# Job Turnover and the slope of the Phillips Curve

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**Abstract**

This paper relates the observed flatter Phillips Curve to the increased bargaining power of employers in the labour market. Traditionally it

## 1 Introduction

The Phillips Curve is central to macroeconomics but its shape has been questioned recently. The strong short run relationship between inflation and output (or unemployment) seems to have vanished in the aftermath of the 2008 financial crisis: unemployment increased and then fell sharply, while inflation remained low and positive. The relationship seems to have broken down. This would suggest that the short-run Phillips curve has become flatter, as evidenced by Blanchard et al. (2015) or Ball and Mazumder (2014).

The idea of a vertical, or near-vertical long-run Phillips Curve, has also been questioned. In a recent Peterson policy brief (2016), Blanchard argues that the long run Phillips curve has become flatter, largely due to inflation expectations anchoring at zero or low levels. As such, there would be a real trade-off between output and inflation in the long run. Some explanations such as menu costs and anchored expectations have been put forward, but they either lack microfoundations or tractability, which would be useful for welfare analysis. Others relate it to globalisation (see Carney, 2017).

This paper, instead, relates these evolutions to job turnover. The advantage of this microfoundation is that it is more observable and more tractable. As we shall see in the next subsection, there has been a secular trend in job turnover and other features of the labour market over the past decades (see Haldane, 2016). This paper shows how it can explain the evolution of the Phillips curve: a flatter long run curve which is no longer vertical or near vertical. And in the short run, the curve will look flatter than if turnover is properly accounted for. The optimal monetary policy, in terms of inflation target and stabilisation, are then derived.

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## Job turnover

In the New Keynesian wage Phillips curve models, such as the one pioneered by Erceg, Henderson and Levin (2000), workers (or unions) set staggered wages optimally. Current (wage) inflation depends on future (wage) inflation expectations as well as the output gap. In the log linear approximation, the coefficient of future inflation is $\beta$, the riskless discount factor.

However, when there is a significant probability that a worker quits, or that he will be fired and replaced by someone else, the net present value of his job will be discounted with a lower factor than the risk-less discount factor. It is important to distinguish layoffs and (personal) dismissals because persons who quit or are dismissed are replaced and hence count as turnover, while layoffs diminish employment and are not replaced by new hires. [1] This probability of turnover makes the wage setting decision, and hence the wage Phillips curve, less forward looking.

Figure 1 comes from the job tenure survey from the OECD, for people aged $25 - 54$. The proportion of people less than a year into their job is a good indicator of yearly job turnover, though temporary contracts probably overstate the figure. In most countries, there has been an increase in the *less than one year* proportion of workers, which indicates a rising turnover. This can also be seen with the increase of the *less than three years* proportion, which is less sensible to temporary employment. Last, the proportion of people *more than ten years* into thir job has fallen accross most countries. This is highly suggestive of an increased turnover.

The increasing share of temporary contracts, and the recent rise in the "gig economy"(part time contracts, self-employed contractors, zero-hours in Britain) are also likely to weaken collective bargaining in favour of more individual bargaining, as suggested by Haldane (2017). This would suggest lower wages, but also less forward looking decisions, which is the point of this paper. Table 2 shows how the share of temporary contracts has evolved over time (again using OECD data). While the upward trend is not always monotonic and varies in magnitude accross countries, it is relatively strong, especially in countries like France, Italy or the Netherlands).

## Calvo meets perpetual youth

A crucial assumption is that when a worker quits (or is dismissed) and is replaced by an entrant worker, the wage stickiness will be (at least partially) transmitted to the entrant. The entrant does not renegotiate its wage immediately, and has to abide by the wage of the previous incumbent it has replaced. Or equivalently, there is no difference between incumbents and entrants in their distribution of

[1]The probability of being laid off should not matter for the worker, if the union acts as an insurance mechanism. Because the union is assumed to split the wage income between employed and unemployed members, the employee does not lose his income when he is laid off. But turnover relates to quits and personal dismissals, not layoffs. And it is not the purpose of the union to insure against these, so the turnover probability is a relevant discount factor.

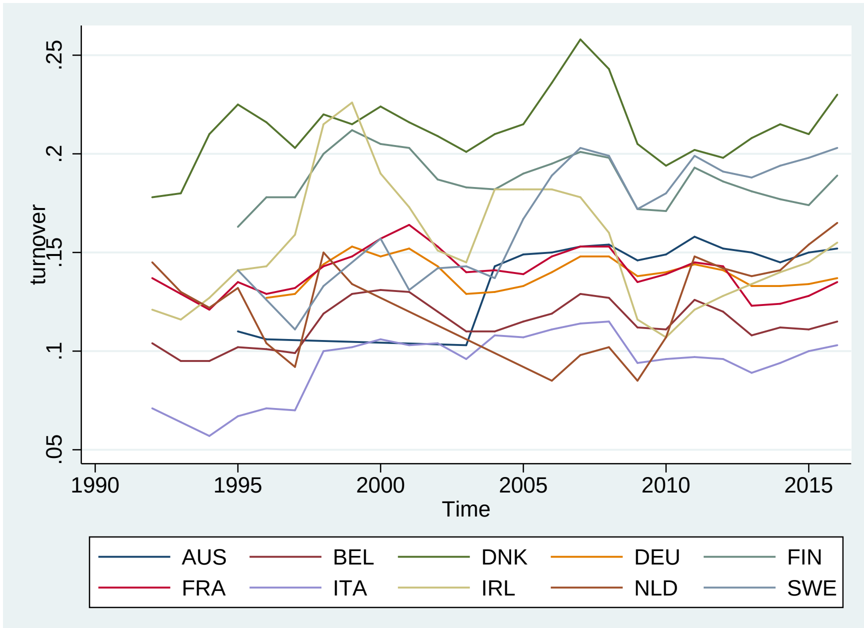


Figure 1: Turnover (OECD job tenure survey)

wages. Assuming wage rigidity for new hires is crucial in models such as Hall (2005) or Gertler and Trigari (2009), who combine wage and labour search frictions. Gertler, Huckfeldt and Trigari (2016) find no evidence that the wage of new hires is more cyclical than for existing workers. Galuscak et al. (2012) find similar results for 15 EU countries.

This model of entry has some perpetual youth flavour as in Blanchard (1985). As hinted by Weil (1989), the crucial feature in these models is as much the probability of death of the agent, as the stream of newborns, who don't have a say over decisions made before their birth. [2] Here, when a new worker starts a job, he is bound by the decisions of his predecessor. [3] The externality between existing and new agents creates the extra discounting.

