National Centre of Competence in Research
Financial Valuation and Risk Management

Working Paper No. 768

Portfolio Choice under Uncertainty

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First version: June 2012
Current version: June 2012

This research has been carried out within the NCCR FINRISK project on
"Macro Risk, Systemic Risks and International Finance"

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June 22, 2012

Abstract

During the financial crisis that first hit the U.S. economy and soon became a world crisis, investors reduced their holdings of foreign equities, and, at the same time, they increased their holdings of short-term government bonds. The paper analyzes, within the context of a DSGE model, the hedging properties of foreign bond and foreign equity holdings during a crisis, when the degree of uncertainty is high. I show that uncertainty generates portfolio dynamics and that they differ depending on the source of uncertainty. Investors increase their holdings of foreign government bonds and, at the same time, reduce their holdings of foreign corporate equities, when uncertainty originates from aggregate demand. When instead uncertainty originates from aggregate supply, it is optimal for investors to reduce their holdings of foreign bonds and increase their holdings of foreign equity. These findings are supported by the recently developed theories that consider the collapse of aggregate demand the main cause of the "Great Slump" that started in 2007.

Keywords: International Finance, Capital Flows, Uncertainty.

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1 Introduction

The financial crisis that started in the U.S. in August 2007 and soon spread to the rest of the world has been extensively analyzed. This crisis did not display particularly novel features and in fact economists have pointed out many analogies between the current crisis and the previous ones. Reinhart and Rogoff (2008a, 2008b) empirically document the similarities in the behavior of several variables during past financial crises, including this last one. Eichengreen (2008) also emphasizes how the U.S. crisis, similarly to the 1997-98 Asian crises, has been characterized by lack of transparency, lax bank regulation, and connected lending that allowed some large institutions to enjoy privileged access to borrowed funds.

Nevertheless, the most important difference between this crisis and all the previous ones is that it had the largest economy in the world at its epicenter. The U.S. financial crisis soon became a world crisis and the panic spread across economies. As a result, in this occasion, capital did not massively flow out of the crisis country generating the "sudden stops" and "capital reversals" that have instead characterized previous episodes of turmoil. Foreign investors reduced their holdings of U.S. equities indeed, but, at the same time, they increased their holdings of U.S. Treasuries, to such an extent that the U.S. dollar started appreciating, and the yields on U.S. Treasury bills reached historical lows. The sudden change in the holdings of the different types of assets, including U.S. Treasuries, U.S. equity, and foreign corporate stocks and bonds, has been followed by a stabilization and a return to previous trends in 2009.

The decision of private markets to invest so heavily in the country from which the crisis originated has been explained [1] as a flight to safety. The behavior of investors has been described as the result

[1] Caballero (2009), DeLong (2010) and Cochrane (2010), among many others.

of "the insatiable need to accumulate safe debt instruments"[2]. As international investors realized that U.S. corporate assets could not satisfy their demand, they decided to run to U.S. Treasuries. The reallocation of funds took place across asset classes, rather than across countries. But, what did U.S. investors do during the crisis? They adopted a similar strategy: They dramatically reduced their holdings of foreign equity and, in some countries, they increased their holdings of short-term bonds. So, the question is: Under which conditions are foreign bonds a better hedging instrument than foreign equity? The goal of this paper is to identify the factors that can rationally justify the choice of agents to increase their holdings in foreign bonds and run away from foreign equity, when there is an increase in world uncertainty.

In order to do so I use a DSGE model with uncertainty. Earlier DSGE literature often abstracts from endogenous portfolio choices because of technical difficulties. The standard approach to solve DSGE models in fact relies on first-order approximations around the steady state that prove inadequate to capture the second moments that determine portfolio choices. In a stochastic world, financial assets are differentiated by their degree of risk, and optimal portfolio choices are determined by correlations and variances of stochastic variables.

This paper makes use of the solution method recently developed by Devereux and Sutherland (2006). Their novel procedure consists in the combination of second-order approximations of the portfolio equations of the model with first-order approximations of non-portfolio equations. This allows to determine how the stochastic structure of the model affects the portfolio allocation and it becomes possible to characterize economy's first-order response to stochastic shocks under the optimal portfolio. They also show that first-order dynamics of steady-state portfolio can be obtained by

[2] Caballero (2009).

combining a third-order approximation of the portfolio equation and a second-order approximation of the rest of the model. Time variations in portfolios become relevant for macroeconomic dynamics at the second level of approximation.

I introduce uncertainty in this portfolio problem assuming that the exogenous state variables follow conditionally-linear stochastic processes where variances are modeled through stochastic linear processes. As shown in Benigno et al. (2011), the use of a conditionally-linear approximation, as opposed to the fully-linear approximation used by Fernandez-Villaverde et al. (2009), implies that the process for the exogenous state variables is not approximated and still displays time-varying volatility. Because of this, a second-order approximation of the policy rules is sufficient to identify a "distinct and direct" role for time-varying volatility in affecting the endogenous variables. The same principle applies to the portfolio problem that I study and therefore with the conditionally-linear model I am able to observe the effect of time-varying risk on portfolio dynamics simply using a combination of the third-order approximation of portfolio equations and the second-order approximation of the rest of the model. The use of a fully-linear approximation, like in Fernandez-Villaverde et al. (2009), would instead require a combination of the fourth-order approximation of the portfolio equations of the model and a third-order approximation of all the other equations.

The choice of an optimal portfolio of assets in a multi-period setting with time varying volatility of returns has been studied in the finance literature[3]. However, the macroeconomic setting used here gives the possibility to better understand the relationship between portfolio behavior and macroeconomic variables. Differently from what happens in finance models, the behavior of returns is not exogenously given. Within the context of a full blown general equilibrium model it is in fact possible

[3]Campbell and Viceira (1999), Chacko and Viceira, (2005).

to precisely identify the way in which returns are affected by the fundamentals of the economy and the way in which these returns co-move with consumption. The model shows how time-varying risk is generated by uncertainty in the fundamentals and how volatile fundamentals in turn can affect portfolio holdings. Thanks to these features the analysis of portfolio dynamics can give not only positive, but also interesting normative insights.

The contribution of the paper is twofold: First, I show that uncertainty is a source of portfolio-dynamics that can contribute to explain, together with the other sources already identified in the literature, deviations of the portfolio from its steady-state. My findings suggest that the time-varying volatility of the fundamentals affects portfolio dynamics through two different channels: Time-varying volatility first affects the covariance between consumption and asset returns and, additionally, it determines an interaction between the size of the shocks that hit the state variables and the change in returns' variances and covariances. Depending on the shock that we consider, we can have that a time-varying variance pushes the portfolio exactly in the same direction of the shock to the state variable, amplifying the final effect, or, alternatively, can offset the effect of the shock and determine different adjustments of the portfolio.

Second, in the numerical solution of the model, I show under which conditions it is rational for investors to increase their holdings of foreign government bonds and, at the same time, reduce their holdings of foreign equity, in response to an increase in global uncertainty.

In the two country DSGE model that I use I assume that there is trade in both goods and financial assets. Each country is endowed every period with one good, and trade is justified by the features of final consumption that is a bundle of domestic and foreign goods. The international asset portfolio includes two types of securities: stocks and bonds. Each country issues one government bond and one equity, denominated in local goods. There are three sources of shocks: one preference shock that is common to the two economies, two endowment shocks and two government spending shocks. Furthermore I proxy the increase in uncertainty with the introduction of uncertainty shocks, i.e. I allow the variances of the shocks to be time-varying. The number of shocks is larger than the number of assets. International financial markets are incomplete.

Investors that have the availability of equities and bonds, choose their portfolio with two main goals in mind: They want to smooth their consumption and, in order to do so, they want to have a well diversified portfolio with assets that display a low degree of co-movement and low variance. When uncertainty shocks hit, the way in which real variables covary with asset returns changes. As a consequence, agents need to re-adjust their portfolios until when the shock has disappeared.

My main findings are that the response of the portfolio to an increase in uncertainty crucially depends on the source of uncertainty itself. If uncertainty comes from aggregate demand, it is always optimal for agents to increase their holdings of foreign bonds. They reduce their holdings of foreign equity if the increase in uncertainty comes from government spending, while they increase their holdings of foreign equity if uncertainty comes from preferences. If instead the source of uncertainty is aggregate supply, agents find it optimal to increase their holdings of foreign equity and reduce their holdings of foreign bonds.

This finding suggests that the movements of capital that took place during the crisis are compatible with an increase in uncertainty coming from aggregate demand. This result is supported by those theories that identify the collapse in demand as the main cause of the slump experienced by the U.S. and by many other economies during the crisis[4]. On the one hand, the explosion of the

[4] Hall, (2011).

housing bubble and the dramatic reduction in consumer spending in housing and durables can be described as an increase in the volatility of consumers' preferences. On the other hand, the inability of governments to realize effective fiscal policies during the crisis can be considered a big source of uncertainty that did not help the recovery and that might have affected investors' portfolio choice. The model is also able to give reason of other episodes of capital movements observed in the past: A combination of aggregate demand and aggregate supply shocks is also able to replicate the massive capital outflows that have been observed in the late 90's.

The rest of the paper is structured as follows. Section 2 gives a more detailed description of capital movements during the crisis. Section 3 presents the model set-up. In section 4 I briefly show the solution method, in section 5 I calibrate the model and discuss the results. Section 6 concludes.

2 A Look at the Crisis

In this section I show some more details on the movement of capital that took place at the beginning of the crisis. After 2007 foreign holdings of U.S. equities registered a dramatic reduction, while foreign holdings of U.S. Treasury bonds increased.

It is well known that a large component of foreign holdings of U.S. Treasury bonds is represented by official holdings. Furthermore, it has been documented (Warnock, 2010) that private agents mainly hold short-term U.S.Treasuries, while governments mainly invest in long-term U.S. Treasuries. Figure 1 shows that private and official investors' holdings of Treasuries followed different paths after the panic: Private investors increased their holdings of U.S. Treasuries after 2007, but after 2008 they started reducing them. Official investors, instead, went on investing in U.S. Treasury bonds. Private holdings of short-term Treasuries could have been used as a hedge against the risk, until the panic was over. After 2008 things went back to normality and investors gradually started increasing their holdings of U.S. equity.

But what happened to U.S. holdings of foreign assets? As Figure 2 shows, there was a strong reduction in U.S. holdings of foreign equity and a milder reduction in U.S. holdings of foreign bonds, while foreign direct investment kept increasing. But if we carefully look at U.S. holdings of foreign bonds, we can observe a difference in the behavior of short and long-term bond holdings. Figure 3 shows a classification of the countries that experienced an increase in U.S. holdings of their short-term debt, in millions of U.S. dollars. In some cases, U.S. investors reduced their holdings of both long-term and short-term bonds, but, in some other cases, they only reduced their holdings of long-term foreign bonds, while they increased their holdings of short-term foreign bonds. For example, U.K., Ireland and Greece, registered a reduction in the U.S. holdings of their debt. However in the case of other countries, like Germany, Canada or Norway, there was a reduction in U.S. holdings of long-term debt, but, at the same time, an increase in U.S. holdings of short-term debt. The only exception is Switzerland that instead registered an increase in U.S. holdings of long-term debt and a reduction in U.S. holdings of short-term debt. Also developing countries like Brazil and Thailand registered an increase in U.S. holdings of their short-term debt. Some developing countries had in fact been implementing rigorous fiscal and monetary policies in order to recover from their own crises. When the crisis hit the U.S., they were therefore characterized by relatively strong fundamentals that might have increased the attractiveness of their assets.

The choice of U.S. and foreign investors to increase their holdings of foreign short-term debt shows a certain symmetry in the behavior of investors during the crisis: As uncertainty increased, both U.S. and foreign investors used foreign short-term debt to protect themselves. This justifies my choice to use a two-country symmetric model in the rest of the paper[5].

