Imbalances and Capital Allocation Puzzle in Chinese Provinces

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Preliminary Version

March 2012

Abstract

Although a large literature on differences in development between coastal and inner provinces exists, surprisingly little attention has been devoted to external imbalances within China in terms of investment, saving, net exports or international trade flows. We shed light on this issue by using three indicators: net exports, international trade and a proxy for interprovincial trade. The contribution of this descriptive paper is to establish some empirical facts pertaining to regional external imbalances in China and the factors driving them.

First, general issues related to data quality, aggregation properties and cross-sectional patterns are discussed. In a second step, we apply the methodology of Gourinchas and Jeanne (2011) to impose some structure on the analysis of imbalances. It consists in introducing an investment and saving wedge in a standard small open economy model and estimating their value necessary to match average investment rate and relative flows. By relating the productivity catch-up parameters and the cumulated relative flows, a clear capital allocation puzzle appears as both are negatively related: provinces which catched up experienced capital outflows. Investment wedges are positively related to productivity. This pattern is in contradiction with the standard model and stands in sharp contrast with the results of Gourinchas and Jeanne. As in the original paper, the saving wedges are the key factor driving the capital allocation puzzle and are negatively related to the productivity catch-up parameters.

We gather usual suspect explanatory variables of high Chinese savings mentioned in the literature and take a first look at some basic correlations and regressions. The structure of investment in fixed assets (state-owned, collective or foreign owned) plays a key role in explaining the cross-provincial variability of investment and saving wedges. Factors related to the economic and trade structure like the share of exports stemming from multinational enterprises or the share of the secondary sector partially account for differences in saving wedges.

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1 Introduction

1.1 Motivation

For nearly a decade, global imbalances have been on the top of the discussion agenda. The obvious policy implications fostered sustained research involving many instruments modern economics provides. While much has been learnt, the profession unsurprisingly came with many competing and sometimes complementary theories. A great deal of explanations, predictions and potential measures have been proposed over the years with near no political or institutional consequences. There probably isn't many prominent economists who haven't expressed themself on the topic. This issue experienced a regain of public attention in the wake of the Great Recession. Over the last decade, the litterature disproportionately discussed the issue through the lens of US interests. At the onset of the Great Rebalancing, observers and experts increasingly lay the focus on China.

With the rise of the national economy, Chinese regions gained in importance and most continental provinces truly are GDP equivalent to big developing countries in terms of PPP international dollar[1]: Sichuan overtook Malaysia, Yunnan roughly corresponds to Vietnam and Henan is comparable to Thailand. More developped coastal regions long have the weight of small industrialized countries: Shandong and Jiangsu stand for Switzerland, Zhejiang for Austria and Fujian for Ireland. If their size is still small, they have become highly integrated in the world economy: Jiangsu exports roughly as much as Taiwan while Zhejiang is comparable to Thailand. The output of Guangdong in the Pearl River Delta is expected to overtake Indonesia in the next years and is on the same scale as South Korea in total export terms. These regions thus play a key role in the world economy as well as in global imbalances.

Although a large literature on differences in development between coastal and inner provinces exist, surprisingly little attention has been devoted to within-China external imbalances, for example in terms of investment, saving, net exports or international trade flows. A better knowledge of the empirical facts could well pave the way for further works including theoretical modelling and a better understanding of the mechanisms involved. The contribution of this descriptive paper is to establish some empirical facts pertaining to regional external imbalances in China and the factors driving them.

To start with, we provide a review of the China specific literature on imbalances. In the second section, we discuss the quality and aggregation properties of output, net exports and international trade data on the regional level. In section three, we update and corroborate our findings on the evolution of cross-sectional patterns of provincial output over the last six decades with the ones of the literature. We come up with three indicators of provincial external imbalances and discuss their cross-sectional patterns over the last decades. In the fourth section, by applying the Gourinchas and Jeanne framework to Chinese provinces, we want to shed light on the drivers of provincial external imbalances by identifying the investment and saving wedges necessary to mimick observed investment rates and relative cumulated capital

[1] http://www.economist.com/content/all_parities_china

flows over the 1984-2009 period. In the next part, we gather some data on potential explaing factors and look at some basic correlation and regression patterns. In section six and seven, we check the robustness of our using alternative data and model extensions. We conclude in section eight. The appendix contains some tables, variables used in the econometric part and mathematical proofs.

1.2 Literature review

Much of the discussion focused on the special position of the US in the world financial system and the related exhorbitant privilege it delivered (Gourinchas and Rey, 2005; Gourinchas et al., 2010; Habib, 2010). The well-documented other side of the coin was dubbed the saving glut phenomenon (Bernanke, 2007). While the saving behaviour of resources rich countries such as oil producers seemed to be explanable in the framework of the mainstream models, East Asian patterns were somewhat puzzling. Some didn't hesitate to newly qualify the actual world financial system as a new version of Bretton Woods with the US being the Center country providing growth impetuses (Dooley and Folkerts-Landau, 2003). The new mercantilism argument (Bergsten, 2010) provided US lobbies and politicians with an easy argument to justify possible trade retaliations. By and large, recent research has shown that export-led growth cases in point such as South Korea and Japan and to a certain extent China rather relied on financial than monetary mercantilism (Aizenman and Lee, 2008). The systematic hoarding of foreign reserves that followed the 1997 Asian crisis historically rather seems to be related to the exposure to potential financial crisis and sudden stop than exchange rate manipulation (Aizenman and Lee, 2007). As a whole, evidences for export-led growth in China is weak and could well explain the resilience of the economy following the 2001 and 2007-2008 crisis (Anderson, 2007).

As often in economics, data issues complicate the discussion. Hausmann and Sturzenegger (2006) provoked a hot debate on whether the US net foreign position is far better than official statistics suggest given non-measured exports of dark matter under the form of liquidity service, insurance and venture-capitalist services. In general, global imbalances measurement itself is biased by the origin-country system of capturing exports and imports: a value-added focus would enable to better understand integrated world supply chains and would shed new light on trade flows (Lamy, 2010). On the Chinese side, large measurement errors of the current account driven by hidden capital inflows via underestimated returns on foreign investment and misreporting of exports/imports are supected since the mid 2000s (Zhang, 2008). If this is true, it shouldn't come as a surprise that the world has been running a current account surplus over the past few years (IMF, 2011).

The misallocation problem of capital is obviously related to global imbalances. Gourinchas and Jeanne (2011) observed that developing countries whose productivity grew faster had capital outflows between 1980 and 2000. By introducing an investment and a saving wedge in a standard neoclassical model, they were able to identify the saving wedge as the key driver of this pattern. The capital flows conundrum could be explained by differences between industrialized and emerging markets pertaining to entry costs, real returns on investment,risks, availability of investment projects and assets and the predominant role of government in the accumulation of foreign reserves (Dadush and Stancil, 2011). Indeed, the puzzle is less stringent once one accounts for public aid flows and reserve accumulation. Private capital seems to behave more in accordance with the standard model (Alfaro et al., 2011).

The importance of public flows in the discussion is obvious as soon as one takes a glance at the structure of the Balance of Payment of China: since the beginning of the opening policy in the 1990s, portfolio investment (transfers of shares and bonds) and other investment (bank accounts and loans) have been stationary. The authorities sticked to their policy of capital controls for these components while allowing massive inflows of direct investment. At the same time, increasing current account surpluses have been registered. We propose to dubb them the surplus brothers[2] . The only way to absorb the huge capital surpluses and FDI inflows without liberalizing the rest of the financial account has been to accumulate foreign reserves. As discussed in Yongding (2007b), it certainly isn't compatible with an efficient allocation of resources: instead of relying on cheaper domestic saving, private firms rely on more expensive foreign flows which add to the oversupply of domestic savings[3]. The surplus brothers are then invested in low or even negative yield US assets which continue to finance the US private and public spending spree. The People's Bank of China partly sterilized money inflows by selling domestic assets but there is no doubt the surplus brothers foster the formation of asset bubbles (Yongding, 2007a).

In contrast to recent developments, the initial period of the transformation to a market economy was accompanied with large outflows of financial capital due to high domestic financial transaction costs, political uncertainty and inappropriate exchange rate policy (Gunter, 1996). It is only recently that flows reversed and inofficial capital began to flow into China as confidence in the government increased (Ljungwall and Wang, 2008). Recently, a new strand of literature focused on risk sharing and capital flows inside China. Using Household survey and provincial data, Xu (2008) found that risk sharing across prvinces over 1980-2004 was lower than among regions in the US or Canada but still higher than across industrial countries. These results were confirmed inHo et al. (2010) who used macroeconomic provincial expenditures data over the 1953-2006 period. Interestingly, consumption risk sharing didn't improve following the reforms. In Li (2010), savings and investment have indeed been strongly related between 1978 and 2006. To our knowledge, it is the first work in which huge variations across provinces in external positions are noticed by computing the average of provincial net exports over GDP, albeit in a side remark.

By focusing extensively on the initital and late reform period, Chan et al. (2011) found considerable improvement in private capital mobility across China between 1978-1992 and 1993-2006. By estimating government saving and investment, they showed in a cointegrating

[2] Yongding (2011) named this development twin surpluses. The danger lies in confusing it with the original meaning of twin deficits.

[3] Off course, technology transfers and integrated value supply chains of international firms make FDI attractive.

framework that government may have alleviated barriers to capital mobility. In the more developed costal provinces, private capital mobility already developed in the 1990s while in other regions it only started to improve in the 2000s. Government transfers seem to have promoted capital mobility in the more backward regions.
All in all, capital mobility is low in China and the degree of financial integration even seems to have decreased in the 1990s (Boyreau-Debray and Wei, 2004). Investment by private firms seems to respond positively to marginal capital productivity while investment through government budget and state-owned financial institutions are negatively related to it. Thus, it seems the government systematically allocates capital away from more productive towards less productive regions.

A good grasp of the structure of the Chinese internal financial system is fundamental to better understand potential factors driving imbalances.Bai (2006) observes that the bulk of the financial system is dominated by official banks. Stock market is still a financial midget and the bond market is dominated by government assets. Private firms mostly have to rely on retained earnings and FDI or trade credits for their financing. Prasad and Wei (2007) compare the Chinese strategy with other East Asian systems and show China's long term commitment to FDI as the main source of financial inflows. The low international debt positions and accumulation of foreign reserves enable to defend the peg and would be welcome to bail out the inefficient banking sector. China has perfectly integrated the fact that fast financial integration without good macro policies and institutions may lead to sudden stops.

2 Data

2.1 General issues

Most existing studies take Chinese data at face value without questioning their quality. A small circle of scholars addressed this issue. The question of data quality is as old as the Communist Party itself: The "Great Leap Forward" famines of 1958-1962 were a direct consequence of overreports of grain output by local officials for fear of the Anti-Rightist movement (Cai, 2000). The resilience ot the Chinese economy during the 1997 Asian crisis raised scepticism about growth figures (Smith, 2001).

Output figures mostly came under close scrutiny: Adams and Chen (1996) reassessed real GDP growth over the 1978-1994 period using the elasticity of energy consumption. It halved official growth figures. In the same spirit, Rawski (2001) argued official 1997-2001 GDP figures have been overestimated and propose Airline travel as an alternative measure. Rawski and Mead (1998) found data massively overestimated Chinese farm workers (the notorious phantom farmers) from 1979 to 1993 causing a large bias in sectorial output estimations. Following official instructions of the NBS, Holz (2004b) reconstructed private consumption between 1997 and 2001 but the methodologies he used rarely matched official data and time series patterns were not credible. Unfortunately, he wasn't able to repeat the exercise with other expenditure components of the National Accounts due to the limited data availability.

Rawski (2000) points to the long Chinese tradition of literacy and record-keeping of the socialist state. Central planning necessitates an extensive array of data. Officials have little interest in neglecting data collection as they are an important tool for planning and development. Chow (2006)opines that statistics are by and large reliable. Still, there is a general agreement that the 1990s saw a near collapse of the statistical system. Xiaolu and Lian (2001) observe that during the era of central planning and the initial phase of the reforms, dominant SOEs[4] managed their production according to official plans which made data collection easy and transparent. Incentives to overreport GDP growth originated in the 1990s as this indicator became a central criterion in assessing performance of local officials. The NBS conducted periodic inspections and reported violations of statistical standards at various administrative levels. Discrepancies in methodology across regions partly account for it. Logically, direct collection by the NBS teams are more reliable then level-by-level reporting (Cai, 2000). For example, peasant income has to undergo four administrative layers before ending up on the Bureau's desk. Incentives for misreporting aren't obvious: some regions may benefit from substantial help if they don't live up to expectations while other may be penalized (Cai, 2000). The fact that in some cases *"officials make statistics and statistics make officials"* has been suggested by former officials of the NBS themselves (Rawski, 2000).

On the one side, the domination of the Communist party seems to guarantee a stability over time of institutions or at least of statistical bureaus. Reliable data collection should build the socle on which a centrally planned economy develops. On the other side, Mao's reforms commonly known as the Great Leap Forward and the Cultural Revolution led to major disruptions in the economy, institutions and social capital, not to mention the difficulties of recording the take off of the private sector since the 1990s. Over the sample, the boundaries of some regions have been modified and it is clear these events potentially impair data quality[5]. Most of the numerous administrative experiments following the victory of the People's Liberation Army in 1949 were later reversed.

