

Interactions of Commitment and Discretion in Monetary and Fiscal Policies

by

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Appendix

A Microfounded model

We consider a one-country general equilibrium monetary model as in Oliver J. Blanchard and Nobuhiro Kiyotaki (1987). There are N goods in the economy, which are imperfect substitutes, and money. Each good is produced by a producer who acts as a monopolistic competitor facing a downward sloping demand curve and chooses the nominal price and the level of production of her good. Production makes only use of labor and, since labor supply is elastic, production is endogenously determined. Each producer is also a consumer, who derives utility from the consumption of all goods and real money balances but derives disutility from the effort put in production. Producer-consumer (producer for short) j has the following period utility function

$$U_j = \left(\frac{C_j}{\gamma}\right)^\gamma \left(\frac{M_j/P}{1-\gamma}\right)^{1-\gamma} - \left(\frac{d}{\beta}\right) Y_j^\beta, \quad \gamma \in (0, 1), \quad d > 0, \quad \beta \geq 1, \quad (\text{A.1})$$

where the variable C_j is a real consumption index

$$C_j = N^{\frac{1}{1-\theta}} \left[\sum_{z=1}^N C_{zj}^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}, \quad \theta > 1, \quad (\text{A.2})$$

where C_{zj} is the j -th individual consumption of good z and θ is the elasticity of substitution across goods. The price deflator for nominal money is the consumption-based money price index corresponding to the consumption index (A.2)

$$P = \left[\frac{1}{N} \left(\sum_{z=1}^N P_z^{1-\theta} \right) \right]^{\frac{1}{1-\theta}}, \quad (\text{A.3})$$

where P_z is the price of good z . The interpretation of equations (A.1) to (A.3) is completely standard – see Blanchard and Kiyotaki (1987). We will focus throughout on a single period so as to avoid carrying around time subscripts and will ignore intertemporal linkages by assuming that both private agents and the government do not borrow or lend. Given our assumptions, these intertemporal linkages would not affect equilibrium allocations in a multiperiod version, with the exception of optimal taxes: with deadweight losses from fiscal policy, a benevolent fiscal authority would like to smooth taxation over time. Here we assume that the deadweight losses, namely α , are small in absolute value and ignore the tax-smoothing motive.

Producer j has the following budget constraint:

$$\sum_{z=1}^N P_z C_{zj} + M_j = P_j Y_j (1 - \tau) - PT + \overline{M}_j \equiv I_j, \quad (\text{A.4})$$

which says that nominal consumption expenditure plus the demand for money must equal nominal income. It is assumed that taxes τ are proportional to sales; individuals also pay per-head taxes PT and have an initial holding of money, \overline{M}_j . Hence, nominal income is equal to nominal after-tax revenues from selling the produced good, minus per-head taxes, plus the initial money holding. Both τ and T can be either positive or negative.

There is a government that runs fiscal policy and a central bank that runs monetary policy in this economy. The government has the budget constraint:

$$I_g \equiv \sum_{j=1}^N P_j Y_j \tau (1 + \alpha(\tau)) + NPT = 0. \quad (\text{A.5})$$

$\alpha(\tau)$ are deadweight losses inherent in fiscal policy; we assume that $\alpha(\tau) = \alpha \geq 0$ when $\tau < 0$ and $\alpha(\tau) = -\alpha \leq 0$ when $\tau > 0$: the government wastes a fraction α of its budget, whether it levies sale taxes or it gives sale subsidies. Government resources, I_g , come from sale or per-head taxation and are redistributed to the producers-consumers net of the deadweight loss $\alpha\tau$. In this paper we only consider sale taxes. Other types of fiscal policies are possible: government spending to purchase goods or supply-side policies, financed via debt or distortionary taxation of income. Different fiscal policies would have different implications on output, prices and the parameters we derive at the end of this appendix. The working paper version, Avinash Dixit and Luisa Lambertini (2000), discusses the case of government spending to purchase goods financed with lump-sum taxes. Notice that money supply does

not enter the government budget constraints: the monetary and the fiscal authorities do not share (A.5).

The solution of this model is briefly sketched here. The first order condition with respect to C_{zj} and M_j , respectively, imply

$$C_{zj} = \left(\frac{P_z}{P}\right)^{-\theta} \frac{\gamma I_j}{NP}, \quad (\text{A.6})$$

$$M_j = (1 - \gamma)I_j. \quad (\text{A.7})$$

As usual, the demand for each good is linear in wealth and depends on its relative price with elasticity $-\theta$. The demand for money is also linear in wealth.