In what follows I assume that one country is the U.S. and the other one is the rest of the world.

3 The Model

This is an infinite horizon, two-country open economy model. There are two ex-ante symmetric countries,"Home" (H) and "Foreign" (F), each one populated by a representative household who consumes and trades a portfolio of financial assets. Each country is endowed with one good and issues stocks and government bonds, which are internationally traded. The international portfolio therefore consists of four assets: two equities and two government bonds. I allow for three types of shocks in each country: preference shocks, endowment shocks, and government spending shocks. The number of shocks is larger than the number of assets available: Financial markets are incomplete.

3.1 Preferences

The household in country $i = H, F$ chooses consumption C^i to maximize the inter-temporal utility function:

$$\max E_t \sum_{s=0}^{\infty} \theta_s \frac{(C^i_{t+s})^{1-\sigma}}{1-\sigma} \tag{1}$$

[5]In this version of the model I assume that the two countries have the same size. Size effects could be important in quantitatively determining steady-state portfolios, but they should not affect the qualitative predictions of the symmetric model, as shown in Ghironi et al. (2009).

where $\sigma > 0$ is the coefficient of relative risk aversion and θ_s is the endogenous discount factor. The discount factor is defined as follows:

$$\theta_{t+s} = \theta_t \beta(C_t^i), \tag{2}$$

with $\theta_0 = 1$ and $\beta(C_t^i) = \omega C_t^{i^{-\eta}}$, where $0 \leqq \eta < \rho$ and $0 < \omega C_t^{i^{-\eta}} < 1$. The endogenous discount factor is used as a stationarity inducing device[6].

The Home final consumption basket combines Home and Foreign goods:

$$C_t^H = [\kappa_t^{1/\theta}(c_t^{HH})^{(\theta-1)/\theta} + (1-\kappa_t)^{1/\theta}(c_t^{FH})^{(\theta-1)/\theta}]^{\theta/(\theta-1)} \tag{3}$$

where c^{ij} is the amount of consumption good received by country i and consumed by country j. κ_t is a time varying preference parameter defined as $\kappa_t = \kappa \exp(\upsilon_t)$, where υ_t is the preference shock that follows an AR(1) process: $\upsilon_t = \phi^\upsilon \upsilon_{t-1} + \varepsilon_t^\upsilon$[7]. The parameter $\kappa \in (0,1)$ measures the share of local spending in consumption. If $\kappa > 0.5$, the agent is said to have a "home bias" preference in consumption.

Similarly, the consumption bundle for the Foreign country depends on Home and Foreign goods, and the Foreign preference parameter is $\kappa_t^* = \kappa \exp(-\upsilon_t)$. The Foreign consumption basket is:

$$C_t^F = [(\kappa_t^*)^{1/\theta}(c_t^{FF})^{(\theta-1)/\theta} + (1-\kappa_t^*)^{1/\theta}(c_t^{HF})^{(\theta-1)/\theta}]^{\theta/(\theta-1)}. \tag{4}$$

[6] See Schmitt Grohe and Uribe, (2003).

[7] The introduction of preference shocks generates incomplete markets, since the number of assets is smaller then the number of shocks. Furthermore the assumption of preference shocks during a crisis is not unrealistic: During a crisis agents for example prefer to postpone purchases of some goods, like consumer durables, while there are some other types of goods, like food, to which they cannot renounce.

A positive preference shock at the same time increases the consumption of the Home good and reduces the consumption of the Foreign good in both countries. The aggregate consumer price index for Home, P_t^H, is defined as:

$$P_t^H = [\kappa_t (p_t^H)^{1-\theta} + (1-\kappa_t)(p_t^F)^{1-\theta}]^{\frac{1}{1-\theta}}, \tag{5}$$

where p_t^H and p_t^F are the prices of Home and Foreign goods. Similarly, Foreign price index is defined as:

$$P_t^F = [\kappa_t^* (p_t^F)^{1-\theta} + (1-\kappa_t^*)(p_t^H)^{1-\theta}]^{\frac{1}{1-\theta}}. \tag{6}$$

In this economy the law of one price holds. If $\kappa > 0.5$, Purchasing Power Parity (PPP) does not hold, and the real exchange rate is defined as the ratio between the Foreign price index over the Home price index: $Q_t = \frac{P_t^F}{P_t^H}$.

3.2 Endowments

This is an endowment economy: In each period the two countries receive an endowment of two different goods, Y^H and Y^F. The stochastic endowment follows an $AR(1)$ process:

$$\hat{Y}_{t+1}^i = \phi^Y \hat{Y}_t^i + \varepsilon_{t+1}^{Y^i}. \tag{7}$$

As in Coeurdacier and Gourinchas (2011), I assume that only a fraction δ of the endowment can be capitalized and distributed to stockholders as dividend, while the remaining fraction $1-\delta$ is distributed to agents and can be interpreted as labor income.

3.3 The Government

The public sector of country i issues at time t government bonds B^i_{t+1} denominated in the domestic good. The real price of the bond at time t is z^i_t and its return is one unit of good i at time $t+1$. The government collects lump sum taxes T^i in order to finance government spending that, by assumption, entirely falls on country i good. The budget constraint of the government is:

$$z^i_t B^i_{t+1} = \frac{p^i_t}{P^i_t} B^i_t + \frac{p^i_t}{P^i_t} G^i_t - T^i_t, \tag{8}$$

where G^i is government spending in country i. Government spending follows an AR(1) process:

$$\hat{G}^i_{t+1} = \phi^G \hat{G}^i_t + \varepsilon^G_{t+1}. \tag{9}$$

3.4 Financial Markets

Each country i issues two types of assets: government bonds and stocks. There is a bond denominated in the Home good and one denominated in the Foreign good: Buying one unit of the Home (Foreign) bond at time t delivers one unit of Home (Foreign) good in the following period. Each country also issues a stock that represents a claim to a share δ of country i endowment. The supply of each share is normalized to unity. Let S^{ij}_{t+1} denote the number of shares of stock i held by country j household at the end of period t, while B^{ij}_{t+1} represents holdings of bonds issued by country i and held by country j. The budget constraint for the home economy at time t is:

$$nfa_{t+1}^{H} = (Y_t^H - G_t^H)\frac{p_t^H}{P_t^H} - C_t^H + \gamma_t^F nfa_t^H + (\gamma_t^F - \gamma_t^H)q_{t-1}^H S_t^{HF}$$
$$+(R_t^F - \gamma_t^F)z_{t-1}^F B_t^{HF} - (R_t^H - \gamma_t^F)z_{t-1}^H B_t^{HH}, \tag{10}$$

where nfa^H are net foreign assets, and they are defined as:

$$nfa_{t+1}^{H} = z_t^F B_{t+1}^{FH} + q_t^F S_{t+1}^{FH} - q_t^H S_{t+1}^{HF} - z_t^H B_{t+1}^{HF}, \tag{11}$$

where z^i is the price of the bond issued by country i, while q^i is the price of the equity issued by country i. γ^H and γ^F are the returns on stocks issued by Home and Foreign and defined in terms of Home consumption:

$$\gamma_t^H = \frac{(P_t^H q_t^H + p_t^H d_t^H)}{P_t^H q_{t-1}^H}, \tag{12}$$

$$\gamma_t^F = \frac{(P_t^H q_t^F + p_t^F d_t^F)}{P_t^H q_{t-1}^F}, \tag{13}$$

where d_t^i are dividends that are equal to δY_t^i. R_t^H and R_t^F are the returns on Home and Foreign bonds in terms of Home consumption, and they are defined as:

$$R_t^H = \frac{p_t^H}{P_t^H z_{t-1}^H}, \tag{14}$$

and

$$R_t^F = \frac{p_t^F}{P_t^H z_{t-1}^F}. \tag{15}$$

Foreign agents solve a similar portfolio allocation problem and their budget constraint is:

$$nfa_{t+1}^F = (Y_t^F - G_t^F)\frac{p_t^F}{P_t^F} - C_t^F + \gamma_t^H nfa_t^F + (\gamma_t^H - \gamma_t^F)q_{t-1}^F S_t^{FH}$$
$$+(R_t^H - \gamma_t^H)z_{t-1}^H B_t^{FH} - (R_t^F - \gamma_t^H)z_{t-1}^F B_t^{FF}, \quad (16)$$

where Foreign net foreign assets nfa_{t+1}^F, are defined as:

$$nfa_{t+1}^F = z_t^H B_{t+1}^{HF} + q_t^H S_{t+1}^{HF} - q_t^F S_{t+1}^{FH} - z_t^F B_{t+1}^{FH}. \quad (17)$$

3.5 Market Clearing Conditions

The resource constraints are

$$c_t^{HH} + c_t^{HF} + G_t^H = Y_t^H \quad (18)$$

and

$$c_t^{FF} + c_t^{FH} + G_t^F = Y_t^F. \quad (19)$$

If we define the value of Home equity held by the two countries in terms of Home consumption, $q^H S^{HH}$ and $q^H S^{HF}$, as a_t^{HH} and a_t^{HF} the market clearing condition for the asset issued by Home economy is:

$$a_t^{HH} + a_t^{HF} = q_t^H, \quad (20)$$

since the total number of equities is normalized to unity, $S_t^{HH} + S_t^{HF} = 1$. Similarly, by defining holdings of Foreign equity in terms of Home consumption, the market clearing condition for Foreign equity is:

$$a_t^{FF} + a_t^{FH} = q_t^F. \tag{21}$$

In the case of bonds, we have:

$$B_t^H = b_t^{HH} + b_t^{HF}, \tag{22}$$

$$B_t^F = b_t^{FF} + b_t^{FH}, \tag{23}$$

where B_t^i is the total number of bonds issued by country i at time t, and $b^{ij} = B_t^{ij} z_{t-1}^i$, is the value of country i bonds defined in terms of country i consumption.

3.6 Uncertainty Shocks

Recently there has been an increasing interest in the analysis of the effects of uncertainty on macroeconomic variables[8]. Differently from the previous portfolio literature, I look at the effect of uncertainty shocks on the steady-state portfolio and on its dynamics. I introduce uncertainty through the assumption that the variances of the shocks that hit the economy are time-varying and can deviate from their steady-state value according to the following stochastic processes[9]:

$$\sigma_{t+1}^{2Y} = \rho^Y \sigma^{2Y} + (1 - \rho^Y)\sigma_t^{2Y} + \eta^Y u_{t+1}^Y, \tag{24}$$

$$\sigma_{t+1}^{2G} = \rho^G \sigma^{2G} + (1 - \rho^G)\sigma_t^{2G} + \eta^G u_{t+1}^G, \tag{25}$$

[8]For example see Benigno et al.(2011), Bloom (2009), Bloom, Floetotto and Jaimovich (2009), Fernandez-Villaverde, Guerron-Quintana, Rubio-Ramirez and Uribe (2009).

[9]As in Benigno et al. (2011), I assume that the exogenous state variables follow conditionally-linear stochastic processes, where the variances of the primitive shocks follow stochastic linear processes.

$$\sigma_{t+1}^{2v} = \rho^v \sigma^{2v} + (1-\rho^v)\sigma_t^{2v} + \eta^v u_{t+1}^v, \tag{26}$$

where u_{t+1}^i is an identically and independently distributed process with mean zero and unitary variance. σ^{2i} are the steady state values of the variances, with $i = Y, G, v$.