Another difficulty arises from the frequent re-definition of variables. Reporting category changes over time cause statistical breaks in time series. Unfortunately, they aren't systematically indicated (Holz and Lin, 2001). Until 1993, China used the Material Product System typical for planned economies (Campbell, 1985). The economy only consisted of socialist productive enterprises and households. Services (most of them were free) and smaller "independent" enterprises are part of the non-productive sector and add no value to output. The introduction of the System of National Accounts enabled to better keep track of the growing economy. The problems of the 1990s to capture the rapid growth in private productive units led to two major revisions of the statistical system (1993 and 1998). The revised laws reduced the role of reporting system in favor of censuses as basis for revision of yearly data (Holz, 2004a). Holz 1986argue that data falsification at higher levels of statistical bureaucracy are

[4]State-owned enterprises

[5]For example, Guangxi won access to the sea in gaining a part of Guangdong's coastline from 1952 to 1955. From 1969 to 1979, the autonomous region of Inner Mongolia was split and integrated into surrounding provinces. More recently, Chongqing gained its independence from Sichuan in 1997.

less likely than at the local level. Data are rather incomplete than inaccurate and regular revisions improve the quality. Variations in enterpise coverage and definition are at the core of discrepancies. There is little evidence that provincial data are worse than national ones: the 2004 economic census validated provincial GDP data and invalidated national ones (Holz, 2008).

There are many reasons to doubt the exactness of the NBS data. Do we have to go as far as proposing a US led estimation of Chinese economic aggregates as in Scissors, Reuters 2012 ? Probably not. There are certainly noisy but with the exception of some obervations in the 1990s, experts agree on the fact that quality has been improving and is still better than countries with comparable income level[6].

All the data used in this project stem from the *National Statistical Yearbooks* of the People's Republic of China and of the *Provincial Statistical Yearbooks* of the 22 provinces, 5 autonomous regions and 4 directly controlled municipalities of Mainland China[7]. The special administrative regions of Macau and Hong Kong won't be considered. Figures at best cover the period stretching from the arrival in power of the Communist Party in 1949 to the present time.The *China Data Center* of the University of Michigan[8] gathered main information in electronic form and made the content of the yearbooks conveniently available. The Center reported data values as soon as they were published in the corresponding yearbook. Although results have been sometimes subject to revision in later years, they didn't systematically adapt past values. Thus, a discrepancy sometimes arises between National Accounts or trade statistics figuring in the database and the later official yearbooks of some provinces providing revised data. For most provinces, online statistical Yearbooks are only available for the 90s and 00s. Even for those years, results are incomplete over time and accross provinces. For these reasons, we will rely on the Center's database. In a later section, we plan to check to which extent revisions affect our conclusions.

2.2 Aggregation properties

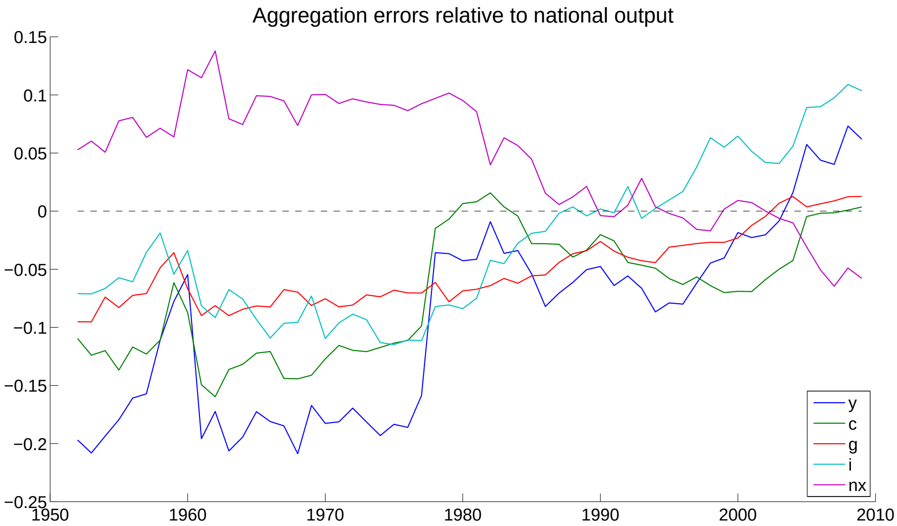
Before starting with some empirical facts, we focus on the aggregation properties of the data. Intuitively, the sum of provincial aggregates should be roughly equivalent to national values, keeping in mind that measurement errors and sample gaps probably hinder any perfect match. The question of interest is whether errors appear to be random. This is an indispensable step towards checking the quality of the aggregates and having a better grasp of consequences on our later findings. To get an intuitive error indicator, we substract national values from aggregated provincial ones and weigh the result with national GDP for each year. A positive result means the aggregated provincial data underestimate national values. Inevitably, as the data availability across regions improves over time, we expect initial gaps to disappear or at least to fade towards the end of the sample. The errors of GDP are decomposed into private

[6] In 2011, the IMF estimated ppp gdp per capita at 8394$, still less than Tunisia (9557$) or Jamaica (9003$).

[7] The autonomous regions are Tibet, Xinjiang, Guangxi, Inner Mongolia and Ningxia. The cities of Beijing, Tianjin, Shanghai as well as the region of Chongqing are municipalities.

[8] http://chinadataonline.org/

consumption, government consumption, investment and net exports errors. Thus, for each variable, we obtain an indicator of the relative aggregation error compared to national values.



Aggregated provincial output is massively underestimated compared to national values with an average of 16.65% of national gdp in the part of the sample preceding economic reforms (1952-1978). Nevertheless, it is partly compensated given that some heavyweight provinces only have available GDP data from 1978 onward[9]. As an intuitive mean of taking these gaps into account, we substract their 1978 weight from the average error and are left with a value of 5.09% which is in line with the corresponding indicator from the 1980s to the early 2000s[10]. Thus, between 1952 and 1999, a sizeable chunk of national output hovering around 5% isn't captured in the provincial statistic. Surprisingly, this pattern reversed during the following period: provincial aggregates topple national values in 2004 and end up overestimating them by 6.19% of GDP. Obviously, an unidentified factor has been at work in the last decade that either led provinces to report too high output, national statistics to report a too low output or a mix of both.

By turning to the elements of output, one can better appreciate the key components responsible for this marked trend. Private consumption error is suddenly reduced as Guangdong, Sichuan and Ningxia appear in 1978, followed by Hainan in 1979, Jiangxi in 1980 and Tibet in 1992. After having reached roughly the same value as its national counterpart in the early 80s, it followed a U-pattern reaching -7.01% in 1999 and came back to a neutral position of 0.36% by 2009. As for government consumption, the positive trend from under- to more precise estimation is clearly visible and doesn't seem to be much influenced by the arrival

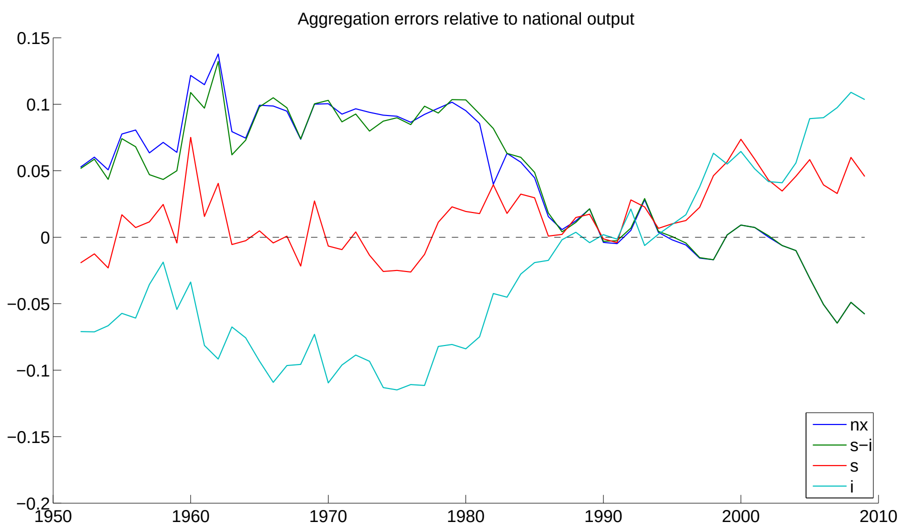
[9]There is a high heterogeneity in the regions with incomplete GDP sample: at reporting start in 1978, Guangdong and Sichuan accounted for no less than 5.38 and 5.12% of national output. The impact of smaller provinces is negligible: Hainan and Ningxia make up about 0.54 and 0.36% of production in 1978. Tibet only starts in 1992 with an impact of 0.15%.

[10]Not correcting for the absence of Tibet, the average errors amount to 5.45% between 1979 and 1999.

of the bigger provinces in the sample[11]. Thus, consumption in general doesn't seem to have played a major role in the recent output error trend.

Although the bulk of already mentioned incomplete provinces all enter the sample at one fell swoop in 1978[12], no markant slump in investment errors is observed. This indicator rather follows a clear positive trend as it reversed from more than -11% in the 1970s to nearly 0 a decade later and continued on its rise. In 2009, the overestimation amounted to 10.35% of GDP. Errors in investment thus largely triggered off the trend in output errors.

In a next step, we turn to a decomposition of net export errors in investment and saving component. The investment error reversal reflects itself in a major shift in net exports errors: provincial values had been overestimated at an average rate of 8.81% of GDP from 1952 to 1980 before reaching 0 in the early 90s and ended up underestimating national ones by 5.78%. Errors in savings have no clear trend over the entire sample. Nevertheless, a slight underestimation mean of 1.61% is observed. The bias grew in the last decade to reach 4.57% in 2009 and partly compensated the effect of the huge investment errors on net exports. Because of the asymmetry in data availability, the net exports we computed from savings and investment (s-i on the graph) don't exactly correspond to their direct reported values (nx). This is less of an issue as the discrepancy weeds out as we approach the end of the sample reflecting improvement in data availability.

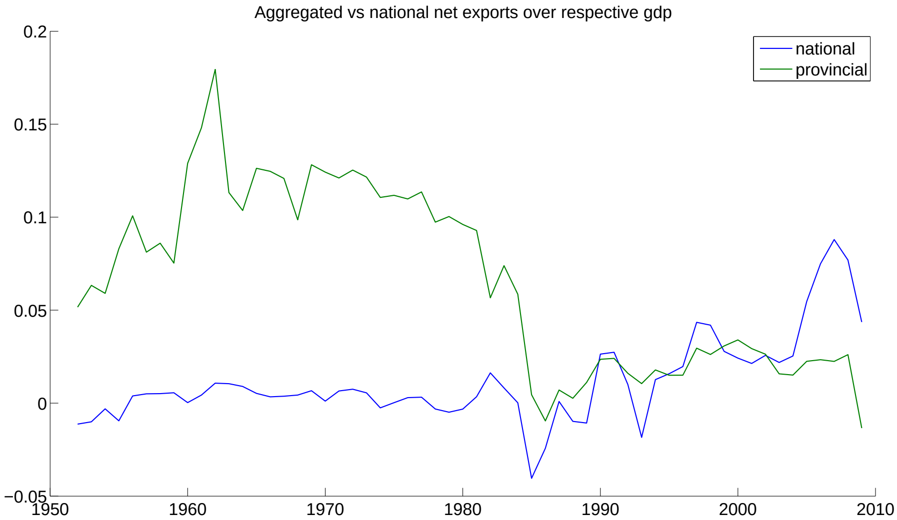


Obviously, provincial data have poor aggregation properties. The investment error reversal is particularly troubling as it contaminates output and biases net exports far more than errors in savings. Being a small part of output compared to other components of the national account, net exports are severely affected. By comparing aggregated with national net exports, we end

[11] Apart from the fact that Hainan starts one year later (1980), there are no differences in sample entry compared to private consumption.

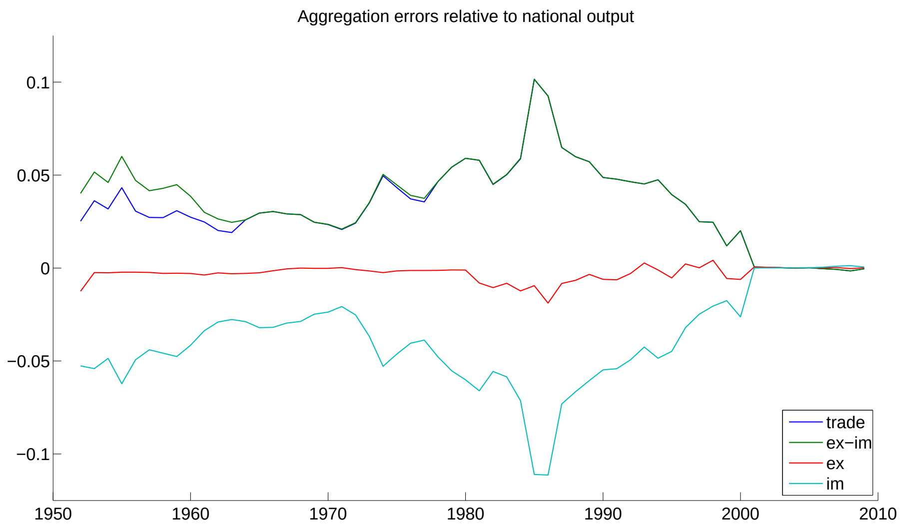
[12] Guangdong, Sichuan, Ningxia, Jiangxi and Hainan start in 1978 while Tibet shows up in 1992.

up with a far too low but still positive values for aggregate provincial data towards the end of the sample. By dividing both net export measures by their respective output, we notice the bias is far more blatant for the years before the onset of economic reforms when provincial net exports were far too high. In fact, the best match extends from the mid 80s to the early 00s.