Let $W \equiv \gamma I / (NP)$, where $I \equiv \sum_{j=1}^N I_j$. The demand facing producer z can be obtained by aggregating individual demand over consumers

$$Y_z^d = \sum_{j=1}^N C_{zj} = \left(\frac{P_z}{P}\right)^{-\theta} W. \quad (\text{A.8})$$

The price, and therefore output, chosen by producer j is found by maximizing her indirect utility function

$$U_j = (1 - \tau)W^{\frac{1}{\theta}} Y_j^{\frac{\theta-1}{\theta}} - T + \frac{\bar{M}_j}{P} - \left(\frac{d}{\beta}\right) Y_j^\beta$$

with respect to the relative price, which gives

$$\frac{P_j}{P} = \left[\frac{\theta d}{(\theta - 1)(1 - \tau)} W^{\beta-1} \right]^{\frac{1}{1+\theta(\beta-1)}}. \quad (\text{A.9})$$

The higher the demand W and the disutility of effort d , the higher the relative price set by producer j .

Suppose the parameters d, θ, β are stochastic with variances $\sigma_d, \sigma_\theta, \sigma_\beta$, respectively; for simplicity, we normalize $\sigma_\beta = 1$ and assume that these stochastic variables are independent. We consider a particular model of staggered-price setting, a discrete-time variant of a model proposed by Guillermo A. Calvo (1983) and used by Michael Woodford (2002). In this model, a fraction $0 < \phi < 1$ of goods prices remain unchanged each period, while new prices are chosen for the other $1 - \phi$ goods; for simplicity, the probability that any given price will be adjusted in any given period is assumed to be independent of the length of time since the price was changed and independent of what the good's current price may be. This implies that, in any period, a fraction ϕ of the prices is given from the past and constant; we denote

the preset price of the z -th good as \bar{P}_z . A fraction $1 - \phi$ of the prices is set freely after uncertainty is resolved and we denote the price of the z -th good \tilde{P}_z . Then, the price level is

$$P^{1-\theta} = \left[\phi E \bar{P}_z^{1-\theta} + (1 - \phi) \tilde{P}_z^{1-\theta} \right]. \quad (\text{A.10})$$

The first term on the right-hand side is the average of the pre-set prices. The second term is the newly set price this period; because each producer that chooses a new price for its good faces exactly the same decision problem, which we will solve later, the optimal price \tilde{P}_z is the same for each of them. We define aggregate output as

$$Y \equiv \sum_{j=1}^N \frac{P_j Y_j}{P} = WN. \quad (\text{A.11})$$

In our model, fiscal policy consists in a production subsidy. The government levies its revenues by per-head taxes $T > 0$ and redistributes the revenues via a production transfer $\tau < 0$. An expansionary fiscal policy is a reduction in τ . It is easy to show that

$$W = \frac{\gamma}{N} \left[Y(1 + \tau\alpha) + \frac{\bar{M}}{P} \right], \quad Y = \frac{\gamma}{(1 - \gamma)(1 - \frac{\gamma\alpha}{1-\gamma}\tau)} \frac{\bar{M}}{P}, \quad (\text{A.12})$$

The relative price level can be easily derived from $W = Y/N$ and plugging the result in (A.9).

A fraction ϕ of producers do not get a chance to update their prices and simply keep the prices they had set in the past, \bar{P}_z . The fraction $1 - \phi$ of producers who set new prices choose \tilde{P}_z optimally to maximize their expected indirect utility. We now proceed to find the optimal price.

Let $\mu = \log \bar{M}$, $\pi = \log P$, $\bar{\pi}_j = \log E \bar{P}_j$, $\tilde{\pi}_j = \log \tilde{P}_j$, $y = \log Y$, $x = -\tau$. The log of the optimal price satisfies the following log-linear approximation

$$\tilde{\pi}_j = (1 - \phi\eta) \left[\pi_j + \frac{\phi\eta}{1 - \phi\eta} \bar{\pi}_j \right], \quad (\text{A.13})$$

where η is the personal discount factor and π_j is the optimal price for the current period only. Intuitively, the newly set price is an average of the price that is optimal in the current period, given the current realization of the stochastic shocks and policy, and of the price that is expected to be optimal in the future. The latter depends on the expected realization of shocks and policies and, thanks to the law of large number, is equal to the average of the preset prices already existing in the economy. We first find $\bar{\pi}_j$, which is the price that maximizes

the future expected indirect utility. Under the assumption that $(1-\tau)W(\theta-1)\bar{P}_j^{-\theta}P^{\theta-1}$ and $dW^\beta\theta\bar{P}_j^{-\theta\beta-1}P^{\theta\beta}$ are lognormally distributed and after several manipulations, the first-order condition with respect \bar{P}_j gives

