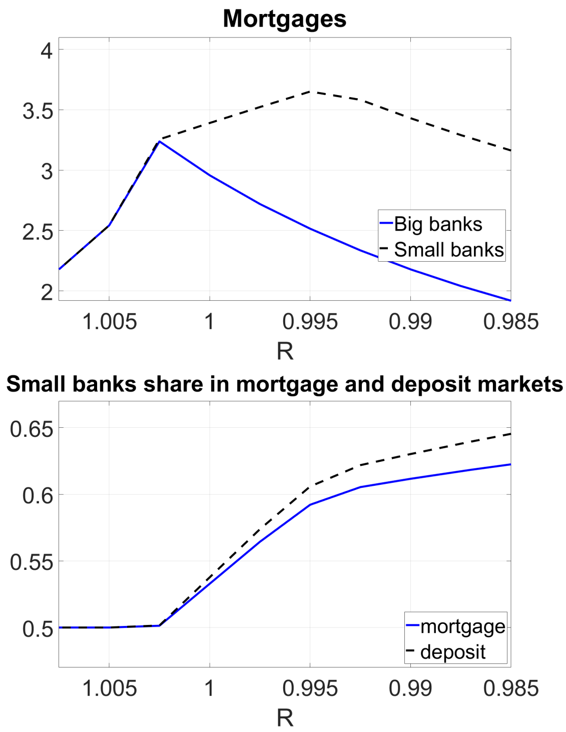
$$(\varphi_j^c + CC)L_j \leq N_j, \; j = G, D.$$

where CC represents the CCyB requirement and is the same across all banks, so the total required risk-weighted capital ratio is equal to the level $\varphi^c + CC$. To match the fact that the CCyB has been implemented since 2013, we set CC equal to zero for $R > 1.0025$ and then to 0.02 for $R < 1.0025$.

Figure 11 shows the effect of the CCyB on the lending side. Comparing to the baseline case, we notice that mortgage lending and small banks' gain in market share are smaller with the CCyB. The CCyB makes the capital requirement stronger, thus imposing stricter limits on their mortgage lending. For small banks, the capital constraint starts binding at a higher policy rate, which limits their mortgage expansion and their market share increase. The CCyB helps to reduce credit expansion as the policy rate falls; it also plays a role in containing small banks' market share increase.