Data reporting international trade flows on the provincial level expressed in US dollar will enable us to focus exclusively on imbalances of provinces relative to the world. Exports and total trade are available. Unfortunately, no specific data on provincial national account aggregates in dollar such as output exist. With the help of national trade values available in both currencies, we derive an implicit exchange rate and use it to transform these data into Renminbi[13]. The aggregation properties of the data are checked by dividing the difference between provincial and national value by national GDP (all in US dollar).

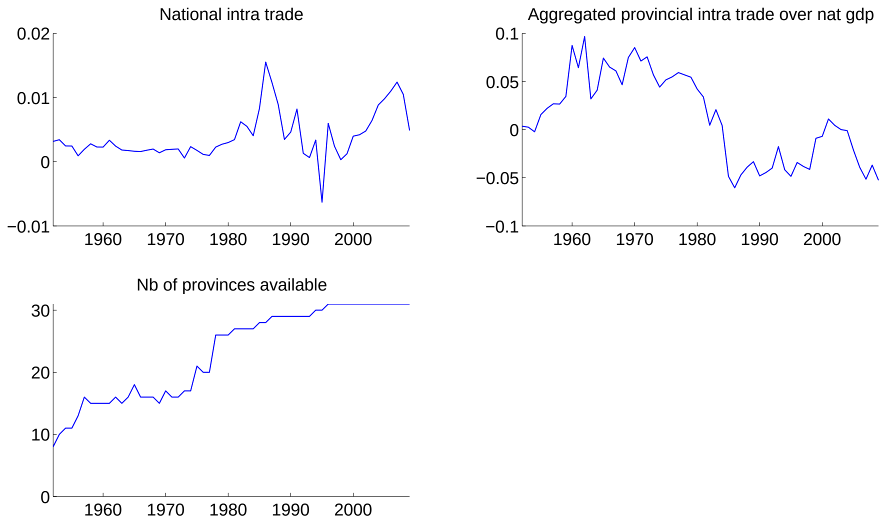
[13]Some regional Yearbooks made results in RMB available as well but only for recent decades. To keep the sample as large as possible and avoid to provoke distortions related to the potential use of different exchange rates, we apply the national trade exchange rate to all provincial results.



The data availability is definitely worse than for net exports and we thus refrain from commenting pre-1978 patterns[14]. Exports seem to be far better recorded than imports which are far too low. As a consequence, the provincial net trade saldo is on average 3.30% underestimated relative to national output. Estimations improved in the 90s. In the last decade, aggregates correspond to their national counterparts[15].
Sadly, only 7 out of 31 provinces have data covering the entire period preceding the reforms. The later sample looks better with only 5 provinces having incomplete data[16].

An approximation of intra-Chinese trade is realisable by using data on provincial net exports and trade. The net exports indicator captured all goods and services flowing in and out of the province meaning no difference was made between international and inter-provincial trade. The international flows proxy exclusively focuses on exchanges with the rest of the world. By substracting them from the net exports, we get a rough measure of intra (or inter-provincial) trade. The implicit exchange rate computed earlier is used. The data availability problem is more acute than in the two preceding part due to the linear combination of the two variables, particularly in the earlier decades. Still, starting in 1979 only five provinces have sample gaps.

[14] Even at this later point, Fujian, Hainan, Chongqing and Shanxi have no recorded values.
[15] Two outliers in relative trade balance are far-off the normal path and we suspect a recording error. Shanghai went through the roof from -20.43% to 180.66% in 2004. In 2005, it is back at -4.41%. In the given data, the exports are bigger than the sum of exports and imports, leading to negative imports. In the same spirit but in another direction, Xinjiang plummeted from 21.12% to -338.39% in 2008 to get back on track at 12.83% in 2009. This bump is caused by a jump in imports in dollar data. After having consulted the Provincial Yearbook Statistics, they were obviously typos and we took the official (revised) values for these two points.
[16] Fujian, Hainan, Chongqing, Tibet and Shaanxi.



A few checks derived from simple theoretical considerations will enable us to better appreciate this approach. To begin with, the intra-trade indicator using national values shouldn't deviate persistently from 0. Indeed, no alarming difference between both is observable as an average of 0.39% of GDP over the entire period is obtained. The highest transitory discrepancies amount to 1.55% in 1986, -0.63% in 1995 and 1.24% in 2007. Another check lies in simply adding the balance of all provinces: the sum of them should be approximately 0 if intra trade is sensibly estimated. Unfortunately, the poor data availability means we won't get balanced results anyway. Only half of the sample is available until the mid-70s. More data were collected with the onset of economic reforms. In the first part of the sample, we have an excess aggregate provincial value of 4.92% of national GDP in average. The pattern flips in the reform era with a mean of -2.08%. Our indicator thus only partly captures intranational trade, even with a complete sample (available from 1996 onward).

This discrepancy is worrying but there are some sound potential reasons to rationalize it.
Firstly, in the provincial National Accounts in Renminbi, exports and imports of good and services are calculated at free on board (fob) while in the international trade statistic imports are at cost, insurance and freight terms (cif). Thus, the value of imports contains services such as insurance and transport costs not included in the first net exports measure. Trade balance in dollar is therefore lower and our indicator of intra trade too high.
Secondly, the NBS boasts foreign trade statistics capture some income flows such as aid or contributions from overseas Chinese[17]. Following the IFS standards, they would rather be categorized in the current transfers of the current account which has been positive and steadily rising in China since the mid 90s. For most provinces, the saldo should logically be positive as well implying a too high trade balance compared to the first concept of net exports and therefore too low intra trade measures. In 2008, current transfers were making up 12.7% of

[17] Although the official definition clearly mentions that gifts, aid and contributions of special administrative regions and oversea Chinese are included in these statistics, we have found no clue of them in the figures ventilated by composition which are available for some regions and China. This indicator seems to capture only trade movements. In this case, the following point would be irrelevant.

the trade balance of China[18].

Thirdly, provinces at the end of the production path containing major shipment zones such as harbors tend to have overestimated exports. This is a consequence of the known issue of the difficulty in adequatly measuring trade flows as transfers of final goods are better captured than intermediate goods. Consequently, East Coast export hubs probably have too high international exports and too low internal balances. This argument is only relevant if imports are concerned to a lesser extent as coastal provinces could possibly rather rely on a mix of imports from hinterland regions and foreign countries and export predominantly to the rest of the world. Clearly, more information is needed on that issue.

The second and third effects probably largely outweigh the first one, especially in the trade-intensive coastal regions. It could potentially explains the negative aggregated value of intra trade observed since the mid 80s.

By using provincial input-output tables, we would be able to check to which extent our approximation makes sense. Unfortunately, no comprehensive yearly time series are easily available. Provincial surveys exist for 1987, 1992, 1997, 2002 and 2007 but are considered highly sensitive for commercial and national security reasons. Naughton (2003) and Poncet (2002) used them to assess domestic market integration. We have been able to get flow of funds data over 2000-2007 for the province of Henan and found our interprovincial indicator to have mostly negligible errors.

As a consequence of the problems encountered in this section, one has to remain cautious by interpreting the results of this study, particularly with statements concerning absolute values. Keeping that in the back of our mind, we will rather concentrate on the relative ranking of provinces and cross-sectional pattens. The errors shouldn't invalidate our conclusions as long as they occurred andomly or in all provinces with the same relative intensity. At this point, we have no choice but to rely on a fingers-crossed argument.

3 Some empirical facts on imbalances

3.1 Cross-sectional patterns of output

There is a large literature on the level and dynamics of output and income inequalities in China. Fleisher and Chen (1997) found a conditional convergence of per capital production across provinces from 1978 to 1993[19]. Historically, a large part is attributable to the rural-urban income gap (Chang, 2003). As noticed by Kanbur and Zhang (1999), the inland-coastal contribution to total inequality hugely increased in the 1990s. Zhang et al. (2001) note that Western and Eastern China seemed to converge to different steady states. The evolution of regional clusters relative to national values is nicely presented in Yao and Zhang (2001): since the reforms started, the East Coast region grew faster while Central China lost momentum and converged toward backward Western China. An acceleration of the process

[18] IFS database for mainland China.

[19] They controlled for physical investment share, employment growth, human capital investment, FDI and coastal location.

is observed since the mid 1990s. A short description of the Maoist development strategy is provided by Yang (2002): the official goal was to eradicate regional industrial disparities and investments were mostly promoted in interior provinces for strategic reasons. Ideally, each province had to be able to survive in autarchy. At the same time, the low prices imposed on agricultural production and resources harmed interior provinces. Jian et al. (1996) found that real income among provinces was stable from 1952 to 1965, diverged during the Cultural Revolution (1965-1978) but began to equalize in the initial reform period thanks to a rise in rural productivity. Disparities started to grow again with the integration of the coastal provinces in the world economy in the 1990s. Thus, it seems spillover effects from coastal regions haven't reduced inequalities at least in the 1990s as discovered by Brun et al. (2002). Not surprisingly, productivity in coastal regions is stronger (Chen et al., 2009).

The factors explaining these differences are numerous. Demurger (2001) used provincial data over 1985-1998 and identified differences in geographical location and infrastructure endowment (transport and communication). Preferential policies also played a key role (Demurger et al., 2002) as coastal regions were the first to benefit from special tax treatments and FDI friendly legislations. Over the 1981-2005 period, openness boosted productivity growth in coastal provinces(Jiang, 2011). Human capital seemed to differ according to the economic structure: from 1997 to 2006, more developed coastal provinces benefited more from tertiary education while interior provinces were more sensitive to primary and secondary education. At the firm level, Mody and Wang (1997) used data on 23 industrial sectors in seven coastal regions from 1985 to 1989: existing strength in high quality human capital and infrastructure were important explaining factors as the new open door policy and SEZs[20] began to take hold. Liu and Li (2006) found that growth imbalances over 1984-1998 were strongly related to the financial sources or ownership types of capital. Domestic bank loans and foreign-owned enterprises are important in coastal provinces while state appropriation or state owned enterprises rule the roost in inner provinces.

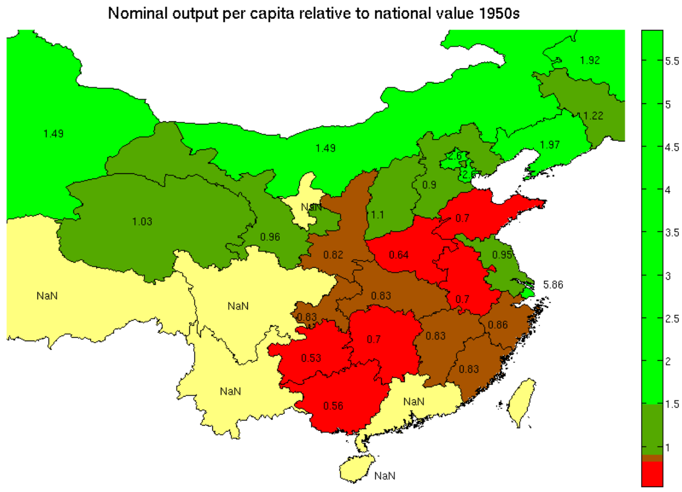
Nominal gdp data are available from 1952 onward for 26 out of 31 regions. For each period, we divide gdp per capita by national value to facilitate comparison over time. The population data used stem from the Data Center and the corresponding Statistical Yearbooks[21]. They are local estimates less precise than general periodic census results but have the advantage of better capturing dynamics as they are collected on a year to year basis[22]. We will not linger on their good aggregation properties. Provincial consumer price indexes are available for all but one province from 1985 onward[23]. We denominate the deflated nominal values real per capita GDP (in 1984 Renminbi).

[20] Special economic zones

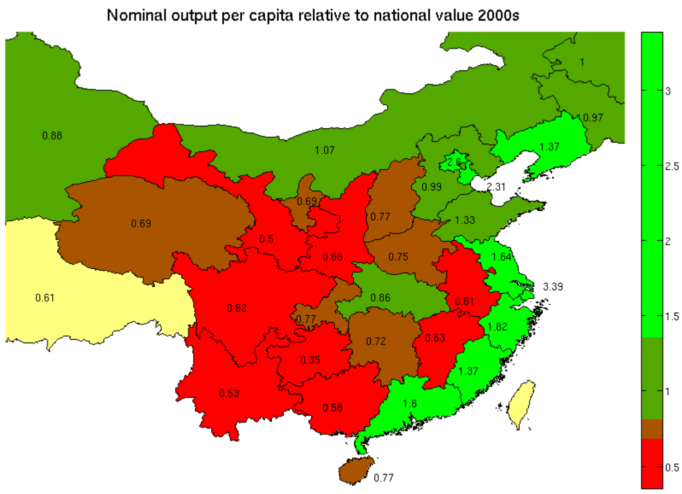
[21] Only short gaps are reported: Yunnan starts in 1953 and Tibet lacks any data from 1953 to 1957.