## Related literature

Snower and Tesfaselassie (2017) derive a positive optimal long run inflation target in the presence of job turnover, but they do not investigate the short run properties much. Bilbiie, Ghironi and Melitz (2012; 2016) as well as Bilbiie, Fujiwara and Ghironi (2014) look at the optimal long run monetary policy in similar setup: sticky prices with firm entry and exit. In their model, the exit

[2]In the positive sense, the death probability creates the lower discount factor, but in the normative sense, the externality is caused by the stream of new workers.

[3]Or if wages are set by a union, it only cares about the welfare of its *existing* members.

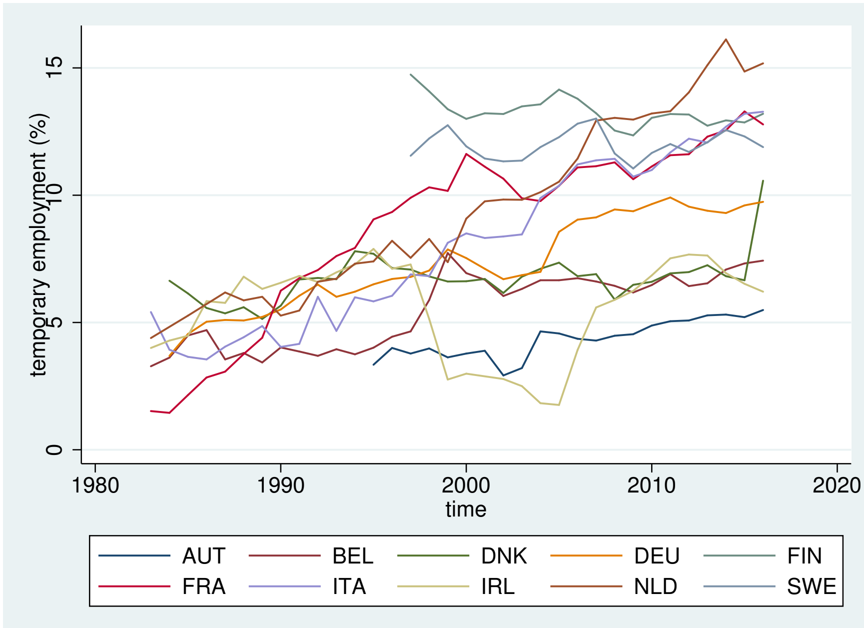


Figure 2: Share of temporary employment (OECD)

probability affects the Phillips curve and the optimal long run Ramsey policy. While these papers use a Rotemberg instead of a Calvo framework, and inflation offsets different long run distortions, the intuition, as well as the assumption that new workers cannot reset their wage, is largely the same [4]. But this paper shows how turnover leads to a flatter long run Phillips curve, and a *perceived* flatter curve in the short run. It also explains how the optimality of *price targeting* is broken, compared to the classical result in Woodford (2003), Benigno and Woodford (2004), or Gali (2008). Last, it shows how optimal short run policies are affected.

Different explanations have been put forward for the recently flatter Phillips Curve. Ball and Mazumder (2011) suggest that with menu costs, price changes will be less frequent when inflation is low, and the resulting Phillips Curve will be flatter. Blanchard (2016) relies on anchored inflation expectations. My approach has the advantage of tractability and observable micro-foundations, which allow for a welfare analysis. While the labour market has been highlighted as a possible driver of the flatter Phillips Curve (see Haldane, 2017 or chapter 2 of the October 2017 World Economic Outlook), no proper model has been suggested yet. The idea of a global Phillips Curve – inflation reacting to global not domestic conditions – has also been floated (eg. Carney, 2017), but again

[4] In my Calvo framework, workers adopt the wage distribution of existing workers. In a Rotemberg setup, it is assumed that new workers (or firms) take the existing symmetric wage (or price), and are not free to choose their starting wage (price) optimally

without a proper underlying model.

This paper also belongs to the stream of literature that reassesses the New Keynesian model in light of the Great Recession and the Zero Lower Bound. While this paper introduces an extra discount factor in the Phillips curve, other papers have introduced a discount factor in the Euler equation instead, to explain the *forward guidance puzzle*. In McKay, Nakamura and Steinsson (2016) this is due to incomplete financial markets, while in Del Negro, Giannoni and Patterson (2013), it comes from a Blanchard-Yaari model of perpetual youth for households which is similar to this paper (where it applies to workers). The interaction between a discounted Phillips curve and a discounted Euler equation has been partially studied by Gabaix (2016).

Last, this paper is related to the literature on the optimal level of inflation, which does not solely rely on the Phillips curve. In their handbook chapter (2011), Schmitt-Grohe and Uribe document such other motives for positive inflation. If the price stickiness exhibits a quality bias (Schmitt-Grohe and Uribe, 2009), then a positive inflation will simply ensure that the hedonic price level remains constant. If wages are more rigid downwards than upwards, positive inflation will make relative wage adjustments easier (Olivera, 1964; Akerlof, Dickens and Perry, 1996; Kim and Ruge-Murcia, 2009). A positive amount of inflation might also be useful to increase the nominal interest rate safely above zero, in case the zero lower bound needs to be avoided (Adam and Billi, 2006; Reifschnieder and Williams, 2000).

The paper is organized as follows: Section 2 builds a New Keynesian model with sticky wages, as well as job turnover. The non linear Phillips curve is derived and linearly approximated. Section 3 investigates and estimates the prediction of a flatter Phillips curve in the short, middle and long run. Last, Section 4 solves the welfare maximization problem, both in the non linear (steady state inflation) and quadratic setups (optimal stabilisation).

# 2 The model

## 2.1 A microfounded model

The model of wage rigidities closely follows Gali's (2008) notations, with monopolistic competition in the labour market. There is a continuum of wage-setting worker types, indexed by $j \in [0, 1]$.