(a) Baseline

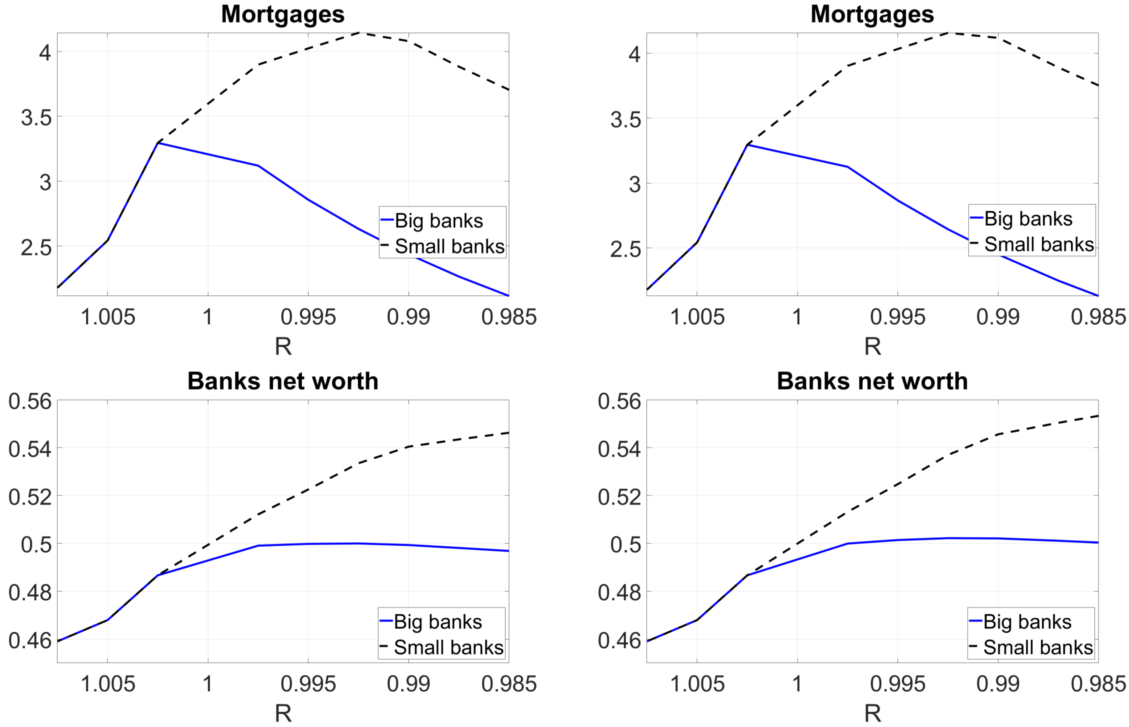


(b) WIth CCyB implemented

Figure 11: The effect of Countercyclical capital buffer

7.2 Negative Rate Exemption on Reserves

When the SNB brought its policy rate into negative territory, it contemporaneously set an exemption threshold for sight deposit account balances. By setting the threshold at 20 times the minimum reserves, the SNB de facto relieved practically all balances from negative interest rates. Nevertheless, the exemption threshold could be reduced and therefore become binding for banks. To show the impact of this exemption policy, we assume that when the policy rate falls into negative territory, the net return on the safe assets held by banks remains zero rather than becoming negative. Figure 12 plots mortgage lending and net worth without (benchmark) and with the exemption policy. The exemption policy is effective for $R <= 1$. It shows that this policy mainly cushions banks from the NIRP.



(a) Baseline

(b) With exemption

Figure 12: Exemption of reserves from NIRP

The exemption policy raises bank net worth once the policy rate is negative. However, we find very little effect on mortgage lending for either type of banks. These results suggest that the exemption on excess reserves makes banks more resilient but it is ineffective in curbing mortgage growth when the policy rate is negative.

8 Conclusions

To summarize, we provide a two-period equilibrium model consisting of two types of bank to evaluate the effect of the reduction in the monetary policy rate as well as the asymmetric tightening of capital requirements on mortgage lending across banks. The reduction in the monetary policy rate from its initial positive level to zero generates a housing boom, namely an increase in mortgages and in real house prices; A tightening in capital requirements is able to generate a reduction of mortgages and house prices. A combination of them is necessary to generate a mortgage expansion and an increase in house prices as experienced in Switzerland since 2008. Additionally, an asymmetric tightening of capital constraint can rationalize different bank lending behavior. It is the tighter capital constraint on big banks that limits their capacity to lending and thus generate an increase in the mortgage and deposit share of small banks, which is consistent with the data.

However, we are concerned about that market share concentration in small banks implies larger default risk in this sector. To maintain financial stability and mitigate excessive credit growth, an uniformed CCyB requirement was introduced, we find that it can help to constrain mortgage lending, and correct the effect of decreasing policy rate and a tightening of capital regulation on market share concentration by a little. However, default risk still largely builds up in small banks. Making financial system safer is one of the key targets a continuously strengthened capital requirement tries to hit, from this point of view, we may need to reflect on it. A complementary revise on risk weights or a specific restriction on housing purchase may be needed in this situation to cope with the unintended credit boom.

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A Mathematical derivations

This section presents how we obtain the equilibrium conditions.