$$\bar{\pi}_j = \chi_0 + \bar{e}E\pi + (1-\bar{e})E\mu + \bar{f}E\tau \quad (\text{A.14})$$

with

$$\chi_0 = \frac{1}{E[1+\theta(\beta-1)]} \left\{ E \left[\log d + \log \frac{\theta}{\theta-1} + (\beta-1) \left(\log \frac{\gamma}{N(1-\gamma)} \right) \right] + \frac{1}{2}(\text{Var}_0 - \text{Var}_1) + \text{Cov}(\mu, \beta) + \text{Cov}(\pi, (\theta-1)(\beta-1)) - \text{Cov} \left(\tau, \frac{\alpha\gamma\beta}{1-\gamma} \right) \right\},$$

$$\text{Var}_0 = \text{Var} \left[\log \left(dW^\beta\theta P^{\theta\beta} \right) \right]$$

$$\text{Var}_1 = \text{Var} \left[\log \left((1-\tau)W(\theta-1)P^{\theta-1} \right) \right]$$

$$\bar{e} = \frac{E[1+(\theta-1)(\beta-1)]}{E[1+\theta(\beta-1)]}, \quad \bar{f} = \frac{[1-\gamma-\gamma\alpha E(\beta-1)]}{(1-\gamma)E[1+\theta(\beta-1)]},$$

where $\text{Var}_0, \text{Var}_1$ are constants. Now we find π_j , which is the price that maximizes the current period indirect utility. This is given by

$$\pi_j = \chi_1 + e\pi + (1-e)\mu + f\tau \quad (\text{A.15})$$

with

$$\chi_1 = \frac{1}{1+\theta(\beta-1)} \left\{ \log \frac{\theta d}{\theta-1} + (\beta-1) \left[\log \frac{\gamma}{N(1-\gamma)} \right] \right\},$$

$$e = \frac{1+(\theta-1)(\beta-1)}{1+\theta(\beta-1)}, \quad f = \frac{[1-\gamma+\gamma\alpha(\beta-1)]}{(1-\gamma)[1+\theta(\beta-1)]}.$$

The price level in the economy is an average of preset and newly changed prices; log-linearization of (A.10) gives

$$\pi = \phi\bar{\pi}_j + (1-\phi)\tilde{\pi}_j.$$

Using (A.13), we can write the price level as

$$\pi = \rho\bar{\pi}_j + (1-\rho)\pi_j, \quad \rho = \phi[1+(1-\phi)\eta]. \quad (\text{A.16})$$

It is useful to write the price level as a function of monetary and fiscal policy. Then

$$\pi = m + cx \quad (\text{A.17})$$

with

$$m = \frac{1}{1 - e(1 - \rho)} \left\{ \rho[\chi_0 + \bar{e}E\pi + (1 - \bar{e})E\mu + \bar{f}E\tau] + (1 - \rho)[\chi_1 + (1 - e)\mu] \right\},$$

$$c = \frac{-(1 - \rho)[1 - \gamma + \gamma\alpha(\beta - 1)]}{(1 - \gamma) \{ \rho[1 + \theta(\beta - 1)] + (1 - \rho)(\beta - 1) \}} < 0.$$

m is the monetary policy variable and it is an increasing function of μ . A fiscal expansion reduces the price level. Output is derived from (A.12) and (A.16) and it is given by

$$y = \bar{y} + b(\pi - \bar{\pi}_j) + ax \tag{A.18}$$

with

$$b = \frac{\rho[1 + \theta(\beta - 1)]}{(1 - \rho)(\beta - 1)} > 0, \quad a = \frac{1}{\beta - 1} > 0, \quad \bar{y} = \log N + \frac{1}{\beta - 1} \log \frac{(\theta - 1)}{\theta d}.$$

An expansionary fiscal policy has an expansionary effect on output if $a + bc > 0$; this condition is satisfied as long as the deadweight loss of fiscal policy is small, namely $\alpha < [(1 - \gamma)(1 - \rho)] / \{ \gamma \rho [1 + \theta(\beta - 1)] \}$.