### Households and firms

Let me first look at the household. A worker of type $j$ maximizes a utility

$$\mathrm{E}_0 \sum_{t \geq 0} \beta^t U(C_t(j), N_t(j)) \tag{1}$$

The period utility function $U$ is separable in consumption and labour. The utility of consumption $C$, $u(C)$, is a concave function with inverse elasticity of intertemporal substitution $\sigma$, while the disutility of labour $N$, $v(N)$ is convex with an inverse Frisch elasticity $\phi$. The utility from consumption and disutility from labour are scaled by a parameter $\lambda$:

$$U(C_t(j), N_t(j)) = u(C_t) - v(N_t(j)) = \frac{C_t^{1-\sigma}}{1-\sigma} - \lambda \frac{N_t(j)^{1+\phi}}{1+\phi} \tag{2}$$

Perfect competition is assumed in the goods market. The production function has diminishing returns to labour $N_t$, with a labour elasticity $(1-\alpha)$:

$$Y_t = N_t^{1-\alpha}$$

Labour is a CES aggregate of the labour of each type $j$, with a wage elasticity of substitution $\epsilon$:

$$N_t = \left[\int_0^1 N_t(j)^{1-1/\epsilon} dj\right]^{\frac{\epsilon}{\epsilon-1}}$$

The aggregate wage index $W_t$ is

$$W_t = \left[\int_0^1 W_t(j)^{1-\epsilon} dj\right]^{\frac{1}{1-\epsilon}}$$

The amount of labour of type $j$ employed by firm $i$ is

$$N_t(j) = \left(\frac{W_t(j)}{W_t}\right)^{-\epsilon} N_t$$

Worker $j$ maximizes the expected utility (1) subject to the budget constraint

$$P_t C_t(j) + Q_t B_t(j) = B_{t-1}(j) + (1-\tau_t) W_t(j) N_t(j) + D_t + T_t$$

where $\tau_t$ is a proportional labour tax (or subsidy) on his labour compensation $W_t(j)N_t(j)$, $D_t$ is the dividend from owning a diversified portfolio of firms, and $T_t$ is a lump sum transfer (or tax) from the government. New bonds $B_t(j)$ can be bought or sold at price $Q_t$, the stochastic discount factor of the household. Balanced government budget in each period ($T_t = \tau_t W_t N_t$), as well as zero net supply of bonds, ensures that consumption and output are equal in each period:

$$P_t C_t = W_t N_t + D_t = P_t Y_t$$

With perfect competition for goods, prices are equal to marginal costs, or

$$P_t = MC_t = W_t \frac{N_t^{\alpha}}{1-\alpha}$$

Hence the real wage is linked to output as

$$\Omega_t = (1-\alpha) Y_t^{-\frac{\alpha}{1-\alpha}}$$

With decreasing returns to scale, firms make a profit $D_t = \alpha P_t Y_t$.

As in Erceg et al. (2000) or Gali (2008), let us assume markets with complete contingent claims for consumption but not leisure. This ensures full consumption smoothing accross agents.

**Lemma 1** *With complete markets, there is full consumption smoothing:*

$$\forall (t,j), \qquad C_t(j) = C_t = Y_t$$

*The Euler equation of consumption pins down the riskless discount factor*

$$Q_t = \mathrm{E}_t \beta \frac{P_t}{P_{t+1}} \frac{u'(C_{t+1})}{u'(C_t)} = \beta \frac{P_t}{P_{t+1}} \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \tag{3}$$

*The labour supply decision for a worker $j$ in problem (1) is equivalent to maximizing the following quantity in each period*

$$u'(Y_t) \frac{(1-\tau) W_t(j) N_t(j)}{P_t} - \lambda \frac{N_t(j)^{1+\phi}}{1+\phi} \tag{4}$$

**Distortions and dispersions**

Let us define the first-best and flexible outcomes. Using the utility and production function, the first-best level of output is

$$\bar{Y} = \left( \frac{1-\alpha}{\lambda} \right)^{\frac{1}{\sigma + \frac{\phi+\alpha}{1-\alpha}}}$$

**Lemma 2** *In the flexible outcome, the real wage $\Omega = \frac{W}{P}$ is a markup $\mu$ above the marginal rate of substitution of the worker:*

$$\mu = \left( \frac{\epsilon}{(\epsilon - 1)(1-\tau)} \right)$$

*The flexible-wage output is*

$$\tilde{Y} = \left( \frac{1-\alpha}{\lambda \mu} \right)^{\frac{1}{\sigma + \frac{\phi+\alpha}{1-\alpha}}} = \bar{Y} \left( \frac{1}{\mu} \right)^{\frac{1}{\sigma + \frac{\phi+\alpha}{1-\alpha}}}$$

The markups depend on the wage elasticity – with a high elasticity, the markup is close to 1. But it also depends on the wage tax $\tau$. A positive tax creates an additional wedge, but a subsidy can offset the inefficiency caused by the finite wage elasticity. Unless the subsidies fully offset the wedges ($\mu = 1$), the flexible output will be inefficiently low as $\tilde{Y} < \bar{Y}$.

With staggered wages, the wage dispersion will be costly in terms of welfare. When wages are heterogeneous, the aggregate number of hours must increase to produce the same amount of goods.

**Lemma 3** *The aggregate utility function can be written*

$$\int_0^1 U(C_t, N_t(j))dj = \tilde{Y}^{1-\sigma} \left[ \frac{\left(\frac{Y_t}{\tilde{Y}}\right)^{1-\sigma}}{1-\sigma} - \frac{\frac{1-\alpha}{1+\phi}\Delta_t \left(\frac{Y_t}{\tilde{Y}}\right)^{\frac{1+\phi}{1-\alpha}}}{\mu} \right] \tag{5}$$

*with the wage dispersions*

$$\Delta_t = \int_0^1 \left(\frac{W_t(j)}{W_t}\right)^{-\epsilon(1+\phi)} dj \geq 1 \tag{6}$$

## 2.2 Sticky wages and the Phillips Curve

### Worker discounting

A fraction $\theta$ of workers have sticky wages, and a fraction $\delta$ keeps their job from one period to another; the two are independent. The discount factor accounts for the price and the firms survival probabilities $\theta$ and $\delta$. Instead of maximizing the discounted sum of expression (4) with a discount factor $\beta$, the applicable rate of time preference will be $\beta\theta\delta$: the disutility of labour – attached to a wage and a worker – is discounted by $\beta\theta\delta$, while the labour compensation is discounted by $\theta\delta Q_t$.

It is assumed that when a worker is replaced, the new worker cannot automatically renegotiate his wage. Instead, he faces the same probability of sticky wages than existing workers. If they were completely free to choose new wages, the effect would die out; but as long as the new wage partly takes into account the wage of existing workers, the effect would be lessened but not die out. This gives a discrepancy between the joint survival probability $\theta\delta$ of the optimal wage setting decision, and the true wage stickiness $\theta$ that is featured in the dynamics of the aggregate wage and dispersion. This is the cause of the flatter wage Phillips curve[5]. As mentioned before, evidence in Gertler et al. (2016) or Galuscak et al (2012) tends to support this assumption.

It is also possible to think about the case where it is the union which sets the wage of workers of type $j$, and the union insures workers against layoffs but not quits or dismissals. When a worker quits, or is dismissed, we can assume that he leaves his labour type and finds a different occupation, where wages are set by a different union. As such, if the union maximizes the utility of its *existing* members, employed or not, it will have a short discounting horizon. And it will not take into account the utility of *future* members, because they do not belong to this union *yet*.