Savers: Their dynamic programming is given by

$$
\begin{aligned}
max \quad & lnC_{s,1} + \frac{j_m}{1-\eta}(Liq)^{1-\eta} + \beta_s lnC_{s,2} \\
& + \mu_{s,1}(W_s - C_{s,1} - B - D - M) + \mu_{s,2}(R^d D + RB + M - C_{s,2} - T + \pi_b) \ .
\end{aligned}
$$

The first-order conditions relative to $C_{s,1},\ C_{s,2}$, B and D are:

$$
\begin{aligned}
\frac{1}{C_{s,1}} &= \mu_{s,1} \ , \\
\beta_s \frac{1}{C_{s,2}} &= \mu_{s,2} \ , \\
\mu_{s,1} &= j_m (Liq)^{-\eta - \frac{1}{\xi_m}} \alpha_b B^{-\frac{1}{\xi_m}} + \mu_{s,2} R \ , \\
\mu_{s,1} &= j_m (Liq)^{-\eta - \frac{1}{\xi_m}} D^{-\frac{1}{\xi_m}} + \mu_{s,2} R^d \ ,
\end{aligned}
$$

Borrowers: Their dynamic programming is given by

$$
\begin{aligned}
max \quad & lnC_{b,1} + j_h \ln h + \beta_b lnC_{b,2} + \mu_{b,1}(L + W_b - C_{b,1} - q_{h,1} h) \\
& + \mu_{b,2}[(1 - G(\frac{R^l L}{q_{h,2} h}))q_{h,2} h - (1 - F(\frac{R^l L}{q_{h,2} h}))R^l L - C_{b,2}] \ .
\end{aligned}
$$

The first-order conditions relative to $C_{b,1},\ C_{b,2}$, h and L: are:

$$
\begin{aligned}
\frac{1}{C_{b,1}} &= \mu_{b,1} \ , \\
\beta_b \frac{1}{C_{b,2}} &= \mu_{b,2} \ , \\
\mu_{b,1} q_{h,1} &= \frac{j_h}{h} + \mu_{b,2}[(1 - G(\bar{\omega}))q_{h,2} + G^{'}(\bar{\omega})\frac{R^l L}{h} - F^{'}(\bar{\omega})\frac{(R^l L)^2}{q_{h,2} h^2}] \ , \\
\frac{\mu_{b,1}}{\mu_{b,2}} &= (1 - F(\bar{\omega}))R^l - F^{'}(\bar{\omega})\frac{(R^l)^2 L_t}{q_{h,2} h} + G^{'}(\bar{\omega})R^l \ ,
\end{aligned}
$$

where $\bar{\omega} = \frac{R^l L}{q_{h,2} h}$.

Banks: First, we solve banks' optimal problem in the baseline setting of variable market power, they choose their rates and implied quantities to maximize their next-period net worth:

$$
\begin{aligned}
max \ & \beta_s \{[(1-\mu)\frac{G(\bar{\omega})}{\bar{\omega}} R^l + (1 - F(\bar{\omega}))R^l_{i,j}]L_{i,j} + RS_j - R^d_{i,j} D_{i,j}\} \\
& + \lambda^c_{i,j}\{[(1-\mu)\frac{G(\bar{\omega})}{\bar{\omega}} R^l + (1 - F(\bar{\omega}))R^l_{i,j}]L_{i,j} + RS_{i,j} - R^d_{i,j} D_{i,j} - \varphi^c_{i,j} L_{i,j}\} \\
& + \lambda^l_{i,j}\{S_{i,j} - \varphi^l D_{i,j}\} + \lambda^b\{D_j + N_{j,0} - L_j - S_j\} \\
& + \lambda^m\{L_{i,j} - 0.5(\frac{R^l_{i,j}}{R^l_i})^{-\xi_l}(\frac{R^l_i}{R^l})^{-\rho} L)\} + \lambda^d\{D_{i,j} - 0.5(\frac{R^d_{i,j}}{R^d_i})^{-\xi_d}(\frac{R^d_i}{R^d})^{-\rho} D)\}
\end{aligned}
$$

