International Interest Rates and Housing Markets

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Abstract

Current account deficits and housing prices have shown a strong positive correlation from the mid-90s to 2007. This paper studies the effect of a decrease in the international interest rate and in the downpayment requirement to buy a house during that period on the joint behavior of the current account and housing prices. To this end, I build a life-cycle heterogeneous agents, small open economy model with two goods: tradable (non-housing) and non-tradable (housing). I calibrate the model to replicate selected aggregate statistics of the U.S. economy and I compute the transition after the decrease in the interest rate and in the downpayment. The model is able to match some relevant facts: the boom and the bust (after 2007) in the housing market, where the bust happens without a reversal in the interest rate, as data show; the increase in the homeownership rate; the simultaneous boom - and bust - in non housing consumption; and the coexistence of borrowing from abroad with a current account deficit along the transition.

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1 Introduction

"In my view... it is impossible to understand this crisis without a reference to the global imbalances in trade and capital flows that began in the latter half of the 1990s."
Ben S. Bernanke (2009)

Current account deficits and housing prices have been positively correlated between the mid-90s and 2007[1]. This period has been characterized by: first, the huge size of housing market booms compared with previous experiences, see for example André (2010); and second, the existence of "global imbalances"[2], a particular event of this period of time. In this paper, I argue that these two facts are related and help us to explain the overall dynamics in house prices and consumption in the last 20 years.

Figure 1 and Figure 2 show the evolution of employment and value added in construction, respectively, for the U.S., Spain, Germany and Japan[3]. Both variables were increasing for Spain and the U.S., and decreasing for Germany and Japan. Hosing prices, in Figure 3, follow the same pattern. Figure 4 depicts "global imbalances" for these four countries. Summarizing this evidence, countries with current account deficit, Spain and the U.S., experienced a housing market boom and the opposite is true for Japan and Germany.

Most of the literature hinge on preference shocks to the demand for housing to generate a housing boom in the economy, and so account for this positive correlation. See for example Punzi (2012), Ferrero (2011), or Gete (2010). These papers develop a two country model in which one of the economies experiences a positive shock to the demand for housing. The implications are: in one hand, an increase in the expenditure share of housing together with a decrease in non-housing consumption; and, in the other hand, an increase in the international

[1] See Punzi (2007), Aizenman and Jinjarak (2009), André (2010), Gete (2010), or Ferrero (2011).

[2] Global imbalances are large and persistent current account deficits run by some countries (e.g. the U.S., Spain) and, simultaneously, current account surpluses in other countries (e.g. Japan, Germany, emerging Asia, some oil exporting countries). See Obstfeld and Rogoff (2010).

[3] More countries could be added as it has been documented by others. See Punzi (2007), Aizenman and Jinjarak (2009), André (2010), Gete (2010), or Ferrero (2011) for the same evidence in other OECD countries, non-OECD, or emerging economies.

interest rate after the shock. Furthermore, the implications for the housing bust in these economies, that occur after a reversal in the preference shock, would imply a current account surplus, an increase in non-housing consumption, and so a bust in housing prices.

However, these predictions are inconsistent with empirical evidence. As Figure 4 shows, current account deficit start to decrease but is still big and negative during the years of the bust. Figure 5 shows personal consumption expenditures in the U.S. from the beginning of the 90s to 2010. Non-housing consumption mimicked the housing market boom and bust (and also by a decrease in savings during the boom period, see André (2010) or Iacoviello (2011)) with an annual growth rate of around a 3% over all the period before 2007, and negative growth after 2007.

Another important characteristic over those years is a big increase in the homeownership rate experienced in the economies where housing market boomed. This component of the housing demand means an extensive margin increase in the demand for housing. As it can be seen in Table 1, homeownership rates increased in Spain and U.S. and was fairly constant in Japan and Germany. The papers mentioned above do not model the housing tenure decision. The existence of rental markets accounts for the proportion of the economy with no access to borrowing since owned houses can be used as collateral for credit. So, new homeowners will have access to credit.

In this paper I develop a theory of housing boom and bust with the following ingredients: small open economy, to analyze shocks to the international interest rate; life-cycle heterogeneous agents model, with housing tenure decision, in order to account for the extensive margin increase in housing demand; and residential land, modeled following Davis and Heathcote (2005) and needed to produce new houses. As Davis and Heathcote (2007) shows, land governs housing price dynamics.

Two channels fuel a housing boom in this environment: cheap credit and financial innovation. Both of them as given in the model.

The first channel is a decreasing trend in international interest rates. As Figure 6 shows, there was a permanent decrease in interest rates during the period under analysis. The reason for this decrease has been studied in some papers together with the existence of global imbalances. Caballero et al. (2008) or Mendoza et al. (2009) study, under different hypothesis, the observed fact of low interest rates, and attribute it to the huge savings showed up in the international capital markets during the 90's[4]. As Caballero et al. (2008) claims, the long-run real interest rate has been steadily declining over the last decade, despite the efforts from

[4] As the "saving glut" hypothesis (Bernanke (2005)) suggests.

central banks to rise interest rates - the "Greenspan's Conundrum". So, I will assume that, because of exogenous reasons from the point of view of developed economies, international interest rates went down over that period.

The second channel is institutional. Strong financial innovation (development of housing equity withdrawal, subprime loans, development of securitization), liberalization in mortgage markets, and government support to increase the homeownership rate, implied a decrease in the downpayment requirement to buy a house in the U.S.[5]. Table 2 shows evidence for the loan-to-value ratio (LTV) in the U.S. These data refers to LTV ratios for first time home-buyers, the marginal buyers most affected by borrowing constraints. The increase in the LTV ratios was huge over all the period in the U.S..

A housing boom in the model presented in this paper, after a decrease in the interest rate and in the downpayment requirement to buy a house, will imply an increase in housing demand - at both margins -, together with an increase in real house prices and labor in construction. The decrease in the downpayment requirement to buy a house makes possible to some renters to become homeowners. The economy will borrow from abroad because borrowing becomes cheaper and because more expensive houses are used as collateral. This produces a current account deficit. The economy will move labor from the tradable (non-housing) to the non-tradable (housing) sector and will run a trade deficit to fulfill the demand for non-housing consumption. As time goes by, and households start to reach their desired stock of housing, the demand for new houses cools down, decreasing housing prices and labor in construction: a bust in the housing market. Non-housing consumption decreases for two reasons: the economy must pay its debt; and the decrease in housing prices makes homeowners poorer than before. This is consistent with a boom in non-housing consumption and its bust. Moreover, the bust in the economy happens without a reversal in low interest rates. Thus, the model also provides some insights for the bust period based on fundamentals.

Now, I offer a brief review of the literature. Gete (2010) argues that preference shocks and a desire for smooth consumption (across goods) can generate a correlation between house prices and capital inflows. He shows that consumption smoothing across tradable (non-housing) goods and non-tradable (housing) goods can lead to a positive correlation between house prices and current account deficits. With an exogenous increase in the home country preference for housing, productive inputs in the home country are reallocated toward housing production, so that housing consumption can rise.

Other papers, like Punzi (2012) or Ferrero (2011), also rely on higher domestic demand

[5]See for example Chambers et al. (2009), Doms and Krainer (2007), or Green and Wachter (2005).

to drive both house prices and capital inflows in the same direction, but they do so through different mechanisms. For example, Punzi (2012) investigates the ability of borrowing constraints with housing as collateral to account for this negative correlation after the preference shock. Punzi (2012) also studies other shocks such as an increase in the loan-to-value ratio and productivity shocks.

There are three papers, to the best of my knowledge, similar to this one in adopting a small open economy model and analyzing an interest rate shock. Adam et al. (2011) departs from rational expectations in an asset pricing model with learning. They find that real house prices and current account deficits will rise together. On the contrary, my model is perfect foresight and generates a boom and a bust - with no reversal in the interest rate - in housing markets.

Kiyotaki et al. (2011), and Garriga et al. (2012) aim to explain housing prices but not the joint behavior of current account balances and housing market variables. Kiyotaki et al. (2011) is quantitatively-oriented and their model would need a reversal in the interest rate to generate a bust in housing prices. The main difference to the recent paper Garriga et al. (2012) is that they follow Adam et al. (2011) and the bust is explained by a reversal in the beliefs about future interest rates. Another important difference is that they develop a representative agent model without rental markets. In the model presented here I show how, even with rational expectations, the economy would generate a bust in housing markets and this can be consistent with the current account evolution.

There are alternative channels in the literature to generate a housing boom like the interaction between credit market conditions and house prices. For example, in Favilukis et al. (2010), house prices rise in the boom period because of a relaxation of credit constraints and a decline in housing-related transactions costs, both of which reduce risk premia. Conversely, the reversal of the financial market liberalization raises housing risk premia and causes the housing bust.

This paper is also connected to the literature trying to explain the evolution of the homeownership rate. In some papers, for example Chambers et al. (2009), the decrease in the downpayment requirement in a close economy would increase interest rate with, even, a decrease in the homeownership rate. As I will show, in a small open economy, a decrease of the downpayment can explain the increase in the homeownership rate over this period, even after an increase in housing prices, because international interest rates can not be affected.

The rest of the paper is organized as follows. The model economy is presented in Section 2. In Section 3, the benchmark model is calibrated to the U.S. economy. Simulations are performed and discussed in Section 4, and a brief summary concludes the paper in Section 5.

2 The Model Economy

I investigate a life-cycle small open economy model populated by heterogeneous agents with three sectors of production. The model strategy follows Gervais (2002), Díaz and Luengo-Prado (2008) and Díaz and Luengo-Prado (2010) in studying different issues of the demand for housing in life-cycle closed economies. The main differences stem from the focus of this paper on the ability of an open economy to run a trade deficit; in the feature that housing consumption is considered as a non tradable good and non housing consumption as a tradable one; and in the existence of three sectors of production: consumption/tradable sector, residential structures sector, and housing/non tradable sector. Housing sector supplies new non tradable houses in the economy by combining residential structures and land where land is a fixed factor of production.

2.1 Households

2.1.1 Preferences

The economy is populated by overlapping generations of individuals with finite lives and age $j \in \{1, ..., J\}$. The utility function in period t of a new born individual is a CES utility function over consumption of housing services (x_t^j) and non housing consumption goods (c_t^j):

$$\sum_{j=1}^{J} \beta^{j-1} u(c_{t+j-1}^j, x_{t+j-1}^j) = \sum_{j=1}^{J} \beta^{j-1} \frac{\left(((1-\theta)(c_{t+j-1}^j)^{\frac{\varepsilon-1}{\varepsilon}} + \theta(x_{t+j-1}^j)^{\frac{\varepsilon-1}{\varepsilon}})^{\frac{\varepsilon}{\varepsilon-1}}\right)^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}}$$

where $x_t^j = f_t^j + h_t^j$ is housing services, f_t^j being services coming from renting a house, and h_t^j being housing capital (services coming from owning). Renting and owning are perfect substitutes. I assume that one unit of housing capital generates one unit of services, $x_t^j = h_t^j$. c_t^j is non housing consumption.

2.1.2 Housing Capital and Housing Services

An individual must pay in advance at least a minimum downpayment requirement to buy (and thus own) a house. This downpayment requirement is given by a fraction γ of the value of the house. The remaining cost can be financed by borrowing against the house, with $(1-\gamma)$ giving the maximum loan-to-value ratio. Housing capital is subject to some degree of indivisibility. This is modeled by assuming a minimum size of housing investment, $\underline{h}$.

An individual can rent housing services as an alternative to be a home-owner. Renting housing services has two advantages over owning: first, it allows individuals to consume housing services and thus avoid the downpayment requirement; and second, rent houses are not subject to the same indivisibility as owner-occupied housing.

The price of one unit of housing services in terms of consumption goods is denoted by $r_t^f p_t^h$. Where p_t^h is the price of a house in terms of consumption and r_t^f represents the fraction of that price that an individual has to pay for renting.

Housing capital depreciates at rate δ_h. But rented houses depreciates at a rate δ_f, where $\delta_f > \delta_h$. The different depreciation cost is a result of a moral hazard problem that occurs in rental markets as renters decide on how intensely to utilize a house. The market rate for rental services will incorporate the moral hazard problem and renters have to pay a premium reflecting the additional maintenance cost.

2.1.3 Income Dynamics

The life of each individual consists of a working period and a retirement period. These stages are separated by an exogenously given mandatory retirement age, denoted by j^*. Individuals are endowed with one unit of working time in each period of their working lives which they supply inelastically to the both sectors using labor as an input. An age-j individual's unit of working time is transformed into z_j efficiency units of labor. Each unit of effective labor is paid the wage rate ω_t. This specification allows for individuals to differ both across and within generations. At any point in time, the average productivity level is fixed at unity. The measure of the entire population is also normalized at unity.