3.7 Optimality Conditions

The first-order conditions for country i with respect to equities and bonds are:

$$C_t^{i\eta-\sigma} = \omega E_t(C^{i}{}_{t+1}^{-\sigma}\gamma_{t+1}^H), \tag{27}$$

$$C_t^{i\eta-\sigma} = \omega E_t(C^{i}{}_{t+1}^{-\sigma}\gamma_{t+1}^F), \tag{28}$$

$$C_t^{i\eta-\sigma} = \omega E_t(C^{i}{}_{t+1}^{-\sigma}R_{t+1}^H), \tag{29}$$

$$C_t^{i\eta-\sigma} = \omega E_t(C^{i}{}_{t+1}^{-\sigma}R_{t+1}^F), \tag{30}$$

4 Model Solution

It is well known that in open economy macro literature, the optimal portfolio is indeterminate. Standard approximation methods that use only first-order approximations imply that certainty equivalence holds, and, as a result, all assets are perfect substitutes. In order to overcome this indeterminacy problem, Devereux and Sutherland (2006, 2010a)[10], provide a solution method that allows to determine steady-state portfolios. Their solution method is characterized by a two part solution: a second-order approximation of portfolio equations, in combination with a first-order

[10]From now on DS.

approximation of the rest of the model. In this way it is possible to determine how the stochastic structure of the model affects the portfolio allocation, and it is possible to characterize the economy's first-order response to stochastic shocks under the optimal portfolio.

Equations (7), (9), (10)-(15), (24)-(26), (27)-(30), along with their foreign equivalent, may be solved in order to determine the path of quantities $\{C_t^H, C_t^F,\}$, prices $\{q_t^H, q_t^F, z_t^H, z_t^F, Tot_t, Q_t\}$, rates of return $\{\gamma_t^H, \gamma_t^F, R_t^H, R_t^F\}$, and the vector of steady-state asset holdings $\{a^{HH}, a^{HF}, a^{FF}, a^{FH}, b^{HH}, b^{HF}, b^{FF}, b^{FH}\}$.

In Appendix A I find the steady-state value of Foreign equity holdings and Foreign bond holdings. Steady-state Foreign equity and Foreign bond holdings are a function of the parameters of the model and of the steady-state variances of the shocks. Agents' goal is to hold a portfolio able to hedge themselves against consumption risk. The steady-state share of each asset depends on the co-movement between consumption and the return of that asset, but also on the co-movement between consumption and the return of the other available asset, and on the variances and covariances of both asset returns.

4.1 Portfolio Dynamics

DS (2010a) show that a combination of third-order approximations of the portfolio equations and second-order approximations of the rest of the model delivers the path followed by the portfolio, when the economy is hit by a shock to the state variables. They show that deviations of exogenous state variables from their steady-state are a source of portfolio changes. Here I show that uncertainty shocks can be an additional source of portfolio dynamics. Not only the shocks of the model, but also their time-varying variances can contribute to explain deviations of the portfolio from its steady-state.

Traditionally DSGE models rely on the assumption of homoscedastic shocks, but recently the attention has been focused on the effects of time-varying variances on the business cycle. As in Benigno et al. (2011), I rely on the assumption that the exogenous state variables follow a conditionally-linear stochastic process in which the variance of the primitive shocks is modeled through a stochastic linear process. As shown in Benigno et al. (2011), the first-order approximation of the solution is consistent with a conditionally-linear model, where the process for the exogenous state variables is not approximated and therefore displays time-varying volatility. They show that under this assumption the second-order approximation of the model is sufficient to capture the effects of uncertainty. It follows that when this principle is applied to the portfolio problem, first-order portfolio dynamics generated by time-varying volatility can be then observed through a combination of second and third-order approximations of the model. The use of a conditionally-linear model greatly simplifies the solution of the portfolio problem that would have otherwise required a combination of fourth and third-order approximations of the model.

The third-order approximations of the portfolio equations, as obtained in DS (2010a), are:

$$E_t[-\sigma \hat{C}^D_{t+1}\hat{\gamma}^D_{t+1} + \frac{\sigma^2}{2}(\hat{C}^{D(2)}_{t+1}\hat{\gamma}^D_{t+1}) - \frac{\sigma}{2}(\hat{C}^D_{t+1}\hat{\gamma}^{D(2)}_{t+1})] = 0, \tag{31}$$

and

$$E_t[-\sigma \hat{C}^D_{t+1}\hat{R}^D_{t+1} + \frac{\sigma^2}{2}(\hat{C}^{D(2)}_{t+1}\hat{R}^D_{t+1}) - \frac{\sigma}{2}(\hat{C}^D_{t+1}\hat{R}^{D(2)}_{t+1})] = 0, \tag{32}$$

where $\hat{C}^{D(2)}$ is the second-order approximation of the difference between Home and Foreign consumption and $\hat{\gamma}^{D(2)}$ and $\hat{R}^{D(2)}$ are the second-order approximations of the difference in Home and

Foreign returns on equites and bonds. Plugging in the equilibrium conditions (31) and (32) the expressions for the second-order approximations of $\hat{C}^{D(2)}$, $\hat{\gamma}^{D(2)}$ and $\hat{R}^{D(2)}$ you find the equations that describe first-order dynamics of Home holdings of Foreign assets, $\hat{a}_t$ and $\hat{b}_t$.

Time-varying volatilities are, together with state variables, a source of portfolio dynamics. Differently from the benchmark two-asset case, where the shock and its variance play two distinct roles in the determination of portfolio dynamics, in the four-asset case changes in the optimal portfolio also depend on the interaction between the shock and its variance. The time-varying volatility of the fundamentals affects portfolio dynamics through two different channels: Time-varying volatility first affects the covariance between consumption and asset returns and, additionally, it determines a change in the variances and covariances of asset returns that interacts with the size of the shock that hits the state variables. As it will be shown in the next paragraph, depending on the shock that we consider, we can have that a time-varying variance pushes the portfolio exactly in the same direction of the shock, amplifying the final effect, or, alternatively, can offset the effect of the shock and determine different adjustments of the portfolio.

5 Numerical Results

5.1 Calibration

The portfolio solution of the full model is a highly complicated expression that can only be described numerically. The parameter values are presented in Table 1. The discount factor, β [11], is set equal to 0.96, in order to have a steady state real interest rate of 4%. Agents in both countries maximize the

[11] The steady-state discount factor $\beta = \omega \bar{C}^{-\eta}$

same utility function. The constant risk aversion parameter, σ, is set to 4, since empirical evidence suggests for this parameter a range between 3 and 5. In the choice of the persistence parameters of the shocks I follow the estimations of Gali and Monacelli (2005) for the endowment shock and Lubik and Schorfheide (2005) for the government spending shock. As in Devereux and Sutherland (2010b), I assume that demand and endowment shocks have the same persistence. The standard deviation of all the shocks is equal to 0.01%, and all the shocks are *i.i.d.*. The share of endowment $1-\delta$ that is used as labor income is 0.7 as typically suggested by empirical evidence. I analyze the special case in which the home bias parameter κ equals 0.5. This choice simplifies the solution of the model and the interpretation of the results. Under this assumption the PPP holds and we do not observe changes in the real exchange rate. Bonds' return differentials are proportional to changes in the terms of trade. The total number of bonds is normalized to 60% GDP, and the compatible ratio of taxes over GDP is 0.046.

5.2 Results

5.2.1 Steady-State Portfolio

In this economy there are four assets and five shocks. Markets are incomplete. As the number of assets available in the economy increases, the steady-state portfolio becomes a highly complicated expression that depends on the parameters of the model. When each country is allowed to issue more than one asset, all the returns of the assets issued by the same country depend on the fundamentals of the country itself and therefore it becomes necessary to study how these fundamentals affect the returns and how the returns co-move. Agents' goal is to hold assets that allow them to smooth consumption and, in order to do that, they want to hold assets with returns that do not positively co-move.

The returns offered by the stocks and the bonds issued by country i depend on the fundamentals of country i. In expected terms all the assets should offer the same returns, but, ex-post, returns can differ because of the unexpected shocks that hit the economies. Stock returns depend on the value of endowments, i.e., on real endowments and on their prices. Bond returns, instead, depend only on the price of the endowment. For reasonable parameter values, stock returns are positively related to endowment shocks, government spending shocks and preference shocks. Bond returns, instead, are negatively related to endowment shocks, but positively related to the other two shocks. Intuitively, endowment shocks increase the amount of endowment available, but have a negative effect on relative prices. This explains the divergence between the behavior of stock and bond returns. When all the other shocks are involved, they act only on prices and because of that we observe that returns move all in the same direction.

Steady-state Home holdings of Foreign assets depend on the co-movement between consumption and the return of each asset, and on the co-movement between stock and bond returns [12].

In the case without demand shocks markets are complete and agents can build a portfolio that is able to fully equalize Home and Foreign consumptions[13]. My model assumes that government

[12] The co-movement between consumption and asset returns is defined as $E(\hat{C}\hat{\gamma}) = \eta_{CY}\eta\gamma_{\varepsilon^y}\sigma_y^2 + \eta_{CG}\eta\gamma_{\varepsilon^g}\sigma_g^2 + \eta_{Cv}\eta\gamma_{\varepsilon^v}\sigma_v^2$. It is the sum of three components: the product between consumption elasticity to endowment shocks and return elasticity to endowments shocks, multiplied by the variance of endowments; the product between consumption elasticity to government spending shocks and return elasticity to government spending shocks, multiplied by the variance of government spending; the product between consumption elasticity to preference shocks and return elasticity to preference shocks, multiplied by the variance of preferences. In the determination of steady-state portfolio shares the three components act individually. Therefore in the analysis of each volatility, when I mention the covariance of consumption with asset returns, I refer each time to one component of this covariance, depending on the variance that I am considering.

[13] As shown in Coerdacier and Gourinchas (2011), when the number of shocks equals the number of assets, rank and spanning conditions are satisfied and it is possible to replicate the efficient risk-sharing allocation up to the first order.

spending entirely falls on Home goods and, even without assuming any home bias in consumption, in the complete market case features complete home bias in equity and bond holdings, when the elasticity of substitution between Home and Foreign goods, θ, is larger than one [14]. In this case, in fact, the elasticity of consumption to endowment is positive and the elasticity of consumption to government spending is negative. Equity returns positively react to both endowment and government spending shocks, while bond returns go down with endowment shocks and instead increase with government spending. These elasticities therefore guarantee that a portfolio that is completely biased towards holdings of Home assets is enough to smooth agents' consumption. They can in fact use Home bonds when the endowment shock hits and Home equity when the government shock hits. When instead the parameter θ is smaller than 1 things substantially change: Consumption increases with government spending shocks and decreases with endowment shocks. Furthermore, the returns of the two assets increase with government spending and decrease with endowment shocks. Since agents want to smooth consumption and, at the same time, hold assets that do not positively co-move, their only solution is to hold also Foreign stocks.

In the incomplete market case that includes also demand shocks, my numerical analysis shows that the optimal portfolio for the Home country is such that Home stocks represent 60% of its total stock holdings, while Foreign stocks represent the remaining 40% of its stock holdings. Home also holds a short position in Foreign bonds equal to 3% of GDP, while its bond holdings are completely biased towards Home bonds [15]. The steady-state portfolio is a function of the parameters of the model, including the variances of the shocks, the degree of risk aversion and the persistences of the

[14] Also Berriel and Bhattarai (2012) find the same result.

[15] As documented by Lane and Shambaugh (2009) and by Coeurdacier and Gourinchas (2011), there is large heterogeneity across countries in the currency denomination of their bond holdings. Anyhow, they find that advanced countries mainly invest in their own currency debt.

shocks.

The steady-state variances of the three shocks that hit each economy generate different hedging motives that affect the steady-state portfolio in an important way.