We take first-order conditions with respect to $L_{i,j}$ and $R^l_{i,j}$:

$$(\beta_s + \lambda^c_{i,j})\{[(1-\mu)\frac{G(\bar{\omega})}{\bar{\omega}}R^l + (1-F(\bar{\omega}))R^l_{i,j}] - R\} - \lambda^c_{i,j}\varphi^c_{i,j} - \lambda^l_{i,j} + \lambda^m_{i,j} = 0 \ ,$$

$$\begin{aligned}(\beta_s + \lambda^c_{i,j})[(1-F(\bar{\omega}))]L_{i,j} - \lambda^m_{i,j}\{-\xi_l 0.5(\frac{R^l_{i,j}}{R^l_i})^{-\xi_l-1}(\frac{R^l_i}{R^l})^{-\rho}L\frac{1}{R^l_i} \\ + (-\rho+\xi_l)0.5(R^l_{i,j})^{-\xi_l}(R^l)^{\rho}L(R^l_i)^{-\rho+\xi_l-1}\frac{\partial R^l_i}{\partial R^l_{i,j}}\} = 0 \ .\end{aligned}$$

Plugging the local mortgage demand function (17) into the first-order condition to $R^l_{i,j}$ we get

$$(\beta_s + \lambda^c_{i,j})[(1-F(\bar{\omega}))]L_{i,j} - \lambda^m_{i,j}\{-\xi_l L_{i,j}(R^l_{i,j})^{-1} + (-\rho+\xi_l)L_{i,j}(R^l_{i,j})^{-1}s^l_{i,j}\} = 0 \ .$$

This implies

$$\lambda^m_{i,j} = \frac{(\beta_s + \lambda^c_{i,j})[(1-F(\bar{\omega}_{t+1}))]R^l_{i,j}}{\xi_l - (-\rho+\xi_l)s^l_{i,j}} \ . \tag{37}$$

Then substituting this $\lambda^m_{i,j}$ into the first-order condition to $L_{i,j}$ yields the optimal mortgage rate chosen by the bank:

$$R^l_{i,j} = \underbrace{\frac{1}{1-\frac{1}{\epsilon^l_{i,j}}}}_{\text{markup}}\frac{1}{1-F()}[R - (1-\mu)\frac{G(\bar{\omega})}{\bar{\omega}}R^l + \underbrace{\frac{1}{\beta_s + \lambda^c_{i,j}}(\varphi^c_{i,j}\lambda^c_{i,j} + \lambda^l_{i,j})]}_{\text{regulation}} \ , \tag{38}$$

where

$$\epsilon^l_{i,j} = \xi_l(1-s^l_{i,j}) + \rho s^l_{i,j} \ .$$

Similarly, by combining first-order conditions with respect to $D_{i,j}$ and $R^d_{i,j}$ we get the optimal deposit rate set by the bank:

$$R^d_{i,j} = \underbrace{\frac{1}{1-\frac{1}{\epsilon^d_{i,j}}}}_{\text{markdown}}[R + \underbrace{\frac{1}{\beta^s + \lambda^c_{i,j}}(1-\varphi^l)\lambda^l_{i,j}]}_{\text{regulation}} \ , \tag{39}$$

where

$$\epsilon^d_{i,j} = \xi_d(1-s^d_{i,j}) + \rho s^d_{i,j} \ .$$

Second, we repeat this exercise in a setting of constant market power where there will be no influence on the CES composite rate in the location, $\frac{\partial R^l_i}{\partial R^l_{i,j}} = 0$. Now $\epsilon^l_{i,j} = \xi_l$ and $\epsilon^d_{i,j} = \xi_d$, thus the implied first-order conditions become

$$R^l_{i,j} = \underbrace{\frac{1}{1-\frac{1}{\xi_l}}}_{\text{markup}}\frac{1}{1-F()}[R - (1-\mu)\frac{G(\bar{\omega})}{\bar{\omega}}R^l + \underbrace{\frac{1}{\beta_s + \lambda^c_{i,j}}(\varphi^c_{i,j}\lambda^c_{i,j} + \lambda^l_{i,j})]}_{\text{regulation}} \ , \tag{40}$$

$$R^d_{i,j} = \underbrace{\frac{1}{1-\frac{1}{\xi_d}}}_{\text{markdown}} \underbrace{[R + \frac{1}{\beta^s + \lambda^c_{i,j}}(1-\varphi^l)\lambda^l_{i,j}]}_{\text{regulation}} . \tag{41}$$