[5]. In their Rotemberg setup, Snower and Tesfaselassie (2017) (or Bilbiie Ghironi and Melitz, 2012;2016) assume that new workers (or firms) start with the symmetric wage (price) of existing workers (firms). It is similar to here: entrants are bound by incumbents

### The non linear Phillips curve

When a worker is free to set a wage $w_t(j)$, he seeks to maximize the discounted sum of the wage compensation minus the disutility, defined in expression (4).

$$\mathrm{E}_t \sum (\theta\beta\delta)^{T-t} \left[ u'(Y_T) \frac{(1-\tau_T) w_t(j) N_T(j)}{P_T} - \lambda \frac{N_T(j)^{1+\phi}}{1+\phi} \right]$$

**Lemma 4** *The re-optimizing price $w_t^*$ is :*

$$\left(\frac{w_t^*}{W_t}\right)^{1+\phi\epsilon} = \frac{\mathrm{E}_t \sum (\theta\beta\delta)^{T-t} \mu_t \left(\frac{W_t}{W_T}\right)^{-\epsilon(1+\phi)} \lambda N_T^{1+\phi}}{\mathrm{E}_t \sum (\theta\beta\delta)^{T-t} \left(\frac{W_t}{W_T}\right)^{1-\epsilon} \Omega_T u'(Y_T) N_T} = \left(\frac{K_t}{F_t}\right) \tag{7}$$

*with recursive terms $F_t$ and $K_t$*

$$F_t = (1-\alpha) Y_t^{1-\sigma} + \theta\beta\delta \mathrm{E}_t F_{t+1} \Pi_{t+1}^{\epsilon-1} \tag{8}$$

$$K_t = \mu_t \lambda Y_t^{\frac{1+\phi}{1-\alpha}} + \theta\beta\delta \mathrm{E}_t K_{t+1} {\Pi_{t+1}}^{\epsilon(1+\phi)} \tag{9}$$

This is where the job survival probability, $\delta$ plays a role, compared to the standard model. $\delta$ is an extra factor, appearing here in the worker's discounting, through the recursive $F_t$ and $K_t$. In the recursive equation, $F_t$ depends on the expected future value $\mathrm{E}_t F_{t+1}$, multiplied by the inflation and a discount factor $\theta\beta\delta$. The exact same phenomenon occurs for the recursive term $K_t$. The $\delta$ makes these two terms less forward looking than in the standard model, and it makes the wage Phillips curve flatter, as we will see with the linear approximation.

In each period, only a fraction $(1-\theta)$ of wages are re-optimized at the value $w_t^*$, while a fraction $\theta$ still follows the previous distribution of wages, with an aggregate $W_{t-1}$.Using the definition of the aggregate wage, the wage level $W_t$ is a weighted aggregate of the previous wage level $W_{t-1}$ and the current optimal wage $w_t^*$: [6]

$$W_t^{1-\epsilon} = \theta W_{t-1}^{1-\epsilon} + (1-\theta)(w_t^*)^{1-\epsilon}$$

This provides the dynamics for the wage inflation and dispersion

$$\frac{1-\theta {\Pi_t}^{\epsilon-1}}{1-\theta} = w(\Pi_t) = \left(\frac{w_t^*}{W_t}\right)^{1-\epsilon} = \left(\frac{F_t}{K_t}\right)^{\frac{\epsilon-1}{1+\phi\epsilon}} \tag{10}$$

$$\Delta_t = \theta \Delta_{t-1} {\Pi_t}^{\epsilon(1+\phi)} + (1-\theta) w(\Pi_t)^{\frac{\epsilon(1+\phi)}{\epsilon-1}} \tag{11}$$

### Linear quadratic setup

Although we will look at the optimal steady state level of inflation that the non linear model yields, it is useful to derive a linear quadratic approximation around a zero inflation steady state. In the flexible price steady state, there is

[6] Importantly, the new hires follow existing wages, so that turnover $\delta$ doesn't play a role in this law of motion of the aggregate wage

no inflation ($\Pi = 1$), and no dispersion ($\Delta = 1$). The steady state values $\tilde{Y}$, $\bar{\Omega}$, $\bar{F}$ and $\bar{K}$ are easy to pin down. Let us define the percentage deviation of each variable: $\pi_t = \log \Pi_t$, and $d_t = \log \Delta_t$. Similarly $y_t$, $\omega_t$, $f_t$ and $k_t$ denote log deviations of the capital-letter variables from the steady state.

**Proposition 1** *The linear wage Phillips curve is*

$$\pi_t = \kappa y_t + \beta\delta \mathrm{E}_t [\pi_{t+1}] \tag{12}$$

*with* $\kappa = \left(\frac{\phi+\alpha}{1-\alpha} + \sigma\right) \frac{(1-\theta)(1-\theta\beta\delta)}{\theta} \frac{1}{1+\phi\epsilon}$

This linear wage Phillips curve is broadly similar with the standard wage Phillips curve in a model of price and wage stickiness. Current wage inflation positively depends on the output gap and future expected wage inflation, and negatively on the real wage. However, two differences stand out. The coefficient $\kappa$ is slightly different as it features the parameter $\delta$. But most importantly, future inflation is discounted by $\beta\delta$ instead of simply $\beta$. In terms of intuition, this is because $\beta\delta$ is now the discount factor that is applicable to the job tenure of the worker.

# 3 A flatter Phillips curve

## 3.1 Predictions of the model

### Non vertical long run Phillips curve

The long run version of (12) implies a flatter long-run Phillips curves, and it is no longer vertical or nearly vertical as without turnover:

$$\bar{\pi} = \frac{\kappa}{1-\beta\delta}\tilde{Y}$$

When $\delta$ is smaller than 1, $\kappa$ increases slightly. However the increasing effect on the denominator $(1-\beta\delta)$ largely dominates. This means that long run inflation will depend less strongly on the long run output gap, and the curve is not as vertical.