During the retirement period ($j > j^*$) households receive a retirement pension, b_t, until the end of their lives. The retirement pensions are paid through the social security system and collected from income taxes on labor.

Another source of income comes through individuals asset holdings. Individuals accumulate wealth because life-cycle reasons and to meet the downpayment required to buy a house. Individuals have access to three assets to accumulate wealth: housing, business capital equity, and deposits at financial institutions.

From a household's perspective, deposits at financial institutions and business capital equity are equivalent. A zero-profit condition guarantees that the rates of return on these assets are equalized. As a result, the sum of deposits at financial institutions and business capital equity constitute a single financial asset, denoted a_t^j.

Homeowners also receive rents from the proportion of total land they have and needed to residential development. This proportion of land is exogenously given each period from the point of view of homeowners. And the amount of land they have is a proportion of their housing stock. This issue is explained in more detail in Appendix A.

2.2 Financial Institutions

Financial institutions receive individuals deposits and make use of it for three activities: finance loans issued to homeowners, purchase residential capital, and borrow/lending in the international capital market. Financial institutions use the same linear technology as homeowners to produce housing services, which they rent out to individuals who do not own a house. Financial institutions borrow from/lend to abroad through the possibility of accessing international capital markets given a fixed interest rate, r_t^*. They have to satisfy the demand for credit from individuals at this interest rate.

Financial institutions, just as homeowners, also receive rents from the amount of land suitable for residential investment they have. This amount is proportional to the stock of housing they buy for renting. See Appendix A for an explanation.

2.3 Technology

2.3.1 Residential Structures and Consumption Sector

Output by residential structures and the consumption sector is produced using a Cobb-Douglas production technology in each sector:

$$f^i(K_t^i, N_t^i) = A_i(K_t^i)^{\alpha_i}(N_t^i)^{1-\alpha_i}$$

where $i \in \{c, s\}$ refers to a specific sector, with c being consumption sector and s residential structures sector. A_i is a technology parameter, K_t^i is the total amount of business capital used in each sector and N_t^i represents the share of the working population employed in each sector. The capital factor share, α_i, is different for each sector with residential structures sector being more labor intensive, $\alpha_s < \alpha_c$. Each period the stock of business capital depreciates at a rate δ_k. The price of residential structures in terms of the consumption good, which is normalized at unity, is equal to p_t^s.

I assume perfect mobility of factors between sectors such that wages (ω_t) and the price for business capital (r_t^k) is equal in both sectors.

2.3.2 Housing Sector

I am following Davis and Heathcote (2005) in modeling houses. I assume that a constant acreage of new land suitable for residential investment is sold by homeowners and financial institutions to the firms producing houses. In order to produce new houses residential investment must be combined with land. Homeowners and financial institutions own an amount of the acreage of land proportional to the housing stock they own. Each period one new acreage of land appears and it is sold to firms. This acreage is normalized to one.

Real estate developers combine new residential structures with newly-availabe land to produce new houses according to a Cobb-Douglas technology:

$$f^h(X_t^s, L_t) = (X_t^s)^{1-\phi}(L_t)^\phi$$

where X_t^s is the total amount of residential structures used in the production of new houses and L_t represents the amount of land employed. The share of land in the production of new houses is denoted by ϕ. The price of a new house and the price of land, both in terms of consumption good, are represented by p_t^h and p_t^l, respectively.

2.4 Government Expenditure

τ_y is a proportional tax rate on labor income and the return on financial assets. Each period the entire proceed from taxation on financial assets is used to finance government expenditures. The entire proceed form taxation on labor income is given back to individuals when they are retired as a pension. The government thus maintains a balanced budget every period.

2.5 Household's Decision Problem

Households decide consumption (c_t^j), housing services (x_t^j), the housing capital stock for the next period (h_{t+1}^{j+1}) and the amount of financial assets (a_{t+1}^{j+1}) by solving this problem:

$$v_t^j\left(a_t, h_t; i\right) = \max_{\left\{c_t^j, x_t^j, h_{t+1}^{j+1}, a_{t+1}^{j+1}\right\}} \left\{u(c_t, x_t) + \beta v_{t+1}^{j+1}\left(a_{t+1}, h_{t+1}; i\right)\right\} \tag{1}$$

$$s.t. \quad c_t + r_t^f p_t^h f_t + a_{t+1} + p_t^h h_{t+1} \leq$$

$$\leq z^i(1-\tau_y)w_t + b_t^{j>j^*} + (1+(1-\tau_y)r_t^a)a_t + (1-\delta_h)p_t^h h_t + p_t^l l(h_t, h_{t+1}) \tag{2}$$

$$a_{t+1} \geq -(1-\gamma_t)p_t^h h_{t+1} \tag{3}$$

$$h_t \geq \underline{\mathrm{h}} \ otherwise \ h_t = 0 \tag{4}$$

$$x_t^j = f_t^j + h_t^j \tag{5}$$

Equation (2) is the budget constraint. The term $p_t^l l(h_t, h_{t+1})$ in the budget constraint represents that households hold land in this economy. Households have the proportion of land relative to the amount of their housing capital[6]. Equation (3) is the borrowing constraint. Equation (4) is a constraint for the minimum house size available in the housing market. Equation (5) is the value of housing services.

So, in this environment owning is preferred to renting because of three reasons: first, there is a preference tax treatment for saving in housing rather that in financial assets; second, owning a house allows households to borrow using their housing stock as collateral; and third, the depreciation rate of a rented house is bigger that the one for a house owned, as it is explained in section 2.1.2..

2.6 Financial Institutions' Decision Problem

Financial intermediaries issue loans each period and buy residential capital using the proceeds from deposits they accept and by accessing international capital markets. They have access to the international capital markets through a bond at an international interest rate. They receive payments for rental accommodations, from selling land to the housing sector and receive the interest on loans issued, and pay interests on deposits and on international bonds. The problem of a new financial institution in period t is as follows:

$$\begin{aligned} \Psi(F_t, B_t, A_t, K_t) &= \max_{\{F_{t+1}, B_{t+1}, A_{t+1}, K_{t+1}\}} \Big\{ r_t^f p_t^h F_t + p_t^l l(F_t) + X_t^A + \\ + r_t^k K_t - X_t^K &- p_t^h X_t^f - X_t^B + \frac{1}{1+r_t}\Psi(F_{t+1}, B_{t+1}, A_{t+1}, K_{t+1}) \} \end{aligned}$$

$$s.t. \quad X_t^K + p_t^h X_t^f + X_t^B \leq r_t^f p_t^h F_t + p_t^l l(F_t) + \ X_t^A + r_t^k K_t \tag{6}$$

[6] An explanation of how households hold the proportion of land relative to their homes is offered in Appendix A.

$$X_t^A = A_{t+1} - (1 + r_t^a)A_t$$

$$X_t^B = B_{t+1} - (1 + r_t^*)B_t$$

$$X_t^f = F_{t+1} - (1 - \delta_f)F_t$$

$$X_t^K = K_{t+1} - (1 - \delta_k)K_t$$

where F is the amount of houses rented to households, A is the deposits by households, B is the international borrowing/lending and K is the business capital rented to firms.

From this problem I get the dynamics for the rental price:

$$r_t^f p_t^h = (1 - \phi)[(1 + r_t)p_{t-1}^h - (1 - \delta_f)p_t^h]$$

and the zero-profit condition: $r_t = r_t^* = r_t^a = r_t^k$.

2.7 Recursive Competitive Equilibrium

I am interested in the transition of the model since I need to replicate a trade deficit together with borrowing from abroad as it was the case for the U.S. during the period under study. To replicate both facts at the same time it is necessary to evaluate the transition since in a steady state, and without any kind of exogenous growth, a country will be paying debt interest through exports with a small enough international interest rate. For a definition of the steady-state competitive equilibrium see Appendix B.

Denote $q = \{a_t, h_t, i\}$, q ∈ Q.

Definition *A* ***recursive competitive equilibrium*** *for a given government policy, τ_y, downpayment requirement, γ_t, and an age-dependent measure of agents type, $\lambda_j(q)$, is a collection of relative prices $\left\{p_t^h, p_t^s, p_t^l, r_t^f, r_t, w_t\right\}$, a collection of functions for the household problem $\left\{v_t^j(q), c_t^j(q), f_t^j(q), h_t^j(q), a_t^j(q)\right\}$, a value function for financial institutions $\Psi\left(F_t, B_t, A_t, K_t\right)$, and aggregate quantities for the whole economy $\left\{Y_t^c, Y_t^h, Y_t^s, X_t^s, L_t, K_t^c, K_t^s, N_t^c, N_t^s, F_t, B_t, A_t\right\}$ such that:*

1. *Inputs are priced competitively every period.*

2. *Given* τ_y, γ_t *and prices, the functions* $\left\{v_t^j(q), c_t^j(q), f_t^j(q), h_t^j(q), a_t^j(q)\right\}$ *solve the dynamic program from the household problem.*

3. *Given prices and the function* $\Psi(F_t, B_t, A_t, K_t)$, $\{F_{t+1}, B_{t+1}, A_{t+1}, K_{t+1}\}$, *solves the financial institutions' problem.*

4. *Individual and aggregate decisions are consistent:* $C_t = \sum_{j=1}^{J} \int_Q c_t^j d\lambda_j(q)$, $H_t = \sum_{j=1}^{J} \int_Q h_t^j d\lambda_j(q)$, $F_t = \sum_{j=1}^{J} \int_Q f_t^j d\lambda_j(q)$, $A_t = \sum_{j=1}^{J} \int_Q a_t^j d\lambda_j(q)$.

5. *The government maintains a balanced budget every period:*

$$G_t + b_t = \sum_{j=1}^{J} \int_Q \left[\tau_y w z_t^j + \tau_y r a_t^j\right] d\lambda_j(q)$$

   where $b_t = \sum_{j=1}^{J} \int_Q b_t^j d\lambda_j(q) = \tau_y w_t \overline{N}_t$.

6. *Labor market clears every period:* $N_t^c + N_t^s = \overline{N}_t$.

7. *Capital market clears every period:* $K_t^c + K_t^s = K_t$.

8. *Land market clears every period:* $L_t = \overline{L}_t$.

9. *Residential structures market clears every period:* $X_t^s = Y_t^s$.

10. *Housing market clears every period:*

$$Y_t^h = X_t^h + X_t^f$$

   where $X_t^h = H_{t+1} - (1 - \delta_h) H_t$.

11. *Trade balance is determined every period:*

$$TB_t = Y_t^c - C_t - X_t^k - G_t$$

   where $X_t^k = K_{t+1} - (1 - \delta_k) K_t$.

12. *Net foreign asset position is determined every period:*

$$B_{t+1} = TB_t + (1 + r_t^*) B_t$$

2.7.1 Characterization

The price for structures becomes:

$$p_t^s = \frac{A_c}{A_s}\frac{\alpha_c^{\alpha_c}(1-\alpha_c)^{1-\alpha_c}}{\alpha_s^{\alpha_s}(1-\alpha_s)^{1-\alpha_s}}\left(\frac{w_t}{r_t+\delta_k}\right)^{\alpha_c-\alpha_s}$$

From this equation, after a shock to the international interest rate, the effect over the price of structures depends on the difference between the capital shares of the consumption sector and the residential structures sector ($\alpha_c - \alpha_s$). As it will be shown in the calibration section, it is true for the U.S. economy that the residential structures sector is more labor intensive than the consumption sector. This means that after an exogenous decrease in the international interest rate, wages will increase, since labor becomes relatively scarce, and as a consequence of the bigger capital share in the consumption sector, this decrease in the international interest rate will imply an increase in the price of residential structures.

Housing prices become:

$$p_{ht} = \frac{p_{st}^{1-\phi} p_{lt}^{\phi}}{(1-\phi)^{1-\phi}\,\phi^{\phi}}$$

From this equation for housing prices it can be inferred that after a positive shock (interest rate and downpayment requirement) to the demand for housing, an increase in the demand for land will occur, and, given the fixed supply of land in the economy, the price of land will increase. Both the increase in the price of structures and the increase in the price for land will increase housing prices in this model.

3 Calibration

The benchmark model is calibrated for the U.S. economy as a closed economy in my initial steady state. After the shocks I allow the economy to have access to international borrowing and lending and to have a trade balance different from zero. Thus, U.S. economy becomes a small open economy during the transition. I present the calibration of my benchmark economy in the following order: demographics and labor income distribution, technology, and preferences and market arrangements.