[22] In the Center's database, the population of Chongqing had wrongly been added to the one of Sichuan from 1989 to 1996. We correct it and use disentangled values for both provinces.

[23] As usual, the province of Tibet starts reporting them later. In most of our maps, it will be considered as non reported value color because of the short sample available.

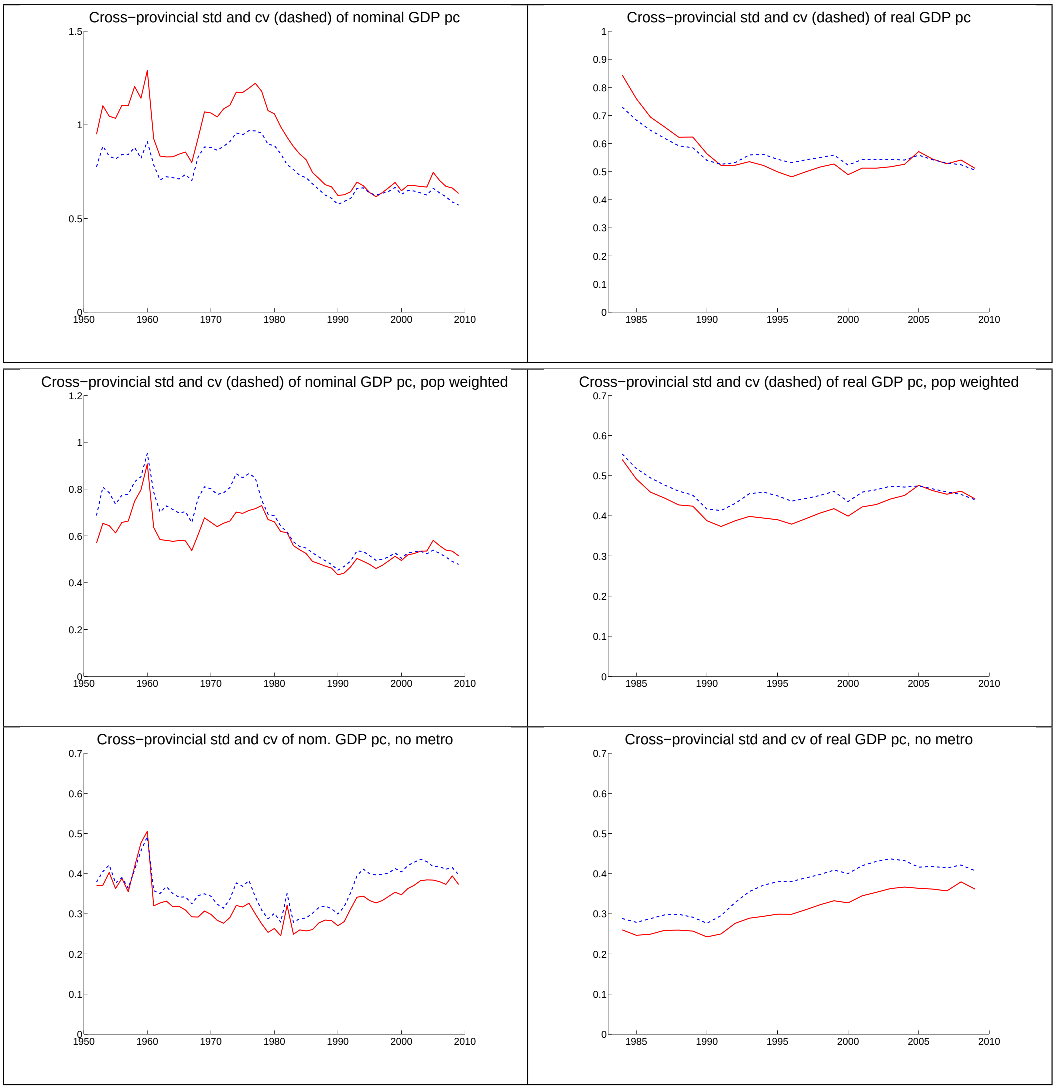


In the 50s, a clear North-South divide in output per capita relative to national value is observed. The metropolises stand out as the richest provinces. The wealth of territorial provinces was principally based on natural resources: Manchuria boasted important coal, gas, oil and mineral deposit reserves and was to become the main industrial base of China for the next decades. Inner Mongolia and Shanxi benefited from coal extraction as well. In the Far-West, Xinjiang, Qinghai and Gansu took advantage of their oil reserves. This broad pattern persisted until the progressive opening up of the economy in the 80s. Then, a shift of production capacities to the East Coast happened over the following decades. The impressive gap between the city-provinces and national values has steadily been narrowing.



As a result of the export-led development, the wealthiest provinces are now located on the coastline. Guangdong in the Pearl River Delta obviously benefits from the proximity of Hong-Kong and Macau. Another production hub is the Yangtze River Delta comprising Shanghai, the North of Zhejiang and the South of Jiangsu. Higher in the North, the region bordering the Bohai sea is another industrial hotbed with Beijing, Tianjin, Hebei, Shandong and Liaoning. Still, most resources abundant regions continue to have relatively comfortable positions relative to Southern and Central West provinces which badly ranked[24]. Central China is below national values as well, albeit to a less dramatic extent with the exception of Anhui and Jiangxi which are probably suffering from the proximity of more competitive coastal hubs.

To shed light on the evolution of disparity in output across provinces over time, we compute the cross-sectional standard deviation of the relative nominal and real gdp for each year. The practice of dividing them by the national values enable us to ignore the tremendous growth experienced by all of these economies and focus on the disparities relative to the nation as a whole.



The disparity indicator dropped dramatically in the aftermath of the Great Leap Forward experiment and the slump it provoked. Obviously, provinces were levelled down so that a

[24] Agrarian and mountainous regions like Guizhou (0.35), Yunnan (0.53), Sichuan (0.62) and Guangxi (0.58) lagged behind in terms of output per capita relative to national values.

certain convergence took place. Pre-reforms discrepancies were reached by the end of the Cultural Revolution. Then, we report a sustained decrease in standard deviation as the first reforms began to take effect[25]. Cross-provincial disparities in relative gdp staid relatively constant in the 90s and 00s with some occasional minor peaks. As an alternative indicator, we always compute the coefficient of variation (standard deviation divided by mean) using absolute values (dashed line). The pattern is similar but the decrease less marked.

We recompute standard deviation and weigh provinces and the cross-sectional mean according to the relative population on the grounds that convergence of some heavily populated and economically important provinces should have a deeper impact than small ones. The order of magnitude of relative variations is similar[26]. Interestingly, we now denote a slight increase in the last two decades. Following Demurger et al.(2002), we check whether the decrease in variability survives to the exclusion of Beijing, Tianjin and Shanghai. The difference is striking: both indicators show a sustained increase in discrepancies across provinces since the onset of economic reforms. The city-provinces thus greatly influenced the discussed patterns.

A potential factor biasing our finding lies in the apparition of four new provinces in the sample at the onset of economic reforms in 1978[27] and potential differences in the inflation rate. The same indicators computed for cpi deflated GDP starting in 1984 (full sample except Tibet) corroborates our earlier findings (graphs on the right).

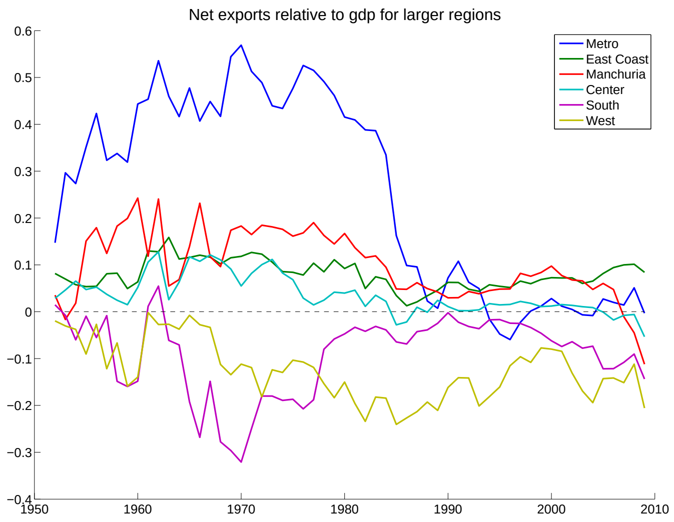
3.2 Cross-sectional patterns of flow indicators

In the Provincial Accounts statistics we have been using, each entity is considered as a country with its own GRP (gross regional product) and GRP components. Goods and services crossing the boundaries of one province have thus to be reported in net exports, independently of whether their final destination is the rest of the world or another province[28]. A good grasp of the internal distribution of provincial net exports over output is of paramount importance to have a better understanding of the imbalances in inter-provincial and international trade.

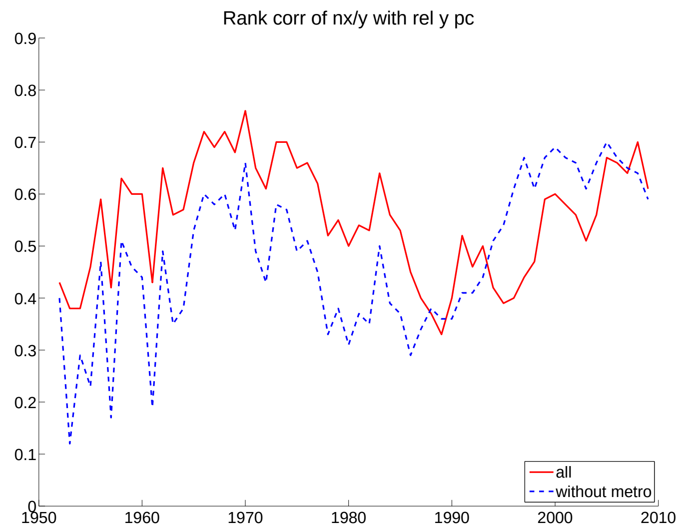
A clear pattern emerges from the comparison of different periods in the sample: prior to the reforms, more industrialized Manchuria[29] and Metropolises[30] mostly already had large surpluses. By the end 2000s, theses regions had turned into net importers. Central China provinces[31] have had roughly balanced position over the last decades. Interestingly, East Coast regions[32] already had large surpluses averaging more than 10% of their gdp in the 1960s and 1970s. Apparently, the initial years of the market economy era brought them to a near neutral position by the mid-1980s. Since the opening up to foreign trade, this region

[25] Our indicator experienced a 49% decrease from 1977 to 1990.
[26] Drop of 41% from 1977 to 1990.
[27] These are Guangdong, Hainan, Sichuan and Ningxia.
[28] Unfortunately, no additional information on the exact computation of provincial net exports has been found. Definitions made available by the Statistical Bureaus are identical across provinces and are similar to those usually used in international trade.
[29] Liaoning, Jilin and Heilongjiang.
[30] Beijing, Tianjin and Shanghai.
[31] Henan, Hubei, Hunan, Jiangxi, Anhui and Shanxi.
[32] Hebei, Jiangsu, Zhejiang, Fujian, Shandong and Guangdong.

steadily increased its surpluses to reach more than 10% in 2008. Southern China[33] markedly reduced its deficits in the 1970s and 1980s but lost its positive momentum in the 1990s. The West[34] always had large negative net exports all along.

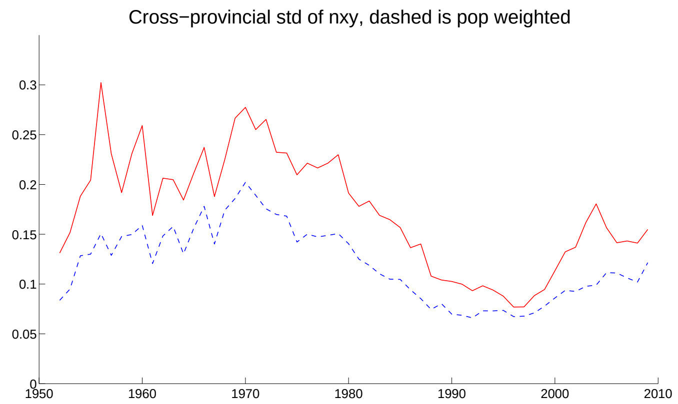


Historically, high savings over gdp and net export surpluses seem to have been related to the relative provincial output per capita as the relatively wealthier regions had large surpluses. The relationship dwindled somewhat during the initial reform years and has been on the rise since the 1990s. A similar correlation indicator for aggregate saving over gdp with relative gdp per capita would hover around a value of 0.7. The corresponding correlation measure of relative investment with gdp is of near 0 on average.



[33]Guangxi, Hainan, Guizhou, Yunnan, Chongqing and Sichuan.
[34]Inner Mongolia, Shaanxi, Qinghai, Xinjiang, Ningxia, Gansu and Tibet.