**Property 1** *In the long run Phillips curve between inflation and output of the form $\bar{\pi} = \chi\tilde{Y}$, the coefficient $\chi$ decreases with turnover ( $\delta$ falls):*

$$\chi = \left(\frac{\phi+\alpha}{1-\alpha} + \sigma\right) \frac{(1-\theta)}{\theta(1+\phi\epsilon)} \frac{1-\theta\beta\delta}{1-\beta\delta}$$

Because the linear equation is only an approximation of a highly non-linear model, it is useful to see the impact of turnover on the non linear long run Phillips Curve. In steady state the price and wage inflation must be equalized: $\Pi = \Pi$. Taking the steady state in equations (8), (9) and (10), output can be written in terms of inflation

**Lemma 5** *The non linear long-run Phillips curve is*

$$\left(\frac{Y}{\tilde{Y}}\right)^{\frac{\phi+\alpha}{1-\alpha}+\sigma} = \left[\frac{1-\theta\beta\delta\Pi^{\epsilon(1+\phi)}}{1-\theta\beta\delta\Pi^{\epsilon-1}} w(\Pi)^{-\frac{1+\phi\epsilon}{\epsilon-1}}\right] \tag{13}$$

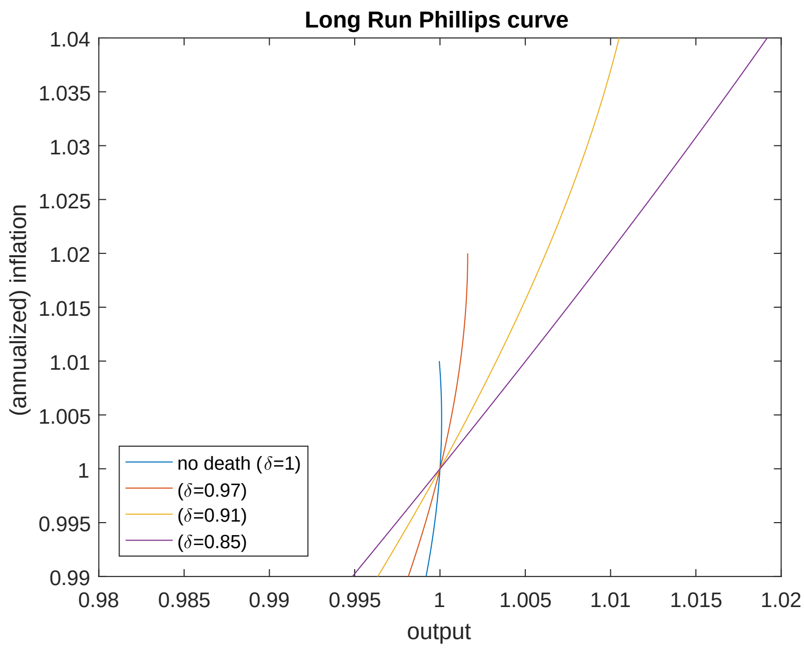


Figure 3: Non linear long run Phillips curve for different values of $\delta$

Figure (3) displays the output level $Y$ associated to a long run (annualized, price and wage) inflation $\Pi$. When $\Pi = 1$, $Y = 1$ (the flex price case). As $\Pi$ increases, there is a limited output gain, at least to the first order. With turnover ($\delta < 1$), the long run trade-off is flatter than in the normal case without. This was true for the linear approximation of the curves around zero inflation, and it is also true for the non linear case.

### Short and middle run

In equation (12), the coefficient of the output gap does not fall with more turnover (a fall in $\delta$). The coefficient $\kappa$ is (slightly) decreasing in $\delta$, so it increases when the survival probability falls. The intuition is that with a lower discount factor, more weight is put on current economic conditions, so inflation reacts more strongly to current output. However, let us look at two cases where the Phillips curve would be perceived as flatter. Equation (12) can be iterated forward:

$$\pi_t = \kappa y_t + \beta\delta \mathrm{E}_t\left[\pi_{t+1}\right] = \kappa \sum_{k\geq 0} \left(\beta\delta\right)^k \mathrm{E}_t y_{t+k}$$

Let us assume that the output gap is serially correlated:

$$y_t = \rho_y y_{t-1} + u_t$$

with $u_t$ a mean-zero disturbance. Then we can write inflation as

$$\pi_t = \frac{\kappa}{(1-\rho_y \beta\delta)} y_t$$

**Property 2** *The slope of a traditional Phillips Curve displaying only current inflation and output, $\pi_t = \tilde{\kappa} y_t$ , will depend on the ratio*

$$\frac{(1-\theta\beta\delta)}{(1-\rho_y \beta\delta)}$$

*As long as $\rho_y > \theta$ (the output gap being more persistent than wages), the slope will decrease when $\delta$ falls (turnover increases).*

Let us also look at an estimated New Keynesian Phillips curve with a restricted $\beta$, if the turnover is not accounted for. Using the assumptions above,

$$\pi_t - \beta \mathrm{E}_t [\pi_{t+1}] = \kappa y_t - \beta(1-\delta)\mathrm{E}_t [\pi_{t+1}]$$

$$\pi_t - \beta \mathrm{E}_t [\pi_{t+1}] = \kappa \left[ y_t - \beta(1-\delta) \sum_{k \geq 1} (\beta\delta)^k \mathrm{E}_t y_{t+k} \right]$$

**Property 3** *The estimated slope in this case will be*

$$\kappa^* = \frac{cov(\pi_t - \beta \mathrm{E}_t [\pi_{t+1}], y_t)}{var(y_t)} = \frac{(1-\beta\rho_y)}{(1-\rho_y \beta\delta)} \kappa$$

*As long as $\rho_y > \theta$ (the output gap being more persistent than wages), the slope will decrease when $\delta$ falls (turnover increases).*

This is the case in the empirical estimates of Gali and Gertler (1999), where they use marginal costs instead of the output gap. They estimate $\pi_t = \lambda mc_t + \beta E\pi_{t+1}$. The estimated coefficient of marginal costs, $\lambda$, depends on the assumption about the coefficient of future inflation, $\beta$. When this coefficient is restricted to $\beta = 1$, the estimated value of $\lambda$ is smaller than when there is no restriction and $\beta$.takes a lower value.

**Remark** We have to assume here that the output gap is more persistent than sticky wages ($\rho_y > \theta$) in order to generate a downward bias in the traditional PC, and the restricted New Keynesian PC. This is not difficult as $\rho_y \approx 0.95$ in the US for example. However, such an assumption would not be necessary in a Rotemberg setup. In such a setup, the coefficient $\kappa$ does not depend on turnover. Assuming $y_t = \rho_y y_{t-1} + u_t$ as before, $(1-\rho_y \beta\delta)$ increases when $\delta$ falls, so the traditional and restricted New Keynesian slopes are always smaller with turnover.

## 3.2 Empirical results

I rely on data from the OECD to test a wage Phillips curve between inflation and cyclical unemployment[7]. I have 21 countries, between 1996 and 2014 (or fewer years for some countries). Cyclical unemployment $u_t$ is defined as unemployment minus the NAIRU, or structural unemployment. Wage growth is the yearly percentage increase in nominal compensation per worker. For turnover, I rely on the job tenure survey. While the proportion of worker who have been in their job for less than a year is not a perfect metrics for the rate of yearly job turnover, it is nevertheless a relatively good indicator. Therefore my turnover variable $\tau_t$ is the proportion of worker between 25 and 54 who have been in their job for a year or less.