The calibration involves parameters associated with preferences ($\beta, \theta, \sigma, \varepsilon$), the income tax rate ($\tau_y$) as well as the downpayment fraction (γ), and parameters associated with technology ($A_c, A_c, \alpha_c, \alpha_s, \phi$) as well as depreciation rates ($\delta_h, \delta_f, \delta_k$). The distribution of productivity levels within and across generations also needs to be specified. The U.S. in 1994 is the target for the experiment.

3.1 Demographics and Labor Income Distribution

A model period is taken to correspond to one full year. Individuals are assumed to live for $J = 60$ model periods. One can think of members of a new generation as being born at real-life age 24 (model period one) and having an expected age of death of 83 years (60 model periods). The retirement age is set at age 63 (model period 40).

The distribution of productivity levels directly controls the labor endowment process and thus labor income. I calibrate this process using the CPS survey for 1994. More precisely, I calculate the mean labor income for each quintile in the data at each age and assign this value directly to the five individuals making up the population in the model. The normalized labor income profile for each individual, each representing a quintile, is shown in Figure 7.

The retirement age is obtained using the same data. The median labor income for the entire population becomes zero at age 63 (model age 40). When individuals become retired they start to draw their pension collected from taxes on labor income over their working life.

3.2 Technology

I need to construct measures of output by the consumption sector and by the residential sector, capital, the stock of houses and their investment counterparts $(Y^c, p^hY^h, K, p^h(H + F), X^k, p^h(X^h+X^f))$. I use data from the National Income and Product Accounts (henceforth NIPA) and the Fixed Assets Tables (henceforth FAT), both from the Bureau of Economic Analysis. I define capital as the sum of non-residential private fixed assets plus the stock of inventories plus consumer durables. Investment in capital, X^k, is defined accordingly[7]. $p^h(H + F)$ is private residential stock in the data and $p^h(X^h + X^f)$ is private residential investment. I define output in consumption sector as labor income plus income from non-residential capital, $Y^c = F(K^c, N^c) = wN^c+rK^c = C+I^k+G$, output in the residential sector is labor income plus income from non-residential capital plus land income $Y^h = F(X^s, L^c) = wN^s + rK^s + p^lL = I^h + I^f$, and total output is the sum of output from each final sector, $Y = Y^c + p^hY^h$, or measured GDP minus imputed housing services[8]. I do not make any imputation to output for government owned capital since our focus is on privately held wealth.

The business capital share for residential structures sector I use is $\alpha_s = 0.132$, and I take this from Davis and Heathcote (2005). I also take from Davis and Heathcote (2005) the land

[7] I include net exports in my measure of capital investment since the benchmark economy is a closed economy.

[8] C is output in the consumption sector minus the sum of investment in physical capital and government expenditures.

share in new housing, $\phi = 0.106$. Calculations in Davis and Heathcote (2005) are in the same context as the model presented here.

I proceed as Cooley and Prescott (1995) and calculate the implied share of capital in output in the consumption sector, which is $\alpha_c = 0.26$. The capital- output ratio (K/Y) is 1.66 and the housing-output ratio $((p^h(H+F))/Y)$ is 1.07[9]. I set the depreciation rate of capital, δ_k, so that it matches the investment to capital ratio in NIPA, $\delta_k = 0.12$. The implied steady state interest rate is 3.4 percent.

The value of the implied capital share in the consumption sector may seem low, but it is not very different from typical values in the literature when given as a function of GDP instead of output. GDP is output plus the imputed value of housing services: $GDP = Y + (r+\delta_h)p^h H + r^f p^h F$. The capital-GDP ratio (K/GDP) is 1.53, the housing-GDP ratio $(Kp^h(H+F))/GDP$ is 0.98, and the aggregate ratio $(K + p^h(H+F))/GDP$ is 2.51. The resulting share of capital income to GDP is 31.52 percent, just slightly lower than that estimated by Prescott (1986).

The technology level for consumption sector (A_c) and residential structures sector (A_s) is such that $A_s < A_c$ as suggested by evidence. I set $A_c = 2$ and $A_s = 0.9$ to replicate the aggregate ratio $(K + p^h(H+F))/GDP = 2.51$ and the housing-output ratio of 1.07.

The minimum house size is such that the homeownership rate in the economy is 64 percent[10]. With minimum size equal to 1.4775 for owner-occupied houses and given a downpayment fraction of 20 percent, the model replicates the homeownership rate for the U.S..

I borrow the values for the depreciation rates from Díaz and Luengo-Prado (2010), given that my benchmark economy is a closed economy and those values are consistent with general equilibrium. The values are $\delta_h = 0.0424$, $\delta_f = 0.0483$, and $\delta_k = 0.12$. I also borrow the income tax rate τ_y from Díaz and Luengo-Prado (2008) with a value of 0.2.

3.3 Preferences and Market Arrangements

The value of the discount factor is chosen to make the capital-output ratio equal to 1.66. It should be noted that capital refers to the total amount of capital, which includes housing and business capital, and that output corresponds to the sum of output goods and the value of housing services. The discount factor which achieves the desired capital-output ratio is 0.9757.

The share of housing services in total expenditures is controlled by θ. I set this parameter in order to replicate the ratio $p^h(H+F)/C$ in the U.S. economy. In the data this ratio is

[9] All figures I report are averages in NIPA/FAT for the sample period 1954-1994.

[10] Data from United States Statistical Abstract and Housing Vacancies and Homeownership (CPS/HVS) for 1994.

equal to 1.4, and the θ that matches it is 0.0765. Whit this value for θ I also replicate the ratio $p^h(H+F)/GDP$ in the U.S. of 0.98.

I set the risk aversion parameter in the utility function at $\sigma = 0.5$, since this is the value usually employed in business cycle literature. This parameter determines the inter-temporal elasticity of substitution. The last parameter referred to preferences is ε. This value determines the intra-temporal elasticity of substitution between consumption goods and housing services. Usually the literature on housing in closed economies uses a Cobb-Douglas utility function in order to reconcile the fact that different estimations of this parameter have a lot of variability in the literature, from above and to below one[11]. Recent studies that rely on structural analysis in order to estimate those parameters suggest that housing services and consumption goods are complements. See for example Bajari et al. (2010) or Li et al. (2009). So I set this parameter at a value smaller than one (and conduct some sensitivity analysis) in order to see how this value affect the results. I choose a conservative value of $\varepsilon = 0.9$, as in Gete (2010).

I require a minimum downpayment of 20 percent. Thus individuals can borrow up to 80 percent of the value of the house. While in reality households may be able to acquire houses with lower downpayment, it is also the case that these households face higher marginal borrowing costs (including a higher interest rate and the purchase of mortgage insurance). To keep the model tractable, the downpayment parameter is the same for all consumers and the borrowing rate is not a function of γ.

All parameter values for the benchmark calibration are summarized in Table 3.

4 Results

4.1 The Benchmark Economy

The typical behavior of individuals in the initial steady state can be broken down into three categories of individuals: poor and lower middle class (first and second quintile), middle-class (third quintile) and upper-middle class (fourth and fifth quintiles). The behavior of poor and lower middle-class individuals is quite simple: they hardly own their home, they set up cooperatives[12] in which a small proportion of households (around 17-20% in the first quintile and 50-55% in the second) own the house in which they live, and always consume a small amount of housing services. Middle-class individuals initially consume small amounts of

[11] See Fernandez-Villaverde and Krueger (2011) for a discussion.

[12] Explained in Appendix A.

housing services and consumption in order to save enough to eventually become homeowners. When they become homeowners, at age 33, they move into the smallest possible house. At that time, these individuals are constrained both by the downpayment and the minimum house size constraints. During their first few years as homeowners, they use all their extra income to increase consumption and pay down their mortgage. As they get wealthier, they eventually move into bigger houses.

The consumption level of young upper middle-class individuals is also constrained as they accumulate wealth to cover the downpayment on a house. After 2-3 years accumulating wealth, at age 29-30, they move into the largest house their downpayment can afford. Unlike lower middle-class individuals, they are not constrained by the minimum house size: they keep living in the largest house their downpayment can afford for 2-3 years. After this constrained period, they increase housing services and consumption.

All individuals accumulate assets during their productive years which they deplete to provide for consumption once retired instead of having a retirement pension. Homeowners thus partially revert to debt financing their house (rather than holding the entire asset as equity) in order to consume goods as well as housing services during the last few periods of their life. Without uncertainty, all individuals die with zero net worth.

4.2 Experiment

I evaluate the behavior of the model after a permanent and unanticipated shock to the international interest rate and to the downpayment requirement to buy a house. I want to evaluate the joint effect of these two shocks and identify the contribution of each one separately. Since the model is calibrated to the U.S. economy the size of the shocks will be taken from data for this country. As I explained before, I will evaluate the transition of the model after both shocks.

The initial steady state will be the closed economy case with trade balance and borrowing from abroad equal to zero. I will compute a long run case with the final values for the international interest rate and for the downpayment requirement in which the country will be borrowing from abroad and paying debt interests by exporting goods, so a sustainable debt. Then I will compute the transition between these two points and look at different variables. A detailed explanation of the computational procedure is in Appendix A.

I set the shock for the international interest rate from the value obtained in the initial steady state to 0.6 percentage points smaller. This value is the median of the annual decreases in the long run interest rate on government bond in U.S. since 1991 to 2007. I choose the median instead of the mean of the annual decreases because of the high variability over all the period in this interest rate. This value is taken from the data showed in Figure 6. So, my quantitative targets in this experiment will be the median of the changes in variables over all the period. Of course, it is work in progress to see how the model respond to a decrease over all the years but the main insights of the model should not change. Moreover, it is easier to understand the mechanisms of the model with a once and for all change in the exogenous variables.

I set the shock to the downpayment requirement in the same way, that is the median of its annual decrease over all the period. This will imply a fall from its value in the initial steady state of 0.2 to a 0.18. The size of these magnitude are in line with the evidence presented in the introduction.

4.2.1 The Joint Effect

In this section I investigate the effect of both shocks at the same time. Figure 8 shows both shocks, implemented once and for all.

Both shocks drive up the demand for housing services - at both margins. The interest rate decrease makes, in one hand, mortgages[13] cheaper, and, in other hand, more attractive to save in a house than in financial assets. The downpayment shock makes it easier to access to the housing market for households that would rent otherwise, and makes it possible to buy bigger houses for previous owners.

The first result is plotted in Figure 9 to show that the model is consistent with the evolution of housing prices and the current account balance. The model generates a housing prices boom in the period of the shock together with a current account deficit. The increase in housing prices accounts for 80% of the median annual increase in housing prices for the period under study. The model exaggerates the current account deficit (around a 10% of the GDP) in the period of the boom since there is no adjustment cost and all variables move freely in the period of the shock.

Figure 10 shows that the model is able to replicate the behavior of the extensive margin increase after both shocks even with an increase in housing prices. The increase in the homeownership rate in this period is around a 1% increase in line with the annual increase in the homeownership for the U.S. economy since 1994. The housing stock increases in the period of he boom and it is still increasing, at a smaller rate, after some periods. The increase in

[13]The model presented can be rewritten to include a simple mortgage into the definition of financial assets. See Gervais (2002).

housing prices raises the value of the collateral during the boom, and so, there is an increase in non-housing consumption because of two reasons: the increase in the value of the collateral and the decrease in the interest rate. As it can be seen, the increase in non-housing consumption is consistent with the annual median increase in personal consumption expenditures for the U.S. economy, around a 3%. The overshooting in non housing consumption is related with the small open economy assumption, but the quantitative response in the non housing consumption boom period is determined also by the increase in the value of the collateral. Thus, after the boom in non-housing consumption there is a bust because of two reasons: the debt must be paid, since it is a small open economy borrowing from abroad in the period of the boom; and because of a decrease in the value of the collateral, since housing prices start to decrease after the boom in the first period.

Figure 11 represent the ability of an open economy to replicate what is observed in data. An open economy moves resources from the tradable to the non-tradable sector (labor in construction), thus increasing the production of the non-tradable good (housing production). Since there is an increase in the demand for non-housing consumption, running a trade deficit allows the economy to satisfy this demand. Gete (2010) points out this result, but here the reason is the negative shock to the international interest rates and no a positive shock to the demand for housing.

In Figure 12 I show the decomposition of housing prices together with the rent-to-price ratio. As it can be seen, the price of structures increases once and for all. This is because a converse result of the Stolper-Samuelson theorem. After the decrease in the interest rate, the capital-labor ratio of the economy increases, causing wages to increase. Since structures are more labor intensive than the consumption sector, this increase in wages makes the price of structures increase. Land prices increase a lot in the period of the shock, and then start to decrease. The pressure on land is bigger in the period of the shock and then decreases gradually over time. This effect is what governs the evolution of land prices and is what move housing prices following the same pattern. The existence of land modeled as in Davis and Heathcote (2007) make it possible for housing prices to be demand driven.

