Sudden Spikes in Global Risk[1]

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Abstract

Recent episodes (October 2008, May 2010, August 2011) have witnessed huge spikes in equity price risk (implied volatility). Apart from their large size, several features characterize these risk panics. They are global phenomena, shared among a broad set of countries. There is substantial variation though in the extent to which individual countries are impacted, while the impact bears little relation to financial linkages with the epicenter of the crisis. In addition there is usually not a large shock to fundamentals that sets off these panics. We provide an explanation for these risk panic features in the context of a two-country model that allows for self-fulfilling shifts in risk.

1 Introduction

The financial panic in the Fall of 2008 lead to a sharp drop in equity prices all around the world. It is hard to disconnect this steep fall from the enormous spikes in risk as measured by implied volatility (e.g., VIX in the United States). Figure 1 shows implied volatility indices for 12 countries.[1] In the United States the VIX quadrupled during the Fall of 2008. But this was hardly a phenomenon limited to the United States. The sharp increase in the VIX was seen in all the countries in our sample, including both industrialized and developing countries. Large spikes in implied volatility indices again occurred in May 2010 (Greek sovereign debt crisis) and in July/August of 2011 (U.S. debt debate and intensifying European debt crisis). In both cases the increase in risk was again a global phenomenon shared broadly across all countries.[2] The aim of this paper is to shed light on what causes such spikes in risk on a global scale.

Apart from the global nature of these spikes in risk, four other features stand out from these recent episodes that demand an explanation. First, there is considerable variation across countries in the extent to which risk increases. This can be seen from Figure 2, which reports the percentage increase in the implied volatility indices during the three recent spikes in risk. For example, during the 2008 panic the increase in risk ranged from 109% in South Africa to 350% in Switzerland. In May 2010 the numbers ranged from 52% in Mexico to 230% in Canada. Second, the relative extent to which countries are affected by these global risk panics varies substantially across episodes. For example, in May of 2010 Canada saw a spike in risk almost twice as high as Germany (even though it was set off by European sovereign debt concerns), while in 2008 Germany was somewhat more affected than Canada.

A third feature that stands out regarding these recent risk episodes is that it is hard to connect them to a similarly large sudden change in fundamentals. For example, Greek sovereign debt did not suddenly become large in May of 2010. Similarly, the downgrade of U.S. debt by S&P on August 6, 2011, was certainly

[1]Data sources are reported in Appendix C.

[2]Beyond the recent crises, there is a substantial econometric literature which has shown that asset price volatility co-moves significantly across countries, especially during high volatility periods. See for example Edwards and Susmel (2001), Beirne et al. (2009) or Diebold and Yilmaz (2009). Soriano and Climent (2006) provide a survey of this literature.

not an event containing new information about fundamentals. And the mortgage losses that ultimately generated the financial crisis in the Fall of 2008 had been gradual and started more than a year prior to the panic.

Finally, it is hard to connect the extent to which countries were affected by the 2008 panic to financial linkages they had to the epicenter of the crisis, which was clearly the United States. As documented by Rose and Spiegel (2010) and Kamin and Pounder (2010), there is no evidence that countries with larger cross-border financial linkages to the United States were more affected in terms of either equity prices or business cycles. This casts doubt on explanations of the global nature of these panics resulting from transmission of shocks through financial linkages.

The goal of this paper is to develop a framework that is consistent with all of these risk panic features. The paper builds on Bacchetta, Tille, and van Wincoop (2011), from here on BTW, who develop the concept of risk panics in a closed economy context. BTW show that as long as asset demand depends on asset price risk, self-fulfilling shifts in risk are possible as a result of a circular relationship between the stochastic processes of the asset price and asset price risk. During a risk panic, a weak fundamental can suddenly take on the additional role of a coordination device for the self-fulfilling shift in risk. BTW show that the weaker the fundamental, the larger is the panic.

We extend this idea to a two-country framework. The theory is consistent with the various risk panic features discussed above. The global nature of the panic is a result of a news event that makes a particularly weak macro fundamental somewhere in the world (e.g. Greek sovereign debt in 2010) the sudden focal point of fear everywhere. This news event is a pure sunspot in the model. There is no actual shock to macro fundamentals during the risk panic. The macro variable that becomes the focal point of fear was already weak prior to the panic and does not change at the moment of the panic.

We show that the extent to which equity prices and risk change as a result of the panic can differ substantially across the two countries. It depends both on the fundamental hedging properties of the assets and a self-fulfilling aspect. While there is a precise solution for the world stock price during a panic, the extent to which individual countries are affected depends on self-fulfilling beliefs about the covariance with the global asset payoff. This explains not only why countries may be affected differently, but also why this could vary across crises.

The model is also consistent with the lack of a relationship between financial linkages to the epicenter country and the extent to which countries are affected by the panic. Standard transmission channels (financial or otherwise) play no role in our model. Rather, an event that is watched in real time all over the world draws attention to a particularly weak macro fundamental somewhere that then becomes a focal point of fear everywhere.

Some of the recent literature on financial contagion, which we review in Section 2, has explained the co-movement in equity prices during recent crises with the co-movement in premia that measure expected excess returns on equity. This focus is well placed as it is hard to argue that either of the two other equity price determinants, the risk-free rate and expected dividends, can account for the sharp drop in equity prices around the world in 2008. But fluctuations in risk play little or no role in the expected excess returns considered in the literature. Expected excess returns are attributed, for example, to lack of arbitrage as leveraged institutions are faced with binding borrowing constraints.

Apart from the inability to explain the spikes in risk seen in the data, this literature also differs from the approach here in that changes in asset prices are driven by shocks to fundamentals that transmit across countries through financial linkages. In our model the panic is not caused by a change in fundamentals. It is self-fulfilling. And we have shut down all transmission mechanisms in the model.

We want to emphasize that our model is not aimed at providing a full account of what happened during any particular recent episode. It is notable that these episodes were all quite different in nature. The 2008 panic has its roots in the collapse of the housing bubble in the U.S., with large losses for leveraged institutions and bank runs in the shadow banking system. The 2010 panic instead was associated with concerns over sovereign debt in Greece and was not accompanied by a large-scale bank run. The 2011 spike in risk was connected both to political gamesmanship in the U.S. and concerns about the adequacy of another Greek rescue plan. In our model a risk panic can occur based on any event, anywhere in the world, which happens to grab the attention of the markets at that time and sets off fear on a global scale. It is no wonder that the VIX is often referred to as the "fear factor".

The particular model that we use to derive our results is sufficiently stylized to allow for a closed-form analytical solution. This makes the mechanisms at work easier to understand for what is otherwise a quite difficult topic. We consider a two-country model where investors trade equity claims with exogenous and stochastic dividends. Two simplifying assumptions are an OLG structure and a constant interest rate on bonds.

The only stochastic fundamental is the dividend on equity. The role this fundamental plays during a panic is as a coordination device for a self-fulfilling shift in risk. As emphasized by Bacchetta, Tille and van Wincoop (2010), the precise nature of the macro variable that becomes the focal point for a risk panic is not so important. They show that results are similar when the key macro fundamental is the net worth of leveraged financial institutions, which fits more closely to the 2008 crisis. Focusing on stochastic dividends as the only source of macro shocks has the advantage of making the model more standard and analytically tractable.

The remainder of the paper is organized as follows. Section 2 reviews the recent literature on financial contagion. Section 3 describes the model. Section 4 discusses the solution for the world equity price and global risk panics. Section 5 discusses the solution of the equity prices of the two countries. It focuses on how a global risk panic affects the countries individually. Section 6 connects the implications of the model to the features of recent risk panic episodes discussed above. Section 7 concludes.

2 Recent Financial Contagion Literature

The recent financial crisis has spurred renewed interest in the issue of financial contagion.[3] The vast existing literature[4] is being extended, drawing lessons from recent events. This renewed interest in asset price contagion is not surprising given the large drop in equity prices across the globe in the Fall of 2008. A striking feature however, especially in light of Figures 1 and 2, is that the theoretical literature gives little attention to asset price risk. Co-movement of asset prices is not associated in the literature with co-movements of risk.

Given the importance of leveraged financial institutions during the 2008 crisis, several recent papers have emphasized their role in the contagion of the crisis

[3] There is no precise or agreed-upon definition of contagion, but we think of it as a situation where a shock in one country affects other countries through various transmission channels.

[4] See Dornbusch et al. (2000) or Karolyi (2003) for surveys.

across the globe. Various transmission channels have been emphasized that relate to the balance sheets of leveraged institutions. In the middle of the crisis, Krugman (2008) argued for such a transmission channel. Losses of U.S. leveraged institutions imply a sell-off of both U.S. and Foreign assets, which leads to a further drop in asset prices that amplify these effects. Foreign leveraged institutions that have exposure to the U.S. further contribute to transmission abroad. In the absence of binding leverage constraints, these are essentially wealth effects that are amplified through leverage. The role of such wealth effects in transmission has also been emphasized by Gromb and Vayanos (2002), Kyle and Xiong (2001) and Pavlova and Rigobon (2008). The sell-off of risky assets leads to higher equilibrium expected excess returns, but is not caused by a rise in asset price risk.

Some papers have found transmission to be one for one in settings where the leveraged institutions face binding leverage constraints or other frictions that have an analogous effect. Examples are Devereux and Sutherland (2011), Dedola and Lombardo (2010), Kollmann et al. (2010), Perri and Quadrini (2011) and Mendoza and Quadrini (2010). However, this one-to-one transmission occurs only in models that assume either perfect portfolio diversification across countries or the absence of any portfolio investors other than leveraged institutions.[5]

van Wincoop (2011) has criticized this literature as being sharply at odds with reality. There is significant evidence of portfolio home bias, including for leveraged financial institutions. In addition, leveraged institutions in the U.S. hold at most 20% of all risky assets (and even less in the equity market). It is shown that when a two-country model with leveraged institutions as well as non-leveraged investors is calibrated to the data, transmission is quite limited and the magnitude of asset price changes is small even when faced with massive balance sheet losses of leveraged institutions.

Apart from the unrealistic assumptions needed to get the large price effects and large contagion, the increase in expected excess returns in this literature is unrelated to changes in asset price risk. It is hard to see how the large drop in equity prices can be explained without any relation to the enormous spikes in risk documented in Figures 1 and 2. A few papers do examine the impact of a change in risk associated with asset returns. For example, Fostel and Geneakoplos

[5] See van Wincoop (2011).