Last, when we include substandard borrowers in our model, the Lagrangian of bank's problem becomes

$$\begin{aligned} max\ \beta_s\{[(1-\mu)\frac{G(\bar{\omega})}{\bar{\omega}}R^l + (1-F(\bar{\omega}))R^l_{i,j}]L_{i,j} + [(1-\mu_b)\frac{\tilde{G}(\bar{\omega}^*)}{\bar{\omega}^*} + (1-\tilde{F}(\bar{\omega}^*))]R^*_{i,j}Lb_{i,j} \\ + RS_{i,j} - R^d_{i,j}D_{i,j}\} + \lambda^c_{i,j}\{[N_{i,j} - \varphi^c_j L_{i,j} - 1.3\varphi^c_j Lb_{i,j}\} \\ + \lambda^l_{i,j}\{S_{i,j} - \varphi^l D_{i,j}\} + \lambda^b\{D_{i,j} + N^0 - L_{i,j} - Lb_{i,j} - S_{i,j}\} \\ + \lambda^m\{L_{i,j} - L(R^l_{i,j})\} + \lambda^{mb}\{Lb_{i,j} - (\frac{R^*_{i,j}}{R^*_j})^{-\rho^*}Lb_j\} + \lambda^d\{D_{i,j} - D(R^d_{i,j})\} \end{aligned}$$

The optimal rates $R^l_{i,j}$ and $R^d_{i,j}$ chosen by banks stay the same, and we obtain the first-order condition to $R^*_{i,j}$:

$$(\beta_s+\lambda^c_{i,j})\{[(1-\mu_b)\frac{\tilde{G}(\bar{\omega}^*)}{\bar{\omega}^*}+(1-\tilde{F}(\bar{\omega}^*))](Lb_{i,j}+R^*_{i,j}\frac{\partial Lb_{i,j}}{\partial R^*_{i,j}})-R\frac{\partial Lb_{i,j}}{\partial R^*_{i,j}}\}-\varphi^c_j 1.3\varphi^c_j\frac{\partial Lb_{i,j}}{\partial R^*_{i,j}}-\lambda^l_{i,j}\frac{\partial Lb_{i,j}}{\partial R^*_{i,j}} = 0$$

where $\frac{\partial Lb_{i,j}}{\partial R^*_{i,j}} = -\rho^*(\frac{R^*_{i,j}}{R^*_j})^{-\rho^*-1}\frac{1}{R^*_j}Lb_j = -\rho^*\frac{Lb_{i,j}}{R^*_{i,j}}$. By rearranging this equation we obtain

$$R^*_{i,j} = \frac{1}{1-\frac{1}{\rho^*}}[\frac{\bar{\omega}^*}{\bar{\omega}^*(1-\tilde{F}(\bar{\omega}^*)) + (1-\mu_b)\tilde{G}(\bar{\omega}^*)}][R + \frac{1}{\beta^s+\lambda^c}(1.3\varphi_c\lambda^c + \lambda^l)].$$

B Robustness

This section discusses the generality of our main results by varying a set of key parameters, like growth rate of house prices and quantities of housing supply.

B.1 Growth Rate of House Prices

We fix the rate of growth of house prices from period 1 to 2 to be 0 in our baseline setting. To explore how this growth rate matters, we now calibrate it to be 0.5% and compare the simulated results. See Figure 13, the dynamics of these key variables are similar to that from our baseline calibration. The only difference is that a positive growth rate of house prices raises housing demand and then mortgage lending grows much faster, therefore capital constraint becomes binding earlier for small banks when the policy rate falls to -0.5%.