B Social welfare function

The indirect utility of the representative agent, excluding real balances, with flexible prices is given by

$$U_j = (1 - \tau)Y_j - T - \frac{d}{\beta} Y_j^\beta, \tag{B.19}$$

where the equilibrium level of output for each producer for given τ, α is given by

$$Y_j = \left[\frac{(\theta - 1)(1 - \tau)}{\theta d} \right]^{\frac{1}{\beta - 1}}. \tag{B.20}$$

The fiscal authority decides the production subsidy and per-head taxes according to its budget constraint:

$$T = -Y\tau(1 + \alpha)/N.$$

Notice that the socially optimal subsidy and output level are

$$\tau^{opt} = \frac{\alpha\theta(\beta - 1) - 1}{\theta(\alpha\beta + 1) - 1}, \quad Y_j^{opt} = \left[\frac{(\theta - 1)(1 + \alpha)}{d(\theta + \alpha\beta\theta - 1)} \right]^{\frac{1}{\beta - 1}}. \tag{B.21}$$

With no deadweight losses from fiscal policy ($\alpha = 0$), fiscal policy can achieve the efficient level of output that would arise absent the producers' monopoly power:

$$\tau^* = -\frac{1}{\theta - 1} < 0, \quad Y_j^* = \left[\frac{1}{d} \right]^{\frac{1}{\beta-1}}.$$

With deadweight losses from fiscal policy ($\alpha > 0$), the optimal subsidy is smaller than τ^* and equilibrium output is lower than Y_j^* .

To evaluate the consequences of the interaction between monetary and fiscal policies on the utility of the representative agent, we follow Julio J. Rotemberg and Michael Woodford (1997, 1999) and consider a second-order Taylor series approximation to the objective

$$U = \gamma u(C, M/P; \epsilon) - \sum_{j=1}^N v(Y_j; \epsilon) \quad (\text{B.22})$$

with

$$u(C, M/P; \epsilon) = \left(\frac{C}{\gamma} \right)^\gamma \left(\frac{M/P}{1-\gamma} \right)^{1-\gamma}, \quad v(Y_j; \epsilon) = \left(\frac{d}{\beta} \right) Y_j^\beta.$$

The approximation is made around the level of output $Y_j = \bar{Y}_j$ for each j and the mean values for the exogenous shocks. Here $\bar{Y}_j = Y_j^{opt}$ represents the level of output in an optimal steady state; it is the level of output in an equilibrium with flexible prices and the optimal tax rate τ^{opt} . We only consider the fraction of utility stemming from consumption, γ , because we wish to develop a welfare criterion for the cashless limit of our economy.

We will proceed briefly; for details, see Rotemberg and Woodford (1997, 1999). Let $\epsilon = (d, \beta, \theta)$ denote the complete vector of preference shocks that we normalize so that $E(\epsilon) = 0$ and let a $\bar{\cdot}$ denote steady-state value; a second-order expansion of the first term on the right-hand side of (B.22) is given by

$$\gamma \left[\bar{u} + u_C \tilde{C} + u_\epsilon \epsilon + u_m \tilde{m} + \frac{1}{2} u_{CC} \tilde{C}^2 + \frac{1}{2} \epsilon' u_{\epsilon\epsilon} \epsilon + \frac{1}{2} u_{mm} \tilde{m}^2 + u_{Cm} \tilde{C} \tilde{m} + u_{C\epsilon} \tilde{C} \epsilon + u_{m\epsilon} \tilde{m} \epsilon \right],$$

where $\tilde{C} = C - \bar{C}$, $m \equiv M/P$, $\tilde{m} = m - \bar{m}$, $\tilde{m} = (1 - \gamma)/\gamma \tilde{C}$. At the steady state, $\bar{C} = (1 + \alpha \bar{\tau}) \bar{Y}$, where \bar{Y} is steady state output. After using Taylor expansion, we can substitute for

$$\tilde{C} = \bar{Y} (1 + \alpha \bar{\tau}) \left(\hat{Y} + \frac{1}{2} \hat{Y}^2 \right) + \alpha \bar{Y} (\tau - \bar{\tau}),$$

where $\hat{Y} \equiv \log(Y/\bar{Y})$ and similarly for the other variables. Notice also that, at the steady state, $\bar{m} = (1 - \gamma)\bar{C}/\gamma$, $^1u_C = 1$, $u_m = 1$, $u_{CC} = -(1 - \gamma)/\bar{C}$, $u_{mm} = -\gamma/\bar{m}$, $u_{Cm} = \gamma/\bar{C}$. Since $(\tau - \bar{\tau})$ is $\mathcal{O}^2(\|\epsilon\|)$ and if we neglect terms that are of third or higher order in the deviations of the variables from their steady-state values and independent of policies, we obtain