(2008) present a model where an increase in uncertainty in a developed country's assets leads to a price decline in emerging country asset prices. Schinasi and Smith (2000) examine the impact of a volatility increase in one asset under various portfolio rules and determine when contagion occurs. Blengini (2011) studies the impact of various sources of macroeconomic uncertainty on international portfolio allocation.[6] These papers, however, consider exogenous changes in risk and do not lead to co-movement in asset price risk.

3 A Simple Two-Country Portfolio Choice Model

In this section we describe a simple two-country portfolio choice model. Each country, Home and Foreign, is inhabited by two-period overlapping generations of consumers-investors with mean-variance preferences. They allocate their portfolio between bonds, Home stocks and Foreign stocks.[7] Financial markets are perfectly integrated. The only uncertainty comes from Home and Foreign shocks that affect dividends. In this section, we derive the optimal portfolios and the equilibrium conditions for equity prices. The equilibria themselves are discussed in the next two sections.

3.1 Optimal Portfolios

The model complexity is kept to a strict minimum so that it can be solved analytically. We denote the Home and Foreign countries respectively H and F. In both countries the overlapping generations of investors are born with wealth W. They invest it in equity and bonds and consume the return on their investment when old. The total number of agents is n in the Home country and $1-n$ in the Foreign country.

The bond pays an exogenous constant gross return $R > 1$. Equity consists of a claim on trees with stochastic dividends of respectively $Z_{H,t}$ and $Z_{F,t}$ in the Home and Foreign countries. The per capita capital stock (number of trees) in both countries is K. Equity prices are $Q_{H,t}$ and $Q_{F,t}$. Home and Foreign equity

[6] In a similar vein, Benigno et al. (2012) analyze the impact of various sources of uncertainty on exchange rate movements.

[7] There is no distinction between Home and Foreign bonds as it is a single good economy.

returns from t to $t+1$ are then

$$R_{H,t+1} = \frac{Z_{H,t+1} + Q_{H,t+1}}{Q_{H,t}} \tag{1}$$

$$R_{F,t+1} = \frac{Z_{F,t+1} + Q_{F,t+1}}{Q_{F,t}} \tag{2}$$

The only source of uncertainty in the model is with respect to dividends:

$$Z_{H,t} = \bar{Z} + mA_{H,t} \tag{3}$$

$$Z_{F,t} = \bar{Z} + mA_{F,t} \tag{4}$$

where $\bar{Z}$ is a positive constant, m is a non-negative parameter, and $A_{H,t}$ and $A_{F,t}$ are Home and Foreign macro variables. The formulation of (3) and (4) allows us to vary the fundamental role of the macro variables $A_{H,t}$ and $A_{F,t}$ in affecting asset payoffs. As m becomes smaller, dividends become less affected by fundamental shocks. When $m = 0$, the variables $A_{H,t}$ and $A_{F,t}$ no longer affect dividends. They become pure sunspots that have no fundamental role in the model. The macro variables follow an AR process:

$$A_{i,t+1} = \rho A_{i,t} + \epsilon_{i,t+1} \tag{5}$$

$i = H, F$. The innovations $\epsilon_{H,t+1}$ and $\epsilon_{F,t+1}$ have symmetric distributions with mean zero. Specifically, they can take on the values $-\sigma$ and $+\sigma$ with probabilities 0.5, so that their variance is σ^2. Their correlation is ρ_{HF}.

Investors from both countries born at time t maximize a mean-variance utility over their portfolio return:

$$E_t R^p_{t+1} - 0.5\gamma var_t(R^p_{t+1}) \tag{6}$$

As explained in BTW, the adoption of mean-variance preferences is a simplifying device to make asset demand, and therefore asset prices, depend on future asset price risk. This will also be the case when we introduce financial constraints in an expected utility framework, such as value-at-risk or margin constraints, but the resulting setup will be far more complex. The link between asset prices and asset price risk is key to generating self-fulfilling shifts in risk.

The two countries are perfectly integrated, so that they choose the same portfolio allocation and have the same portfolio returns:

$$R^p_{t+1} = \alpha_{H,t} R_{H,t+1} + \alpha_{F,t} R_{F,t+1} + (1 - \alpha_{H,t} - \alpha_{F,t})R \tag{7}$$

where $\alpha_{i,t}$ denotes the portfolio share invested in equity from country i.

The equity market clearing conditions are

$$\alpha_{H,t}W = Q_{H,t}nK \tag{8}$$

$$\alpha_{F,t}W = Q_{F,t}(1-n)K \tag{9}$$

3.2 Equilibrium Conditions for Equity Prices

Maximization of (6) with respect to $\alpha_{H,t}$ and $\alpha_{F,t}$ gives

$$\alpha_{H,t} = \frac{1}{\gamma}\frac{var_t(R_{F,t+1})E_t(R_{H,t+1}-R) - cov_t(R_{H,t+1},R_{F,t+1})E_t(R_{F,t+1}-R)}{var_t(R_{H,t+1})var_t(R_{F,t+1}) - cov_t(R_{H,t+1},R_{F,t+1})^2} \tag{10}$$

$$\alpha_{F,t} = \frac{1}{\gamma}\frac{var_t(R_{H,t+1})E_t(R_{F,t+1}-R) - cov_t(R_{H,t+1},R_{F,t+1})E_t(R_{H,t+1}-R)}{var_t(R_{H,t+1})var_t(R_{F,t+1}) - cov_t(R_{H,t+1},R_{F,t+1})^2} \tag{11}$$

Write the excess payoff on stocks as $r_{i,t+1} = Q_{i,t+1} + Z_{i,t+1} - RQ_{i,t}$ for $i = H, F$. Substituting the portfolio expressions (10)-(11) into the market clearing conditions (8)-(9) then gives

$$\begin{aligned} & var_t(r_{F,t+1})E_t r_{H,t+1} - cov_t(r_{H,t+1},r_{F,t+1})E_t r_{F,t+1} = \\ & \quad \frac{\gamma nK}{W}\left(var_t(r_{H,t+1})var_t(r_{F,t+1}) - cov_t(r_{H,t+1},r_{F,t+1})^2\right) \end{aligned} \tag{12}$$

$$\begin{aligned} & var_t(r_{H,t+1})E_t r_{F,t+1} - cov_t(r_{H,t+1},r_{F,t+1})E_t r_{H,t+1} = \\ & \quad \frac{\gamma(1-n)K}{W}\left(var_t(r_{H,t+1})var_t(r_{F,t+1}) - cov_t(r_{H,t+1},r_{F,t+1})^2\right) \end{aligned} \tag{13}$$

It is useful to rewrite this system in a way that involves world aggregates. Define the world equity price as $Q_{W,t+1} = nQ_{H,t+1} + (1-n)Q_{F,t+1}$ and the global dividend payment as $Z_{W,t+1} = nZ_{H,t+1} + (1-n)Z_{F,t+1}$. The excess payoff on a world equity claim is then

$$r_{W,t+1} = nr_{H,t+1} + (1-n)r_{F,t+1} = Q_{W,t+1} + Z_{W,t+1} - RQ_{W,t}$$

Writing (12)-(13) jointly in vector notation and then pre-multiplying them with the matrix

$$\begin{pmatrix} var_t(r_{H,t+1}) & cov_t(r_{H,t+1},r_{F,t+1}) \\ cov_t(r_{H,t+1},r_{F,t+1}) & var_t(r_{F,t+1}) \end{pmatrix}$$

gives

$$E_t r_{H,t+1} = \frac{\gamma K}{W} cov_t(r_{H,t+1}, r_{W,t+1}) \tag{14}$$

$$E_t r_{F,t+1} = \frac{\gamma K}{W} cov_t(r_{F,t+1}, r_{W,t+1}) \tag{15}$$

The equilibrium expected excess payoff on equity, which is a risk premium, depends on the covariance with the excess payoff on the world equity claim.[8]

We can write (14)-(15) in terms of equity prices as

$$E_t(Q_{H,t+1} + Z_{H,t+1}) - RQ_{H,t} = \frac{\gamma K}{W} cov_t(Q_{H,t+1} + Z_{H,t+1}, Q_{W,t+1} + Z_{W,t+1}) \tag{16}$$

$$E_t(Q_{F,t+1} + Z_{F,t+1}) - RQ_{F,t} = \frac{\gamma K}{W} cov_t(Q_{F,t+1} + Z_{F,t+1}, Q_{W,t+1} + Z_{W,t+1}) \tag{17}$$

Using (16)-(17) we can solve for the asset prices $Q_{H,t}$ and $Q_{F,t}$ as a function of the state variables $A_{H,t}$ and $A_{F,t}$.

Before solving for the individual asset prices, it is useful to first solve for the global asset price $Q_{W,t}$ as a function of the state variables. This is done in the next section. Taking a weighted average of (16) and (17) with weights n and $1-n$, we get

$$E_t(Q_{W,t+1} + Z_{W,t+1}) - RQ_{W,t} = \frac{\gamma K}{W} var_t(Q_{W,t+1} + Z_{W,t+1}) \tag{18}$$

The expected excess payoff on the global asset price depends on its risk, which is the variance of the global asset payoff.

4 World Equity Price: Multiple Equilibria and Panics

The market clearing condition (18) is analogous to that in the closed economy model of BTW, where agents can invest in a single stock and bonds. Not surprisingly, the nature of the equilibria resulting from (18) is the same as in BTW. Apart from a pure fundamental equilibrium, there are two types of equilibria with self-fulfilling shifts in risk: sunspot-like equilibria and equilibria that allow for a switch between the fundamental and sunspot-like equilibria.

[8] These last two equations imply the familiar capital asset pricing model. Using that $r_{W,t+1} = nr_{H,t+1} + (1-n)r_{F,t+1}$, they can be written as $E_t r_{i,t+1} = \beta_i E_t r_{w,t+1}$ for $i = H, F$, where $\beta_i = cov_t(r_{i,t+1}, r_{w,t+1})/var_t(r_{W,t+1})$.

4.1 Basic Mechanism

To understand why there can be self-fulfilling shifts in risk, assume for the moment that $m = 0$, so that $A_{H,t}$ is a sunspot variable. Assume that the asset price risk $var_t(Q_{W,t+1})$ depends on $A_{H,t}$. Then the asset price $Q_{W,t}$ depends on $A_{H,t}$ and $Q_{W,t+1}$ depends on $A_{H,t+1}$. This in turn implies that the risk $var_t(Q_{W,t+1})$ depends on $A_{H,t}$ when the distribution of $A_{H,t+1}$ depends on $A_{H,t}$. So if agents believe that risk depends on a sunspot, and act upon those beliefs, then indeed it will depend on the sunspot and the beliefs become self-fulfilling.