B.2 The Amount of Housing Supply

In Section 6.5, we allow good and bad borrowers compete for houses, and the total housing supply remains at the level of our baseline setting. A higher level of housing

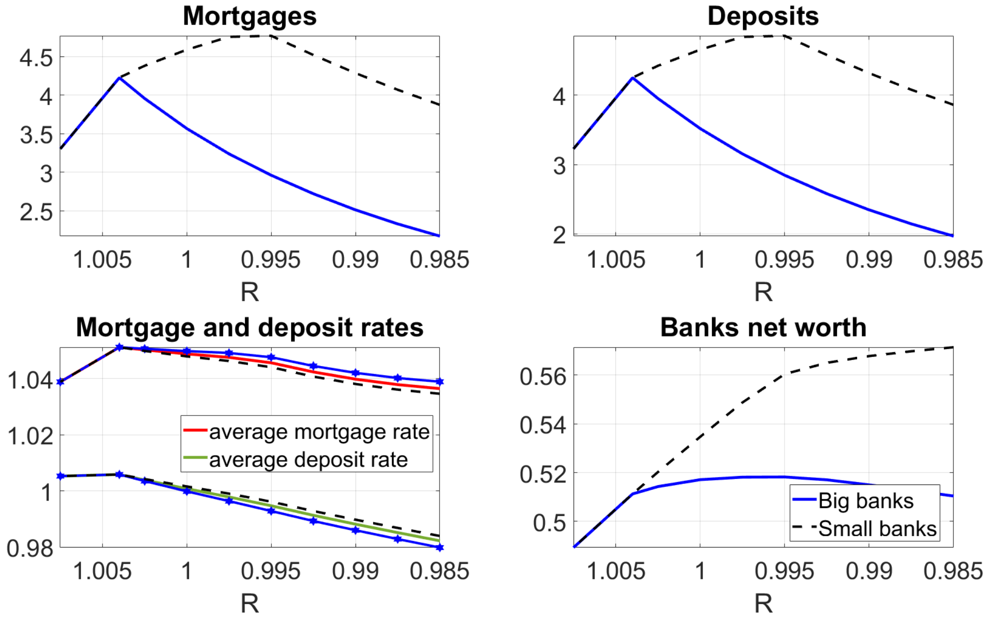


Figure 13: When $g_q = 0.005$

supply to these borrowers may allow them to borrow more from the banks. To explore how a different level of housing supply affect banks' mortgage supply and risk-taking behavior, we set total housing supply to 2 within the framework including bad borrowers. See Figure 14, the result is very close to Figure 10. A larger housing supply implies a lower level of house prices, and banks' lending capacity are still limited by the capital constraints, thus mortgage supply to good and bad borrowers doesn't change much. The main result still holds that banks prefer to increase lending to bad borrowers before capital constraint binds, and small banks take more risk than big banks because of less tightened capital constraint.

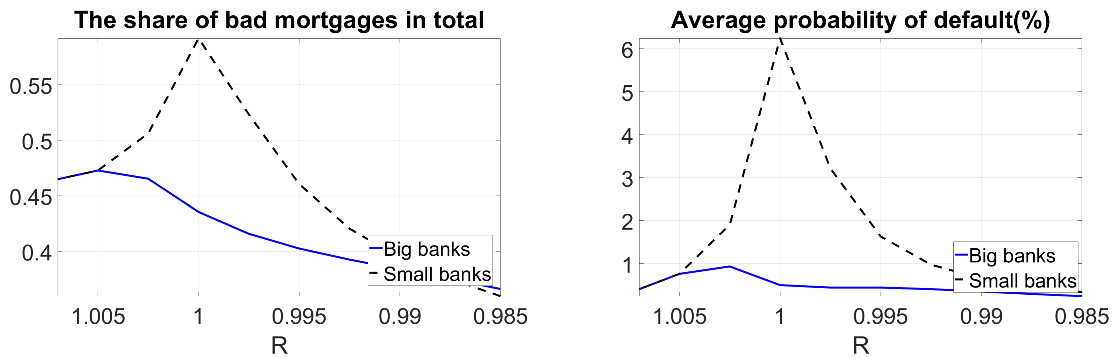


Figure 14: Risk-taking by banks when total housing supply is equal to 2

C Counterfactual Experiments

To understand the relative importance of falling policy rate versus tightened capital regulation, we simulate counterfactual experiments in which we subtract each from the model one at a time.