Rent-to-price ratio decreases in the period of the boom and then increases following the non-arbitrage condition of the financial institutions. This evolution of the rent-to-price ratio is consistent with what we observe in the data for this ratio during the boom period. However, during the bust this price goes in the wrong direction.

It is important to note that the bust happens without a reversal in the interest rates after the boom period, but also without a reversal in the downpayment requirement. The reversal in the downpayment requirement would worsen the bust in housing market variables. If there is evidence for the interest rates to stay low after the bust period, it is also true that there is evidence of a reversal in the downpayment requirement to buy a house.

Figure 13 shows what happens to all the housing variables together and how the model is able to generate a boom and a bust in the housing market. The housing stock shows the biggest increase in the period of the shock, so increasing the pressure on land and increasing housing prices, the economy moves labor to the production of structures in this period. After this boom period, the pressure on land starts to decrease, thus decreasing housing prices and the production of new houses in the economy. Then, the bust happens. This is the mechanism behind the overshooting in housing prices. The evolution of Housing Production, i.e. investment in housing, is the one that puts pressure on land prices replicating the overshooting in this variable. For this reason housing prices drop to a level above the initial steady state. The investment in housing, in a steady state with lower interest rate and downpayment, is bigger than the investment in housing in the initial steady state. Once all new houses have been built, the depreciation of a bigger stock of housing must be built in a plot of land of an equal size each period.

Figure 14 shows how, during the bust period, there is a reversal in the trend of the current account but still a current account deficit. In the period of the shock, the model exaggerates the current account deficit but a big deficit (around 2.5% of GDP) still exists in the first period of the bust and during some periods ahead. The net foreign asset position is deteriorating until the final steady state is reached. Moreover, and consistent with the evidence for this period, the bust happens without a reversal in interest rates.

5 Conclusions

In this paper, I have studied the transition in a life-cycle heterogeneous agents small open economy model in which the economy goes from an initial steady state to a final one with low interest rates and low downpayment requirement to buy a house. The model is able to replicate qualitatively the evolution of the U.S. economy during the housing boom experienced in the 90s and to 2007, and also (qualitatively) the bust period after 2007.

The model is able to replicate some important facts of the U.S. economy such as the boom in non-housing consumption, the increase in the extensive margin demand for housing in this period, and the current account deficit together with borrowing from abroad along the transition. Two characteristics of the model contributes to its successful in replicating those facts: one is that demand drives housing prices in the economy, and the other is the life-cycle heterogeneous agents structure. Households want to build up their desired stock of housing and this generates the pressure over housing prices. The bust in housing prices happens when the pressure of the demand decreases. The degree of heterogeneity in the economy make some households able to access to the housing market because of the decrease in the downpayment requirement to buy a house. The decreasing interest rates allows households to get cheaper credit for consumption with the increasing market value of their homes.

The implication of these assumptions is a surge of housing and non-housing consumption at the beginning of the transition. Everybody would want an expensive house to get access to credit. When the the pressure of new houses over land decreases, the value of houses becomes down, debts must be paid, and the bust happens in the economy. Therefore, current account deficit decreases but remains negative with low interest rates as this seems to be the case for the U.S. after the bust in housing markets. It is true that the downpayment requirement in the U.S. after the bust in housing markets became higher but given the intuition of the model this positive shock to the downpayment requirement would imply a bigger bust in the economy presented here even with low interest rates.

The small open economy assumption allows to have together a decreasing downpayment and an increasing homeownership rate in the economy. In a closed economy, like the one of Chambers et al. (2009), a decreasing downpayment requirement would increase the demand for credit increasing the interest rate in the economy. This could even decrease the homeownership rate of the economy. As I showed here, given international interest rates, this shock would increase homeownership rate even after the increase in housing prices.

Quantitatively the model is able to explain almost a 80% of the median annual increase in housing prices over the period under study. The model show quantitative responses in line with data for the homeownership rate, the decrease in the rent-to-price ratio, the increase of labor in the housing market, and, importantly, the evolution of the current account of the U.S. economy during the bust period in spite of an overestimation in the deficit during the boom period since the model lacks of any kind of adjustment cost.

At the moment, I am developing the paper in three directions. First, I want to analyze each shock separately. A decrease in the interest rate seems to be quantitatively more important than the shock to the downpayment requirement. This result is in line with Kiyotaki et al. (2011). But a decrease in the downpayment is necessary to explain the extensive margin increase in the demand for housing. Also, sensitivity analysis in the intratemporal elasticity of substitution (ε) will be carried out. Second, I am studying the welfare implications for different groups of agents (renters, new homeowners, and previous homeowners). There is a concern on the inequality consequences of the crises, and it is important to understand the dynamics of inequality over the cycle. And third, I am analyzing the role of different tax systems. In the current version of the model, imputed rents are not taxed and mortgage interest payments are fully deductible. Evidence over this period shows a clear preference tax treatment for housing capital. Different tax policies could be evaluated along the transition. Building on Díaz and Luengo-Prado (2008), I am extending the paper to account for this dimension.

The model presented here would be an interesting model to be used in other contexts. Evidence suggests that variations in international interest rates cause similar movements in developing economies. Further research would be why countries like Japan or Germany did not experienced a housing boom over this period since interest rates also decreased in those countries. I think that a two country version of this model could help to answer this question addressing important potential explanations to this pattern, as differences in TFP growth or real exchange rates, as Caballero et al. (2008) suggests. But this is left to further research.

Table 1: Homeownership Rates.

	Germany	Japan	Spain	United States
1990	39	61	76	63.9
2000	41	60	81.34	67.4
2006	40	60.9	85.3	68.8

Source: OECD.

Table 2: Loan to Value Ratio (LTV).

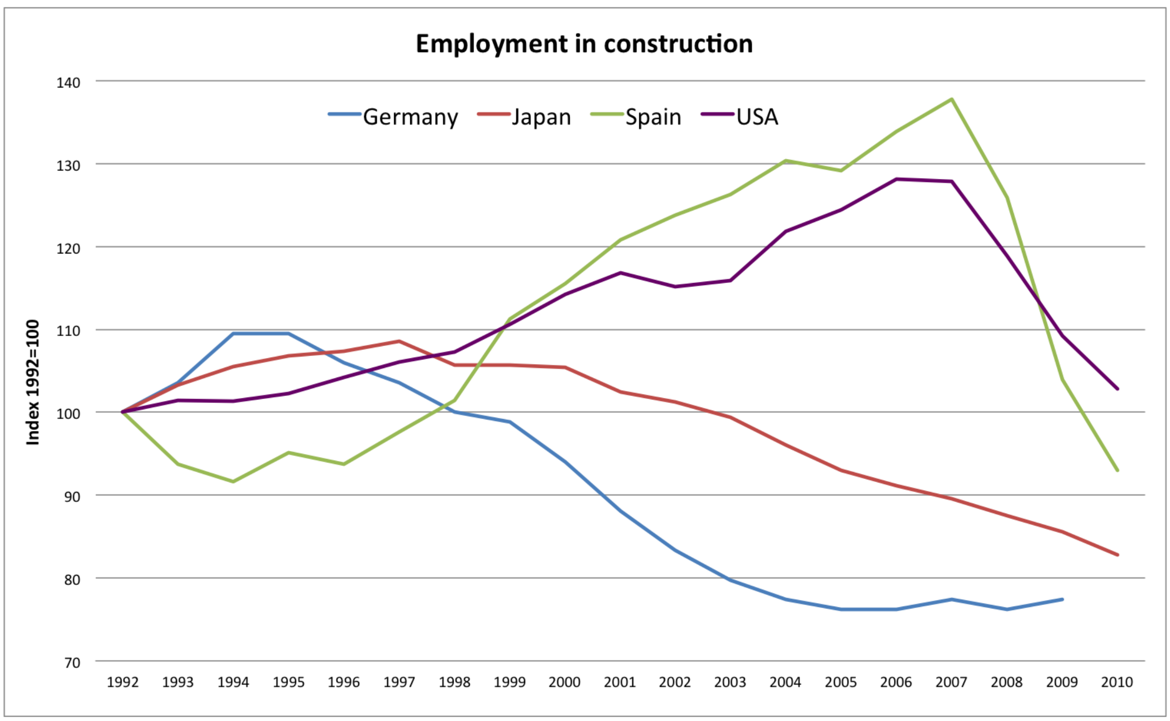
Year	LTV
1995	78.4
1998	86.2
2003	94.4

Source: Duca et al. (2011).

Table 3: Model Parameters.

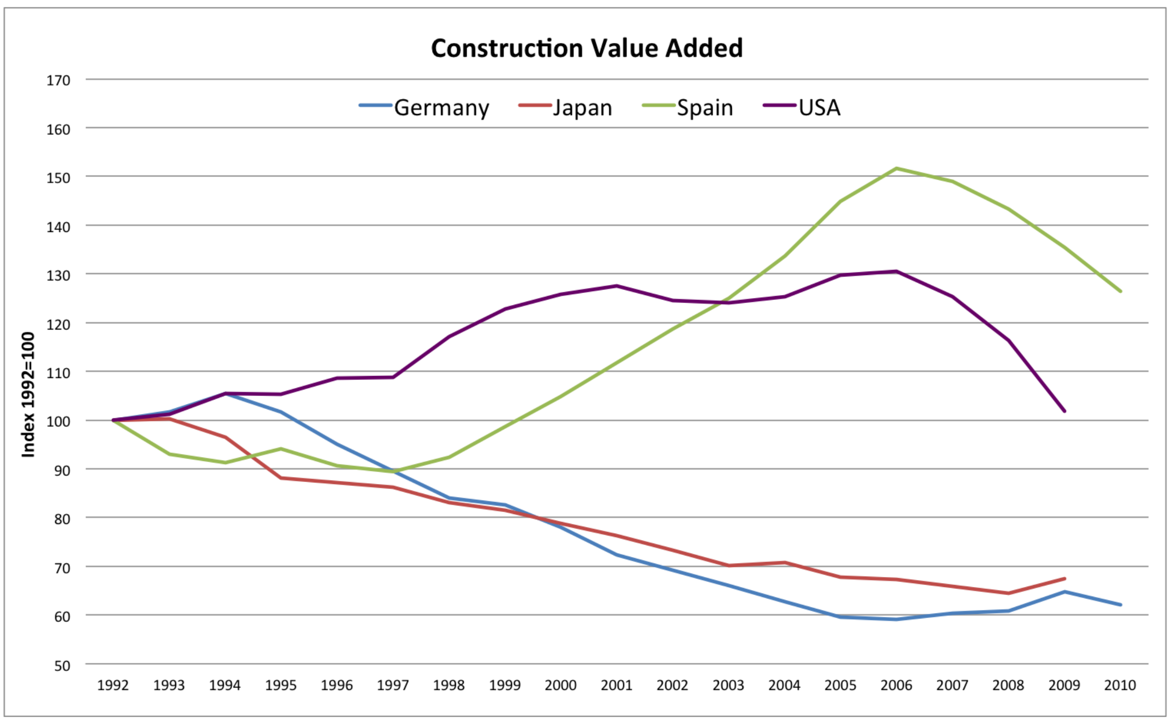
Parameter	Value
β	0.9757
θ	0.0765
σ	0.5
ε	0.9
γ	0.2
$\underline{\mathrm{h}}$	1.4775
A_c	2
A_s	0.9
α_c	0.2616
α_s	0.132
ϕ	0.106
δ_k	0.117
δ_h	0.0424
δ_f	0.0483
τ_y	0.2

Figure 1: Employment in Construction.