BTW show that when $A_{H,t}$ is a macro fundamental there is a sunspot-like equilibrium where the variable plays the dual role of fundamental and of a sunspot-like variable around which self-fulfilling shifts in beliefs about risk are coordinated. In addition there is an equilibrium that involves switches between the fundamental and sunspot-like equilibria. We briefly describe these equilibria, leaving most details to Appendix A and BTW.

4.2 Fundamental and Sunspot Equilibria

We consider solutions for the world equity price of the type:

$$Q_{W,t} = \tilde{Q}_W + v_{WH}A_{H,t} + v_{WF}A_{F,t} - V_W A_{H,t}^2 \tag{19}$$

where $\tilde{Q}_W$, v_{WH}, v_{WF} and V_W are constants to be solved. Using (19), we can compute the expectation and variance of $Q_{W,t+1} + Z_{W,t+1}$. Substituting the result into (18), and equating on the left and the right hand side the constant term, and the terms in $A_{H,t}$, $A_{F,t}$, and $A_{H,t}^2$, allows us to solve for the four unknown parameters. As in BTW, there are two solutions: a fundamental equilibrium with $V_W = 0$ and a sunspot-like equilibrium with $V_W \neq 0$.

In the fundamental equilibrium, the solution is

$$Q_{W,t} = \tilde{Q}_W + \frac{m\rho}{R-\rho}A_{W,t} \tag{20}$$

where $A_{W,t+1} = nA_{H,t+1} + (1-n)A_{F,t+1}$. The world equity price depends on the world dividend, whose impact is larger the higher the persistence ρ of dividend shocks. In this case, preceived risk is constant over time. The coefficient on $A_{W,t}$ goes to zero as we let $m \to 0$, in which case $A_{W,t}$ no longer plays a fundamental role (does not affect the global dividend).

In the sunspot-like equilibrium, beliefs about global risk are time-varying and the Home fundamental $A_{H,t}$ plays the role of a coordination device for beliefs about risk. To see this, we focus on the quadratic term in (19), which captures the role of $A_{H,t}$ as a variable around which beliefs about risk are coordinated. The key parameter V_W is:

$$V_W = \frac{W}{\gamma K} \frac{R - \rho^2}{4\rho^2\sigma^2}$$

As $V_W \neq 0$, the perceived risk in this equilibrium is time-varying and depends on $A_{H,t}$:

$$var_t(Q_{W,t+1}) = (v_{WH} - 2V_W \rho A_{H,t})^2 \sigma^2 + v_{WF}^2 \sigma^2 + 2v_{WF}(v_{WH} - 2V_W \rho A_{H,t})\sigma^2 \rho_{HF} \tag{21}$$

The role of $A_{H,t}$ in coordinating beliefs about risk is entirely separate from its role as a fundamental. This can be seen by letting m go to zero. As $m = 0$, $A_{H,t}$ has no fundamental role and $v_{WH} = v_{WF} = 0$. But V_H does not depend on m, so the role of $A_{H,t}$ in driving time-varying perceptions of risk is unrelated to its fundamental role.

For illustrative purposes, Figure 3 shows both the fundamental and sunspot-like equilibria for a particular parameterization (at the bottom of Figure 3). It shows how the world equity price and risk depend on the Home fundamental $A_{H,t}$. Risk is defined as the standard deviation of $Q_{W,t+1}$ divided by $Q_{W,t}$. In the sunspot-like equilibrium the equity price and risk are clearly much more sensitive to $A_{H,t}$ (risk is constant in the fundamental equilibrium). The additional risk leads to a lower world equity price than under the fundamental equilibrium for all possible values of $A_{H,t}$. At the extreme values of $A_{H,t}$ (on either side) there are very high beliefs about risk and correspondingly low equity prices.

4.3 Switching Equilibria and Global Risk Panics

In a switching equilibrium, there are occasional self-fulfilling switches between low and a high risk states. We assume that the switch is driven by a Markov process. With probability p we remain in the same state and with probability $1 - p$ we switch to the other state.

BTW solve for such switching equilibria. Here we take a somewhat simpler approach, with the benefit of the findings in BTW. As p approaches 1, the low and high risk states approach respectively the fundamental and sunspot-like equilibria. Using this finding, we consider the low risk state to be the fundamental equilibrium and the high-risk state the sunspot-like equilibrium. This is infinitesimally close to the true equilibrium when p is very close to 1. While the probabilities of switching are then very small, we can still consider the impact of a switch. This approach has the advantage that we already know what the low and high risk states are. For larger switching probabilities (lower values of p) a solution can only be obtained numerically, but gives similar qualitative results.[9]

At the moment the risk panic happens there is an immediate increase in perceived risk, coordinated around $A_{H,t}$. In Figure 3 this is the jump in risk from the broken line (fundamental equilibrium) to the solid like (sunspot-like equilibrium). At the time of the panic the fundamental itself does not change. Rather, it suddenly takes on the additional role of a variable around which beliefs about risk are coordinated. As we can see from Figure 3, the increase in risk can be very large, particularly when the fundamental is either very weak or very strong. The latter case is not realistic, but BTW show that the possibility of a large risk panic when the fundamental is strong is specific to the particular process for the fundamental assumed here and does not hold for other types of processes.[10]

Figure 3 shows that the drop in the world equity price can be very large when the panic happens at a time that the fundamental is weak. This is also illustrated in Figure 4, which shows what happens to the world equity price and risk when the panic occurs at the time that the macro fundamental (Home dividend) is at its weakest, i.e., is equal to -0.1.[11] We assume that in period 6 the world economy switches to the high risk state, where it stays until period 10. Before and after that we are in the low risk state. The panic leads to a 48% drop in the world equity price. This is caused by an increase in world equity price risk from 0.4% to 14%. Of course, dependent on the parameters one can get even larger or smaller risk panics. For example, if we lower $\bar{Z}$ from 0.5 to 0.4, the panic leads to a 61%

[9]BTW solve for an exact analytical solution, but only for the case where the process for the fundamentals is Markov rather than an AR process. An earlier version of BTW numerically solves the solution for AR processes when $p < 1$.

[10]One can also avoid a large risk panic under a strong fundamental by making the switching probability itself a function of the fundamental.

[11]The lowest value of A_H is $-\sigma/(1-\rho)$, which is -0.1 for $\rho = 0.9$ and $\sigma = 0.01$.

drop in the world equity price and risk increases from 0.5% to 23%.[12]

5 The Impact of the Global Risk Panic on Individual Countries

The previous section described how global asset prices could collapse because of a risk panic. An important question is how the individual countries are affected by the panic. This is the question that we address in this section.

5.1 Country Prices and Covariance

While the world price is affected by world risk, individual country prices are affected by their covariance with the world price. We can solve equations (16) and (17) forward. Assuming no bubble, this gives:

$$Q_{H,t} = \sum_{i=1}^{\infty} \frac{E_t Z_{H,t+i}}{R^i} - \sum_{i=1}^{\infty} \frac{\gamma K/W}{R^i} E_t cov_{t+i-1}(Q_{H,t+i} + Z_{H,t+i}, Q_{W,t+i} + Z_{W,t+i}) \tag{22}$$

$$Q_{F,t} = \sum_{i=1}^{\infty} \frac{E_t Z_{F,t+i}}{R^i} - \sum_{i=1}^{\infty} \frac{\gamma K/W}{R^i} E_t cov_{t+i-1}(Q_{F,t+i} + Z_{F,t+i}, Q_{W,t+i} + Z_{W,t+i}) \tag{23}$$

The first term on the right hand side of both equations is the present discounted value of dividends. The second term captures the impact of risk on asset prices. $Q_{H,t}$ and $Q_{F,t}$ depend on the present discounted value of the covariance of asset payoffs with the payoff on a global equity claim.

To determine the impact of a risk panic on $Q_{H,t}$ and $Q_{F,t}$, we therefore need to determine the impact on the expected covariances. But the impact on expected covariances themselves depend on the impact of a risk panic on future $Q_{H,t}$ and $Q_{F,t}$. This gives us a loop that leads to multiplicity of equilibria.

We consider the following solutions for $Q_{H,t}$ and $Q_{F,t}$:

$$Q_{H,t} = \tilde{Q}_H + v_{HH} A_{H,t} + v_{HF} A_{F,t} - V_H A_{H,t}^2 \tag{24}$$

[12] While our aim here is certainly not to draw precise quantitative comparisons to recent risk panic episodes, we should point out that the numbers that we reported in Figure 1 for the VIX cannot be directly compared to those reported here for risk. For example, the VIX numbers are risk measures that are multiplied by the square root of 12 in order to annualize them. Therefore a VIX of 80 implies that equity price risk over the next month is 23%.

$$Q_{F,t} \quad = \quad \tilde{Q}_F + v_{FH}A_{H,t} + v_{FF}A_{F,t} - V_F A_{H,t}^2 \tag{25}$$

As in the previous section, we assume that only the Home dividend $A_{H,t}$ can coordinate self-fulfilling shifts in risk.

To determine the parameters in (24) and (25) we take the following steps. First, we conjecture (24) and (25). Then we compute the expectation of $Q_{H,t+1} + Z_{H,t+1}$ and $Q_{F,t+1} + Z_{F,t+1}$ and their covariance with $Q_{W,t+1} + Z_{W,t+1}$. Substituting the result in (16) and (17), we can solve for the 8 parameters in the conjecture (24) and (25). Details of the algebra are left to Appendix B. We first discuss the fundamental equilibrium and then turn to sunspot-like equilibria and switching equilibria featuring risk panics.