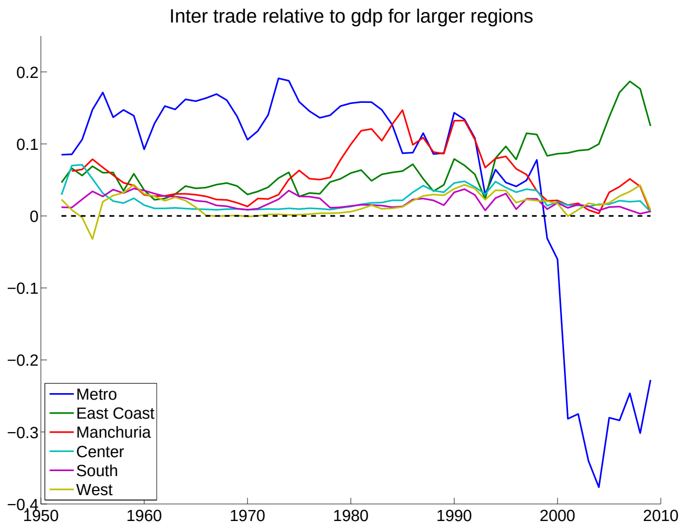
As for the preceding part, we want to discuss the pattern of cross-provincial variability over time. In this section and the following ones, we will not compute the coefficient of variation because it fluctuates hugely near 0 cross-sectional means. The standard deviation of net exports has been volatile but roughly stationary with an average of 0.22 in the pre-reform era. From the 1979 level to 1996, our indicator reports a decrease of 67% but then progressively bounced back to a higher level. Indeed, the 2009 level corresponds to the value reported in the mid-1980s. This U-turn pattern is less marked when considering population-weighted variability[35]. Results are little affected by the exclusion of the city-provinces. The apparition of Jiangxi, Sichuan, Guangdong and Hainan in the sample doesn't modify the general pattern. The surge in standard deviation would be less impressive by removing Tibet and Ningxia.



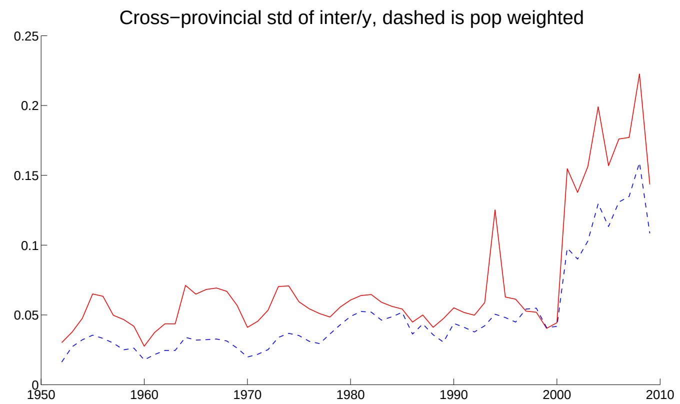
The geographical distribution of international surpluses and deficits for 1979-2009 is clearly different than in the RMB net exports of the preceding part in one aspect: most regional clusters have a positive balance. Not surprisingly, the Metropolises are hugely involved in foreign exchanges: they exhibit a high surplus of 14% of gdp from 1952 to 1990 before experiencing a sharp deterioration which peaked at -38% in 2004. The rising surplus (up to 18%) of the East Coast is not surprising given their involvement in international trade. Note that their positive trend already started before reforms. Manchuria recorded large surpluses in the 1980s and 1990s before converging to a near neutral position. Other clusters are less involved in trade: Central, South and West China had an average small surplus of round 2% between 1970 and 2009.

Thus, there are large differences in the exposure to international trade and position across provinces. Most regions boast a positive average saldo. This wasn't the case in the net exports case where some large negative and positive values were found. It suggests inter-provincial rather than international trade is responsible for the large net export deficits in some provinces.

[35]The 1979-1996 decrease amounts to 55% compared to 67% before. From 1997 to 2009, it increased of 79% against 101% in the computation using similar weights.



The cross-provincial variability is relatively constant and fluctuates around 0.05 until the late 1990s. In the 2000s, we observe a tremendous increase. The population weighted standard deviation has the same dynamics. A big chunk of the jump in variability in 2005 and the subsequent persistent high values are caused by the Metropolises, particularly by the municipality of Beijing which experienced a dramatic fall at the beginning of the 21th century. Little change is observed if we exclude provinces with poor data availability[36]. The 1994 peak originated from a large variation for Tibet.



4 SOE model applied to Chinese provinces

4.1 Model set-up (Gourinchas and Jeanne, 2011)

We assume a small open economy. Time is discrete and there is no uncertainty. A single homogeneous good is produced. The production function is of the Cobb-Douglas type:

$$Y_t = K_t^{\alpha}(A_t L_t)^{1-\alpha}$$

[36]Fujian, Hainan, Chongqing, Tibet and Shaanxi.

Factor markets are competitive. The aggregate BC of the economy is:

$$Y_t = C_t + I_t + R^* D_t - D_{t+1}$$

The dynamics of investment over time is:

$$I_t = K_{t+1} - (1-\delta)K_t$$

The investment or capital wedge τ_k impacts on gross return R_t:

$$(1-\tau_k)R_t = R*$$

The marginal product of capital net of depreciation is (k being capital per capita):

$$R_t = \alpha(k_t/A_t)^{\alpha-1} + 1 - \delta$$

The capital stock per efficient unit of labor is:

$$\tilde{k}_t = \frac{K}{AN} = \tilde{k} = \left(\frac{\alpha}{\frac{R^*}{1-\tau_k} + \delta - 1}\right)^{\frac{1}{1-\alpha}}$$

Countries have an exogenous productivity path bounded from above by the world productivity frontier which grows at rate g^*:

$$A_t \leq A_t^* = A_0^* g^{*t}$$

For a finite period of time, a country could gro faster than the world. The evolution over time of domestic relative to world productivity is captured by the technology catch-up parameter:

$$\pi_t = \frac{A_t}{A_0 g^{*t}} - 1$$

A positive π means the country catches up relative to the world.

Representative HH maximize the following CRRA utility function:

$$\begin{aligned} U_t &= \sum_{s=0}^{\infty} \beta^s N_{t+1} u(c_{t+s}) \\ u(c_t) &= \frac{c_t^{1-\gamma}}{1-\gamma} \end{aligned}$$

subject to the following budget constraint:

$$N_t w_t + N_t z_t = C_t + K_{t+1} - (1-\tau_s)R^* K_t - D_{t+1} + (1-\tau_s)R^* D_t$$

Wages w are equal to the marginal product of labor and z is a lump-sum transfer. The saving wedge τ_s can be interpreted as a tax on capital income. Revenues generated by the wedges ($z_t = \tau_k R_t k_t + \tau_s R^*(k_t - d_t)$) are redistributed in a lump-sum fashion.

The Euler equation is:

$$\begin{aligned} c_t^{-\gamma} &= \beta R^*(1-\tau_s)c_{t+1}^{-\gamma} \\ &= \beta(1-\tau_k)R_t(1-\tau_s)c_{t+1}^{-\gamma} \end{aligned}$$

It is assumed that the rest of the world is composed of steady-state advanced economies with the same preferences and no saving wedge:

$$R^* = \frac{g^{*\gamma}}{\beta}$$

4.2 Provincial capital stock

National and provincial capital stocks are estimated using the perpetual inventory method. The initial gross fixed capital formation is computed using the following formula:

$$F_0 = \frac{1}{T+1}\sum_{t=0}^{T}\frac{f_t}{((\frac{f_T}{f_0})^{\frac{1}{T}})^t}$$

where f is real gross fixed capital formation in 1984 Renminbi[37]. In accordance with the general notation, 1984 is $t = 0$ and 2009 is $t = T = 25$. Business cycles are removed from the initial value by taking the mean of f weighted by its exponented growth rate[38].

The initial capital stock is:

$$K_0 = \frac{F_0}{\delta + k}$$

where δ is the yearly depreciation rate of capital and k is the growth rate of capital. As in Gourinchas and Jeanne (2011), a value of 0.06 is assumed for all provinces for the first parameter. The choice of a region-specific k is more controversial. Usually, a mean of past growth rates is used. Unfortunately, this method is impossible to apply to all regions due to the lack of data on gross fixed capital formation (GFCF) and consumer price index for the pre-1984 period[39]. As an alternative, we use the mean value for the 1984-2009 period. In the literature, the growth of output of past periods is sometimes used as a proxy for capital growth. Here again, the poor output data for some provinces before 1984 restrains us from giving it a try. Using post 1984 output growth would lead to a roughly similar k compared to using GFCF[40].

The functional form of the dynamics governing capital accumulation is:

$$K_{t+1} = I_t + (1-\delta)K_t$$

[37] Technically speaking, one should deflate f by the price of investment in fixed assets. Unfortunately, we were only able to recover it from 1992 onward and therefore stick to CPI.

[38] Suggested by Prof.Dr.Woitek.

[39] Jiangxi, Guangdong, Sichuan and Ningxia report no GFCF values before 1978. As for CPI, half of provinces lack completeness.

[40] For China, the real output growth over the sample period has been of 10% while gross fixed capital formation growth has been of 12%.

By using the initial capital stock and real gross fixed capital formation, we obtain provincial and national time series for capital stock.

In 1984, the initial capital stock to real output ratio is on average a little less than 1.20 with values between 0.58 and 2.29. Our estimation strategy has some obvious shortcomings. Intuitively, relatively rich provinces should have higher K/Y ratios than less developped one. In our case, this is generally true: the provinces with high real per capita gdp in 1984 like Shanghai or Beijing have higher than average capital stock (1.34 and 1.90) while poor regions such as Guanxi or Hunan have lower values (0.75 and 0.58). There are some important exceptions: although being below average in terms of wealth, Western provinces like Xinjiang (2.29), Qinghai (1.76) and Ningxia (1.59) start with high initial values. Towards the end of the time span, the relationship between real gdp per capita and K/Y even turns slightly negative. An explanation lies in the fact that many poorer Western provinces started with low output and experienced huge capital formation flows relative to their economic size. As a consequence, 2009 relative values are higher compared to wealthier regions. On top of that, for some of them, growth in GFCF has remained low causing the initial 1984 value to be high relative to initial output. In 2009, values for relative capital stock are between 1.63 and 3.40 with a mean of 2.40. In spite of tremendous growth in output experienced over the period, the average provincial relative capital stock doubled between 1984 and 2009. Further alternative computations are going to be discussed in a later section.

4.3 Provincial technology and catchup parameters

The time path of productivity A is estimated using the functional form of the production function given in Gourinchas and Jeanne (2011) with $\alpha = 0.30$, employed persons in million as a proxy for L and output by expenditure approach in 100 million Renminbi for Y:

$$\begin{aligned} Y_t &= K_t^{\alpha}(A_t L_t)^{1-\alpha} \\ A_t &= (\frac{Y_t}{K_t^{\alpha} L_t^{(1-\alpha)}})^{\frac{1}{1-\alpha}} \end{aligned}$$

In a next step, all variables are normalized by their starting value. This is legitimate as we are not interested in an econometric estimation of the production function per se but only in the dynamics of productivity over time. In order to smooth out transitory fluctuations, the Hodrick-Prescott filter with smoothing parameter 100 is applied to the time series[41].
The annual growth rate of A can is obtained using filtered data:

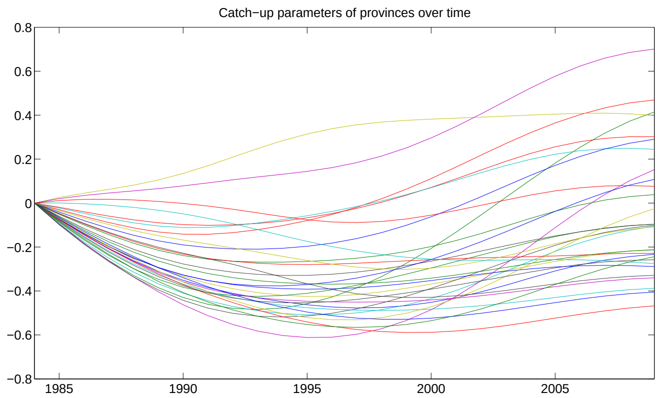
$$g^* = (\frac{A_T}{A_0})^{\frac{1}{T}}$$

[41] Gourinchas and Jeanne (2011) used a higher value of 1600 which filters out more than 70% of cycles lower than 32 years. Although it certainly fits within the long run optic of the model, we stick to the usual macroeconomic value. The reason is specific to the huge transformations of the Chinese economy. The steep rise in TFP experienced in most provinces causes the filter to start with a value far below the data point. As a consequence, TFP growth would be unrealistically high. Furthermore, we are not convinced of using such a heavy filter on a short time series. In the end, it should only affect the absolute values of the catch-up parameters but not their distribution across provinces.