I run two regressions.[8] The first is a short run expectation-based curve:

$$\pi_t = \gamma(\tau_t)u_t + \beta(\tau_t)\pi_{t+1} + v_t$$

where $v_t$ is an error term. $\gamma(\tau_t)$ is expected to be negative, and decrease slightly with turnover $\tau_t$ (a slightly steeper curve). $\beta(\tau_t)$ is positive and smaller than 1, and it should decrease with $\tau_t$. In order to test the effect of turnover on these coefficients, I add the cross terms $(\tau_t \times u_t)$ and $(\tau_t \times \pi_{t+1})$ in the regression. The two estimates are expected to be negative. I also add time and country fixed effects in the regression. Last, to rule out common trends in turnover and the coefficients, I also allow a trend in the coefficients. As such, the equation can be written

$$\pi_{n,t} = \begin{array}{c} \alpha_n + \alpha_t + \gamma_1 u_{n,t} + \gamma_2(\tau_{n,t} \times u_{n,t}) + \gamma_3(t \times u_{n,t}) \\ +\beta\pi_{n,t+1} + \beta_2(\tau_{n,t} \times \pi_{n,t+1}) + \beta_3(t \times \pi_{n,t+1}) + v_{n,t} \end{array}$$

The results are coherent with the predictions of the model. The effect of turnover on the coefficient of future inflation is negative and significant, as predicted. And allowing for a trend in the coefficient does not make turnover insignificant. Contrary to the prediction, the unemployment coefficient increases with turnover (which makes the curve flatter). But this effect was predicted to be small, and in the data the change is positive but insignificant. The flatter unemployment coefficient might be caused by less frequent wage changes as in the menu costs model of Ball and Mazumder (2011): changes in wage will be less frequent as inflation and volatility declined over the past decades.

It is also insightful to look at the case of a restricted $\beta$. If the on future inflation is restricted to the riskless discount factor (about 0.96 yearly), we can see that the coefficient on unemployment is reduced to less than a third, from $-0.3$ to $-0.09$. And the effect of turnover on the unemployment coefficient is magnified under this restriction – in line with my earlier predictions.

[7] It as long been argued (see, eg. Gali and Gertler, 1999; or Gali, 2011) that Phillips curve are easier to estimate with real marginal costs or unemployment than with output

[8] Consistency of my OLS approach requires that unemployment and turnover are exogenous. In particular, they cannot be correlated with any lead or lag of the error term $v_t$ – which also captures variations of the desired wage markup. If there is such a correlation, OLS would be inconsistent, and models such as VAR or GMM could control for the endogeneity issue.

| | (1) | | (2) | | (3) | |
|---|---|---|---|---|---|---|
| $\pi_{t+1}$ | .40*** | 0.96 | 1.26 | 0.96 | 110 | 0.96 |
| | (.055) | | | | | |
| $\tau_t \times \pi_{t+1}$ | | | $-.061^{***}$ | | $-.048^{**}$ | |
| | | | (.017) | | (.017) | |
| $u_t$ | $-.30^{***}$ | $-.089$ | $-.57$ | $-.53$ | 42 | 96 |
| | (.073) | (.082) | | | | |
| $\tau_t \times u_t$ | | | .013 | .026 | .022 | .027 |
| | | | (.021) | (.024) | (.020) | (.024) |
| $\tau_t$ | | | .045 | $-.18$ | .006 | $-.217$ |
| | | | (.11) | (.11) | (.11) | (.11) |
| time trend | | | | | Yes | Yes |
| fixed effects | Yes | Yes | Yes | Yes | Yes | Yes |

Table 1: The NK short run Phillips curve

Now let us look at the medium run Phillips curve. If unemployment is serially correlated of order 1, we saw that the Phillips curve could also be written

$$\pi_t = \tilde{\gamma}(\tau_t)u_t + \tilde{v}_t$$

$\tilde{v}_t$ is the new error term, and $\tilde{\gamma}(\tau_t)$ is predicted to be negative and increasing (a flatter curve) with turnover $\tau_t$. As before, I allow for time and country fixed effects, I include the term $(\tau_t \times u_t)$ in the regression, which is expected to be positive. To rule out common trends in turnover and the coefficient, I again allow a trend in the coefficient. As such, the equation can be written

$$\pi_{n,t} = \tilde{\alpha}_n + \tilde{\alpha}_t + \tilde{\gamma}_1 u_{n,t} + \tilde{\gamma}_2(\tau_{n,t} \times u_{n,t}) + \tilde{\gamma}_3(t \times u_{n,t}) + \tilde{v}_{n,t}$$

| | (1) | (2) | (3) |
|---|---|---|---|
| $u_t$ | $-.38^{***}$ | $-.93$ | $-27$ |
| | (.07) | | |
| $\tau_t \times u_t$ | | .035 | .032 |
| | | (.020) | (.020) |
| $\tau_t$ | | $-.15$ | $-.14$ |
| | | (.09) | (.095) |
| time trend | | | Yes |
| fixed effects | Yes | Yes | Yes |

Table 2: The NK middle run Phillips curve

The effect of turnover on the coefficient is positive, which is consistent with the predictions. It is not significant at the 10% level, but it is not less significant when a trend effect is allowed. As such, these results are consistent with the idea that turnover creates a flatter middle run Phillips curve.

# 4 Price or inflation targeting?

## 4.1 Turnover and price targeting

As we will see, introducing turnover into a standard New Keynesian model has strong implications for the optimal Ramsey policy. Let us first define the aggregate welfare function.

### Welfare function

While workers discount future wages with the probability of job turnover, individuals do not die in my model. Therefore, the aggregate utility function of the social planner is simply the aggregation of each household's utility given in equation (1). Using equation (5), this is

$$\mathrm{E}_0 \sum_{t \geq 0} \beta^t U(C_t, N_t(j)) = \mathrm{E}_0 \sum_{t \geq 0} \beta^t \tilde{Y}^{1-\sigma} \left[ \frac{\left(\frac{Y_t}{\tilde{Y}}\right)^{1-\sigma}}{1-\sigma} - \frac{\frac{1-\alpha}{1+\phi} \Delta_t \left(\frac{Y_t}{\tilde{Y}}\right)^{\frac{1+\phi}{1-\alpha}}}{\mu} \right] \tag{14}$$

In terms of intuition, it is easier to look at the optimality of price targeting in a quadratic setup. Whens steady state distortions are small, the approximation of (14) and (11) bring a quadratic approximation that is not different from the case without turnover. This is because turnover plays no direct role in the utility function, or the dynamics of the dispersions.