$$\bar{C}u_C \left\{ \hat{Y} [1 + (1 - \gamma)g] + \frac{\hat{Y}^2}{2} + \frac{\alpha\bar{\tau}}{1 + \alpha\bar{\tau}} (\tau - \bar{\tau}) \right\}, \quad (\text{B.23})$$

where

$$g \equiv -\frac{u_{C\epsilon}\epsilon}{\bar{C}u_{CC}}.$$

A second-order Taylor expansion of each $v(Y_j; \epsilon)$ gives

$$\bar{Y}_j v_Y \left\{ \hat{Y}_j \left(1 + \frac{v_{Y\epsilon}\epsilon}{v_Y} \right) + \frac{\hat{Y}_j^2}{2} \left(1 + \frac{v_{YY}\bar{Y}_j}{v_Y} \right) \right\}. \quad (\text{B.24})$$

At the optimal production level for the j -th producer when prices are flexible, we have that

$$v_Y = \frac{(1 - \bar{\tau})(\theta - 1)}{\theta} = u_C(1 - \kappa) \quad \text{with} \quad \kappa = 1 - \frac{(1 - \bar{\tau})(\theta - 1)}{\theta},$$

where κ summarizes the overall distortion in the steady-state output level as a result of market power and taxation. We assume that κ is small, specifically of order $\mathcal{O}(\|\epsilon\|)$; substituting this relationship in (B.24), the second term on the right-hand side of (B.22) is approximated by

$$\bar{Y}u_C \left\{ [1 - \kappa + (\beta - 1)q] E\hat{Y}_j + \frac{\beta}{2} [(E\hat{Y}_j)^2 + \text{Var}Y_j] \right\},$$

where

$$q \equiv -\frac{v_{Y\epsilon}\epsilon}{v_{YY}\bar{Y}_j}$$

Using the Taylor expansion to substitute for $E\hat{Y}_j$

$$\hat{Y} = E\hat{Y}_j + \frac{1}{2} \left(1 - \frac{1}{\theta} \right) \text{Var}Y_j$$

and ignoring the terms that are third or higher order, we obtain

$$\bar{Y}u_C \left\{ [1 - \kappa - (\beta - 1)q] \hat{Y} + \frac{\beta}{2} \hat{Y}^2 + \frac{1}{2} \left(\beta - 1 + \frac{1}{\theta} \right) \text{Var}Y_j \right\}. \quad (\text{B.25})$$

Next, we subtract (B.25) from (B.23) and, after rearranging the terms, we obtain

$$U = -\frac{\bar{Y}u_C}{2} \left\{ \hat{Y}^2(\beta - 1) - 2\hat{Y}[g(1 - \gamma) + q(\beta - 1) + \kappa + \alpha\bar{\tau}] + \frac{1 + \theta(\beta - 1)}{\theta} \text{Var}\hat{Y}_j - 2\alpha(\tau - \bar{\tau}) \right\}$$

$$= -\frac{\bar{Y}u_C}{2} \left\{ \theta \frac{\kappa}{1-\kappa} [1 + \theta(\beta - 1)] (\pi - \bar{\pi}_j)^2 + (\beta - 1)(y - y_F)^2 - 2\alpha(\tau - \bar{\tau}) \right\}, \quad (\text{B.26})$$

In deriving (B.26), we have made use of the fact that, under the assumed CES preferences and with the Calvo pricing model,

$$\text{Var} \hat{Y}_j = \text{Var} \log Y_j = \theta^2 \text{Var} \log P_j = \theta^2 \frac{\kappa}{1-\kappa} (\pi - \bar{\pi}_j)^2.$$

In the second line of (B.26), we have rewritten the expression in terms of the output gap $y - y_F$, where $y_F = \log(NY_j^*)$.² The last term in the second line of (B.26) captures the deadweight losses that arise when fiscal policy is used to bring output above its natural rate. (B.26) can be rewritten in terms of the quadratic loss function

$$L^s = \frac{1}{2} \left[(\pi - \bar{\pi}_j)^2 + \theta_F (y - y_F)^2 + 2\delta x \right], \quad (\text{B.27})$$

where

$$\theta_F = \frac{\rho(1-\kappa)}{\theta b \kappa (1-\rho)}, \quad \delta = + \frac{\alpha(1-\kappa)}{\theta \kappa [1 + \theta(\beta - 1)]}.$$

Social welfare is lower the larger the gap between actual and the efficient level of output and the larger the deadweight losses caused by fiscal policy. Notice that $\delta x > 0$ because $\delta > 0$ if $x > 0$ and viceversa. Social welfare is lower the more dispersed prices. The relative weight on the output gap is inversely proportional to slope of the short-run Phillips curve (see equation (A.18)).

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