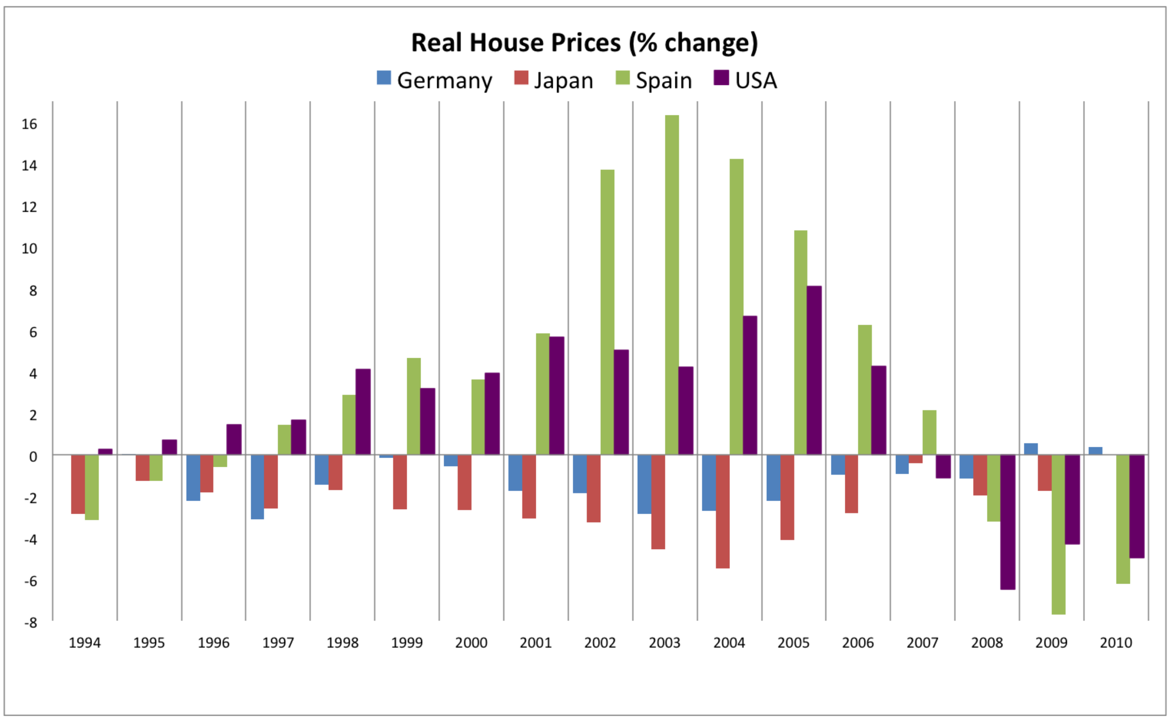
Source: OECD.

Figure 2: Construction Value Added.



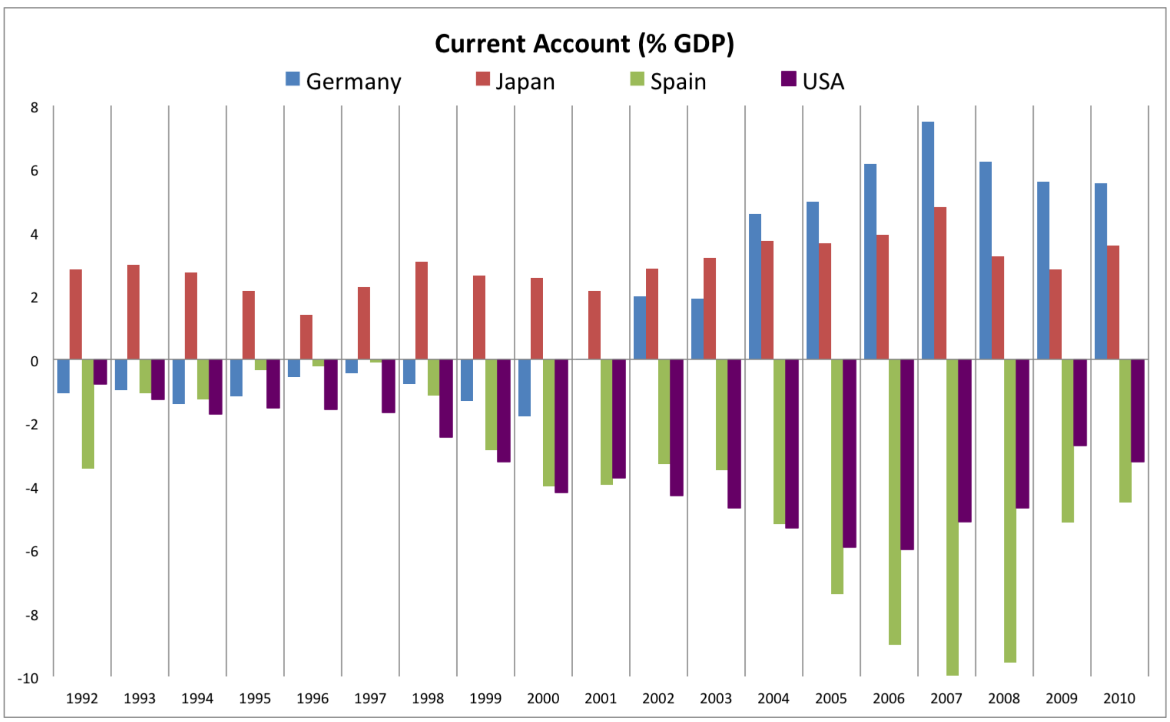
Source: OECD.

Figure 3: Real House Prices (% Change).



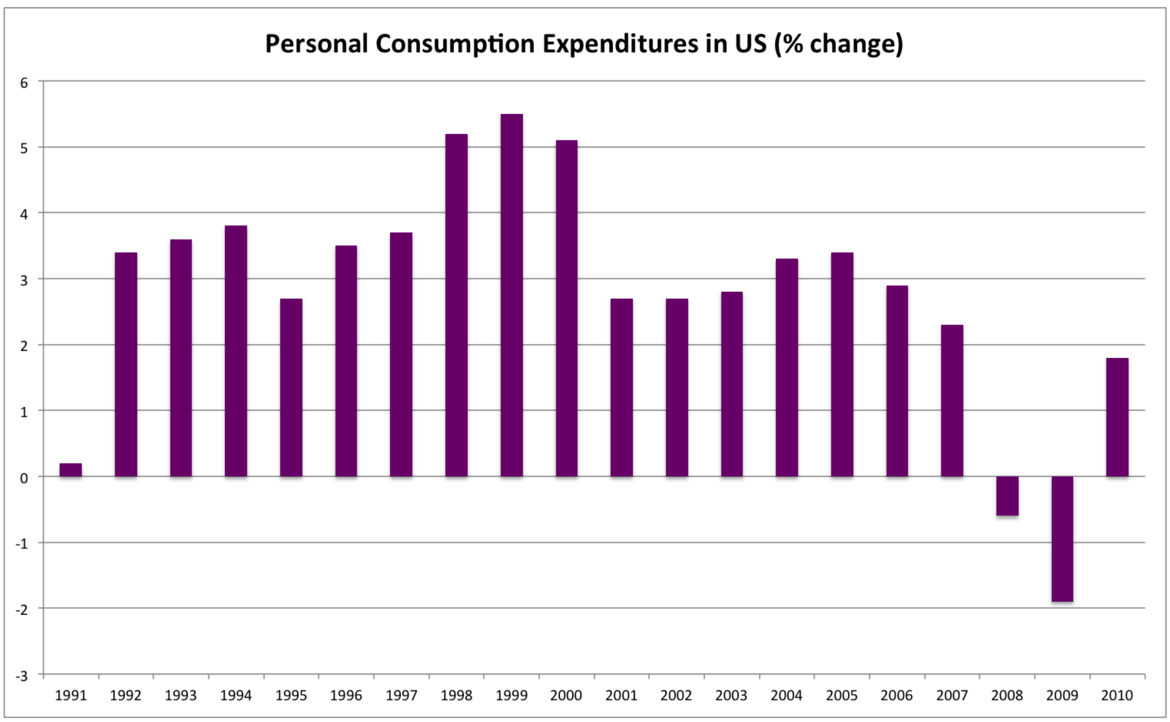
Source: OECD.

Figure 4: Current Account (% GDP).



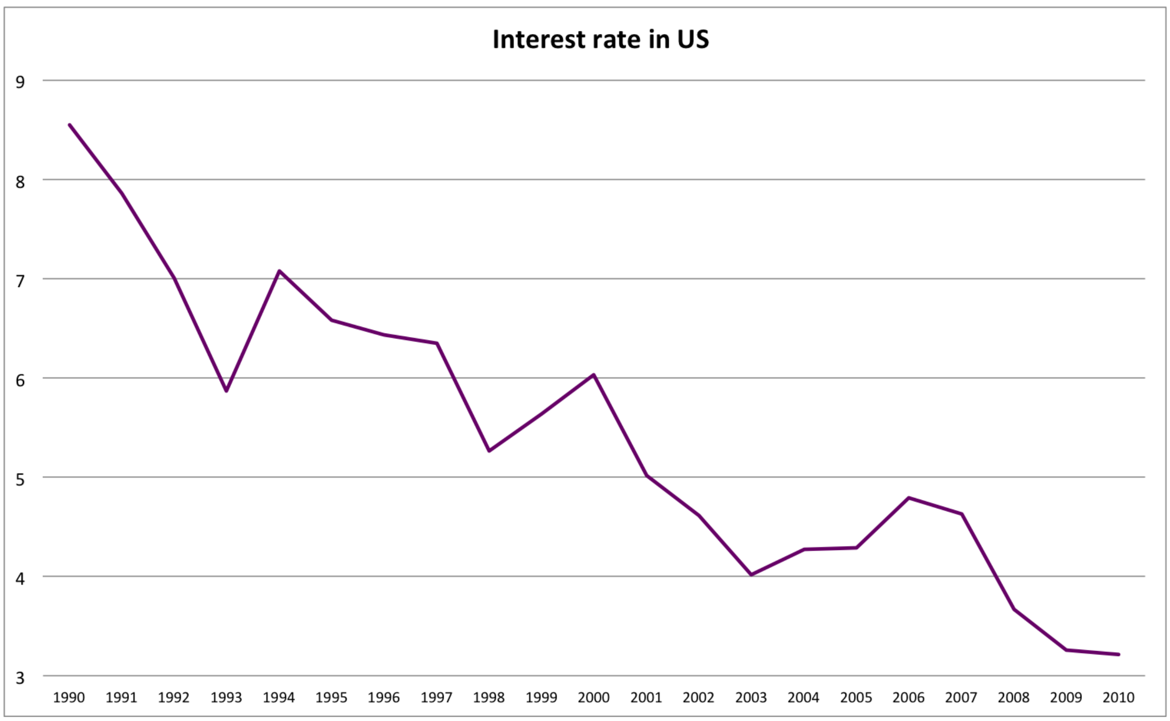
Source: OECD.

Figure 5: Personal Consumption Expenditures (% Change).



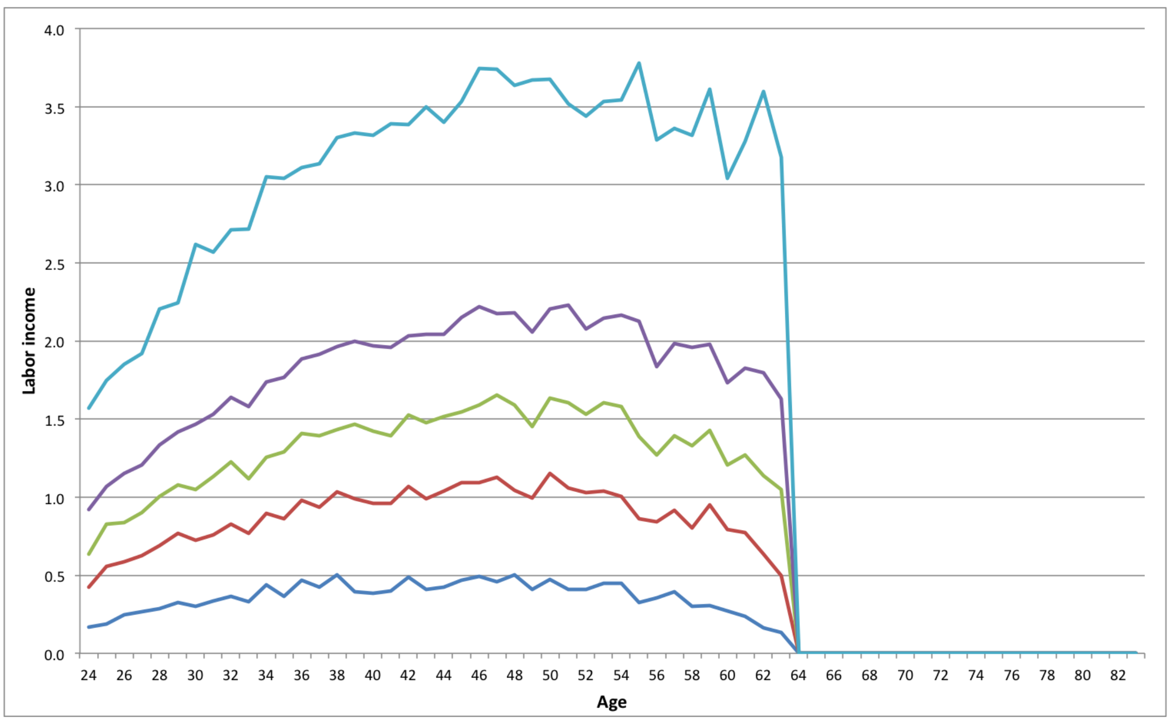
Source: NIPA.

Figure 6: Interest Rates.



Source: OECD.