The volatility of endowments has at the same time several effects: It decreases the covariance between Home stocks and Home bonds, and also the covariance between Home consumption and Home bond returns. It instead increases the covariance between Home consumption and Home stock returns. The reduction in the covariance between Home asset returns could induce agents to keep in their portfolio a larger amount of Home bonds and Home stocks, but, at the same time, the increase in the covariance between Home consumption and Home stock returns could be an incentive for agents to hold a larger share of Foreign stocks and a larger share of Home bonds. The numerical solution shows that actually this last incentive prevails and the optimal portfolio is shifted towards Foreign stock holdings and Home bond holdings when the volatility of endowments is higher than the other volatilities.

Government spending volatility increases the covariance between Home bonds and Home stocks but, at the same time, reduces the covariance between Home consumption and Home asset returns (including both bonds and stocks). Overall, the numerical results suggest that agents use Home stocks to smooth consumption and increase their holdings of Foreign bonds to diversify their portfolio. A relatively high government spending volatility then has a tendency to reduce the steady-state share of Foreign stocks and to increase the share of Foreign bond holdings.

Finally, preference shock volatility increases the covariance between Home stocks and Home bonds, the variance of Home bond and Home stock returns, and it also increases the covariance between Home consumption and Home asset returns. Since the volatility of preferences acts exactly in the same way on consumption and on asset returns and increases their covariance, the only feasible strategy for Home agents is to increase their holdings of Foreign assets. The numerical results in fact show that there is an increase in the steady-state share of Foreign equity and Foreign bonds when the volatility of the preference shock is relatively high.

My findings suggest that the degree of risk aversion affects in a smooth way the steady-state portfolio. As risk aversion increases, the steady-state portfolio is characterized by a larger proportion of holdings of Foreign equity and a smaller fraction of holdings of Foreign bonds. The risk aversion parameter mostly affects the way in which consumption reacts to shocks. When risk aversion increases, the elasticities of consumption to endowment, government spending and demand shocks change. There is in fact a slight reduction in the elasticities of consumption to endowment and preference shocks, η_{cY} and η_{cv}, while there is a stronger increase in the elasticity of consumption to government spending shocks, η_{cG}. Overall there is an increase in the response of consumption to the three shocks. Since the returns of stocks and bonds do not depend on risk aversion, the change in the reaction of consumption to shocks also determines an increase in the covariance between consumption and assets' returns. As a result, as risk aversion increases, Home stocks reduce their ability to be a good hedging instrument for consumption risk and, as a consequence, the steady-state portfolio is shifted towards Foreign equity. Since the covariance between asset returns is positive and independent on risk aversion, the increase in the holdings of Foreign stocks is balanced by a reduction in the holdings of Foreign bonds.

The degree of persistence of the shocks is another factor that affects the steady-state portfolio composition. An increase in the persistence of the endowment shock increases the share of Home holdings of Foreign equity, while it decreases the share of Home holdings of Foreign bonds. This happens because the elasticity of consumption to the endowment shock increases as well as the elasticity of stock returns to endowment shocks. The larger covariance between consumption and stock returns shifts the optimal portfolio towards Foreign stock holdings. Optimal Foreign bond holdings go down even if the covariance between bond and stock returns is reduced.

An increase in the persistence of government spending shocks, instead, increases steady-state holdings of Foreign bonds and reduces steady-state holdings of Foreign equity. This happens because the elasticity of consumption to government spending shocks goes down, while the elasticity of stock returns to the same shock goes up. The elasticity of bond returns is instead unchanged. This implies that there is an increase in the covariance between Home stock and Home bond returns and a reduction in the covariance between Home stock returns and Home consumption. Home stocks become a better hedging instrument than Foreign stocks. The larger covariance between Home asset returns determines a reduction in the share of Home bonds and an increase in the share of Foreign bonds in the steady-state portfolio.

Finally, when the persistence of preference shocks is stronger, the steady-state portfolio increases its bias toward the holdings of Foreign equity and follows an inverse U shape for the holdings of Foreign bonds. A more persistent shock produces an increase in the reaction of consumption to preference shocks and, at the same time, increases the elasticity of stock returns to preference shocks. Foreign stock holdings increase, while Foreign bond holdings are reduced when ϕ^v is equal or larger than 0.6. This happens because when ϕ^v is larger than 0.6, Home stock returns start increasing in a much stronger way and, with them, also the covariance between bond and stock returns. This justifies the willingness of agents to hold a portfolio characterized by a larger share of Home bonds.

5.2.2 Portfolio Dynamics

Portfolio dynamics are the main focus of this paper. Foreign holdings of U.S. assets changed a lot during the crisis. International holdings of U.S. stocks in 2007 inverted the increasing trend that they had been following, going from 20% of U.S. GDP in 2007 to 12% of U.S. GDP in 2008. In the same year international holdings of U.S. Treasuries went from 4.5% of U.S. GDP to 6% of U.S. GDP. The idea of the exercise is to understand the qualitative and quantitative effect that uncertainty can have on portfolio dynamics. When you look at asset holding dynamics, it is worth noting that there is a difference between the benchmark two-asset case and the more complicated four-asset case. In the two-asset case, the shock to the state variables on the one hand, and its variance on the other, play two clearly different roles in portfolio dynamics. Both the shock and the variance generate portfolio dynamics, but they are independent from each other. In the two-asset case, the steady-state portfolio is a weighted average of the product of consumption and return elasticities to the shocks, where the weights are the variances of the shocks themselves. When there is an increase in the variance of one of the shocks, let's say shock i, the related hedging motive becomes more important because the co-movement between consumption and the return generated by shock i gets a bigger weight, and this determines portfolio changes. A time-varying variance changes over time the weight given to these elasticities and, therefore, it requires an adjustment in the optimal portfolio. This means that the time-varying risk biases the portfolio in a particular direction, until when the shock vanishes and the variance goes back to its initial value. In the four-asset case, instead, time-varying variances affect asset holdings in various ways. Besides generating the effects that we also observe in the simpler two-asset case, changes in the variances of the shocks also interact with the size of the shocks. The interaction term depends on the elasticity of consumption to the shock, on the covariance between bond and stock returns, and on the variance of these returns.

In what follows, I analyze the reaction of the steady-state portfolio when the economy experiences increases in the degree of volatility. I use the time-varying volatilities of endowment, government spending and preference shocks as a proxy of uncertainty.

When the uncertainty shock hits endowment volatility, on impact Home holdings of Foreign equity increase by 1.5%. As time passes and the shock vanishes, they smoothly decrease until they go back to their steady-state (Figure 4). Foreign bond holdings instead decrease after an increase in the volatility of the endowment shock. When the shock hits, Home agents short-sell Foreign bonds and their holdings of Foreign bonds are reduced by 0.15%. As variance goes back to its steady-state, Foreign bond holdings go back to their initial value (Figure 6). The speed at which equity and bond holdings go back to their steady-state depends on the value of the autocorrelation of the uncertainty shock: More persistent uncertainty shocks slow down the convergence of the portfolio to its steady-state (Figure 5, Figure 7). An increase in the risk associated to endowment shocks reduces the covariance between bond and stock returns, increases the covariance between Home consumption and Home stock returns and reduces the co-movement between Home consumption and Home bond returns. Even if the co-movement between bonds and stocks is reduced, the consumption smoothing motive prevails and we observe a contemporaneous increase in Home holdings of Foreign stocks and a reduction in Home holdings of Foreign bonds. The increase in the value of the risk-aversion coefficient σ increases the size of portfolio deviations from its steady-state. When the coefficient equals 10 agents increase their holdings of Foreign stocks by 4% and reduce those of Foreign bonds by 0.4%. In order to understand the contribution of uncertainty shocks to portfolio dynamics, I calculated the deviation of asset holdings from their steady-state when the economy is only hit by an endowment shock and the volatility is not time-varying. The impact of an endowment shock on portfolio dynamics is much smaller than before: Home holdings of Foreign stocks increase by 0.01%, while the reduction in Home holdings of Foreign bonds is 0.002%. When the economy is hit by an endowment shock the direction taken by the portfolio is similar to the one observed with uncertainty shocks, but endowment uncertainty amplifies portfolio dynamics.

An increase in the volatility of government spending shocks induces agents to increase Home equity holdings. Foreign equity is in fact reduced by 4% and Foreign bond holdings are increased by 0.15%, as shown in figure 8, figure 9. A government spending shock increases relative Home prices, and therefore Home bond returns and Home stock returns also increase. Since government spending has a negative effect on consumption, an increase in the volatility of government spending shocks reduces the covariance between consumption and both asset returns. The variance of both asset returns goes up, but bond returns become more volatile than stock returns. Therefore agents use Home equity as a hedging instrument to smooth consumption and increase their holdings of Foreign bonds, in order to have assets that do not co-move. When the risk aversion parameter is set to 10, there is a reduction in holdings of Foreign stocks equal to 9% and an increase in the holdings of Foreign bonds equal to 0.3%. Also in this case, uncertainty works as an amplifier in portfolio dynamics. In the absence of time-varying volatility, Foreign equity holdings would decrease only by 0.03%, while holdings of Foreign bonds would go up by 0.005%.

Finally an increase in the volatility of preference shocks results in an increase of both Foreign equity holdings and Foreign bond holdings: Foreign equity holdings increase by 0.8% and Foreign bond holdings go up by 0.08% (figure 10, figure 11). Preference shocks have a positive effect on consumption and on the terms of trade. The increase in the covariance between returns and consumption justifies the increase in Foreign bond and Foreign stock holdings. When σ is 10, Foreign stock holdings and Foreign bond holdings respectively increase by 2% and 0.2%. It is interesting to observe that in the absence of uncertainty shocks, a shock to preferences would still increase Home holdings of Foreign stocks by 0.01%, but it would actually reduce Foreign bond holdings by 0.002%. The absence of shocks to the volatility of preferences would keep the variance of bond returns constant, (instead of increasing it) and therefore agents would prefer to enjoy the increase in the terms of trade, by simply holding a larger amount of Home bonds.

The second order approximation of the model shows that an increase in the variances of the shocks increases the volatility of asset returns and, through them, the volatility of net foreign assets and consumption differentials. Intuitively, compatibly with the considerations done, agents change their portfolio holdings in order to counteract as much as possible the negative effects of higher volatilities on these macroeconomic variables.

An increase in the volatility of endowments increases the variance of Home stock returns and, through this channel, has a negative impact on consumption differentials. The second order approximation of consumption differentials shows that a higher volatility of asset returns reduces consumption differentials because agents prefer to save more and protect themselves from high expected future risk. Agents therefore reduce their exposure to the volatility of Home stock returns shifting their portfolio away from Home stocks and increasing their holdings of Foreign stocks. In other words, in this particular case, agents increase their foreign asset position in order to reduce their exposure to Home volatility. On the other hand, Home bond returns also experience an increase in their volatility. The choice of increasing their portfolio share is determined by the negative covariance of Home bond returns with consumption, but has the drawback of positively affecting the variance of Home consumption.

As already mentioned, government spending volatility increases the volatility of both stock and bond returns. However, since government spending shocks have a negative effect of consumption, agents reduce their holdings of only one of the two assets issued by Home. The numerical analysis shows that actually Home bond returns are more volatile than Home stock returns after an increase in government spending volatility. Consistently with this, we observe that agents reduce their exposure to government spending volatility through a reduction in Home bond holdings and a contemporaneous increase in Foreign bond holdings.

Finally, the negative effect of preference volatility on consumption differential cannot be reduced because Foreign asset returns suffer exactly the same degree of volatility, since the preference shock is common to the two countries. Agents' portfolio choice is only determined by the positive covariance between Home consumption and Home asset returns.