In contrast to Gourinchas and Jeanne (2011), we adopt a new definition of g^* as being the growth of the Chinese productivity frontier instead of the world.
The catch-up parameters are defined in the following way:

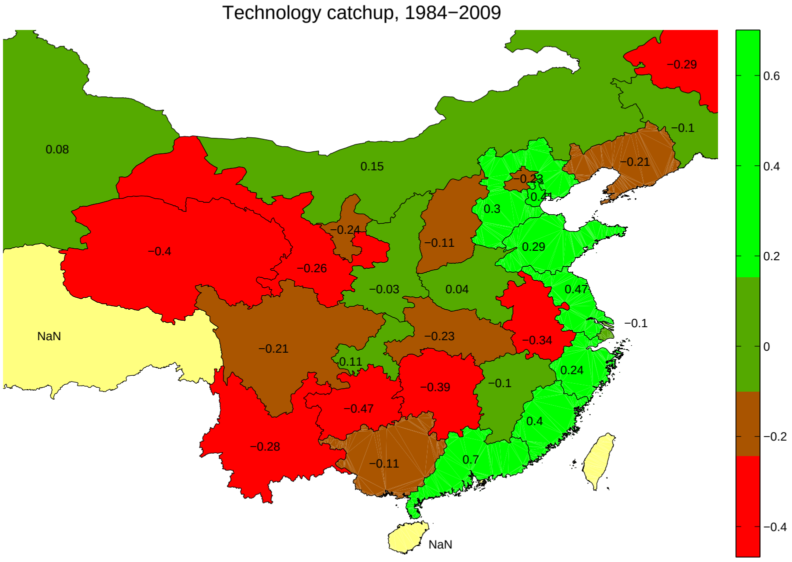
$$\pi_t = \frac{A_t}{A_0 g^{*t}} - 1$$

Note that the last value is defined as the steady-state catch-up which we will use for the estimation of the wedges ($\pi_T = \pi$).
Our empirical value for China is of $g^* = 6.92\%$. We get large differences in average growth rates of productivity across provinces: Guangdong (9.22), Jiangsu (8.58) and Tianjin (8.42) have twice as high values as Guizhou (4.26%).



Here: table 5 in appendix (n, g and catchup for provinces)

Only two provinces have been overtaking the national mean productivity growth during the entire period: Fujian directly started with positive values but it was overtaken by Guangdong. All the remaining regions initially experienced negative catchup from the mid-80s to the mid-90s before riding on a positive trend. There are discrepancies though: many coastal provinces like Hebei, Shandong, Jiangsu and Zhejiang as well as the prefectures of Tianjin and Shangai started to take off in 1993-1994 while others began their ascent only after 1995. Some provinces never recovered the initial decrease and have a flat catch-up over time (for example Anhui, Hubei, Hunan and Yunnan). The geographical distribution of the catch-up parameters is best appreciated using a simple map where provinces are classified per quartile.



Not surprisingly, coastal wealthier regions mostly have positive catch-up parameters. In the Pearl River Delta, Guangdong obviously benefited from investments and shifts of productive capacities of Hong-Kong and Macau. It has the lead with a value of 0.70. In the same spirit, Fujian's cultural and economic ties with Taiwan probably enabled a high growth of productivity over the sample period (0.40). The regions of the Yangtze Delta like Jiangsu (0.47) and Zhejiang (0.24) and have been catching up as well with the exception of a slight decrease for Shangai (-0.1). The last cluster of emerging provinces locates around the Bohai Sea: Tianjin, Shandong and Hebei have values ranging between 0.29 and 0.41. Puzzlingly, Beijing has been falling behind (-0.23). A last rising province is Inner Mongolia which started from large negative values of less than -0.5 to reach 0.15 toward the end of the sample. The old industrial hotbed of Manchuria, although known as being still relatively wealthy, suffered from the opening policy with values between -0.10 and -0.29. The situation in the South is more dramatic as most provinces fell behind relative to national productivity growth although already being among the poorest regions (Guizhou -0.47, Hunan -0.39, Yunnan -0.28). Among Western provinces, only Xinjiang managed to ameliorate its relative position (0.08) while the rest has been experiencing a deterioration (from -0.24 in Ningxia to -0.40 in Qinghai). In the Center and Center-North the situation is more heterogeneous with most provinces having near 0 or slight negative values with the exception of Anhui (-0.34) and the prefecture of Chongqing (0.11).

4.4 Capital flow measures

In Gourinchas and Jeanne (2011), the change in external debt over the sample was approximated by using a measure of initial net external debt[42] and the sum of negative current

[42]They use the difference between the opposite of net international investment position (NIIP) and cumulated errors and omissions.

accounts, both of them transformed in current U.S. dollar. In our case, we unfortunately aren't aware of existing data on provincial net foreign asset position and haven't come up with a good proxy yet. As we start only a few years after the opening up of China, this shouldn't be much of a problem as most of regions are expected to have roughly neutral positions[43]. Turning to the second element necessary to estimate capital flows, we use three narrower measures discussed in preceding sections as no data on provincial current account are available. Variables are deflated (Q) using provincial consumer price indexes[44].

$$\frac{\triangle D}{Y_0} = \frac{D_T - D_0}{Y_0} = -\sum_{t=0}^{T-1} \frac{NX_t}{Y_0} \frac{1}{Q_t}$$

Before we use our three measures of NX, we will have a look at their spatial distribution. Our first proxy is net exports from the Provincial National Accounts. Apart from Tibet and Hainan, our dataset is complete.

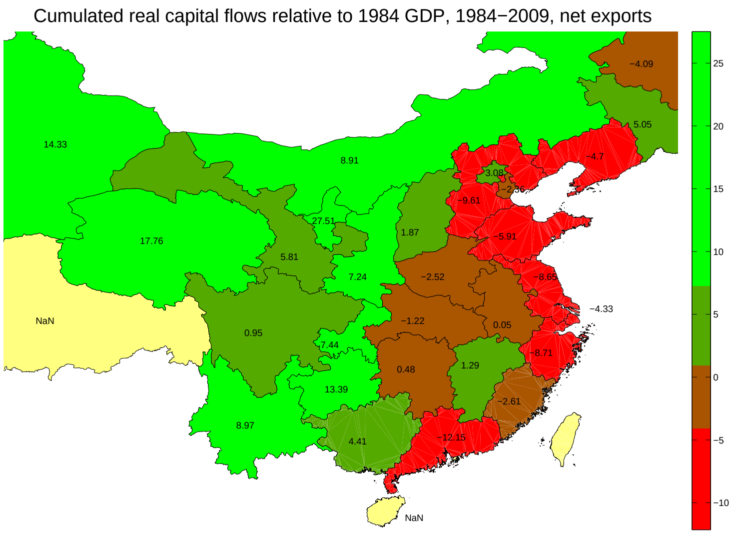


Table 5 in appendix: relative flows

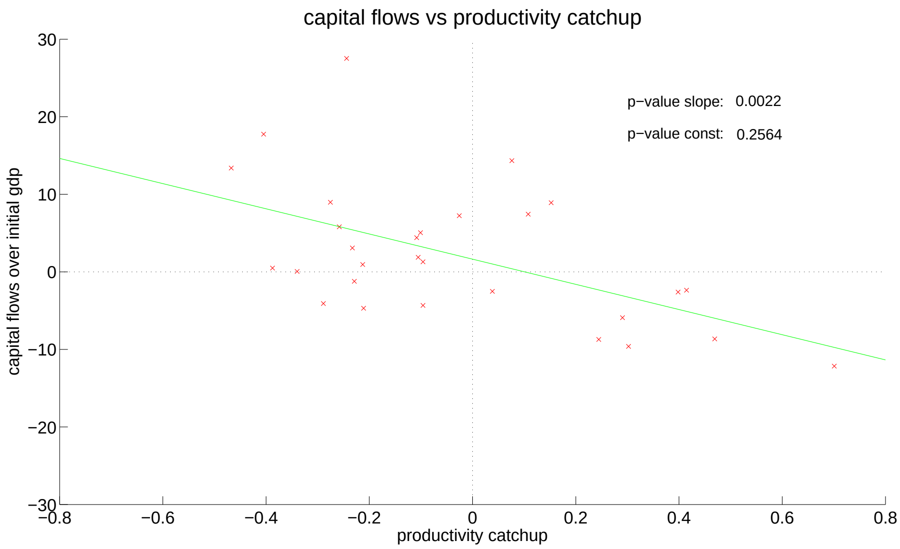
First, massive amounts of capital flows are recorded over the period. While in G&J, values over 1980-2000 typically were between -2 and 1.5 for a sample of 68 developing countries, ours range between -12.15 and 27.51 with a mean of 2.13. The low initial gdp and the tremendous growth experienced afterwards clearly are important factors for these extreme values:

[43] By having a look at values for the People's Republic of China (without Hong-Kong and Macau) computed by Milesi-Ferretti (2006) which were used in the seminal paper, we notice reliable NIIP data only start in 2004. Net foreign assets estimations are available from 1981 onward. Their ratio to gdp hovers around a neutral position in a band of -8.53% and 6.69% until the mid-90s. Then, a clear positive trend pushes NFA to 22.34% of gdp by 2007. As the value for 1984 amounts to 6.69%, one could expect most provinces to have slightly positive NFA but we have no clue of how the distribution would initially look like and don't want to introduce ad hoc effects on such an early stage.

[44] To deflate them properly, one would need a measure of the price of traded goods. G&J used the price of investment goods as a proxy but due to data restriction, CPI is used.

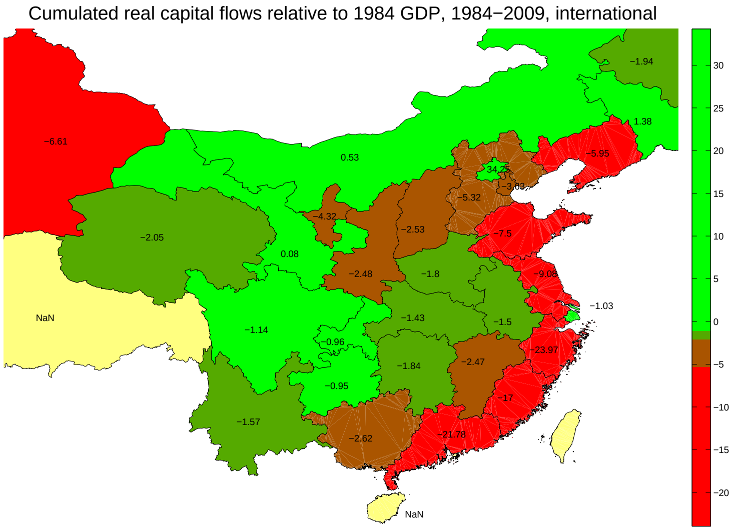
later flows have a disproportionate weight compared to initial (low) output. Coastal export-oriented provinces report sizeable capital outflows with the exception of Beijing (3.08). One step further West, Central China has slight negative or neutral balances. Southern and to a higher extent Western China experienced high capital inflows. Evidence in Manchuria is more disparate as Jilin had capital inflows (5.05) while Liaoning and Heilongjiang had outflows (-4.7 and -4.09).

By plotting the catch-up parameters against the relative flows, a clear pattern emerges: provinces which catched up relatively to national TFP have capital outflows while those which fell behind have capital inflows. Thus, as in G&J on the international level, there seems to be a capital allocation puzzle on the regional level in China[45].



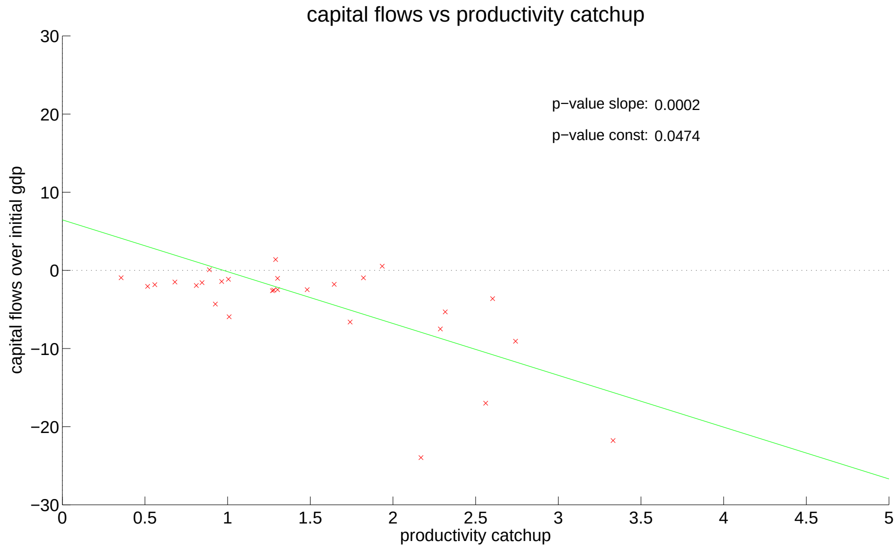
The second NX is computed using data on international trade flows on the provincial level. In order to keep the sample as large as possible, we ignore the fact that Shanxi lacks the 1984 flows and Chongqing the corresponding values between 1984 and 1986. This shouldn't modify the relative distribution because early flows are quite small relative to initial output.

[45]To be fully correct, one should recompute the catch-up values not relative to national figures but to a mix of world and local TFP growth as provincial net exports contain international as well as interprovincial flows. In fact, compared to the rest of the world, all regions have extremely large catch-up values. We find it more convenient to refer to national values to look at cross-sectional patterns. In the robustness checks section, we will see that taking alternative values of g^* only shifts the distribution without modifying the pattern.