In the fundamental equilibrium we have $V_H = V_F = 0$, $v_{HH} = v_{FF} = m\rho/(R - \rho)$ and $v_{HF} = v_{FH} = 0$. The expressions for the equity prices of the two countries then become

$$Q_{H,t} = \frac{1}{R-1}\left(\bar{Z} - \left(\frac{mR}{R-\rho}\right)^2 \frac{\gamma K}{W}\sigma_{HW}\right) + \frac{m\rho}{R-\rho}A_{H,t} \tag{26}$$

$$Q_{F,t} = \frac{1}{R-1}\left(\bar{Z} - \left(\frac{mR}{R-\rho}\right)^2 \frac{\gamma K}{W}\sigma_{FW}\right) + \frac{m\rho}{R-\rho}A_{F,t} \tag{27}$$

where $\sigma_{iW} = cov(\epsilon_{i,t+1}, \epsilon_{W,t+1})$, $i = H, F$, is the covariance between the Home (Foreign) and world dividend innovation. The latter is defined as $\epsilon_{W,t+1} = n\epsilon_{H,t+1} + (1-n)\epsilon_{F,t+1}$. Two points are worth making with regards to this fundamental equilibrium. First, the constant terms depend on the covariance of the dividend innovation with the world dividend innovation. The higher is this covariance, the riskier the asset and therefore the lower the price. Second, equity prices only depend on domestic dividend innovations. Thus, there is no contagion of shocks across countries. This is because we have shut down the regular channels of contagion through the interest rate and wealth. Both are held constant.

Sunspot-like equilibria are described in detail in Appendix B. The impact of the Foreign fundamental $A_{F,t}$ remains the same as in the fundamental equilibrium since the Foreign dividend only plays a pure fundamental role (by assumption). The impact of the Home fundamental $A_{H,t}$ is more complex as it coordinates beliefs about risk in the sunspot-like equilibrium. From the global equity price solution we know that

$$nV_H + (1-n)V_F = \frac{W}{\gamma K}\frac{R-\rho^2}{4\rho^2\sigma^2} \tag{28}$$

However, Appendix B shows that V_H and V_F are not uniquely detetermined. Therefore the change in asset prices and risk during a risk panic is not uniquely determined. There is a continuum of equilibria for different values of V_H and V_F. The only restrictions are that their weighted average corresponds to (28) and that asset prices must be positive for all possible values of the state variables.

This multiplicity is related to the circular relationship between the stochastic processes of the asset prices and the covariance of asset payoffs with the world payoff. Focusing on the quadratic terms of the asset price solution, consider the case where V_H is high and V_F is low. The Home asset price then depends a lot on the quadratic sunspot term (leading to a large price impact during a risk panic), while the Foreign asset price depends little on the quadratic term (small effect of a risk panic).

In this case the covariance of $Q_{H,t+1}+Z_{H,t+1}$ and $Q_{W,t+1}+Z_{W,t+1}$ has a quadratic term that is equal to $4\rho^2\sigma^2 V_H V_W A_{Ht}^2$. When V_H is large, risk will depend very positively on the quadratic term. This in turn implies that the asset price Q_{Ht} will depend very negatively on the quadratic term, which is where we started from. The belief that the Home asset price depends a lot on the quadratic term then becomes self-fulfilling. Similarly, the belief that the Foreign asset price depends little on the quadratic term becomes self-fulfilling as well.

It turns out that the extent to which this indeterminacy affects asset prices and risk depends critically on the fundamental role of macro variables. We first consider the case where the macro variables $A_{H,t}$ and $A_{F,t}$ have a large fundamental role (m is large) and then the case where they play a small fundamental role (m close to 0).

5.2 Large Fundamental Role of Macro Variables

In the case where m is much above zero, so that the macro variables have a large fundamental role, we obtain two results. First, the impact of the indeterminacy on the equilibrium prices is limited. Second, the fundamental hedging properties of the assets are such that the panic affects the Home country more than the Foreign country when the panic happens during a weak Home fundamental.

These two results are illustrated in Figure 5 for the same parameterization as used in Figures 3 and 4. The assumption $m = 1$ implies that the macro variables play an important fundamental role. Figure 5 shows the impact of a risk panic on the equity prices and risk of both countries. As in Figure 4, the panic is assumed to take place when the fundamental is at its weakest ($A_H = -0.1$). Risk is again measured as the standard deviation of the equity price over the next period, divided by its current price. The results are shown as a function of $V_F - V_H$, which is indeterminate. All values are considered for which the Home and Foreign equity prices are positive for all possible realizations of the state space $(A_{H,t}, A_{F,t})$. Clearly, the Figure shows that the indeterminacy associated with $V_F - V_H$ has very little impact on the outcome and that the panic has a much larger impact on the Home country.

The intuition for the limited impact of indeterminacy is somewhat complex. When discussing the indeterminacy above we focused entirely on the quadratic term in A_{Ht} in the covariance between the Home asset payoff and the global asset payoff. But the linear term matters as well. In particular, consider the linear term $v_{WH}A_{H,t+1}$ in the global asset price at $t+1$. The Appendix shows that v_{WH} is negative and proportional to m. The covariance of this term with the quadratic term in the Home asset price at $t+1$ is $-2\rho\sigma^2 v_{WH}V_H A_{Ht}$. If V_H is large, then this is a very positive function of A_{Ht}. This by itself reduces risk during a panic when the panic happens at the time the fundamental is bad (A_{Ht} very negative). This offsets the large increase in risk associated with the quadratic term in the covariance expression. The offset only applies when m is large as v_{WH} is proportional to m.

The intuition for the larger impact of the panic on the Home than the Foreign asset price is a bit more straightforward. It is the result of the hedging property of the assets against risk panics. In particular consider the covariance between the dividend on the Home asset and the quadratic component of the global asset price. This is $-2\rho\sigma^2 mV_W A_{Ht}$, which is very large when the fundamental is weak. Risk on the Home asset therefore rises a lot during the panic. This is because the Home dividend $A_{H,t+1}$ is very positively correlated with $-V_W A_{H,t+1}^2$ when the fundamental is weak. The correlation is smaller for the Foreign asset as long as $\rho_{HF} < 1$.

Not surprisingly, this difference in hedging properties becomes small when ρ_{HF} is close to 1. This is illustrated in Figure 6, which is the same as Figure 5 except that ρ_{HF} is set equal to 0.99 rather than 0.5. In that case the hedging properties of the Home and Foreign dividends are virtually identical and the panic affects the covariance with the global payoff virtually the same across the two assets.

The size of the country that is the focal point for the panic does not matter much for this hedging property. Even if the Home country is small relative to the global economy, it remains the case that the risk panic is much larger in the Home country. For example, setting $n = 0.1$ and $V_H = V_F$, risk increases 55% and 12% in the Home and Foreign country respectively, while their respective equity prices drop by 80% and 46%.

5.3 Weak Fundamental Role of Macro Variables

Next consider the case where m is close to zero. The macro variables $A_{H,t}$ and $A_{F,t}$ then play a very limited fundamental role. This should be interpreted more generally as the case where macro variables have limited impact on payoffs of the assets in normal (non-panic) times. This is not an unreasonable assumption as it is well known that observed macro variables have limited explanatory power for asset prices outside of crisis episodes.[13]

When the macro variables have a weak impact on dividends (m close to 0), the fundamental hedging properties of both assets are weak. The impact of a risk panic on individual asset prices is then primarily driven by indeterminacy. When $m = 0$ the asset prices in the sunspot-like equilibrium are

$$Q_{Ht} = \frac{1}{R-1}\left(\bar{Z} - V_H\sigma^2\right) - V_H A_{Ht}^2 \tag{29}$$

$$Q_{Ft} = \frac{1}{R-1}\left(\bar{Z} - V_F\sigma^2\right) - V_F A_{Ht}^2 \tag{30}$$

There is now only a quadratic term, which captures the impact of a risk panic. As we vary V_H and V_F, keeping their weighted average $nV_H + (1-n)V_F$ unchanged, we can significantly vary the impact of a risk panic on the two asset prices.

This is illustrated in Figure 7, which has the same parameterization as for Figure 5 except that $m = 0$. The panic affects the two countries equally when $V_H = V_F$. But the indeterminacy associated with $V_F - V_H$ now has a big impact on how the panic affects the two countries. There is now a wide range of possible

[13]For equity prices the limited role of public news was first illustrated by Roll (1988). For another asset price, the exchange rate, this disconnect from observed macro fundamentals has received even more attention. In addition, since the early work by Shiller (1981) it is well known that dividend volatility has very limited explanatory power for equity price volatility.

ways in which the global panic may be divided among the two countries. The panic can affect asset prices and risk equally across the two countries or alternatively it could affect one country significantly more than another. The large range of indeterminacy when m is small implies that the model has little predictive power regarding the extent to which individual countries are affected by a panic. This is in line with empirical evidence. Spiegel and Rose (2010) and Kamin and Pounder (2010) have shown that there is little connection between a broad array of macro variables and the extent to which countries have been affected by the 2008 panic.

6 Connection to Recent Crisis Episodes

How well do these results connect to the three recent risk panic episodes discussed in the introduction? We discussed five features related to these episodes. The first is the global nature of the risk panics. This is consistent with the results for both $m = 1$ and $m = 0$. Only for extreme values of $V_F - V_H$ in Figure 7 is the spike in risk relatively small in one of the countries, but still not zero.

The second feature is that the extent to which countries are affected by these risk panics varies substantially. This is again consistent both with $m = 1$ and $m = 0$. In the former case the relative impact on the two countries depends on the hedging properties of the assets, while in the latter case one country may be much less affected than another in a purely self-fulfilling way.

A third aspect of recent risk panic episodes is that the cross-country impact varies across episodes. Theoretically this may be consistent with both $m = 1$ and $m = 0$. The hedging properties of the assets may differ across episodes depending on the macro fundamental that becomes the focal point of a panic. But it appears to be even more consistent with the case where $m = 0$, where the relative impact of countries is entirely driven by self-fulfilling beliefs. There is no reason why these beliefs should be the same across different episodes.

A fourth feature is that the risk panic episodes do not appear to be clearly driven by a large change in macro fundamentals. This is also consistent with our results, where the shift to the high risk state during a risk panic does not involve a change in fundamentals. Once we are in the high risk state asset prices and risk become very sensitive to the fundamental that is the focal point of fear in the market. This is consistent with recent episodes as well. For example, the VIX has fluctuated widely based on any news about Greek bailout packages during the European sovereign debt crisis.

The final stylized fact is that there is little relation between cross-border financial linkages and the extent to which countries are affected. This is relatively easy to explain in our framework. Consider a financial friction τ that reduces the excess payoff on equity from the point of view of investors from the other country. In other words, the excess payoff $r_{H,t+1}$ on Home equity becomes $r_{H,t+1} - \tau$ from the perspective of Foreign investors. Similarly, the excess payoff $r_{F,t+1}$ on Foreign equity becomes $r_{F,t+1} - \tau$ for Home investors. Such frictions have been introduced in many papers and are usually interpreted as a shorthand for a wide range of possible frictions that inhibit cross-border asset holdings.