C.1 Absent Changes in R

First, we set the policy rate R fixed at 0.75%, and still allow capital constraint on big banks to get tighter. Figure 15 shows simulated result against different level of φ_c^G.All the variables stay put initially. When φ_c^G increases to the level 0.21 capital constraint becomes binding for big banks, they start to cut mortgage lending, thus losing market share to small banks. Increased market power leads small banks to raise the markup on mortgage rate, therefore they also decrease mortgage lending from this point, but not as sharply as big banks. On the whole, we find that a binding capital constraint for big banks make them lose market share.

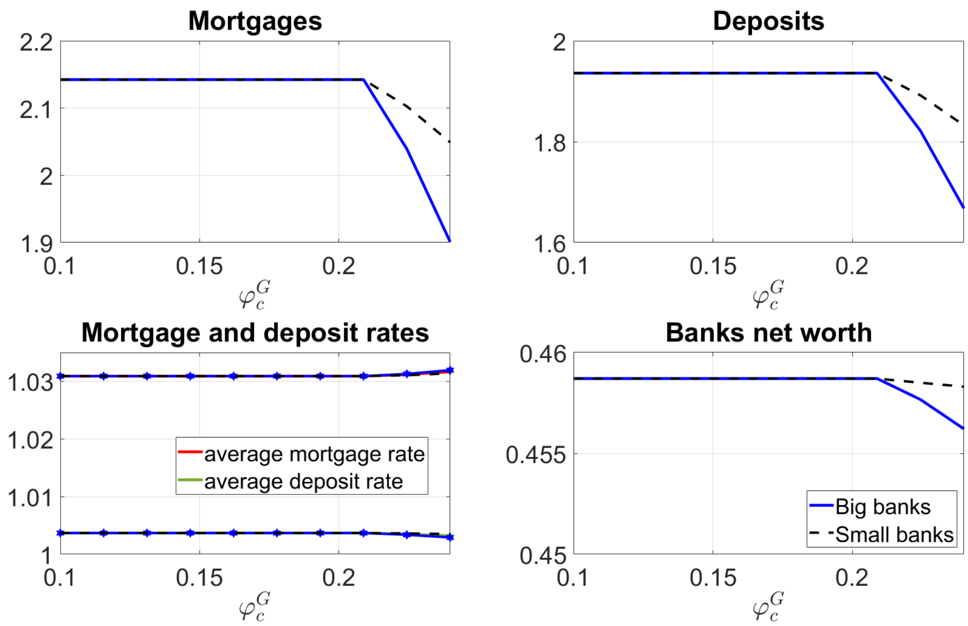


Figure 15: Only φ_c^G increases

C.2 Absent Regulatory Changes in φ_c

Then we simulate our model against different levels of the policy rate without changing φ_c^G and φ_c^D. In Figure 16 we find that as rate falls banks increase mortgage supply. The increasing mortgage lending means a higher default risk, thus banks raise the mortgage rate. Importantly, capital constraints are not binding for big and small banks, both small and big banks behave exactly the same way, and thus there is no shift in market share from one to the other. This suggests that the falling policy rate in our baseline result is essential to generate increased mortgage lending.