The final effect on net foreign assets will depend on the joint change in stock and bond holdings. In order to measure this change we would need at least a third order approximation of the whole model, but intuitively it seems reasonable to think that also in this case the source of uncertainty will matter a lot in the change of net foreign assets. We have observed, in fact, that changes in stock holdings are much stronger than changes in bond holdings. Presumably endowment volatility shocks and preference shocks determine increases in the net foreign asset position of a country, while shocks to government spending volatility should produce changes in net foreign assets with the opposite sign. [16]

[16]Fogli and Perri (2010) find a positive relation between external imbalances and macroeconomic volatility. In their paper, what they call macroeconomic volatility is equivalent to what I call endowment volatility. Their finding therefore seems in line with the results of my model.

6 Conclusion

During the financial crisis that first hit the U.S. economy and soon became a world crisis, investors reduced their holdings of foreign equities, and, at the same time, they increased their holdings of short-term U.S. government bonds. The paper analyzes, within the context of a DSGE model, the hedging properties of foreign bond and foreign equity holdings during a crisis, when the degree of uncertainty is high. Uncertainty is here described as a shock that makes the variances of the model time-varying. Uncertainty shocks are a source of portfolio dynamics, together with deviations of the state variables from their steady-state.

The model is able to qualitatively replicate the change in portfolios observed during the crisis only if uncertainty comes from aggregate demand. As world government spending volatility increases, it is optimal to increase the share of foreign bond holdings and reduce the share of foreign equity holdings. An increase in the variance of the government spending shock increases the volatility of bond returns and the covariance of bond and stock returns, while it reduces the covariance between home consumption and home asset returns. Because of this agents increase their holdings of foreign bonds and, at the same time, increase their holdings of domestic equity. An increase in the volatility of endowments, instead, reduces the covariance between consumption and bond returns and it increases the one between consumption and stock returns. Agents, in response, increase their holdings of domestic bonds and those of foreign equity.

The model suggests that the movement of capital observed during the crisis is compatible with uncertainty shocks coming from the demand side of the market. These results find an important support in the theories that have been recently developed and that identify the collapse in demand as the main cause of the slump experienced by the U.S. and by many other economies during the crisis.

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A Appendix A

A.1 Steady-State Portfolio

As shown by Devereux and Sutherland (2006), a second-order approximation of the portfolio problem is sufficient to capture the different features of assets and tie down a solution for steady-state holdings of foreign assets. The symmetric non stochastic steady state of the model is used as the approximation point for non-portfolio variables. In steady state, $n\bar{f}a^H = n\bar{f}a^F = 0$, $\bar{\gamma}^H = \bar{\gamma}^F$, $\bar{Y} = \bar{G} + \bar{C}$ and $\beta = \frac{1}{\gamma}$. From the second-order approximation of the home country portfolio first order conditions, we get:

$$E_t[\hat{\gamma}^D_{t+1} + \frac{1}{2}(\hat{\gamma}^2_{H,t+1} - \hat{\gamma}^2_{F,t+1}) - \rho\hat{C}_{H,t+1}\hat{\gamma}^D_{t+1}] = 0 \tag{A.1}$$

with $\hat{\gamma}^D = \hat{\gamma^H_t} - \hat{\gamma^F_t}$.

Similarly, for the foreign country:

$$E_t[\hat{\gamma}^D_{t+1} + \frac{1}{2}(\hat{\gamma}^2_{H,t+1} - \hat{\gamma}^2_{F,t+1}) - \rho\hat{C}_{F,t+1}\hat{\gamma}^D_{t+1} - \hat{Q}_{t+1}\hat{\gamma}^D_{t+1}] = 0 \tag{A.2}$$

Subtracting (A.2) from (A.1) , we find one of the two equations that have to hold in equilibrium:

$$E_t[\hat{\gamma}^D_{t+1}(\hat{C}_{H,t+1} - \hat{C}_{F,t+1} - \frac{\hat{Q_{t+1}}}{\sigma})] = 0. \tag{A.3}$$

$$E_t(\hat{\gamma}^D_{t+1}) = -\frac{1}{2}E_t(\hat{\gamma}^2_{H,t+1} - \hat{\gamma}^2_{F,t+1}) + \rho\frac{1}{2}E_t[(\hat{C}_{H,t+1} + \hat{C}_{F,t+1} + \hat{Q}_{t+1})\gamma^D_{t+1}], \tag{A.4}$$

where Q that equals $\hat{P^F} - \hat{P^H}$ is the real exchange rate. We can follow the same procedure for bonds too:

$$E_t(\hat{R}^D_{t+1}(\hat{C}_{H,t+1} - \hat{C}_{F,t+1} - \frac{\hat{Q_{t+1}}}{\sigma})) = 0, \tag{A.5}$$

$$E_t(\hat{R}^D_{t+1}) = -\frac{1}{2}E_t(\hat{R}^2_{H,t+1} - \hat{R}^2_{F,t+1}) + \rho\frac{1}{2}E_t[(\hat{C}_{H,t+1} + \hat{C}_{F,t+1+\hat{Q}_{t+1}})\hat{R}^D_{t+1}]. \tag{A.6}$$

The optimal value of portfolio holdings can be found by solving the first order accurate behavior of $(\hat{C}_{H,t+1} - \hat{C}_{F,t+1})$, $\hat{\gamma}^D_{t+1}$, and $\hat{R}^D_{t+1}$. This requires a first-order accurate solution of the non-portfolio equations of the model, as shown by Devereux and Sutherland (2006). The non-portfolio parts of the model are represented by the law of motion for net foreign assets and by the first order conditions with respect to equities and labor supply. The first-order approximation of the home budget constraint, around a symmetric steady state with zero net foreign assets, is:

$$\hat{nfa}^H_{t+1} + \hat{C}^H_t = y\hat{Y}^H_t - g\hat{G}^H_t + \hat{p}^{HH}_t - \hat{P}^H_t + \frac{1}{\beta}\hat{nfa}^H_t + \frac{a^{HF}}{\beta C}(\hat{\gamma}^F_t - \hat{\gamma}^H_t) + \frac{b^{HF}}{\beta C}(\hat{R}^F_t - \hat{R}^H_t) \tag{A.7}$$

where $g = \frac{G}{C}$ and similarly $y = \frac{Y}{C}$. a^{HF} and b^{HF} represent the steady-state values of foreign holdings of home shares and bonds. Taking differences between the domestic and foreign budget constraint and using the condition $\hat{nfa}^H_t + \hat{nfa}^F_t - \hat{Q}_t = 0$, we get:

$$\hat{nfa}^H_{t+1} = \frac{y}{2}\hat{Y}^D_t + \frac{1}{\beta}\hat{nfa}^H_t - \frac{1}{2}\hat{C}^D_t - \frac{g}{2}\hat{G}^D_t + \tilde{a}\gamma^D_t + \hat{Tot}_t + \hat{Q}^D_t + \tilde{b}\hat{R}^D_t \tag{A.8}$$

where $\tilde{a} = -\frac{a^{HH}}{\beta C}$, $\tilde{b} = -\frac{b^{HH}}{\beta C}$ and $\hat{X}^D_t = \hat{X}^H_t - \hat{X}^F_t$. Terms of trade, Tot are defined as $\hat{p}^H - \hat{p}^F$. The first-order approximations of the first order conditions in differential terms are:

$$\hat{C}_t^D = E_t(\hat{C}_{t+1}^D), \tag{A.9}$$

$$E_t(\hat{\gamma}_{t+1}^D) = 0, \tag{A.10}$$

$$E(\hat{R}_{t+1}^D) = 0, \tag{A.11}$$

The state-space solution is characterized as follows:

$$n\hat{f}a_{t+1}^H = \eta_{aa} n\hat{f}a_t^H + \eta_{aY^D}\hat{Y}_t^D + \eta_{aG^D}\hat{G}_t^D + \eta_{a\xi}\hat{\xi}_t + \eta_{a\vartheta}\hat{\vartheta}_t + \eta_{av}\hat{v}_t \tag{A.12}$$

$$\hat{C}_t^D = \eta_{C^D a} n\hat{f}a_t^H + \eta_{C^D Y^D}\hat{Y}_t^D + \eta_{C^D G^D}\hat{G}_t^D + \eta_{C^D \xi}\hat{\xi}_t + \eta_{C\vartheta}\hat{\vartheta}_t + \eta_{C^D v}\hat{v}_t \tag{A.13}$$

where $\hat{\xi}_t = \tilde{a}\hat{\gamma}_t^D$ and $\hat{\vartheta}_t = \tilde{b}\hat{R}^D$.

$$\hat{\gamma}_t^D = \eta_{\gamma^D \varepsilon^Y}\varepsilon_t^{Y^D} + \eta_{\gamma^D \varepsilon^G}\varepsilon_t^{G^D} + \eta_{\gamma^D \varepsilon^v}\varepsilon_t^v \tag{A.14}$$

$$\hat{R}_t^D = \eta_{R^D \varepsilon^A}\varepsilon_t^{Y^D} + \eta_{R^D \varepsilon^G}\varepsilon_t^{G^D} + \eta_{R^D \varepsilon^v}\varepsilon_t^v \tag{A.15}$$

Substituting (A.12), (A.13), (A.14) and (A.15) into (A.3) and (A.5), we find the system of two equations that have to be solved in order to find the optimal portfolio (a_1^*, b_1^*)

$$E_t[(\eta_{C^D a} n\hat{f}a_{t+1}^H + \eta_{C^D Y^D}\hat{A}_{t+1}^D + \eta_{C^D G^D}\hat{G}_{t+1}^D + \eta_{C^D \xi}\hat{\xi}_{t+1} + \eta_{C^D \vartheta}\hat{\vartheta}_{t+1} + \eta_{C^D v}\hat{v}_{t+1})$$
$$(\eta_{\gamma^D \varepsilon^Y}\varepsilon_{t+1}^{Y^D} + \eta_{\gamma^D \varepsilon^G}\varepsilon_{t+1}^{G^D} + \eta_{\gamma^D \varepsilon^v}\varepsilon_{t+1}^v)] = 0 \tag{A.16}$$

$$E_t[(\eta_{C^D a} n\hat{f}a^H_{t+1} + \eta_{C^D Y^D}\hat{Y}^D_{t+1} + \eta_{C^D G^D}\hat{G}^D_{t+1} + \eta_{C^D \xi}\hat{\xi}_{t+1} + \eta_{C^D \vartheta}\hat{\vartheta}_{t+1} + \eta_{C^D v}\hat{v}_{t+1})$$

$$(\eta_{R^D \varepsilon^A}\varepsilon^{Y^D}_{t+1} + \eta_{R^D \varepsilon^G}\varepsilon^{G^D}_{t+1} + \eta_{R^D \varepsilon^v}\varepsilon^{v}_{t+1})] = 0 \qquad \text{(A.17)}$$

The two following equations that will determine the steady state value of bond and equity holdings:

$$\eta_{C^D Y^D}\eta_{\gamma\varepsilon^Y}\sigma^2_Y + \eta_{C^D G^D}\eta_{\gamma\varepsilon^G}\sigma^2_G + \eta_{C^D v}\eta_{\gamma^D \varepsilon^v}\sigma^2_Y + \eta_{C^D \xi}\tilde{\alpha}(\eta^2_{\gamma\varepsilon^A}\sigma^2_A + \eta^2_{\gamma\varepsilon^G}\sigma^2_G + \eta^2_{\gamma\varepsilon^v}\sigma^2_v)$$

$$+\eta_{C\vartheta}\tilde{b}(\eta_{\gamma\varepsilon^A}\eta_{R\varepsilon^A}\sigma^2_Y + \eta_{\gamma\varepsilon^G}\eta_{R\varepsilon^G}\sigma^2_G + \eta_{\gamma\varepsilon^v}\eta_{R\varepsilon^v}\sigma^2_v) = 0 \qquad \text{(A.18)}$$

$$\eta_{C^D Y^D}\eta_{R\varepsilon^A}\sigma^2_Y + \eta_{C^D G^D}\eta_{R\varepsilon^G}\sigma^2_G + \eta_{C^D \xi}\tilde{\alpha}(\eta_{\gamma\varepsilon^Y}\eta_{R\varepsilon^Y}\sigma^2_A + \eta_{\gamma\varepsilon^G}\eta_{R\varepsilon^G}\sigma^2_G + \eta_{\gamma\varepsilon^v}\eta_{R\varepsilon^v}\sigma^2_v)$$

$$+\eta_{C\vartheta}\tilde{b}(\eta^2_{R\varepsilon^Y}\sigma^2_Y + \eta^2_{R\varepsilon^G}\sigma^2_G + \eta^2_{R\varepsilon^v}\sigma^2_v) = 0 \qquad \text{(A.19)}$$

The two equations (A.18) and (A.19) are functions of the parameters of the model and of the variances of the shocks.