While a larger τ reduces cross-border asset holdings, it has no effect on the magnitude of the global panic and how much it affects the two countries. The friction τ only affects the constant term in the equity price solutions. For both the fundamental and sunspot-like equilibria it changes the constant terms of the Home and Foreign asset prices by respectively $-(1-n)\tau/(R-1)$ and $-n\tau/(R-1)$. Our model therefore does not necessarily generate any relationship between cross-border asset holdings and the extent to which countries are affected by the panic.

More fundamentally, the debate about the role of cross-border asset holdings is relevant when financial contagion is due to the transmission of shocks. Transmission can be larger if cross-border financial linkages are stronger. But in our model the impact of a panic on individual countries does not occur through the transmission of shocks, so that the magnitude of financial linkages is not a determining factor. Rather, what coordinates the panic across countries is an event that suddenly draws attention of investors all over the world to a weak fundamental somewhere. This weak fundamental, by becoming a common focal point of attention by investors everywhere, leads to a widespread self-fulfilling increase in risk perceptions.

7 Conclusion

The paper is motivated by the sharp increases in equity price risk across many countries during several episodes from 2008 to 2011. We have developed a model with self-fulfilling shifts in risk that is consistent with many of the key features of these recent risk panic events. This topic deserves a lot more attention in future research. Not much work has been done in macroeconomics in understanding what drives such enormous changes in risk, let alone their global nature.

Appendix

A Solution World Equity Price

In this Appendix we derive the solution for the world equity price in Section 4. Start with the conjecture (19)

$$Q_{W,t} = \tilde{Q}_W + v_{WH}A_{H,t} + v_{WF}A_{F,t} - V_W A_{H,t}^2 \tag{31}$$

Note that this also includes the fundamental equilibrium as a special case, where $V_{WH} = 0$. We need to compute the expectation and variance of $Q_{W,t+1} + Z_{W,t+1}$. We have

$$\begin{aligned}
&Q_{W,t+1} + Z_{W,t+1} = \\
&\tilde{Q}_W + \bar{Z} + (v_{WH} + nm)A_{H,t+1} + (v_{WF} + (1-n)m)A_{F,t+1} - V_W A_{H,t+1}^2 = \\
&\tilde{Q}_W + \bar{Z} - V_W\rho^2 A_{H,t}^2 + (v_{WH} + nm)\rho A_{H,t} + (v_{WF} + (1-n)m)\rho A_{F,t} + \\
&(v_{WH} + nm - 2V_W\rho A_{H,t})\epsilon_{H,t+1} + (v_{WF} + (1-n)m)\epsilon_{F,t+1} - V_W\epsilon_{H,t+1}^2
\end{aligned} \tag{32}$$

It follows that

$$\begin{aligned}
&E_t(Q_{W,t+1} + Z_{W,t+1}) = \\
&\tilde{Q}_W + \bar{Z} - V_W\rho^2 A_{H,t}^2 + (v_{WH} + nm)\rho A_{H,t} + (v_{WF} + (1-n)m)\rho A_{F,t} - V_W\sigma^2
\end{aligned} \tag{33}$$

and

$$\begin{aligned}
&var_t(Q_{W,t+1} + Z_{W,t+1}) = \\
&(v_{WH} + nm - 2V_W\rho A_{H,t})^2\sigma^2 + (v_{WF} + (1-n)m)^2\sigma^2 + \\
&2(v_{WH} + nm - 2V_W\rho A_{H,t})(v_{WF} + (1-n)m)\rho_{HF}\sigma^2
\end{aligned} \tag{34}$$

Substituting these results into (18), we have

$$\begin{aligned}
&\tilde{Q}_W + \bar{Z} - V_W\rho^2 A_{H,t}^2 + (v_{WH} + nm)\rho A_{H,t} + (v_{WF} + (1-n)m)\rho A_{F,t} - V_W\sigma^2 \\
&-R\tilde{Q}_W - Rv_{WH}A_{H,t} - Rv_{WF}A_{F,t} + RV_W A_{H,t}^2 = \\
&\frac{\gamma K}{W}(v_{WH} + nm - 2V_W\rho A_{H,t})^2\sigma^2 + \frac{\gamma K}{W}(v_{WF} + (1-n)m)^2\sigma^2 + \\
&\frac{\gamma K}{W}2(v_{WH} + nm - 2V_W\rho A_{H,t})(v_{WF} + (1-n)m)\rho_{HF}\sigma^2
\end{aligned} \tag{35}$$

Collecting first terms proportional to $A_{H,t}^2$ and equating coefficients on the left and right hand side, we have

$$V_W(R-\rho^2) = \frac{\gamma K}{W} 4V_W^2 \rho^2 \sigma^2 \tag{36}$$

This has two solutions: $V_W = 0$ (the fundamental equilibrium) and

$$V_W = \frac{W}{\gamma K} \frac{R-\rho^2}{4\rho^2\sigma^2} \tag{37}$$

Next collect terms proportional to $A_{F,t}$ and equate coefficients on the left and right hand side. This gives

$$v_{WF} = \frac{(1-n)m\rho}{R-\rho} \tag{38}$$

This solution is the same in the fundamental and sunspot-like equilibrium.

Next collect terms proportional to $A_{H,t}$ and equate coefficients on the left and right hand sides. This gives

$$v_{WH} = \frac{nm\rho + 4\frac{\gamma K}{W} V_W \rho \left[nm\sigma^2 + (v_{WF} + (1-n)m)\rho_{HF}\sigma^2\right]}{R - \rho - 4\frac{\gamma K}{W} V_W \rho \sigma^2} \tag{39}$$

Substituting the expressions for V_W and v_{WF}, in the fundamental equilibrium we have

$$v_{WH} = \frac{nm\rho}{R-\rho} \tag{40}$$

while in the sunspot-like equilibrium we have

$$v_{WH} = -\frac{m}{1-\rho}\left(n + \frac{R-\rho^2}{R-\rho}\rho_{HF}(1-n)\right) \tag{41}$$

Finally equating the constant terms on both sides, we have

$$\tilde{Q}_W = \frac{1}{R-1}\left(\bar{Z} - V_W\sigma^2 - \frac{\gamma K}{W} v'\Sigma v\right) \tag{42}$$

where Σ is the variance-covariance matrix of $(\epsilon_{H,t+1}, \epsilon_{F,t+1})'$ and $v = (v_{WH} + nm, v_{WF} + (1-n)m)'$.

B Solution Individual Equity Prices

We will derive the equilibrium Home equity price from the market clearing condition (16). The equilibrium Foreign equity price is derived analogously from (17). Using the conjecture (24) for the Home equity price, we have

$$\begin{aligned}
&Q_{H,t+1} + Z_{H,t+1} = \\
&\tilde{Q}_H + \bar{Z} + (v_{HH} + m)A_{H,t+1} + v_{HF}A_{F,t+1} - V_H A^2_{H,t+1} = \\
&\tilde{Q}_H + \bar{Z} - V_H\rho^2 A^2_{H,t} + (v_{HH} + m)\rho A_{H,t} + v_{HF}\rho A_{F,t} + \\
&(v_{HH} + m - 2V_H\rho A_{H,t})\epsilon_{H,t+1} + v_{HF}\epsilon_{F,t+1} - V_H\sigma^2
\end{aligned} \tag{43}$$

It follows that

$$\begin{aligned}
&E_t(Q_{H,t+1} + Z_{H,t+1}) = \\
&\tilde{Q}_H + \bar{Z} - V_H\rho^2 A^2_{H,t} + (v_{HH} + m)\rho A_{H,t} + v_{HF}\rho A_{F,t} - V_H\sigma^2
\end{aligned} \tag{44}$$

and

$$\begin{aligned}
&cov_t(Q_{H,t+1} + Z_{H,t+1}, Q_{W,t+1} + Z_{W,t+1}) = \\
&(v_{WH} + nm - 2V_W\rho A_{H,t})(v_{HH} + m - 2V_H\rho A_{H,t})\sigma^2 + \\
&(v_{WF} + (1-n)m)v_{HF}\sigma^2 + (v_{WH} + nm - 2V_W\rho A_{H,t})v_{HF}\rho_{HF}\sigma^2 + \\
&(v_{WF} + (1-n)m)(v_{HH} + m - 2V_H\rho A_{H,t})\rho_{HF}\sigma^2
\end{aligned} \tag{45}$$

(16) then becomes

$$\begin{aligned}
&\tilde{Q}_H + \bar{Z} - V_H\rho^2 A^2_{H,t} + (v_{HH} + m)\rho A_{H,t} + v_{HF}\rho A_{F,t} - V_H\sigma^2 \\
&-R\tilde{Q}_H - Rv_{HH}A_{H,t} - Rv_{HF}A_{F,t} + RV_H A^2_{H,t} = \\
&\frac{\gamma K}{W}(v_{WH} + nm - 2V_W\rho A_{H,t})(v_{HH} + m - 2V_H\rho A_{H,t})\sigma^2 + \\
&\frac{\gamma K}{W}(v_{WF} + (1-n)m)v_{HF}\sigma^2 + \\
&\frac{\gamma K}{W}(v_{WH} + nm - 2V_W\rho A_{H,t})v_{HF}\rho_{HF}\sigma^2 + \\
&\frac{\gamma K}{W}(v_{WF} + (1-n)m)(v_{HH} + m - 2V_H\rho A_{H,t})\rho_{HF}\sigma^2
\end{aligned} \tag{46}$$

First collecting terms proportional to $A^2_{H,t}$, we have

$$V_H(R - \rho^2) = 4\frac{\gamma K}{W}V_W V_H\rho^2\sigma^2 \tag{47}$$

In the fundamental equilibrium, where $V_W = 0$, it follows that $V_H = 0$. After substituting (37), it follows that (47) holds for all values of V_H in the sunspot-like equilibrium. V_H is therefore indeterminate. It can take on any value, subject of course to the equilibrium being well defined in that the asset prices are non-negative for all values of the state variables.