**Lemma 6** *The second order approximation of the aggregate utility is*

$$U = -\sum_{t \geq 0} \beta^t \left[ \widetilde{\kappa} \frac{(y_t - \bar{Y})^2}{2} + (1-\alpha)\epsilon_w \frac{\pi_t^2}{2} \right]$$

*with* $\bar{Y} = \log \frac{\bar{Y}}{\tilde{Y}}$ *and* $\widetilde{\kappa} = \left(\sigma + \frac{\phi+\alpha}{1-\alpha}\right) \frac{(1-\theta^w)(1-\theta^w \beta)}{\theta^w} \frac{1}{1+\phi\epsilon_w} \neq \kappa$

Contrary to $\kappa$, $\delta$ does not appear in $\widetilde{\kappa}$, which is exactly the same coefficient as in the case with no turnover. This is because the distortion is discounted with the discount factor of the household, where the death shocks play no role.

Let us also assume cost push shocks in the Phillips curve:

$$\pi_t = \kappa y_t + \beta\delta \mathrm{E}_t \pi_{t+1} + u_t$$

with $u_t$ the cost push shock, an error term. We allow it to be an $AR(1)$ process with autocorrelation $\rho_u$ ($\rho_u = 0$ denoting the white noise case).

### The optimality of price targeting

**Proposition 2** *When $\delta = 1$, price targeting is optimal for the Ramsey policy: even with steady state distortions, the long run optimal level of inflation is zero; while inflation reacts to cost push shocks in the short run, this is accompanied* *by deflation in the future, so that there is full mean reversion of the price level. In other words, there is long-run price targeting in response both to long term distortions and short term cost push shocks*

*When $\delta < 1$, price targeting is no longer optimal: long run inflation is non zero if there are steady state distortions; in response to cost push shocks, some deflation in the future offsets the initial response of inflation, but there is no longer full mean reversion of the price level. In other words, price targeting does not hold anymore.*

The intuition is as follows: in the benchmark, by committing to give up some discretion in the future, the planner has some extra discretion in the present correct cost push shocks, or an inefficient steady state. So that price stability is optimal from today's perspective, but there is an incentive to renege tomorrow. With the death shock, firms are less responsive to commitments, so that the current gain in terms of commitment no longer offsets the inefficiency in the future. Thus, even with a credible commitment, inflation will always be used to offset cost push shocks or steady state inefficiencies.

To better grasp the logic, it is useful to compare the Ramsey policy, which is *history dependent*, to an optimal *state dependent* policy. While such a solution is not Ramsey optimal, it features no dynamic inconsistency. We can call this solution *Markovian*, or *Recursively Pareto Optimal* as in Brendon and Ellison (2015). Let us assume that the optimal inflation is a function of the first-best rate of output and the current cost push shock: $\pi_t = \bar{\pi} + \gamma_\pi u_t$.

In such a Markovian setup, the optimal inflation is not zero even without turnover. This is because the long run Phillips Curve is not vertical without turnover, and a very little amount of inflation is welfare improving. On the other hand, the Ramsey policy in this case is to allocate the current and future inflation differently. A high inflation is used in the short run, in exchange for no inflation in the long run. While this is not time-consistent, it is optimal from today's point of view. With turnover, the Markov optimal inflation is higher due to the flatter long run curve. And the Ramsey policy still uses more inflation in the short run, but not zero in the long run.

In response to cost push shocks, the difference between the Markov and Ramsey policy is more important. The Ramsey policy commits to offset current inflation with future disinflation in response to cost push shocks, and this commitments improves the trade off in the short run. Because the Markov policy is not history dependent, it cannot promise future disinflation, and hence mean reversion of the price level. When turnover is introduced, there is no longer full mean reversion of the price level, but it does not impact the Markov policy much.

## 4.2 Long run optimal inflation

In this subsection, we derive the optimal steady state inflation implied by the non-linear model. While a closed form expression was available for the long run Phillips curve, the optimal level of steady-state inflation (for a given amount of

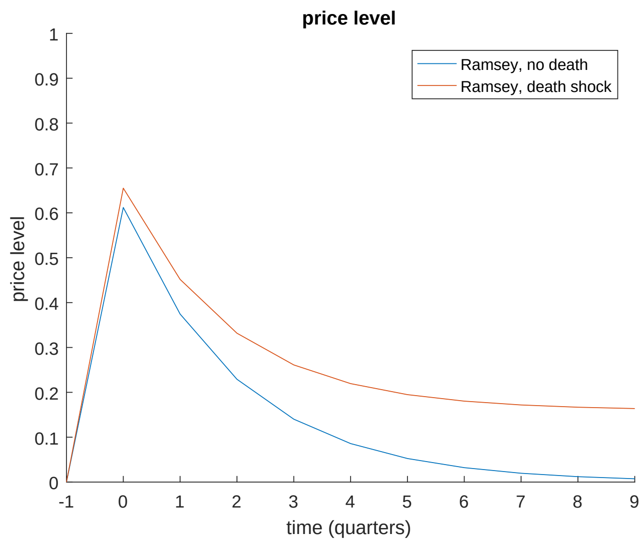


Figure 4: Ramsey and Markov policy in response to wage cost-push shocks

steady state distortions) can only be defined implicitly. As such, it is useful to calibrate most of the parameters, to provide a graphical illustration. As in Gali, let us calibrate $\alpha = 0.25$, $\beta = 0.99$, $\epsilon = 8$, $\theta = 0.66$, $\phi = 0.11$ and $\sigma = 0.16$. Now we need to find values for $\delta$. Let us consider a low turnover scenario ($\delta = 0.95$, or an average duration of 5 years) and an intermediate scenario with $\delta = 0.90$.

It is a well known feature that in the presence of steady state distortions, the optimal Ramsey policy of the New Keynesian model does not bring a constant level of inflation. While there is a small output-inflation trade-off, the Ramsey policy dictates to front-load some of the inflation at the beginning, with a reduced inflation in the future. This brings the classical time inconsistency problem: it is optimal to promise zero or low inflation in the future, while having a higher rate of inflation temporarily. But in the future, there is an incentive to renege on past promises of low inflation

Thus we have two ways to define the optimal long run inflation. One is to look at the long run solution of the Ramsey policy: we solve the dynamic Ramsey problem, with the discounted utility function and the dynamic Phillips curve constraints, and look at the long run solution. But this runs into the issue of inconsistency, and the long run rate of inflation is not optimal for the current period.

If the aim is to have a constant rate of inflation that is applicable both to the short and long run, we can instead look at the long run constraints, and maximize utility subject to them. As such, we are restricting ourselves to the set of constant inflation rates. Instead of solving the dynamic problem and restrict to the time-invariance solution, we impose time-invariance before solving the maximization.