B Appendix B

In what follows I show the analytical solution for the optimal portfolio and its dynamics, when there are uncertainty shocks in the simpler two-asset case. This is an infinite horizon, two-country open economy model. There are two ex-ante symmetric countries,"Home" (H) and "Foreign" (F), each one populated by a representative household who consumes and trades a portfolio of financial assets. There are two different goods: H good and F good. Final consumption is a CES aggregate of the two goods. The international portfolio consists of two assets: Home equity and Foreign equity. I allow for two types of shocks in each country: endowment shocks and government spending shocks. I also allow for uncertainty shocks: The variances of both endowment and government spending shocks are indexed by time. They stochastically move period by period according to their autoregressive processes. The number of shocks is larger than the number of assets available: This implies that financial markets are incomplete.

B.1 Preferences

The household in country $i = H, F$ chooses consumption C^i to maximize the intertemporal utility function:

$$\max E_t \sum_{s=0}^{\infty} \beta^s \left(\frac{(C^i_{t+s})^{1-\sigma}}{1-\sigma}\right) \tag{B.1}$$

with coefficient of relative risk aversion $\sigma > 0$. The Home final consumption basket combines home and foreign goods:

$$C^H_t = [1/2^{1/\theta}(c^{HH}_t)^{(\theta-1)/\theta} + 1/2^{1/\theta}(c^{FH}_t)^{(\theta-1)/\theta}]^{\theta/(\theta-1)} \tag{B.2}$$

where c^{ij} is the amount of consumption good produced by country i and consumed by country j. The Foreign consumption basket is:

$$C_t^F = [1/2^{1/\theta}(c_t^{FF})^{(\theta-1)/\theta} + 1/2^{1/\theta}(c_t^{HF})^{(\theta-1)/\theta}]^{\theta/(\theta-1)}. \tag{B.3}$$

The aggregate consumer price index for Home, P_t^H, is defined as:

$$P_t^H = [1/2(p_t^H)^{1-\theta} + 1/2(p_t^F)^{1-\theta}]^{\frac{1}{1-\theta}}, \tag{B.4}$$

where p_t^H and p_t^F are the nominal prices of final home and foreign goods, denominated in the Home currency. Similarly, Foreign price index in Home currency is defined as:

$$P_t^F = [1/2(p_t^F)^{1-\theta} + 1/2(p_t^H)^{1-\theta}]^{\frac{1}{1-\theta}}. \tag{B.5}$$

In this economy the law of one price holds. Purchasing Power Parity (PPP) also holds, since I assume no home bias.

B.2 Stochastic Processes

The endowment and government spending shocks follow an AR(1) process:

$$\hat{Y}_{t+1}^D = \phi^Y \hat{Y}_t^D + \varepsilon_{t+1}^{Y^D} \tag{B.6}$$

$$\hat{G}_{t+1}^D = \phi^G \hat{G}_t^D + \varepsilon_{t+1}^{G^D} \tag{B.7}$$

where Y^D and G^D denote the log of the endowment and the government spending shocks. $\hat{Y}^D$ is the difference between the Home endowment of Home good and the Foreign endowment of Foreign good. Similarly, $\hat{G}^D$ is the difference between Home and Foreign government spending. The innovations to the log processes (B.6) and (B.7), $\varepsilon_{t+1}^{Y^D}$ and $\varepsilon_{t+1}^{G^D}$, are identically and independently distributed with mean zero and variance σ_{t+1}^{2Y} and σ_{t+1}^{2G}, respectively.

Furthermore, I assume that the variance of the shocks is time-varying. When an uncertainty shock hits the economy at time $t+1$, the variances of $\varepsilon_{t+1}^{Y^D}$ and $\varepsilon_{t+1}^{G^D}$ depart from their steady-state and follow a mean reverting process:

$$\sigma_{t+1}^{2Y} = \rho^Y \sigma^{2Y} + (1-\rho^Y)\sigma_t^{2Y} + \eta^Y u_{t+1}^Y \tag{B.8}$$

$$\sigma_{t+1}^{2G} = \rho^G \sigma^{2G} + (1-\rho^G)\sigma_t^{2G} + \eta^G u_{t+1}^G \tag{B.9}$$

where u_{t+1}^Y and u_{t+1}^G are identically and independently distributed processes with mean zero and unitary variance. Since ρ^Y and ρ^G are smaller than 1, as time passes, the variances converge to their steady-states σ^{2Y} and σ^{2G}[17].

The budget constraint for the home economy at time t is:

$$nfa_{t+1}^H + \frac{\varphi}{2}(nfa_{t+1}^H)^2 = (Y_t^H - G_t^H)\frac{p_t^H}{P_t^H} - C_t^H + \gamma_t^F nfa_t^H + (\gamma_t^F - \gamma_t^H)q_{t-1}^H S_t^{HF} \tag{B.10}$$

[17]As in Benigno et al. (2011), I assume that the exogenous state variables follow conditionally-linear stochastic processes, where the variances of the primitive shocks follow stochastic linear processes

where nfa^H are net foreign assets, and they are defined as:

$$nfa_{t+1}^H = q_t^F S_{t+1}^{FH} - q_t^H S_{t+1}^{HF}. \tag{B.11}$$

S^{ij} stands for the stock issued by country i and held by country j, while q^i is the price of the stock issued by country i. γ^H and γ^F are the returns on the stocks issued by Home and Foreign and they are defined in terms of the consumption good:

$$\gamma_t^H = \frac{(P_t^H q_t^H + p_t^H Y_t^H)}{P_t^H q_{t-1}^H}, \tag{B.12}$$

and

$$\gamma_t^F = \frac{(P_t^H q_t^F + p_t^F Y_t^F)}{P_t^H q_{t-1}^F}, \tag{B.13}$$

where the dividend paid by country's i equity depends on the endowment of that country in that period. Foreign agents solve a similar portfolio problem and their budget constraint is:

$$nfa_{t+1}^F + \frac{\varphi}{2}(nfa_{t+1}^F)^2 = (Y_t^F - G_t^F)\frac{p_t^F}{P_t^F} - C_t^F + \gamma_t^H nfa_t^F + (\gamma_t^H - \gamma_t^F) q_{t-1}^H S_t^{HF}, \tag{B.14}$$

where nfa^F are net foreign assets for the Foreign country, and they are defined as:

$$nfa_{t+1}^F = q_t^H S_{t+1}^{HF} - q_t^F S_{t+1}^{FH}. \tag{B.15}$$

As in Devereux and Sutherland (2006), I define the Foreign holdings of Home equity:

$$a_t = q_{t-1}^H S_t^{HF} \tag{B.16}$$

B.3 Market Clearing Conditions

The resource constraints are

$$C_t^{HH} + C_t^{HF} + G_t^H = Y_t^H \tag{B.17}$$

$$C_t^{FH} + C_t^{FF} + G_t^F = Y_t^F \tag{B.18}$$

The market clearing condition for Home stocks is:

$$S_t^{HH} + S_t^{HF} = 1. \tag{B.19}$$

The market clearing condition for Foreign equity is:

$$S_t^{FF} + S_t^{FH} = 1. \tag{B.20}$$

B.4 Optimality Conditions

The first-order conditions for country i with respect to equities are:

$$C_t^{i^{-\sigma}} = \beta E_t(C^{i^{-\sigma}}_{t+1} \gamma_{t+1}^H), \tag{B.21}$$

$$C_t^{i^{-\sigma}} = \beta E_t(C^{i^{-\sigma}}_{t+1} \gamma_{t+1}^F), \tag{B.22}$$

As shown by DS (2006), a second-order approximation of the portfolio problem is sufficient to capture the different features of assets and tie down a solution for steady-state holdings of foreign assets. The symmetric non stochastic steady state of the model is used as the approximation point for non-portfolio variables. In steady state, $nfa^H = nfa^F = 0$, $\gamma^H = \gamma^F$, $Y = G + C$ and $\beta = \frac{1}{\gamma}$. From the second-order approximation of the home country portfolio first-order conditions, we get:

$$E_t[\hat{\gamma}^D_{t+1} + \frac{1}{2}(\hat{\gamma}^2_{H,t+1} - \hat{\gamma}^2_{F,t+1}) - \sigma \hat{C}_{H,t+1}\hat{\gamma}_{D,t+1}] = 0 \tag{B.23}$$

with $\hat{\gamma}_D = \hat{\gamma}^H_t - \hat{\gamma}^F_t$. Similarly, for the foreign country:

$$E_t[\hat{\gamma}^D_{t+1} + \frac{1}{2}(\hat{\gamma}^2_{H,t+1} - \hat{\gamma}^2_{F,t+1}) - \sigma \hat{C}_{F,t+1}\hat{\gamma}_{D,t+1}] = 0. \tag{B.24}$$

Subtracting (B.24) from (B.23) , we find the equations that have to hold in equilibrium:

$$E_t[\hat{\gamma}^D_{t+1}(\hat{C}_{H,t+1} - \hat{C}_{F,t+1})] = 0 \tag{B.25}$$

$$E_t(\hat{\gamma}^D_{t+1}) = -\frac{1}{2}E_t(\hat{\gamma}^2_{H,t+1} - \hat{\gamma}^2_{F,t+1}) + \sigma\frac{1}{2}E_t[(\hat{C}_{H,t+1} + \hat{C}_{F,t+1})\gamma_{D,t+1}] \tag{B.26}$$