Collecting terms proportional to $A_{F,t}$ gives $v_{HF} = 0$. This holds both in the fundamental and sunspot-like equilibrium. Collecting terms proportional to $A_{H,t}$, we have

$$v_{HH} = \frac{m\rho + 2\rho\frac{\gamma K}{W}(\eta V_H + m\sigma^2 V_W)}{R - \rho - 2\frac{\gamma K}{W}V_W\rho\sigma^2} \tag{48}$$

where

$$\eta = (v_{WH} + nm)\sigma^2 + (v_{WF} + (1-n)m)\rho_{HF}\sigma^2 \tag{49}$$

In the fundamental equilibrium where $V_W = V_H = 0$, it follows that

$$v_{HH} = \frac{m\rho}{R - \rho} \tag{50}$$

In the sunspot-like equilibrium v_{HH} depends on V_H, which is indeterminate.

Finally, collecting constant terms, we have

$$\tilde{Q}_H = \frac{1}{R-1}\left(\bar{Z} - V_H\sigma^2 - \frac{\gamma K}{W}v_1'\Sigma v_2\right) \tag{51}$$

where $v_1 = (v_{WH} + nm, v_{WF} + (1-n)m)'$ and $v_2 = (v_{HH} + m, 0)'$.

Following the same steps for the Foreign market clearing condition (17), in the fundamental equilibrium $V_F = v_{FH} = 0$ and

$$v_{FF} = \frac{m\rho}{R - \rho} \tag{52}$$

In the sunspot-like equilibrium the quadratic coefficient V_F can take on any value, just like for the Home equity price. The only constraints therefore are that $nV_H + (1-n)V_F = V_W$, which is given by (37), and that both asset prices are non-negative for all values of the state variables. v_{FF} takes the same value in the sunspot-like equilibrium as in the fundamental equilibrium, while

$$v_{FH} = 2\rho\frac{\gamma K}{W}\frac{(\eta V_F + (v_{FF} + m)\sigma^2\rho_{HF}V_W)}{R - \rho - 2\frac{\gamma K}{W}V_W\rho\sigma^2} \tag{53}$$

where η is as defined (49). The constant term (in both fundamental and sunspot-like equilibria) is

$$\tilde{Q}_F = \frac{1}{R-1}\left(\bar{Z} - V_F\sigma^2 - \frac{\gamma K}{W} v_1' \Sigma v_2\right) \tag{54}$$

where $v_1 = (v_{WH} + nm, v_{WF} + (1-n)m)'$ and $v_2 = (v_{FH}, v_{FF} + m)'$.

C Implied Volatility Indices

Country	Source
Belgium	Datastream
	Full name: BEL 20 Volatility
	Computed from: BEL 20 Index options, 1 month
Canada	Montréal Exchange
	Full name: MX Implied Volatility Index
	Computed from: CDN S&P/TSX60 Fund, 1 month
France	Datastream
	Full name: CAC 40 Volatility
	Computed from: CAC 40 Index options, 1 month
Germany	Datastream
	Full name: VDAX - NEW
	Computed from: DAX Index options traded at Eurex, 1 month
India	National Stock Exchange of India
	Full name: India VIX
	Computed from: NIFTY Index options, 1 month
Japan	CSFI, University of Osaka
	Full name: CSFI - VXJ
	Computed from: Nikkei 225 Index options, 1 month
Mexico	Mexican Derivatives Exchange
	Full name: VIMEX
	Computed from: IPC options traded at MexDer, 3 months
Netherlands	Datastream
	Full name: AEX Volatility
	Computed from: AEX Index options, 1 month
South Africa	Johannesburg Stock Exchange
	Full name: SAVI Top40
	Computed from: FTSE/JSE Top40 Index options, 3 months
South Korea	Korea Exchange
	Full name: VKOSPI
	Computed from: KOSPI200 Index options, 1 month
Switzerland	Swiss Exchange
	Full name: VSMI
	Computed from: SMI options traded at Eurex, 1 month
U.S.A.	Datastream
	Full name: CBOE - VIX
	Computed from: S&P 500 index options, 1 month

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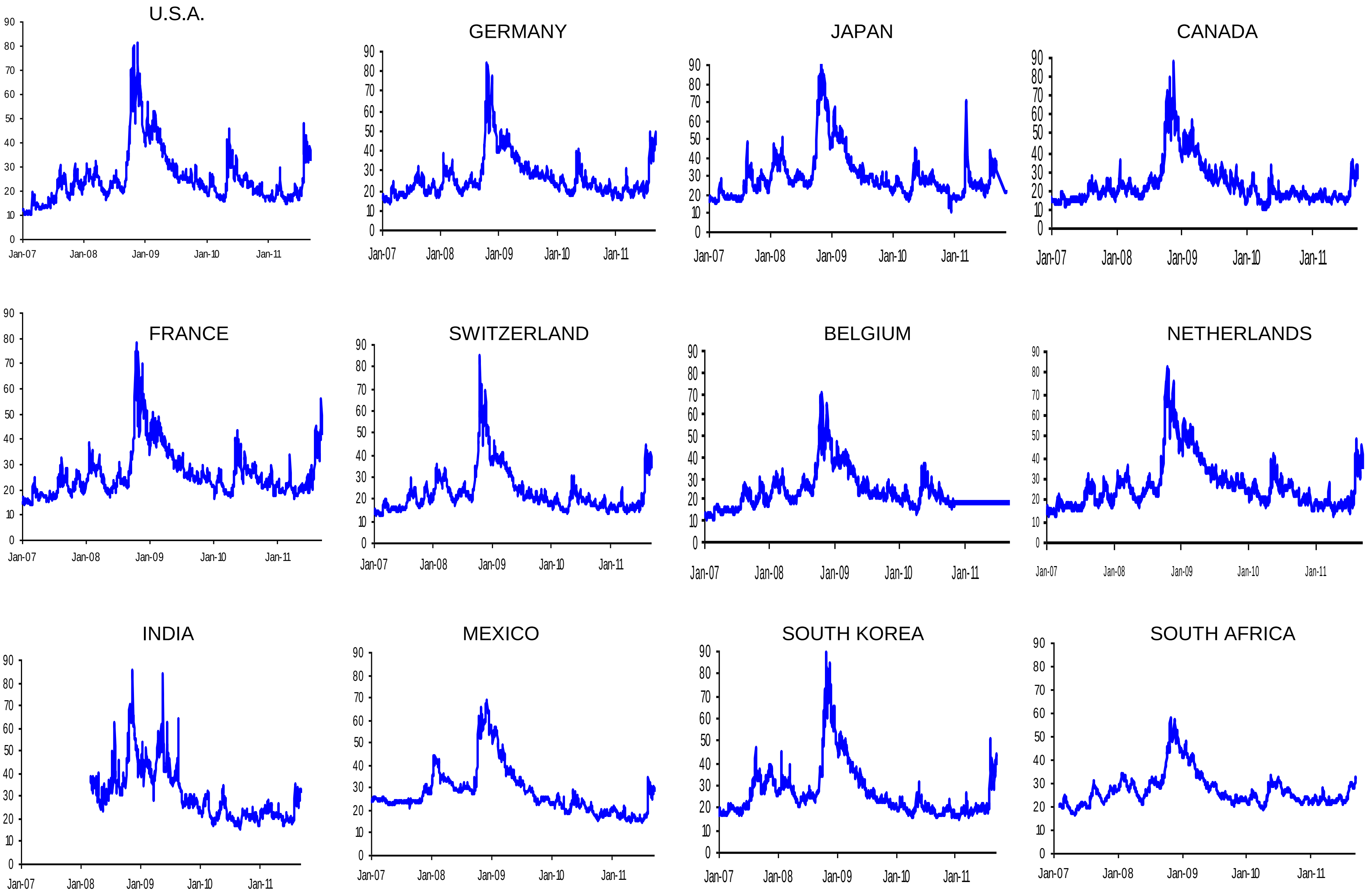
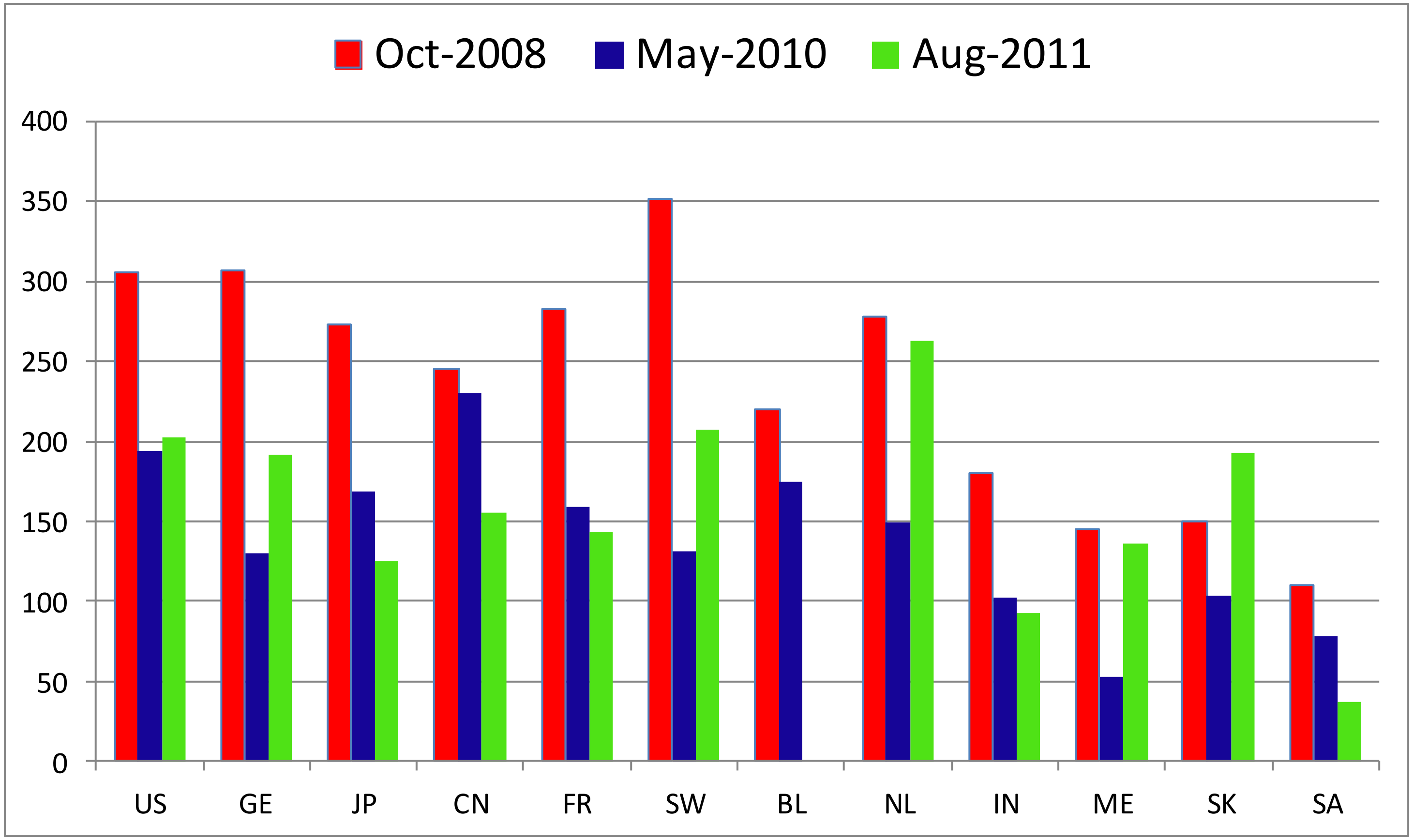
Figure 1: Implied Volatility Indices



Source: Datastream and local stock markets.