Figure 7: Labor Income.



Labor income process for each quintile.

Source: Current Population Survey (CPS).

Figure 8: Interest Rate and Downpayment Shock.

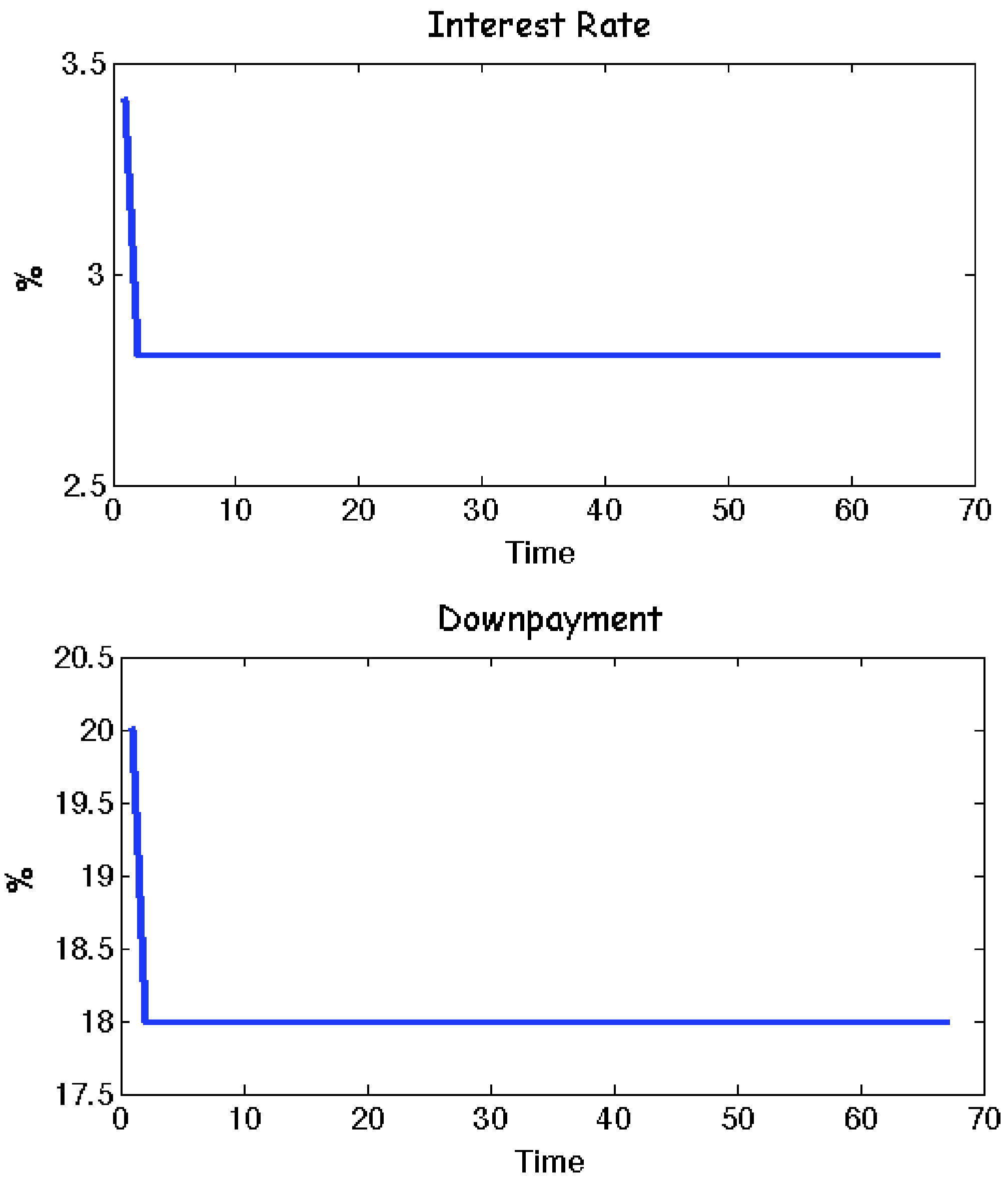


Figure 9: Negative Correlation between House Prices and Current Account Balance.

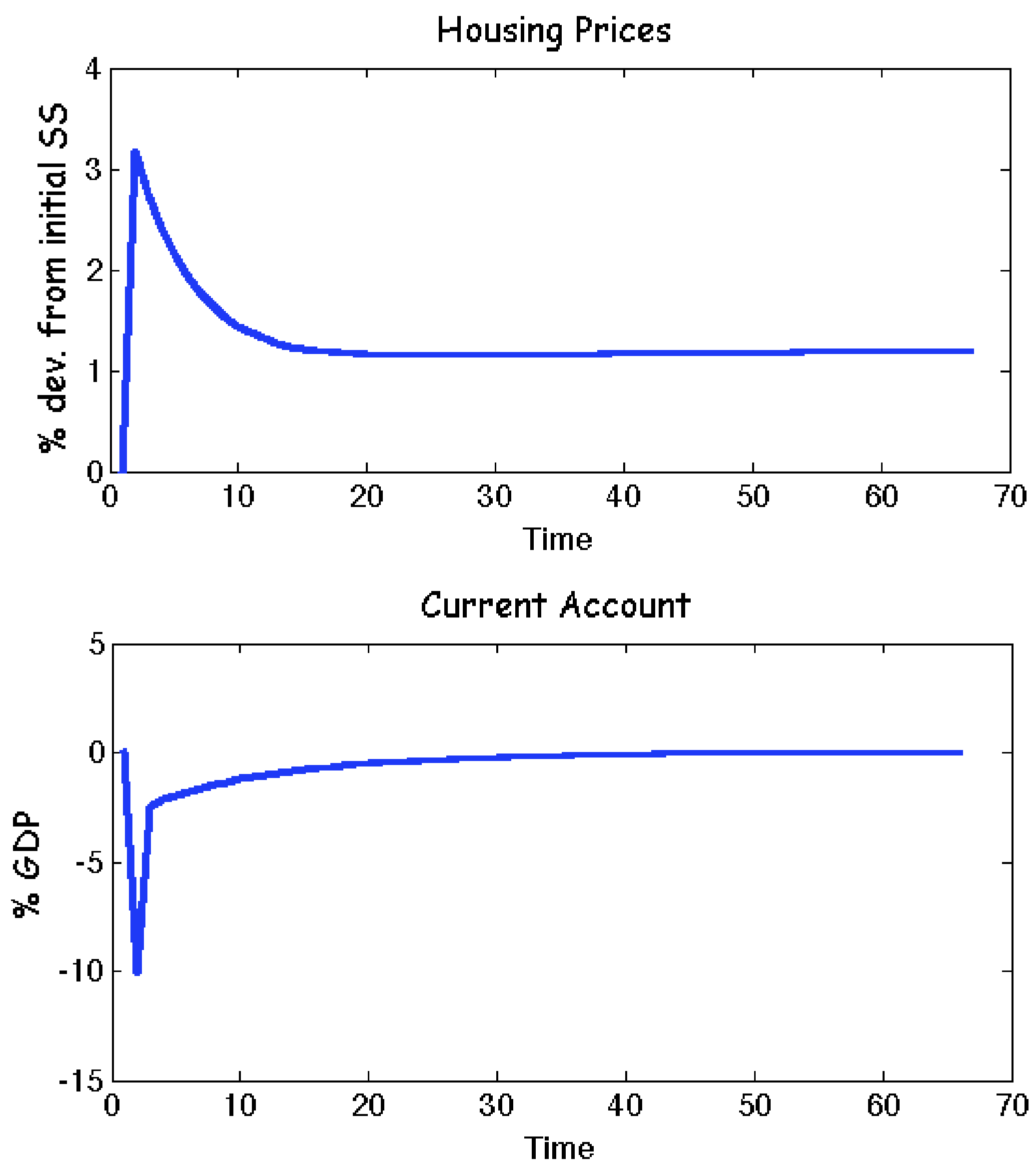


Figure 10: Extensive Margin and Non-Housing Consumption.

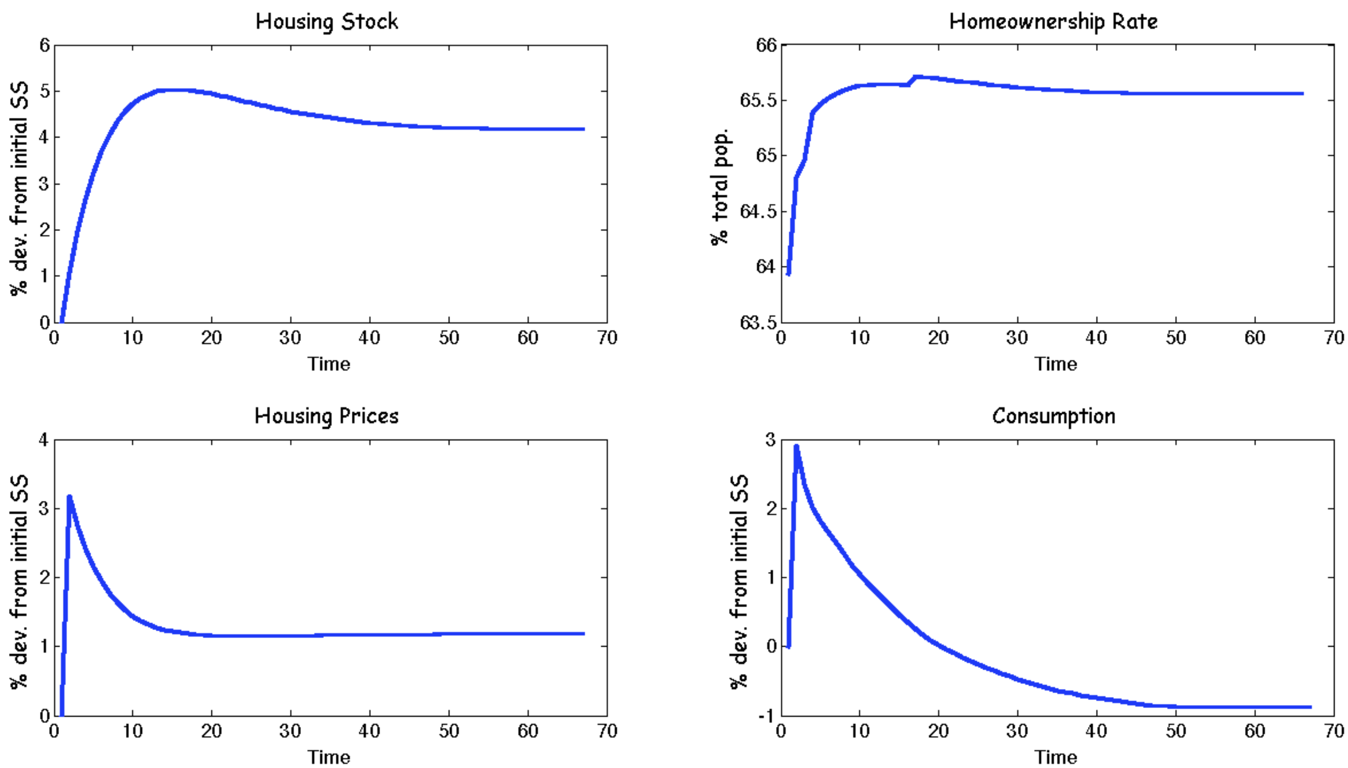


Figure 11: International Interest Rate Shock and the Housing Boom.