Taking a first order approximation of Home and Foreign budget constraints and subtracting the Foreign budget constraint from the Home one, yields the following law of motion for Home net foreign assets:

$$n\hat{f}a^H_{t+1} = \frac{y}{2}\hat{Y}^D_t + \frac{1}{\beta}n\hat{f}a^H_t - \frac{1}{2}\hat{C}^D_t - \frac{g}{2}\hat{G}^D_t + \tilde{a}\gamma^D_t + \frac{y-g}{2}\hat{p}^D \tag{B.27}$$

where

$$\tilde{a} = -\frac{a}{\beta C} \tag{B.28}$$

a being the steady-state value of a_t as defined in (B.16). An increase in the volatility of Home endowment increases Foreign asset's holdings proportionally to the product of the elasticity of consumption to endowment η_{CY} and the elasticity of the return to the endowment shock $\eta_{\gamma\varepsilon^Y}$. Home endowment shocks have a positive effect on Home consumption and on Home returns. The increase in Home endowment volatility increases the covariance between Home consumption and Home asset returns and makes the Foreign asset a better hedging instrument against the endowment risk. Similarly, an increase in Home government spending uncertainty modifies Home holdings of the Foreign asset proportionally to the elasticity of government spending to consumption η_{CG} and the elasticity of government spending to the Home asset return $\eta_{\gamma\varepsilon^G}$. For reasonable parameter values, the sign of η_{CG} is negative, as the intuition would suggest. An increase in government spending crowds out private spending. The fact that returns positively react to government spending shocks implies that their covariance with consumption is negative: An increase in the variance of Home government spending further reduces the covariance between consumption and Home returns. Home assets are therefore a good hedge against government spending volatility. The state-space solution is characterized as follows:

$$n\hat{f}a_{t+1}^{H} = n\hat{f}a_{t}^{H} + \eta_{aY^D}\hat{Y}_t^D + \eta_{aG^D}\hat{G}_t^D + \eta_{a\xi}\hat{\xi}_t \tag{B.29}$$

$$\hat{C}_t^D = \eta_{C^D a} n\hat{f}a_{t}^{H} + \eta_{C^D Y^D}\hat{Y}_t^D + \eta_{C^D G^D}\hat{G}_t^D + \eta_{C^D \xi}\hat{\xi}_t \tag{B.30}$$

$$\gamma_t^D = \eta_{\gamma\varepsilon_Y}\varepsilon_t^{Y^D} + \eta_{\gamma\varepsilon_G}\varepsilon_t^{G^D} \tag{B.31}$$

where $\hat{\xi}_t = \tilde{a}\hat{\gamma}_t^D$. The DS solution for $\tilde{a}$ is:

$$\tilde{a} = -\frac{\eta_{CY^D}\eta_{\gamma\varepsilon^Y}\sigma^{2Y} + \eta_{CG^D}\eta_{\gamma\varepsilon^G}\sigma^{2G}}{\eta_{C\xi}(\eta^2_{\gamma\varepsilon^Y}\sigma^{2Y} + \eta^2_{\gamma\varepsilon^G}\sigma^{2G})}. \tag{B.32}$$

B.5 Portfolio Dynamics

In this section I discuss the role of time-varying variances in portfolio dynamics. DS (2010a) show that a combination of third-order approximations of the portfolio equations and second-order approximations of the rest of the model delivers the path followed by the portfolio when the economy is hit by a shock. It is first of all necessary to notice that the dynamics generated by the shocks are different from the ones generated by their volatility. As shown in Benigno et al. (2011), in the presence of uncertainty shocks the second-order approximation is sufficient to capture the effects of uncertainty. First-order portfolio dynamics can be then generated by a combination of second and third-order approximations of the model. The third-order approximation of the portfolio equations, as obtained in DS (2010a), is:

$$E_t[-\sigma\hat{C}^D_{t+1}\hat{\gamma}^D_{t+1} + \frac{\sigma^2}{2}(\hat{C}^{D(2)}_{t+1}\hat{\gamma}^D_{t+1}) - \frac{\sigma}{2}(\hat{C}^D_{t+1}\hat{\gamma}^{D(2)}_{t+1})] = 0, \tag{B.33}$$

where $\hat{C}^{D(2)}$ and $\hat{\gamma}^{D(2)}$ are the second-order approximations of consumption and return differentials. The second-order approximation of the differential in consumption is:

$$\begin{aligned}\hat{C}^{D(2)}_t = \eta_{C^Da}n\hat{f}a^H_t + \eta_{C^DA^D}\hat{Y}^D_t + \eta_{C^DG^D}\hat{G}^D_t + \eta_{C^D\xi}a\hat{\gamma}^D_t + \\ \eta_{C^Da\gamma^D}n\hat{f}a_t\hat{\gamma}^D_t + \eta_{C^DA^{D(2)}}\hat{Y}^{D(2)}_t + \eta_{C^DG^{D(2)}}\hat{G}^{D(2)}_t + \eta_{C^D\xi^{(2)}}\hat{\xi}_t\end{aligned} \tag{B.34}$$

where ξ_t is now the product between the time-varying portfolio and the first-order approximation of the returns, $\hat{a}_t \hat{\gamma^D}_t$. The second-order approximation of the return differential is instead:

$$\hat{\gamma}_t^{D(2)} = \eta_{\gamma\varepsilon^Y} \varepsilon_t^Y + \eta_{\gamma\varepsilon^G} \varepsilon_t^G + \eta_{\gamma\varepsilon^{Y(2)}} \sigma_t^{2Y} + \eta_{\gamma\varepsilon^{G(2)}} \sigma_t^{2G} \tag{B.35}$$

Plugging (B.34) and (B.35) in (B.33) and solving for $\hat{a}_t$ leads to the following expression:

$$\hat{a}_t = \frac{(3-\sigma)(\eta_{CY}\eta_{\gamma\varepsilon^y} E_t(\sigma_{t+1}^{2Y}) + \eta_{CG}\eta_{\gamma\varepsilon^G} E_t(\sigma_{t+1}^{2G}))}{\sigma\eta_{C^D\xi^{(2)}}(\eta_{\gamma\varepsilon^Y}^2 E_t(\sigma_{t+1}^{2Y}) + \eta_{\gamma\varepsilon^G}^2 E_t(\sigma_{t+1}^{2G}))} + \tag{B.36}$$
$$\frac{1}{\eta_{C^D\xi^{(2)}}}(\frac{3-\sigma}{\sigma}\eta_{C\xi}\tilde{a} - \eta_{Ca} n\hat{f}a_{t+1} + \frac{1}{\sigma}(\phi^G \hat{G}_t^D \eta_{CG} + \phi^A \hat{Y}_t^D \eta_{CY}))$$

where, $E_t(\sigma_{t+1}^{2Y}) = [\rho^Y \sigma^{2Y} + (1-\rho^Y)\sigma_t^{2Y}]$ and $E_t(\sigma_{t+1}^{2G}) = [\rho^G \sigma^{2G} + (1-\rho^G)\sigma_t^{2G}]$.

The resulting expression for the portfolio dynamics $\hat{a}_t$ shows that besides the sources of dynamics already identified by the literature (i.e., the state variables of the model, nfa, G and Y) also the time-varying variances of the model can generate portfolio dynamics around the steady-state.