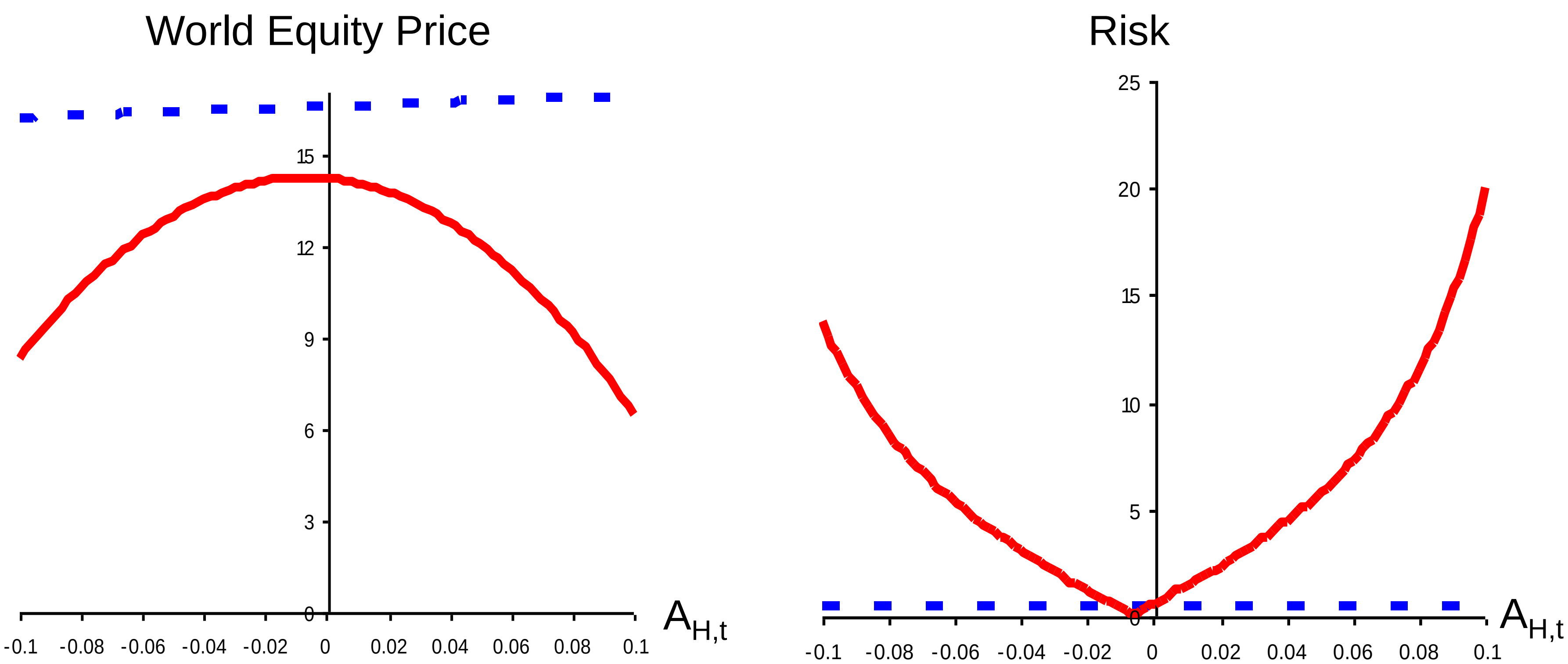
Figure 2 Spikes in Implied Volatility Indices Relative to US*



* Percentage increase in Implied Volatility indices during the panics of October 2008, May 2010 and August 2011. Percentage changes are from the lowest level in the two prior months to the peak level itself. The countries are listed in the same order as in Figure 1.

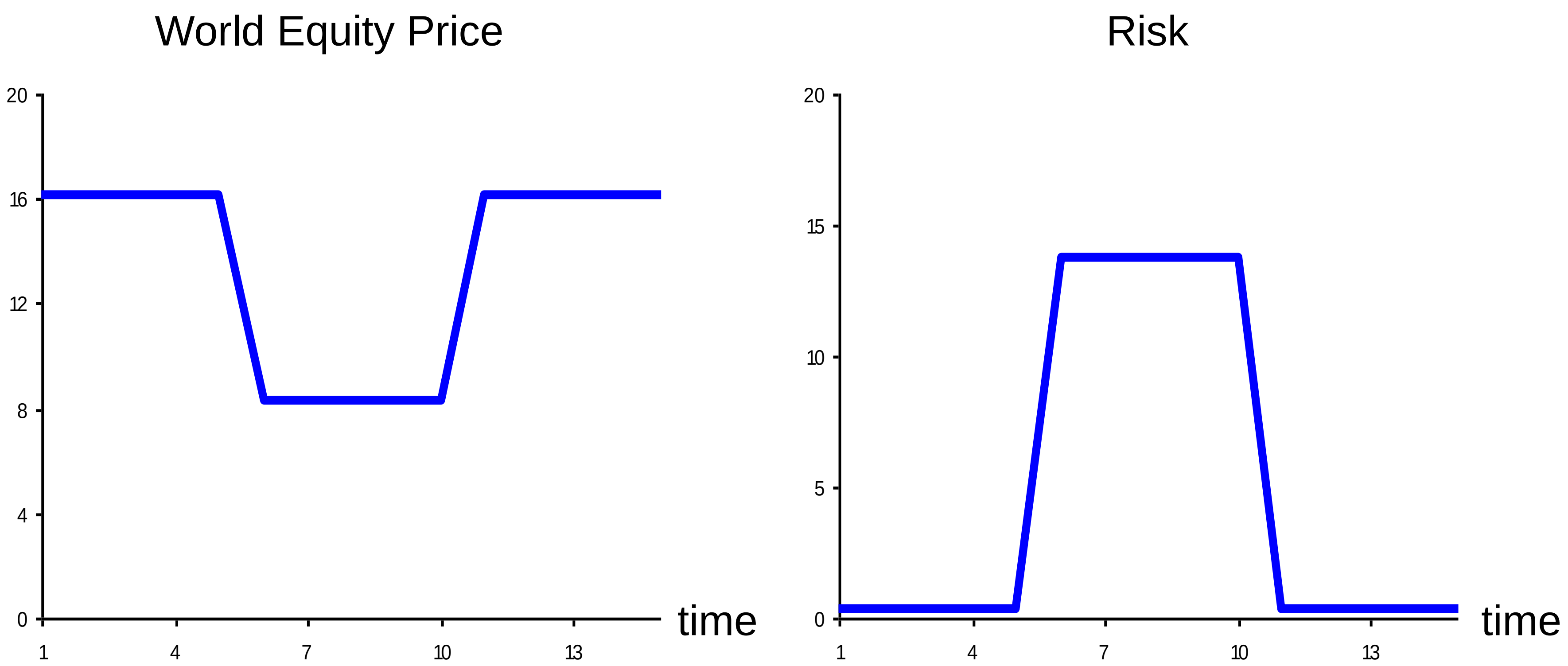
Figure 3 World Equity Price and Risk*

solid = sunspot-like equilibrium; broken = fundamental equilibrium



$\bar{Z}^* = 0.5; \sigma = 0.01; \rho_{HF} = 0.5; \rho = 0.9; \gamma = 1; R = 1.03; n = 0.5; K/W = 1; m = 1$