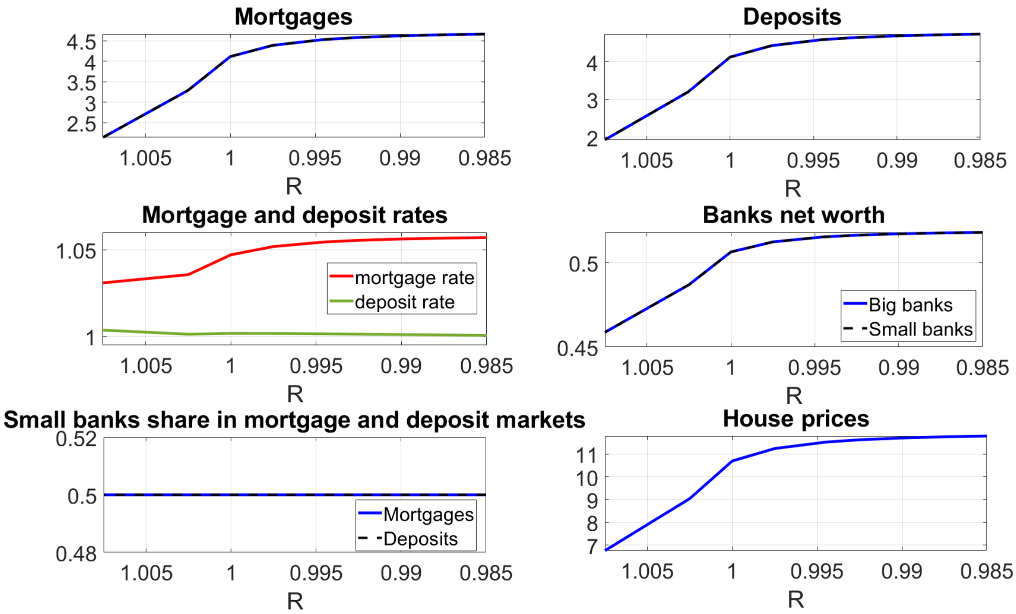


Figure 16: Only R falls

D Capital requirement

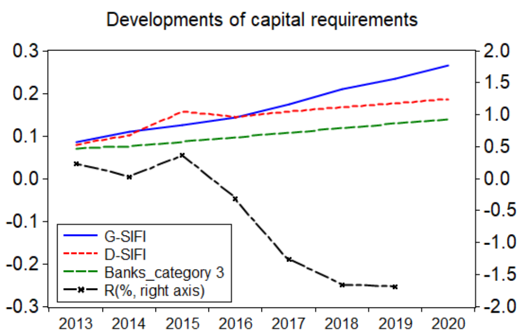
According to Bichsel et al. (2019), capital requirements for the two big banks were tightened significantly, following the progressive introduction of revised international standards (Basel II) and the rescue out of global financial crisis. It is also true for most cantonal banks as their preferential treatment being removed in 2010, and for Raiffeisen banks as additional restrictions regarding capital quality were introduced. We can find that the severity of the capital requirements has been strongly strengthened over the period 2008-2010. Only the extent is different across banks.

The Basel Committee published a reform package in 2010 called Basel III to bolster capital and liquidity requirements. It came into effect in Switzerland since 2013. For the purpose of continuity, we plot the data of total required risk-weighted capital ratio over the period 2013-2020 in the left panel of Figure 17,[15] for G-SIFI (Globally Focused Systemically Important Banks) , D-SIFI (Domestically Focused Systemically Important Banks), , and banks assigned to category 3. G-SIFI are big banks (UBS

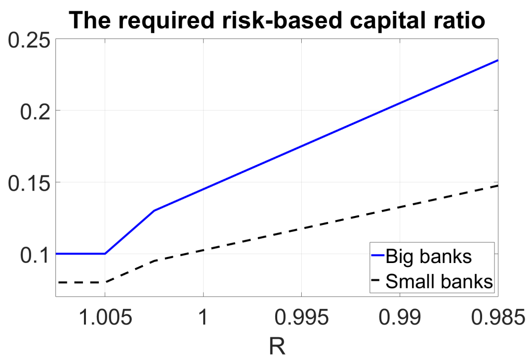
[15] The fully applied capital requirement must be met by 2020, hence we also plot the data for 2020.

and Credit Suisse) we referred to, D-SIFI includes Zurich Cantonal Bank, Raiffeisen and Post Finance, we use data from the first two.[16] For bank assigned to category 3 we use the data from Vaud Cantonal Bank. All the data are from the relevant banks' annual reports. Besides we plot the real interest rates of the same period, as shown by the black dotted line. Two points are worth noting: First, capital constraint has been tightened continuously over the recent years as the policy rate falls, they are negatively related; Second, more regulatory attention is clearly focused on the systemically important financial institutions, especially big banks.

In the model, we assumed that total risk-weighted capital ratio in capital constraint is a decreasing linear function of the real rate, as you can see in the right panel of Figure 17. It can roughly capture the two key features shown in the left panel.



(a) data



(b) model

Figure 17: Capital requirement

[16] PostFinance became a systematically important financial group at a relatively late stage in 2016, so we focus on the other two banks.