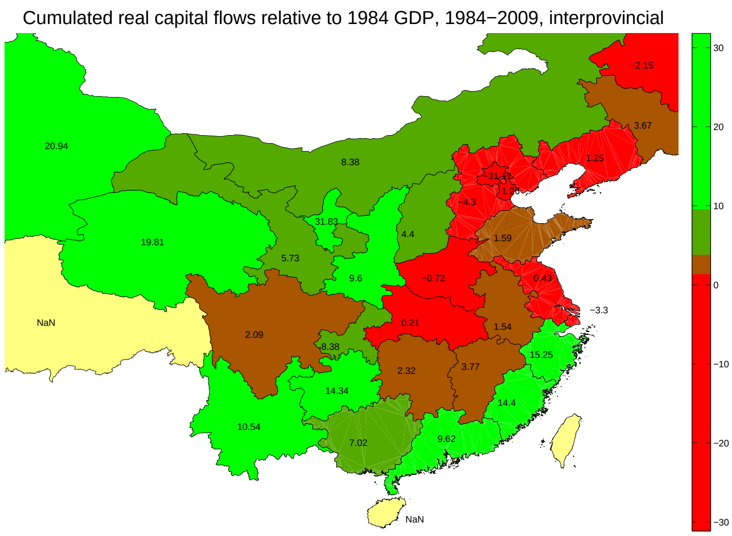


While the range of results is even larger for international flows (from -23.97 to 34.25), only two regions have sizeable capital inflows over the period: Beijing (34.25) and Jilin (1.38). The geographical distribution seems roughly comparable with the East Coast regions recording capital outflows. Some important differences arise for the remaining ones : many Western provinces like Xinjiang (14.33 to -6.61) and Qinghai (17.76 to -2.5) turn into large capital exporters. Thus, the progressive increase in inflows as one moves westward is less clear-cut than before.

As we now focus exclusively on province versus rest of the world trade, we recalculate catch-up parameters using an hypothetical world TFP growth rate of 3% over the 1984-2009 period. Due to their high growth rate, all provinces have largely positive catch-up values. In fact, all should record international capital inflows and lie in the upper-right quadrant above the zero line but it is definitely not the case. As before, we observe a clear negative relationship. Thus, the capital allocation puzzle holds for international flows as well.



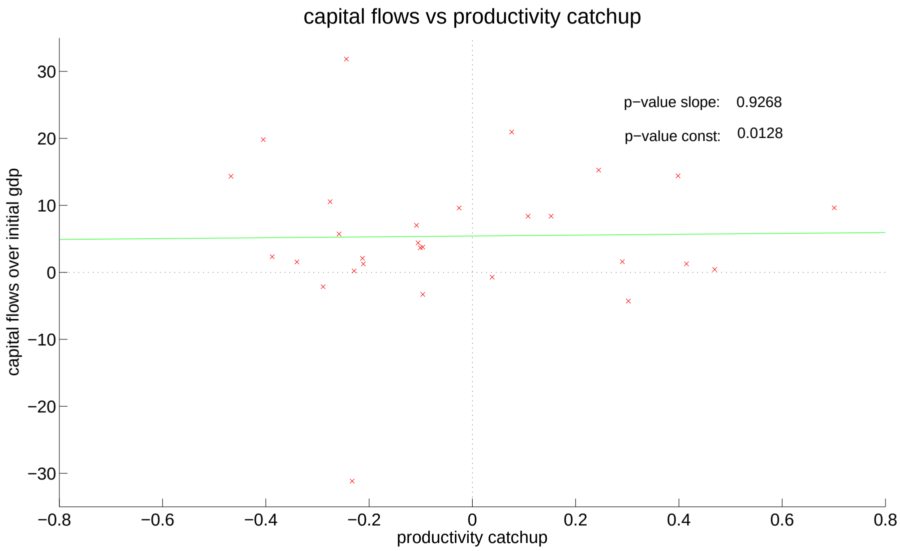
The last flow indicator is the rough approximation of inter-provincial flows. The range of flows (-31.17 to 31.83) is even larger than in the two previous cases.



Some important differences arise in the distribution of relative flows over the period. Only 5 out of 29 regions have experienced capital outflows. It seems the absolute estimated values are too high and some provinces recording small positive values probably have outflows as well. This is not a major issue as we are interested in the relative distribution. Compared to the preceding findings, some new geographical patterns emerge: Beijing (-31.17) confirms its status as a redistributive hub and the entire Bohai Sea region has low or negative values (Hebei -4.3, Tianjin 1.28 and Shandong 1.59). Manchuria is on the lower end of the distribution as well. The Yangtze River Delta (Shangai -3.3, Jiangxi 0.43) and parts of Central

China (Henan -0.72, Hubei 0.21, Anhui 1.54) follow the same pattern. Thus, with the exception of Sichuan (2.09), provinces in the lower part of the distribution form a coherent spatial cluster. A massive part of these flows seems to head toward the South-East export hubs of Guangdong (9.62), Fujian (14.4) and Zhejiang (15.25). As they account for round a fifth of the country's gdp, they should capture a big chunk of intranational flows. Intuitively this makes sense: surrounding provinces provide raw materials and intermediary products to the assembling centers. The fact other coastal provinces further north of Shanghai don't have such massive inflows could possibly be explained by a political will of Beijing to foster economic development in a particular area. South, Western and to a lesser extent the Center North parts of China benefited from large capital inflows as well, in spite of high endowment in mineral resources in some of them.

The foregoing patterns have an influence on the capital allocation puzzle. We plug the relative flows against the same catch-up parameters as for the net exports case. In fact, in a framework of inter-provincial flows, the catch-up relative to Chinese values should be positively related with inflows. Again, it is definitely not the case but we are less far-off the mark than before as there is no relationship between both variables. It is due to the three South-East flows recipient provinces. By omitting them and Beijing, on would obtain a negative (albeit non significant) slope again.



Thus, we register a capital allocation puzzle in chinese provinces using net exports, international and intranational flows.

4.5 Investment wedges

Gourinchas and Jeanne (2011) introduce a wedge on gross returns they dubb capital/investment wedge:

$$(1-\tau_k)R_t = R^*$$

where R corresponds to the marginal product of capital net of depreciation and $R^* = g^{*\gamma}/\beta$ per assumption. They get the following expression for steady-state capital stock per efficient unit of labor:

$$\widetilde{k}^* = \left(\frac{\alpha}{\frac{R^*}{(1-\tau_k)} + \delta - 1} \right)^{\frac{1}{1-\alpha}}$$

Thus, differences in $\widetilde{k}^*$ exclusively arise from different τ_k's. To identify the wedges, they propose a decomposition of average relative investment over the period of the following form (see detailed derivation in appendix):

$$i \;=\; \underbrace{\frac{1}{T}\frac{\tilde{k}^* - \tilde{k}_0}{\tilde{k}_0^{\alpha}}}_{convergence} + \underbrace{g^* \frac{\pi}{T} n \tilde{k}^{*(1-\alpha)}}_{catch-up} + \underbrace{(g^* n + \delta - 1)\tilde{k}^{*(1-\alpha)}}_{depreciation}$$

By implementing a grid-search, the $\widetilde{k}^*$'s and τ_k's enabling to mimick observed i's are identified. By observing the decomposition formula, one easily sees their methodology implies provinces with high relative investment are attributed a high capital per efficient unit of labor. The question is wether this makes sense in our case. For example, the poorer province of Ningxia has a $\widetilde{k}^*$ far bigger than the rich Guangdong (5.22 vs 2.14). The differences in $\widetilde{k}^*$ drive most of the variations in the three channels and determine capital wedges across provinces. As can be seen from the following expression, a higher $\widetilde{k}^*$ implies a lower τ_k.

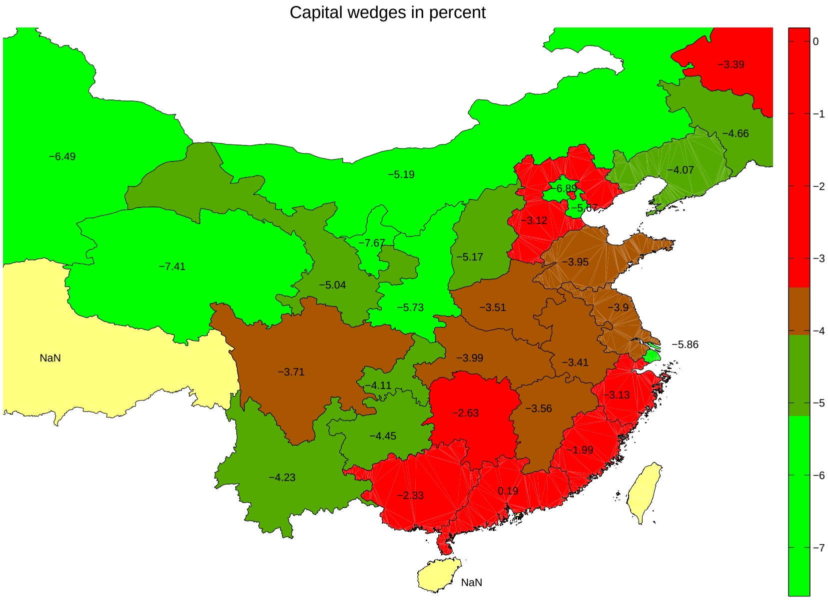
$$\tau_k = 1 - \frac{R^*}{\frac{\alpha}{\widetilde{k}^{*1-\alpha}} - \delta + 1}$$

Table 6 in appendix: all channels, average investment and capital wedges

Not surprisingly, the patterns of the convergence component are pretty similar to the one of i (and $\widetilde{k}^*$). In average, it accounts for nearly a quarter of average investment[46].
For its part, the catch-up channel closely follows the distribution of π: provinces which are lagging behind have large negative values (-5.46% for Qinghai, -4.86% for Guizhou) while we note a positive contribution in regions with high technology growth (5.27 for Guangdong and 4.63 in Jiangsu).
At last, the amount of investment needed to compensate capital depreciation captures the bulk of i (round 80% in average) and is comparable to the spatial distribution of the convergence component and relative investment.

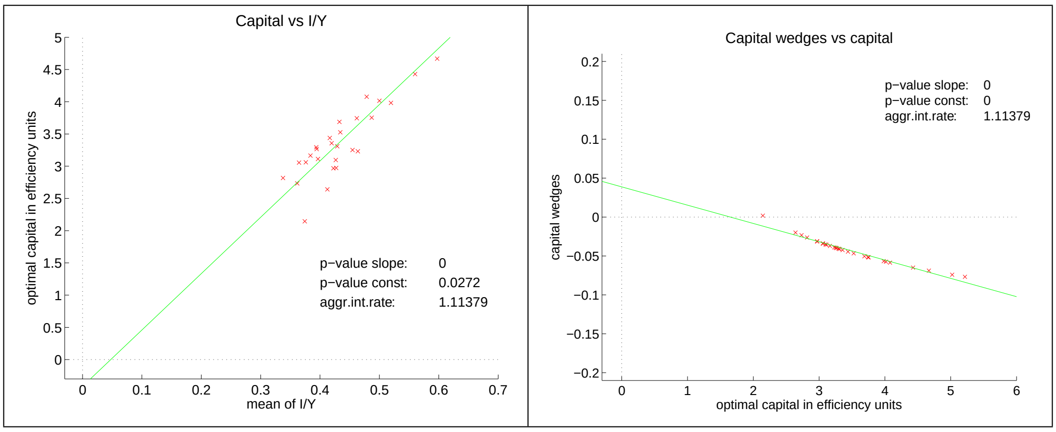
The capital wedges are mostly negative and range from -7.67 to 0.19%. The geographic distribution follows a clear pattern: the West, the Center North and the Metropolises have the highest rate of return (the lowest wedges) while the Center and the East Coast have far lower returns. In fact, the spatial distribution of values look pretty similar to the one of average investment over gdp.

[46] Note that as the catch-up channel can be negative if a province falls behind, the sum of the convergence and depreciation component could be bigger than i.

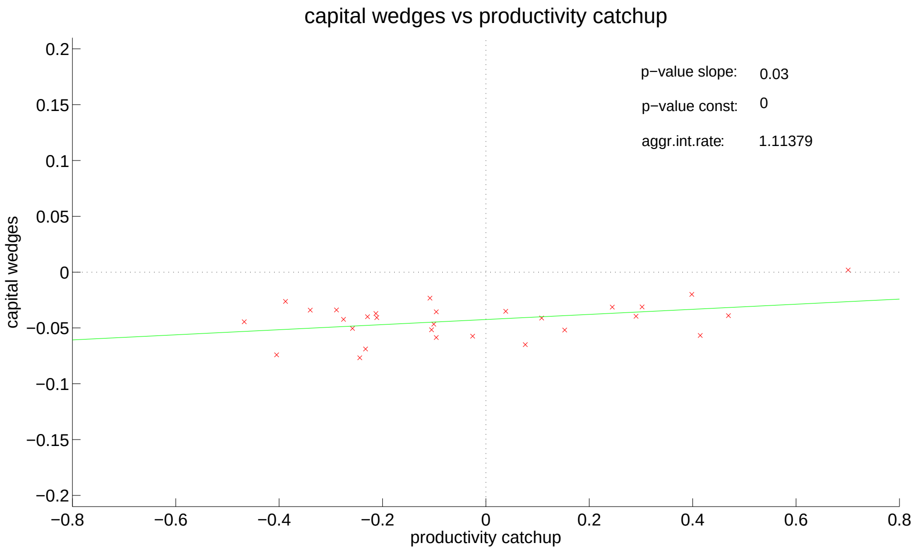


Our results are somewhat counterintuitive. In the original paper, countries with negative catch-up parameter had lower average investment rate than richer countries. Thus, they were attributed a low $\widetilde{k}^*$and a high τ_k, meaning their return on domestic capital was lower than the world interest rate. Remarkably, the authors were able to show that the investment wedges contribute to equalize marginal products of capital. In our case, some relatively poor provinces experimented high investment rates over the period. They were therefore attributed a high $\widetilde{k}^*$ and a low τ_k. The abnormal values of resources abundant regions like Xinjiang, Qinghai, Ningxia or Shaanxi is probably a consequence of the *Western Development Plan* consisting in massive investments in infrastructures (mainly transportation, resources extraction and power generating facilities[47]). Large negative values for Beijing, Tianjin and Shangai are less surprising as they are urban areas and economic magnets.