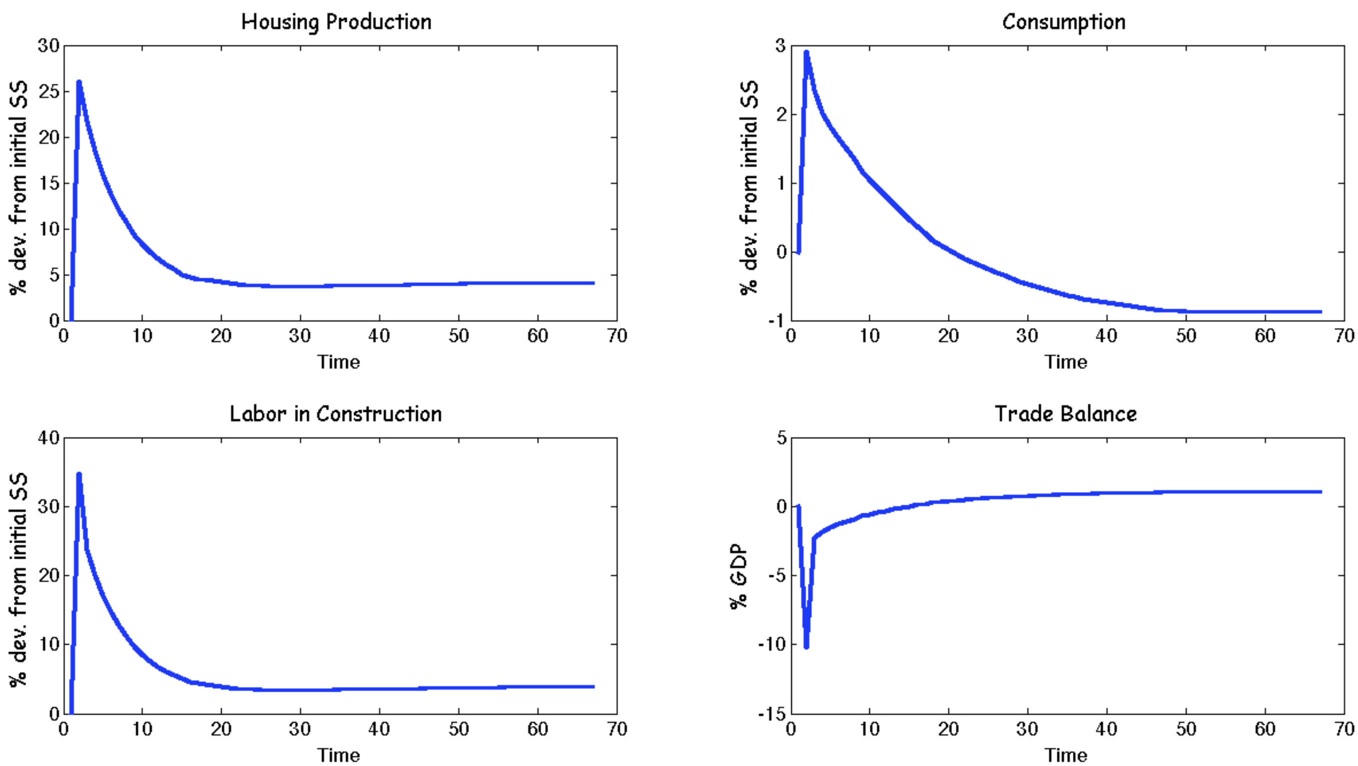


Figure 12: Housing Prices and Land Price.

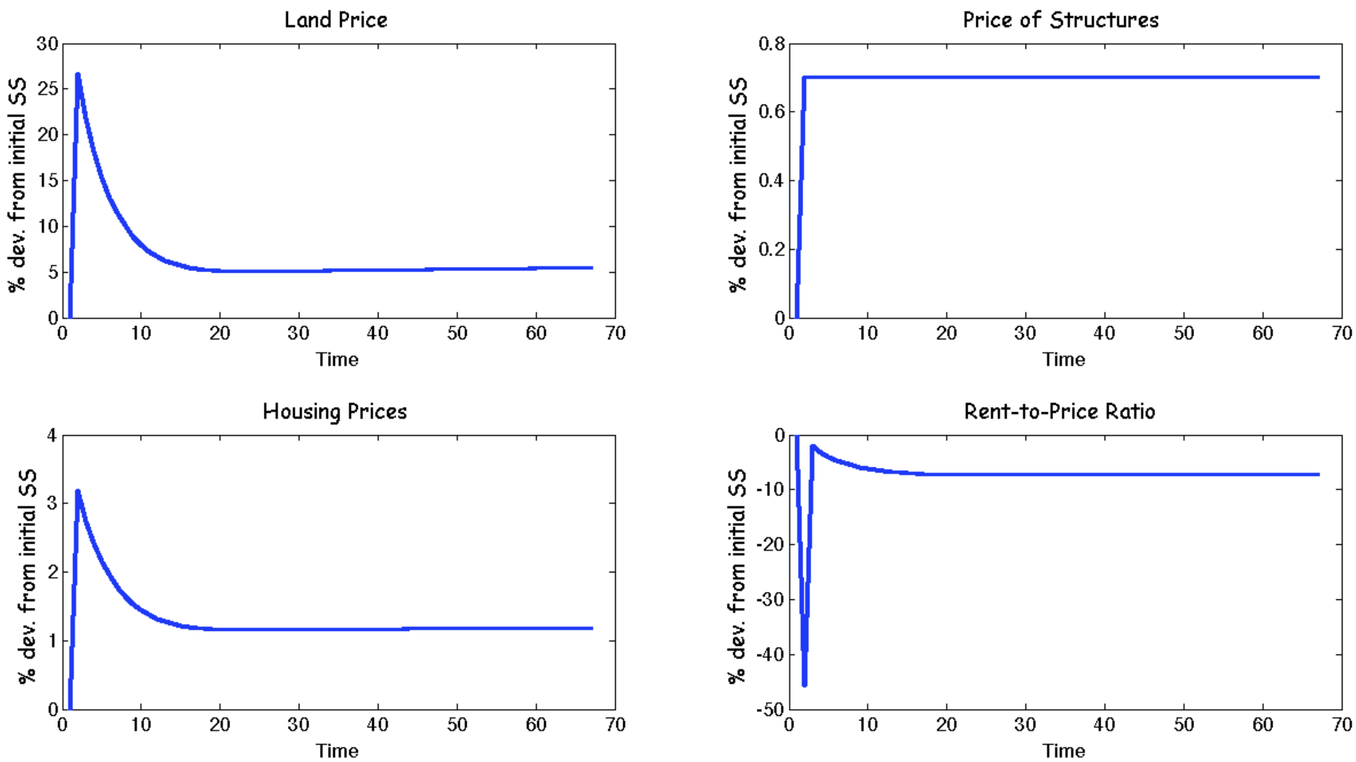


Figure 13: Housing Market.

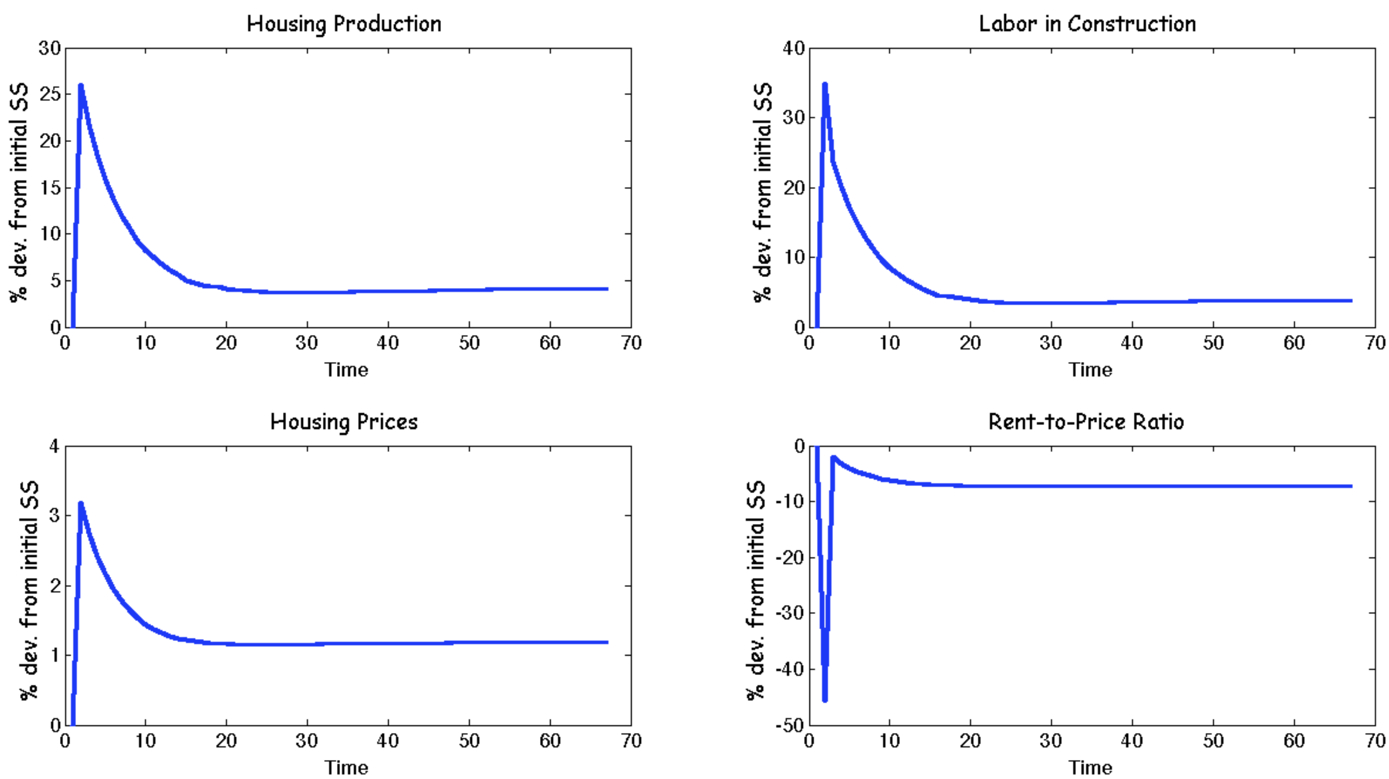
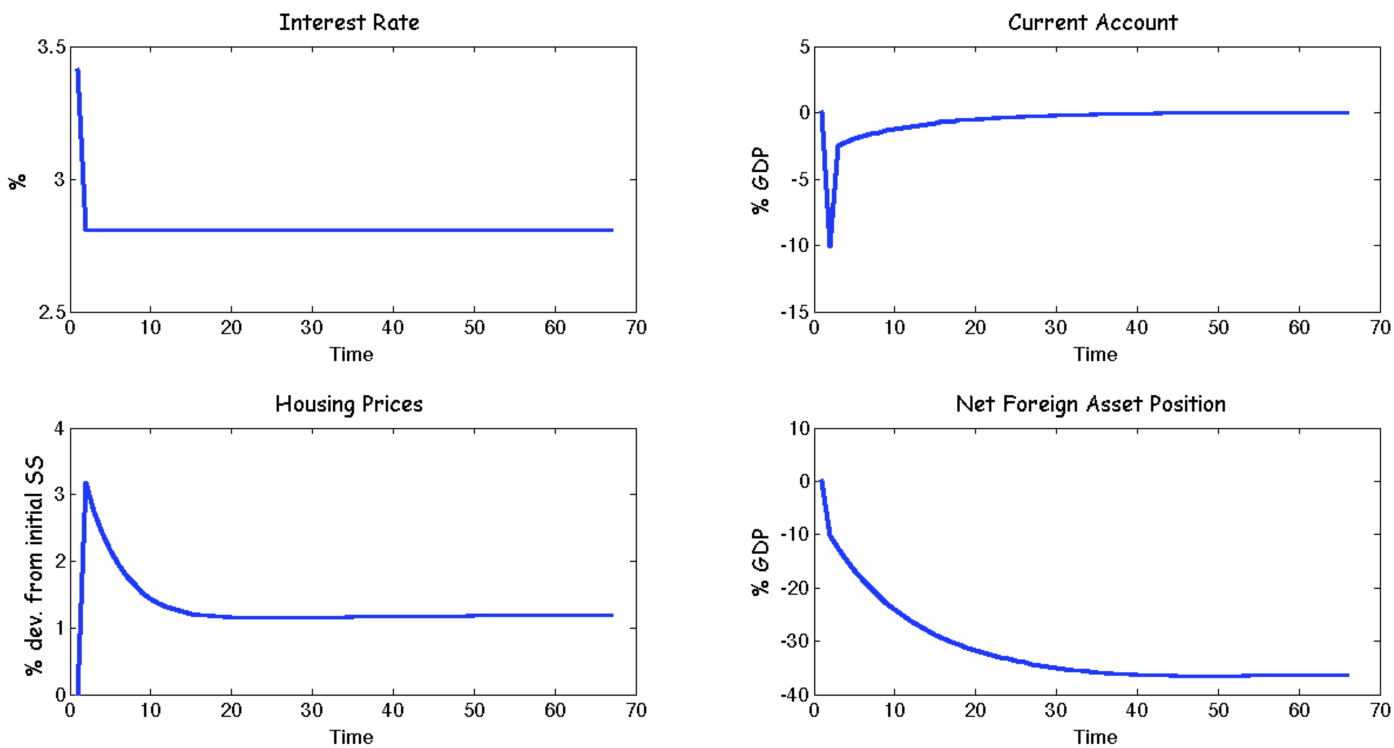


Figure 14: Housing Boom and International Interest Rates.



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Appendix A: Computational Procedures

The first thing to do is to calculate the initial steady state and the final one. The computational procedure for the transition is based on Heer and Maussner (2008).