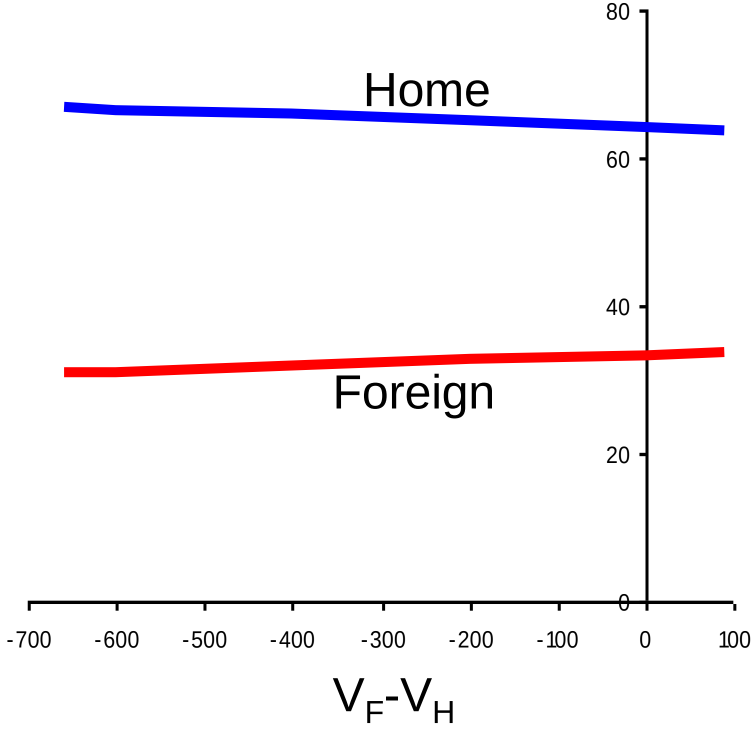
Figure 4 Global Risk Panic*



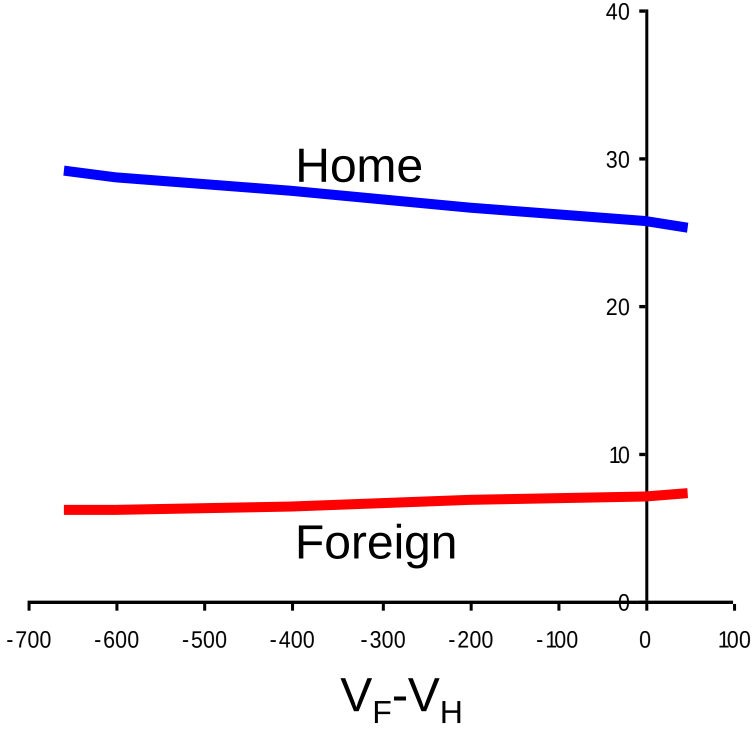
* Same parameterization as Figure 3. Risk panic occurs at $A_{H,t}$=-0.1.

Figure 5 Impact Risk Panic Across Countries—m=1 *

% Drop Equity prices During Panic

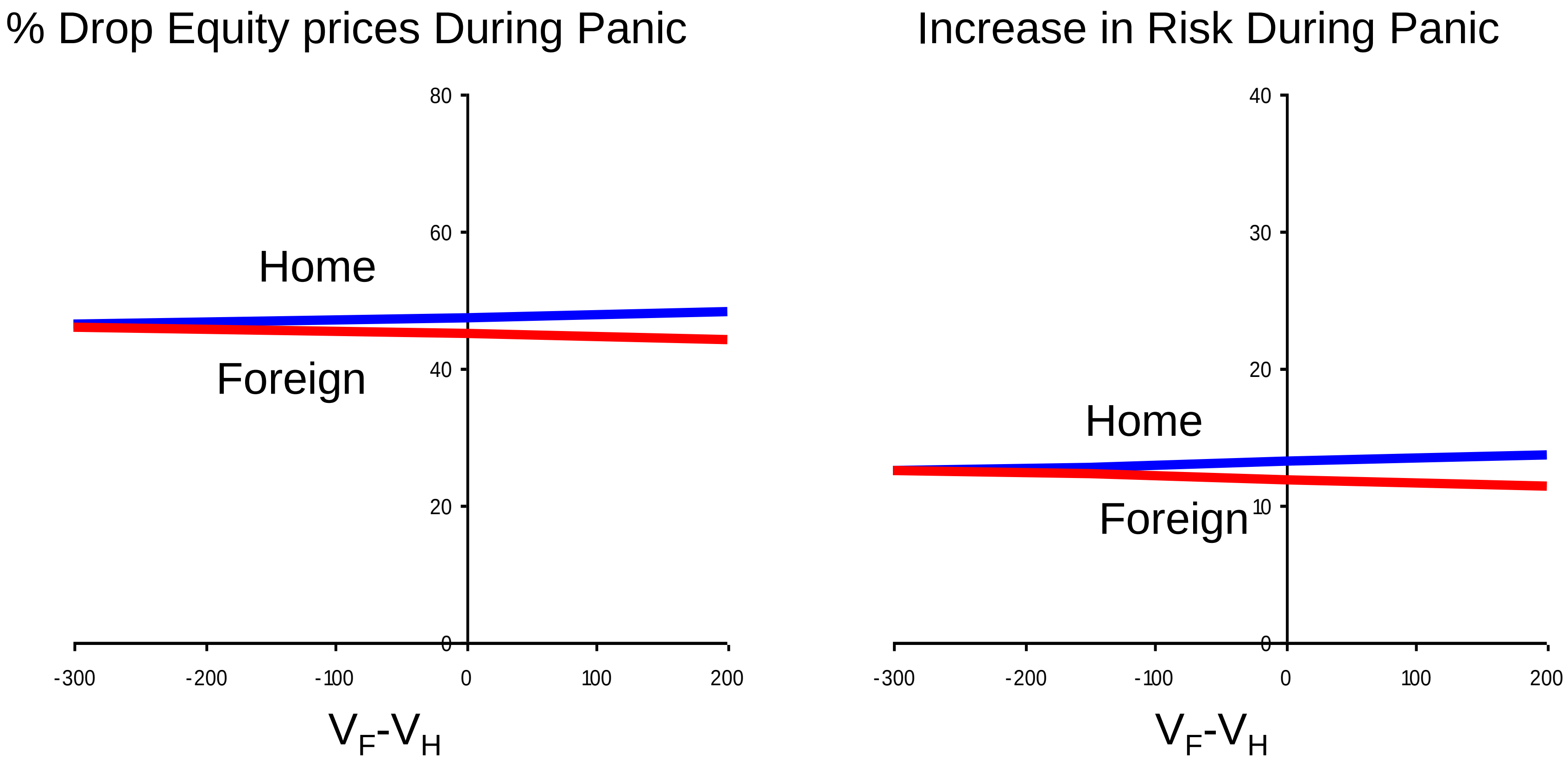


Increase in Risk During Panic



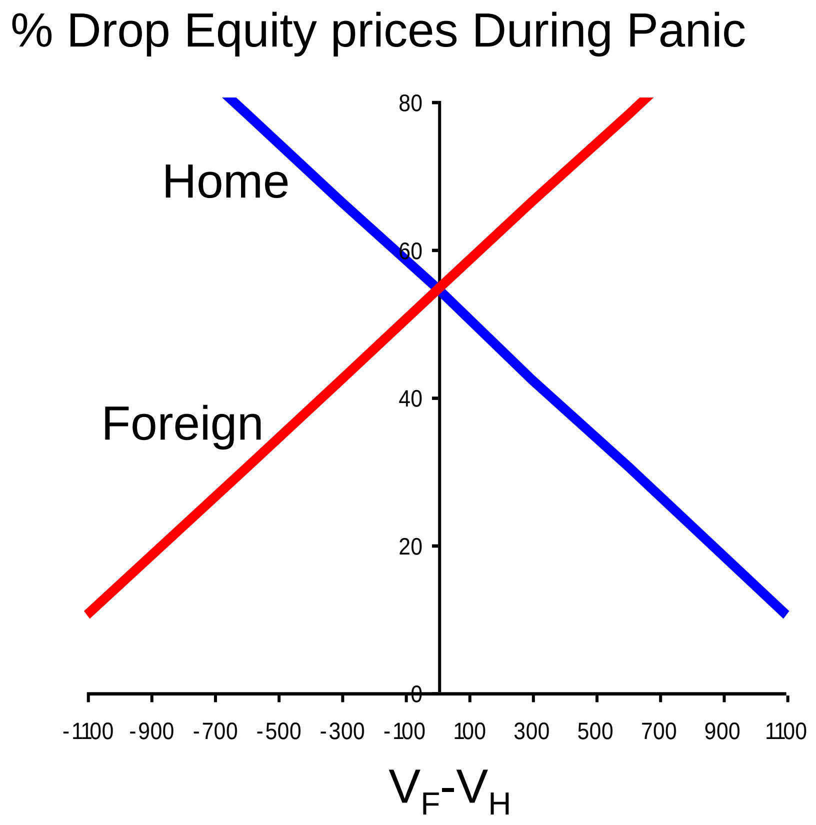
* The parameterization is the same as in Figure 3. The risk panic occurs at $A_{H,t}$=-0.1.

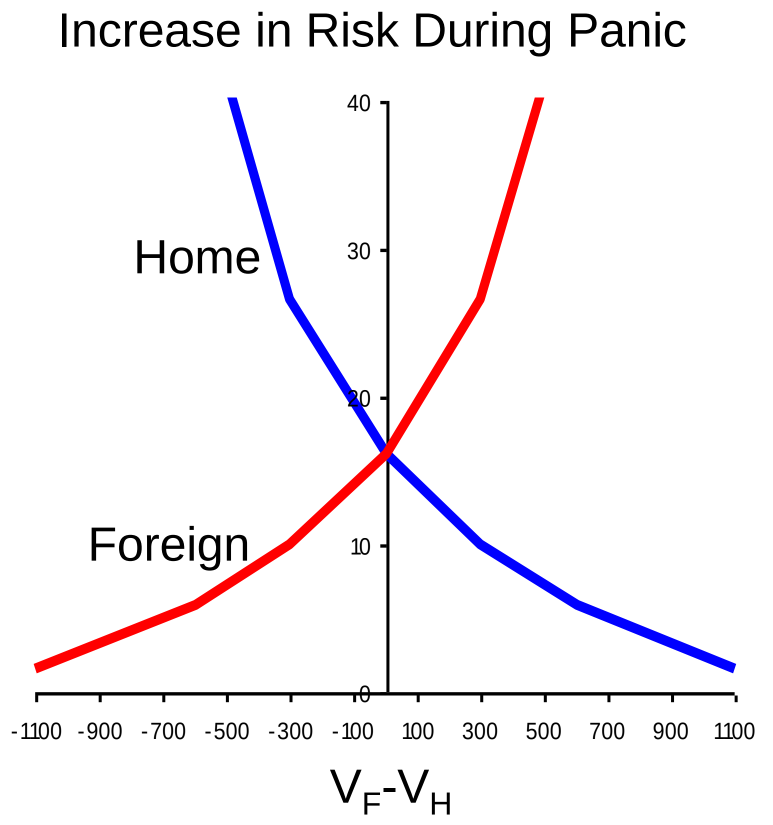
Figure 6 Impact Risk Panic Across Countries— ρ_{HF} =0.99*



* Other than ρ_{HF}=0.99, the parameterization is the same as in Figure 3. The risk panic occurs at $A_{H,t}$=-0.1.

Figure 7 Impact Risk Panic Across Countries—m=0*





* Other than m=0 the parameterization is the same as in Figure 3. The risk panic occurs at $A_{H,t}$=-0.1.