In the case of the time invariant solution, one simply maximizes the per-period objective function of the social planner (5), subject to the long run output inflation Phillips curve (13) and the expression of the long run dispersion

$$\Delta = \frac{(1-\theta)w(\Pi)^{\frac{\epsilon(1+\phi)}{\epsilon-1}}}{1-\theta\Pi^{\epsilon(1+\phi)}}$$

Intuitively, inflation helps to bring output closer to its first-best level – but too much inflation reduces output as the curve is non linear – but it increases the price and wage dispersions, which reduce utility.

$$L = \left\{ \begin{array}{c} \frac{1}{1-\sigma}Y^{1-\sigma} - \frac{1}{\mu}\frac{1-\alpha}{1+\phi}\Delta Y^{\frac{1+\phi}{1-\alpha}} \\ +\Phi_1\left[\ln\left(\frac{1-\theta\beta\delta\Pi^{\epsilon(1+\phi)}}{1-\theta\beta\delta\Pi^{\epsilon-1}}w(\Pi)^{-\frac{1+\phi\epsilon}{\epsilon-1}}\right) - \left(\frac{\phi+\alpha}{1-\alpha}+\sigma\right)\ln Y\right] \\ +\Phi_2\left[\left(1-\theta\Pi^{\epsilon(1+\phi)}\right)\Delta - (1-\theta)w(\Pi)^{\frac{\epsilon(1+\phi)}{\epsilon-1}}\right] \end{array} \right\}$$

As illustrated in figure (5), this brings a positive amount of inflation, even when $\delta = 1$. The optimal inflation increases as $\delta$ decreases.

For the timeless Ramsey policy, we write the full dynamic Lagrangian (with $Y_t$, $\Omega_t$, $K_t$ and $F_t$ renormalized to flex price values).

The social planner maximizes the discounted sum of the per period utilities (5), subject, in each period, to the recursive expressions of $F_t$ and $K_t$ (equations 8 and 9), the ratio $\frac{K_t}{F_t}$ (equation 10), as well as the dynamics of $\Delta_t$ (equations 11).

Intuitively the trade-offs are similar to the time invariant problem: inflation increases output at the first order, but increases the costly price and wage distortions. However, the fully dynamic setting is different from the previously static one. The Lagrangian of the problem writes

$$L = \sum \beta^t \left( \begin{array}{c} \left[\frac{1}{1-\sigma}Y_t^{1-\sigma} - \frac{1}{\mu_t}\frac{1-\alpha}{1+\phi}Y_t^{\frac{1+\phi}{1-\alpha}}\Delta_t\right] \\ +\phi_{1,t}\left[K_t w(\Pi_t)^{\frac{1+\phi\epsilon}{\epsilon-1}} - F_t\right] \\ +\phi_{2,t}\left[F_t - Y_t^{1-\sigma} - \theta\beta\delta \mathrm{E}_t F_{t+1}\Pi_{t+1}^{\epsilon-1}\right] \\ +\phi_{3,t}\left[K_t - Y_t^{\frac{1+\phi}{1-\alpha}} - \theta\beta\delta \mathrm{E}_t K_{t+1}{\Pi_{t+1}}^{\epsilon(1+\phi)}\right] \\ +\phi_{4,t}\left[\Delta_t - \theta\Delta_{t-1}{\Pi_t}^{\epsilon(1+\phi)} - (1-\theta)w(\Pi_t)^{\frac{\epsilon(1+\phi)}{\epsilon-1}}\right] \end{array} \right)$$

After taking the first order conditions, we look at the steady state value of each constraint and multiplier. Figure (5) displays the optimal rate of inflation depending on the amount of steady state distortions, for different values of $\delta$. When $\delta = 1$, we have the classic result of zero inflation in the long run, but it increases as this parameter decreases.

Figure 5 displays the constant and timeless Ramsey steady state inflation depending on the first-best output $\bar{Y} > 1$, for different values of $\delta^w$. The constant policy is in blue while the Ramsey policy is in dashed red. With more frequent death shocks, the optimal level of constant inflation is higher.

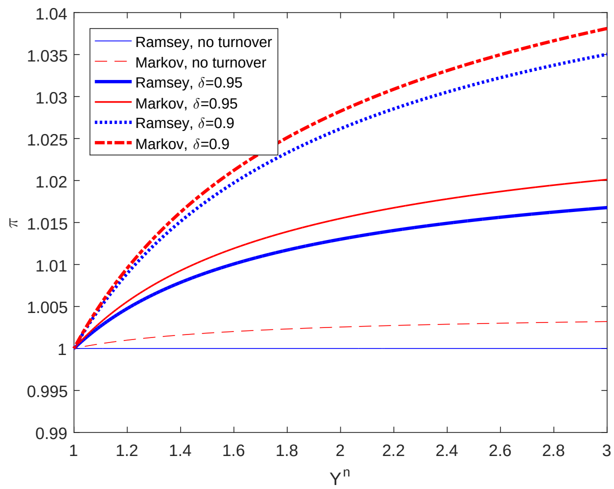


Figure 5: Steady state information without and with turnover

When $\delta = 1$, there is a small level of inflation for the constant policy, but no inflation for the timeless Ramsey policy: this is the optimality of price stability. However, when death shocks are introduced, the optimal level of inflation increases with the output gap, for both the constant and timeless cases. For a large output gap ($\bar{Y} >> 1$) and large death shocks, the optimal annual level of inflation is in the order of $1 - 3\%$ annually.

# 5 Conclusion

This paper constructed a New Keynesian model with Calvo wage stickiness, as well as job turnover. I show how this leads to a Phillips Curve that is far less forward looking. When looking at a medium run Phillips Curve, with persistent output or unemployment disturbances, this can account for a flatter curve. If the coefficient of future inflation is restricted in a standard NK Phillips Curve, this creates a bias on the estimate of the slope of the Phillips Curve, and this bias increases with more turnover. This prediction is tested on OECD data and is not rejected empirically. In the long run, the Phillips Curve is also flatter, and no longer vertical or near-vertical.

I show how turnover breaks the optimality of price stability. Price stability is no longer optimal, and inflation expectations are more anchored than in traditional Phillips curves. As such the optimal Ramsey policy no longer targets the price level in response to cost push shocks. If this turnover is large, and if the steady state distortions are high enough, the optimal level of inflation can reach $1-2\%$ annually. In fact, if there was partial price and wage indexation, the optimal inflation would be higher, or a same amount of inflation would be rationalized by a lower turnover or steady state distortion.

One fruitful avenue of future research would be to investigate the empirics in greater details. Phillips curves can be more informative if we don't impose the restriction that they are vertical or quasi vertical in the long run. Also, a cross section of different sectors, and different types of workers - eg, temporary vs. permanent employees - could provide additional evidence. Another fruitful avenue could be to endogenise this turnover. In such a case, it might be affected by the central bank's decision, and become a policy target on its own.