C Tables and Figures

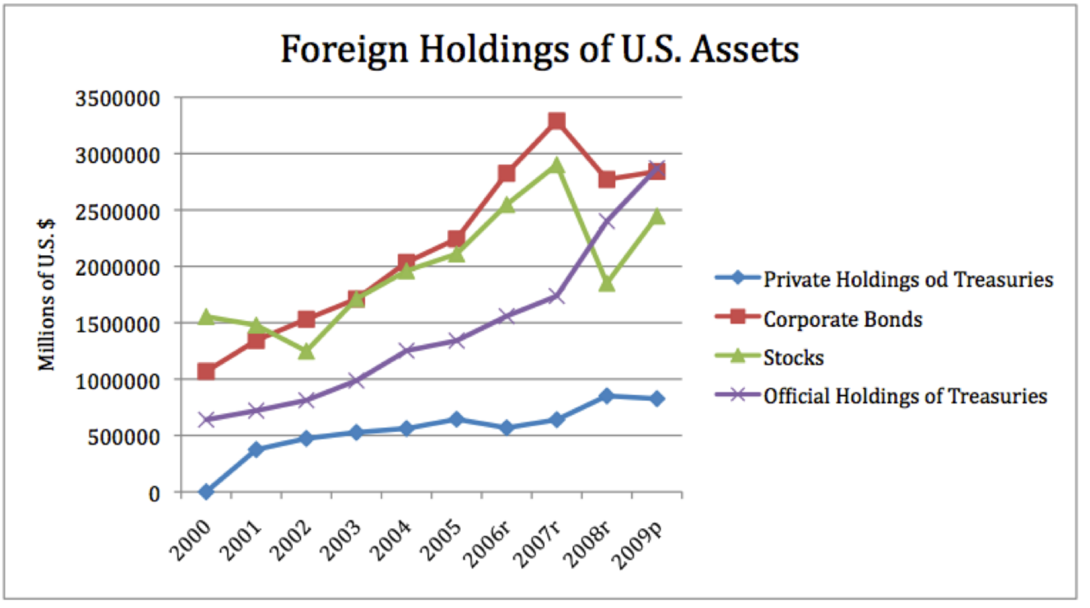

Figure 1: Foreign holdings of U.S. assets

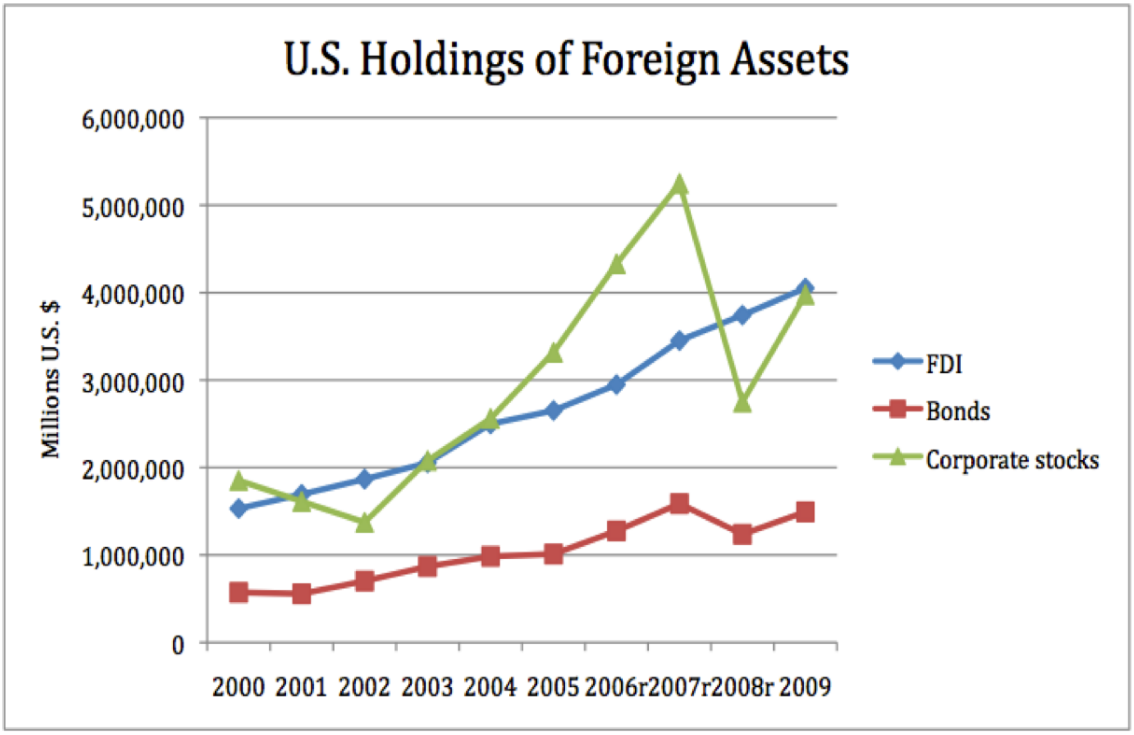

Figure 2: U.S. holdings of foreign assets

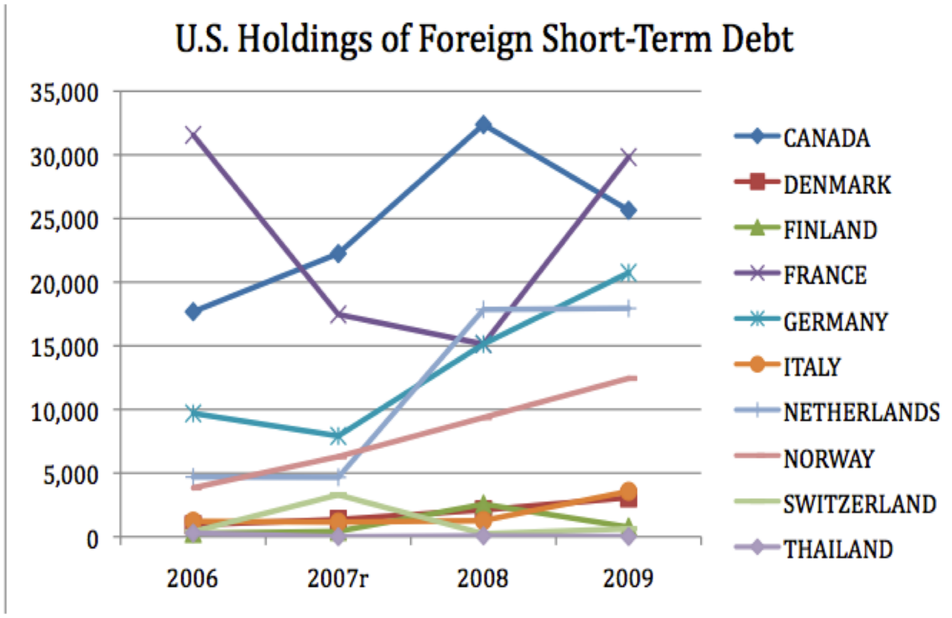

Figure 3: U.S. holdings of foreign short-term bonds

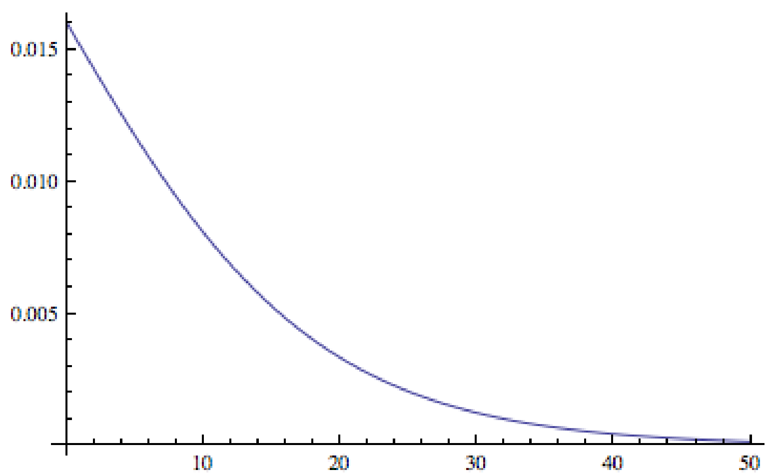

Figure 4: Home Holdings of Foreign stocks when uncertainty hits endowment, big autocorrelation

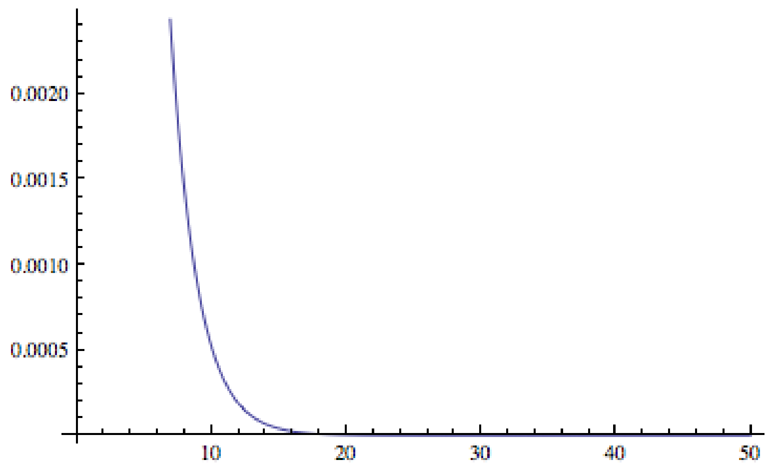

Figure 5: Home Holdings of Foreign stocks when uncertainty hits endowment, small autocorrelation

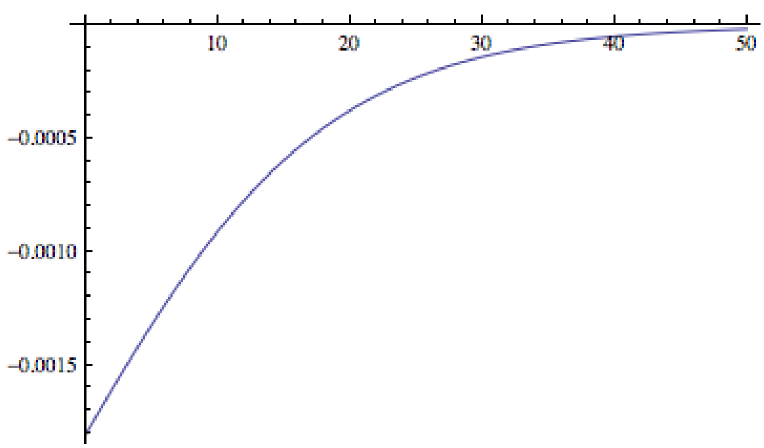

Figure 6: Home Holdings of Foreign bonds when uncertainty hits endowment, big autocorrelation

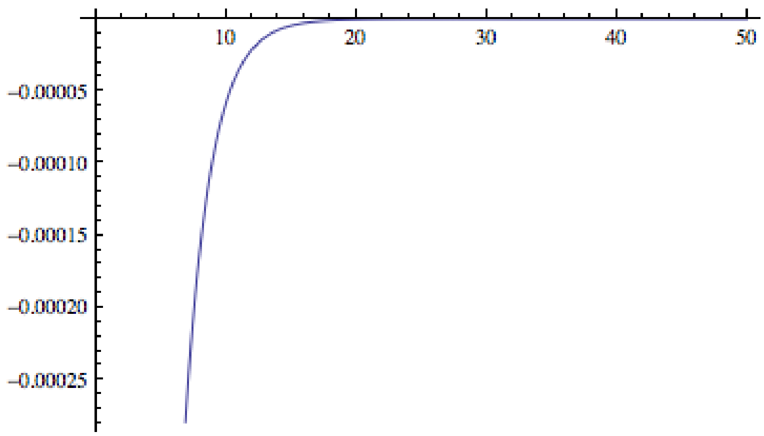

Figure 7: Home Holdings of Foreign bonds when uncertainty hits endowment, small autocorrelation

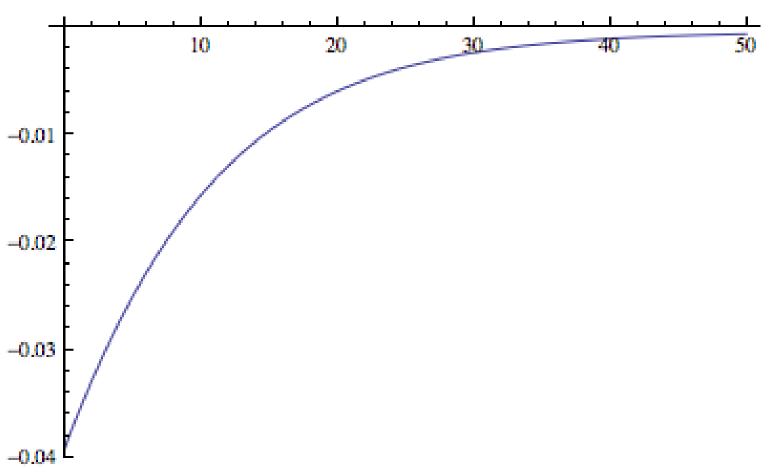

Figure 8: Home Holdings of Foreign stocks when uncertainty hits government spending

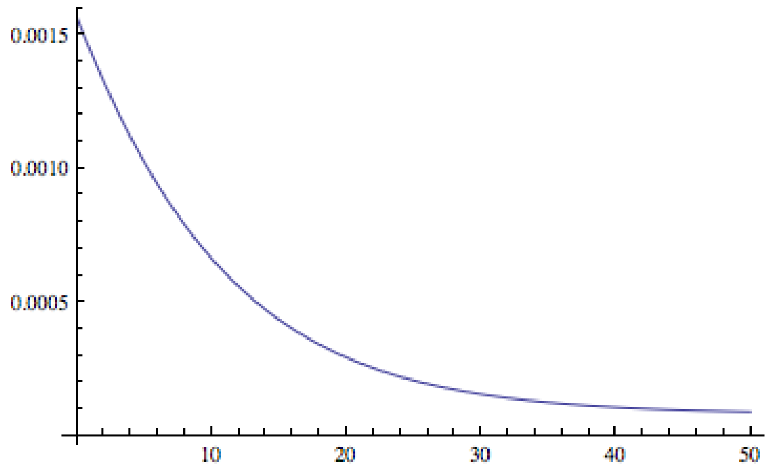

Figure 9: Home Holdings of Foreign bonds when uncertainty hits government spending

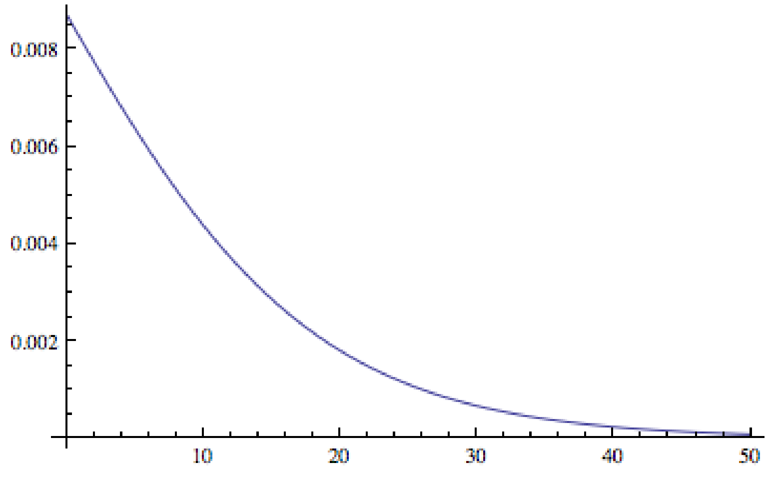

Figure 10: Home Holdings of Foreign stocks when uncertainty hits preferences

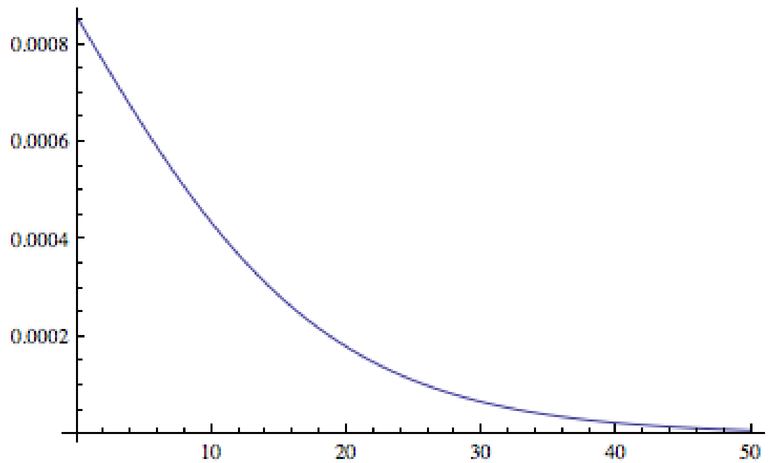

Figure 11: Home Holdings of Foreign bonds when uncertainty hits preferences

Table 1: Parameter Values

	Parameters	*Values*
discount factor	β	0.96
variance Y	σ^{2Y}	0.01^2
variance G	σ_G^2	0.01^2
variance v	σ_v^2	0.01^2
persistence Y	ϕ^y	0.66
persistence G	ϕ^g	0.9
persistence v	ϕ^v	0.66
elasticity of substitution	θ	2
risk aversion	σ	4
persistence σ^{2Y}	ρ^Y	0.9
persistence σ^{2G}	ρ^G	0.9
persistence σ^{2v}	ρ^v	0.9