Households Own Land

The budget constraint is reduced with some assumptions. The first one is relative to the amount of land households hold in the economy. I assume that households have a proportion of the total land in the economy relative to their housing stock, and they receive this amount of land exogenously each period. To have an intuition of this assumption think on the steady state. Each period households have to cover the amount of housing capital depreciated and construct it on the new plot of land that they receive that period. So, rents from land are received by households in a proportional amount to their housing stock. This assumption makes possible to have land in the economy owned by individuals.

In this sense I make use of the Cobb-Douglas production function for houses to derive the relationship between land and the housing stock, and from the marginal product of land I get:

$$p_t^l l(h_t, h_{t+1}) = \phi p_t^h (h_{t+1} - (1-\delta_h)h_t)$$

Using this expression the budget constraint becomes:

$$c_t + r_t^f p_t^h f_t + a_{t+1} + p_t^h(1-\phi)h_{t+1} \leq$$

$$\leq z^i(1-\tau_y)w_t + b_t^{j>j^*} + (1+(1-\tau_y)r_t^a)a_t + (1-\delta_h)p_t^h(1-\phi)h_t$$

Now, I add the term $p_{t-1}^h(1-\phi)h_t$ to both sides of the budget constraint. Rearranging I get:

$$c_t + r_t^f p_t^h f_t + a_{t+1} + p_t^h(1-\phi)h_{t+1} \leq$$

$$\leq z^i(1-\tau_y)w_t + b_t^{j>j^*} + a_t + p_{t-1}^h(1-\phi)h_t + (1-\tau_y)r_t^a a_t - \delta_h p_t^h(1-\phi)h_t + (p_t^h - p_{t-1}^h)(1-\phi)h_t$$

Voluntary Equity

In order to compute the equilibrium of the model, it is convenient to reformulate the household problem. Define voluntary equity as the wealth held less the proportional amount of land, $y_t^j \equiv a_t^j + p_{t-1}^h(1-\phi)h_t^j$. So:

$$c_t + r_t^f p_t^h f_t + y_{t+1} \leq$$

$$\leq z^i(1-\tau_y)w_t + b_t^{j>j^*} + y_t + (1-\tau_y)r_t^a a_t - \delta_h p_t^h(1-\phi)h_t + (p_t^h - p_{t-1}^h)(1-\phi)h_t$$

Where the term $(p_t^h - p_{t-1}^h)(1-\phi)h_t$ refers to the capital gains make by a household because of a change in housing prices from one period to the next.

The Borrowing Constraint

The borrowing constraint is:

$$a_{t+1} \geq -(1-\gamma_t)p_t^h h_{t+1}$$

which, making use of the definition for voluntary equity, can be written in the following way:

$$y_{t+1} + p_t^h \phi h_{t+1} \geq \gamma_t p_t^h h_{t+1}$$

becoming a constraint on next period's net worth:

$$y_{t+1} \geq (\gamma_t - \phi)p_t^h h_{t+1}$$

Solution Method

I follow closely the solution method from Gervais (2002). The state variables for the household problem are the earnings process, the quintile to which households belong to, and voluntary equity, $\{z^j, i, y_t^j\}$. With this reformulation, I deal with one state. This greatly simplifies the problem imposed by the endogenous liquidity constraint in the solution of the household problem.

The household problem can be broken down into intra-period and inter-temporal decisions. The inter-temporal decision consists of choosing the amount of savings to carry over to the next period. Once the inter-temporal decision is made, households choose the amount of composite goods and housing services to consume during the current period, as well as the composition of savings carried over from the previous period.

This possibility derives from the fact that only one state variable is needed to describe the situation of an age-j individual with productivity level z_j. This state variable is todays net worth, or alternatively yesterdays savings. Without uncertainty, the composition of todays savings between different assets is irrelevant since the same composition will result whether the decision is made today or tomorrow. Hence, the only information needed as an individual enters a period is the total amount of savings carried over from the previous period, as opposed its composition between financial assets and housing. In other words, todays amount of savings is chosen knowing that its composition will be optimized tomorrow.

These points become obvious in the following recursive formulation of the households problem. Let $v_t^j(y_t; i)$ denote the value of behaving optimally from period j until period J for an individual who enters period j with net worth y_t, productivity level z, and belonging to quintile i, in each period of time t. Given a net worth position at age j; a household chooses next periods net worth to maximize total future discounted utility. The value function of an age-j individual is defined as:

$$v_t^j(y_t; i) \equiv \max_{\left\{y_{t+1}^{j+1} \in \Gamma\right\}} \left\{G^j(y_t, y_{t+1}; i) + \beta v_{t+1}^{j+1}(y_{t+1}; i)\right\}$$

where Γ is the feasible set from which tomorrows net worth is chosen. The return function of an age-j individual, G^j; is defined as the maximum utility level a household can achieve given todays and tomorrows level of net worth. In other words, the return function is that which solves the following intra-temporal problem:

$$G^j(y_t, y_{t+1}; i) \equiv \max_{\left\{c_t^j, x_t^j, f_t^j, h_t^j, a_t^j\right\}} \{u(c_t, x_t)\}$$

$$s.t. \quad c_t + r_t^f p_t^h f_t + y_{t+1} + \delta_h p_t^h (1-\phi) h_t \leq$$

$$\leq z^i(1-\tau_y)w_t + b_t^{j>j^*} + y_t + (1-\tau_y) r_t^a a_t + (p_t^h - p_{t-1}^h)(1-\phi) h_t$$

$$y_t = a_t + p_{t-1}^h (1-\phi) h_t$$

$$x_t = f_t + h_t$$

$$y_t \geq (\gamma_{t-1} - \phi) p^h_{t-1} h_t$$

$$h_t \geq \underline{\text{h}} \ otherwise \ h_t = 0$$

For the results presented in this paper, I use 200 grid points for voluntary equity and linear interpolation in order to get more accuracy (the grid points are not equally space to maximize efficiency). Households in each quintile are born with zero financial assets ($a^1_t = 0 \ \ \forall i, \forall t$) and zero housing stock ($h^1_t = 0 \ \ \forall i, \forall t$).

Different Households

This economy will have three types of households each one solving her following problem.

Renters: Households with not enough net worth to buy the minimum house size are forced to rent, or some households would prefer to save more time to attain the desired level of owned housing by renting some periods before buying a house. In this case, they solve the following problem:

$$G(y_t, y_{t+1}; i) = \max_{\{c^j_t, f^j_t, a^j_t\}} \{u(c_t, f_t)\}$$

$$s.t. \quad c_t + r^f_t p^h_t f_t + y_{t+1} \leq$$

$$\leq z^i (1 - \tau_y) w_t + b^{j > j^*}_t + y_t + (1 - \tau_y) r^a_t a_t$$

$$y_t = a_t$$

$$y_t \geq 0$$

Home owners: Households with enough net worth to access to a house bigger than the minimum house size solve:

$$G^j(y_t, y_{t+1}; i) = \max_{\{c^j_t, h^j_t, a^j_t\}} \{u(c_t, h_t)\}$$

$$s.t. \quad c_t + y_{t+1} + \delta_h p^h_t (1 - \phi) h_t \leq$$

$$\leq z^i(1-\tau_y)w_t + b_t^{j>j^*} + y_t + (1-\tau_y)r_t^a a_t + (p_t^h - p_{t-1}^h)(1-\phi)h_t$$

$$y_t = a_t + p_{t-1}^h(1-\phi)h_t$$

$$y_t \geq (\gamma_{t-1} - \phi)p_{t-1}^h h_t$$

$$h_t > \underline{\text{h}}$$

Households in the margin (Cooperatives): There are some households with enough resources to buy the minimum house size and there would be constrained by this election. Here I make an assumption by allowing these households to make a convex combination between the minimum house size and the amount of housing services they would rent. The problem with the non-convexity of the minimum house size is that, along the transition and for the calibrated model, I always find some individuals jumping from owning the minimum house size to renting making impossible to clear the housing market. This happens for a very small fraction of individuals and could be solved by some others techniques as linear interpolation in ages. The existence of the minimum house size is key in this model since I do not have adjustment costs in housing capital and I want to model the homeownership rate. In my model, without a minimum house size, all individuals would own a small fraction of housing capital in all periods but the first one, in which they have zero assets by assumption. Since saving in a house has preference tax treatment and allows households to get credit, it is always preferred to renting. Both assumptions are in line with evidence.

By assuming that households can do a convex combination between renting and owning the minimum house, I can solve the problem of cleaning the housing market and have a realistic homeownership rate in the economy. I did some comparisons between the answer of the model with this assumption and without it (in a different calibrated model) and the answer is virtually the same.

The problem I solved for these households is this:

$$G^j(y_t, y_{t+1}; i) = \max_{\left\{c_t^j, x_t^j, q_t^j, f_t^j, h_t^j, a_t^j\right\}} \{u(c_t, x_t)\}$$

$$s.t. \quad c_t + r_t^f p_t^h q_t f_t + y_{t+1} + \delta_h p_t^h (1-\phi)(1-q_t)h_t \leq$$

$$\leq z^i(1-\tau_y)w_t + b_t^{j>j^*} + y_t + (1-\tau_y)r_t^a a_t + (p_t^h - p_{t-1}^h)(1-\phi)(1-q_t)h_t$$

$$y_t = a_t + p_{t-1}^h(1-\phi)(1-q_t)h_t$$

$$x_t = q_t f_t + (1-q_t)h_t$$

$$y_t \geq (\gamma_{t-1} - \phi)p_{t-1}^h(1-q_t)h_t$$

$$h_t = \underline{\mathrm{h}}$$

No more households in the economy will do a convex combination of this kind if it is not with the minimum house size. The reason is that they would never be indifferent between owning and renting in other cases different from the minimum house size.

An intuition for this problem would be to consider it as cooperatives, i.e. that some households, belonging to the same age and quintile, were allowed to establish a cooperative. Then, only some of them would live in the house, while the others would rent. However, they can use this house as a collateral for credit in the capital markets.

An alternative interpretation can be that households deposit their savings in a financial intermediary and that the probability of buying a house depends on the fraction of the downpayment deposited. If a household deposits half of the required downpayment to buy a house, then the household maybe allowed to buy a house with probability one half. If the household does not win the lottery, he does not lose his assets. Next period he will make a new deposit and get a new chance to buy a house.

Appendix B: Stationary Competitive Equilibrium

Denote $q = \{a, h, i\}$, q $\in$ Q.

Definition *A* ***stationary competitive equilibrium*** *for a given government policy,* τ_y*, and downpayment requirement,* γ*, is a collection of relative prices* $\left\{p^h, p^s, p^l, r^f, r, w\right\}$*, a collection of functions for the household problem* $\{v^j(q), c^j(q), f^j(q), h^j(q), a^j(q)\}$*, an age-dependent measure of agents type,* $\lambda_j(q)$*, a value function for financial institutions* $\Psi(F, B, A, K)$*, and aggregate quantities for the whole economy* $\left\{Y^c, Y^h, Y^s, X^s, L, K^c, K^s, N^c, N^s, F, B, A\right\}$ *such that:*

1. *Inputs are priced competitively every period.*

2. *Given* τ_y*,* γ *and prices, the functions* $\{v^j(q), c^j(q), f^j(q), h^j(q), a^j(q)\}$ *solve the dynamic program from the household problem.*

3. *Given prices and the function* $\Psi(F, B, A, K)$*,* $\{F', B', A', K'\}$*, solves the financial institutions' problem.*

4. *Individual and aggregate decisions are consistent:* $C = \sum_{j=1}^{J} \int_Q c^j d\lambda_j(q)$*,* $H = \sum_{j=1}^{J} \int_Q h^j d\lambda_j(q)$*,* $F = \sum_{j=1}^{J} \int_Q f^j d\lambda_j(q)$*,* $A = \sum_{j=1}^{J} \int_Q a^j d\lambda_j(q)$*.*

5. *The government maintains a balanced budget:*

$$G + b = \sum_{j=1}^{J} \int_Q \left[\tau_y w z^j + \tau_y r a^j\right] d\lambda_j(q)$$

where $b = \sum_{j=1}^{J} \int_Q b^j d\lambda_j(q) = \tau_y w \overline{N}$*.*

6. *Labor market clears:* $N^c + N^s = \overline{N}$*.*

7. *Capital market clears:* $K^c + K^s = K$*.*

8. *Land market clears:* $L = \overline{L}$*.*

9. *Residential structures market clears:* $X^s = Y^s$*.*

10. *Housing market clears:*

$$Y^h = X^h + X^f$$

where $X^h = \delta_h H$*, and* $X^f = \delta_f F$*.*

11. Trade balance is determined:

$$TB = Y^c - C - X^k - G$$

where $X^k = \delta_k K$.

12. Net foreign asset position is determined:

$$B = -\left(\frac{TB}{r}\right)$$