[47] Officially launched at the turn of the new millenium, it actually encompasses the Center West, Southern and Center North regions as well. In fact, all regions except the East Coast will be concerned: the Party intends to follow a similar strategy for Manchuria (*Northeast Area Revitalization Plan*) and the Center (*Rise of Central China Plan*).



There is a positive relationship between capital wedges and productivity catch-up: provinces with higher productivity growth have higher distortions and lower returns[48]. This result stands in contrast with Gourinchas and Jeanne (2011) where investment wedges were negatively related to the development in productivity, following the intuitive mechanism that countries with less frictions catch up. Actually, the positive relationship should attenuate the positive correlation of the catch-up parameter and capital inflows predicted by the model and thus make the capital allocation puzzle less stringent. We will come back discuss the possible consequences for our findings in a later section.



4.6 Saving wedges

Gourinchas and Jeanne decomposed the relative capital flows into four channels. A step by step explanation of their method is available in appendix.

The convergence term captures the amount of capital necessary to reach the steady-state capital per efficiency unit of labor:

[48]The level of i doesn't seem to be related with π.

$$\frac{\Delta D^c}{Y_0} = \frac{\tilde{k}^* - \tilde{k}_0}{\tilde{y}_0}(ng^*)^T$$

The second term gathers the cumulated debt inflows required to hold the relative debt ratio constant. G&J dubb it trend growth:

$$\frac{\Delta D^t}{Y_0} = \frac{\tilde{k}_0(ng^*)^T - \tilde{d}_0}{\tilde{y}_0} + \psi(\tau_s)\left[ng^*\phi(\tau_s)\right]^T \frac{\tilde{d}_0 - \tilde{k}_0}{\tilde{y}_0}$$

External borrowing needed to finance domestic investment are captured by the investment channel. Note that a province which is catching up necessitates higher flows:

$$\frac{\Delta D^i}{Y_0} = \frac{\tilde{k}^*}{\tilde{y}_0}(ng^*)^T\pi$$

At last, the saving term captures the intertemporal consumption decision of households: given a positive catch-up parameter, they will borrow on international markets to raise consumption:

$$\frac{\Delta D^s}{Y_0} = \frac{\tilde{w} + \tilde{z}_k}{\tilde{y}_0}\frac{\psi(\tau_s)}{R^*}[ng^*\phi(\tau_s)]^T \sum_{t=0}^{T-1}\left(\frac{ng^*}{R^*}\right)^t \left[\phi(\tau_s)^{t-T}(1+\pi) - (1+\pi_t)\right]$$

As in Gourinchas and Jeanne, $\pi = \pi_T$ and a linear convergence to the steady state catch-up has been assumed ($\pi_t = f(t)\pi$ with $f(t) = min(\frac{t}{T}, 1) \leq 1$). A grid search enables to find the saving wedge necessary in order to reproduce the empirical flows.

Tables 7, 8 and 9 in appendix (channels and saving wedges for three flows measures)

We first focus on the convergence and investment components. They remain similar independently of the type of flows used as they don't depend on the saving wedge. Indeed: given the same assumed $g^*$and thus identical π and R^*, the convergence and investment components are similar independently of the type of capital flows being adopted. Not surprisingly, Beijing, the Western and Center North provinces have high convergence component. Their high relative investment mean they were attributed a more sizeable $\tilde{k}^*$: the correlation between i and $\Delta D^c/Y_0$ is of 0.88. Workforce grow has little effect which is reflected in a lower correlation with the convergence term of 0.45.
The investment channel nearly mimicks the distribution of the catch-up parameter[49].

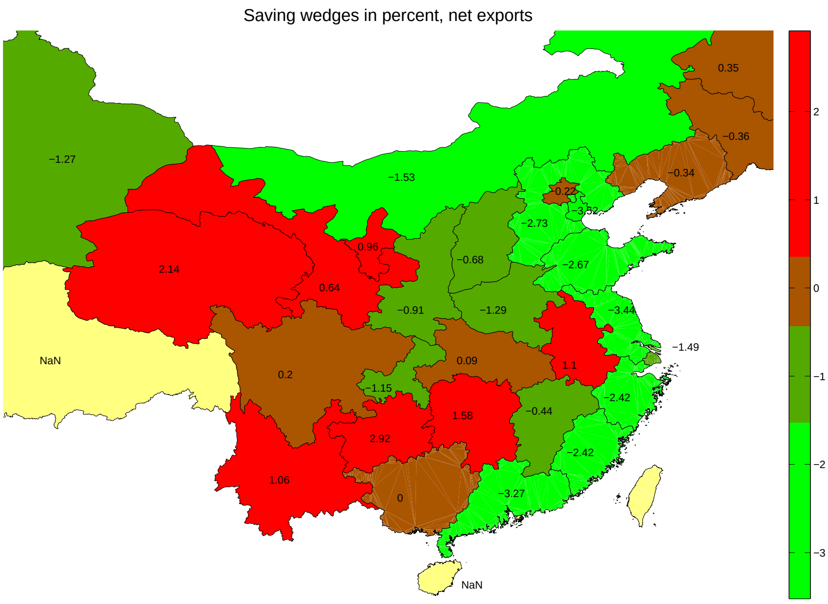
In the following, we stick to the common g^* strategy to facilitate comparison across type of flows used. Compared to the preceding channels, the trend component plays a negligible role. In fact, by setting the initial debt to 0 for all regions, we considerably reduced its importance[50]. Provinces with high catch-up tend to have negative trend channel: the rank correlation is below -0.85 for our three measures of capital flows. The geographical distribution is relatively stable across flow type.

[49]The correlation of both amounts to 0.97.
[50]We haven't found a reasonable proxy for $\tilde{d}_0$ yet.

Table 1: Mean saving wedge for different flows indicator

	Coast	Metro	Center North	Manch	Central	West	South
NX	-2.82	-1.74	-1.04	-0.11	0.01	0.62	1.33
INTER	-3.09	-1.19	-1.56	-0.18	-0.16	-0.75	0.69
INTRA	-2.20	-2.28	-0.94	0.07	0.12	0.82	1.43

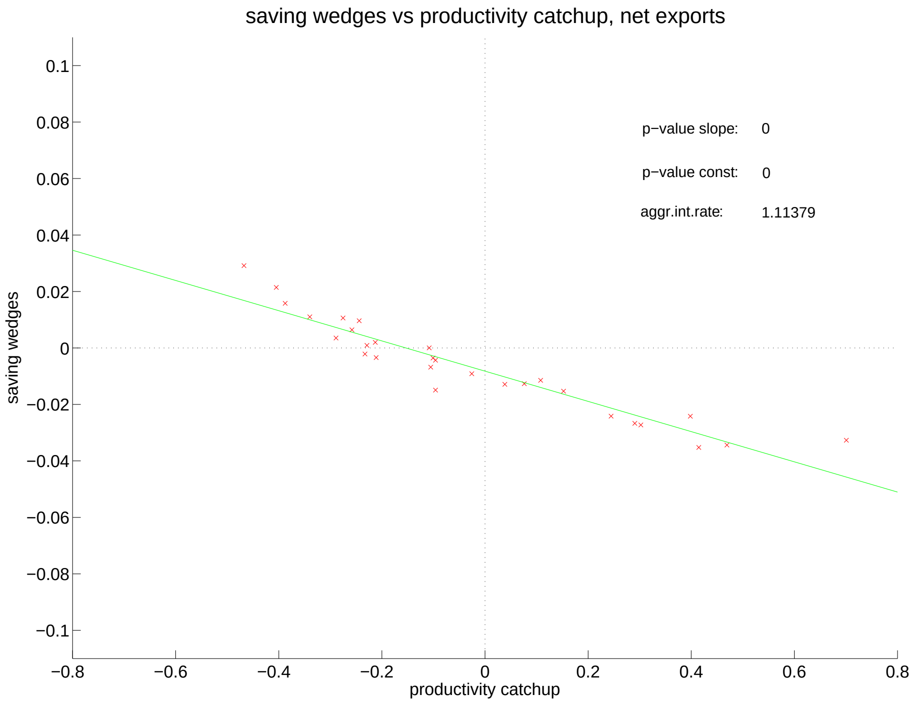
With the convergence term, the saving channel account for the bulk of capital flows. It is in this part that differences in patterns across types of flow data come into play. Using the net export flows, an obvious pattern emerges: From Tianjin to Guangdong, the entire East Coast has a highly negative saving channel. Paradoxically, these regions are exactly the ones that have been catching up compared to China and the rest of the world and are expected to continue on their way. Thus, households should have been borrowing to raise their consumption. The negative net exports resulting from lower savings would be interpreted as capital inflows. As in Gourinchas and Jeanne, it is not the case: these provinces have massive capital outflows compared to the rest of China and the world. The capital allocation puzzle manifest itself through a high negative correlation between the saving channel and the catch-up parameter (-0.87). To trigger off this discrepancy, these provinces are attributed lower or more negative saving wedges[51]: it is *as if* savings were heavily subsidized. Some resource abundant provinces like Inner Mongolia, Xinjiang and Shanxi have a relatively high negative saving term as well. Southern provinces have the most positive values.



This pattern is largely reflected in the distribution of the saving wedges. We use more detailed

[51]The correlation between the saving channel and saving wedge is of 0.91.

provincial clusters than in the preceding part[52]. Without the metropolises, the six coastal provinces have the lowest saving wedges (average of -2.82%). The city-provinces are not far with -1.74%, followed by the Center North (-1.04%) and Manchuria (-0.11%). With a slightly neutral rate (0.01%), Center provinces are heterogeneous: Henan and Chongqing subsidize savings while Hunan and Anhui tax them. In the West, all but Xinjiang (-1.27) have positive saving wedges with Qinghai recording the highest rate (2.14%). With an average of 1.33%, Southern regions have the highest saving wedges. All are positive.
As in Gourinchas and Jeanne, the capital allocation puzzle manifests itself through a very significant negative relationship between productivity catch-up and saving wedge: provinces which are catching up are the ones that implicitly subsidize savings, causing a saving glut which translates in capital outflows. Standard theory would predict provinces with high productivity to experience capital inflows (positive saving wedge). All but seven provinces are in the "wrong" quadrant[53].



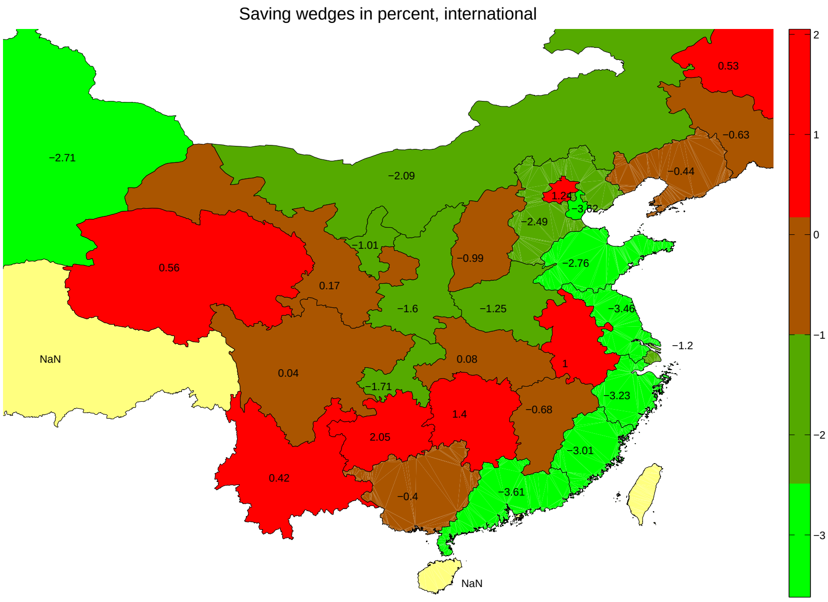
Among these regions, Shanghai is the only one which clearly deviates from the general regression pattern. Compared to Gourinchas and Jeanne, our regression line is less steep, making the relationship less marked. We have a significant negative intercept while they got a near zero one. As we will see in a later section, this is due to our parameters choice.

The spatial distribution of the saving channel using international trade as capital flows is not entirely similar. Value clusters are less homogeneous: Beijing has a small capital inflow

[52]Shanxi, Shaanxi and Inner Mongolia form the Center North, Sichuan and Chongqing are now part of the Center.

[53]Beijing, Shanxi, Liaoning, Jilin, Shangai, Jiangxi and Shaanxi have negative catch-up and saving wedges and are thus compatible with prediction of standard models.

(3.25) instead of a massive outflow (-27.42). The other way round, most resources abundant provinces have higher capital outflows relative to the rest of the world[54]. It is a logical consequence of their high export of oil and minerals. Still, as before, the ranking of flows is highly negatively correlated with the catch-up parameters (-0.81).

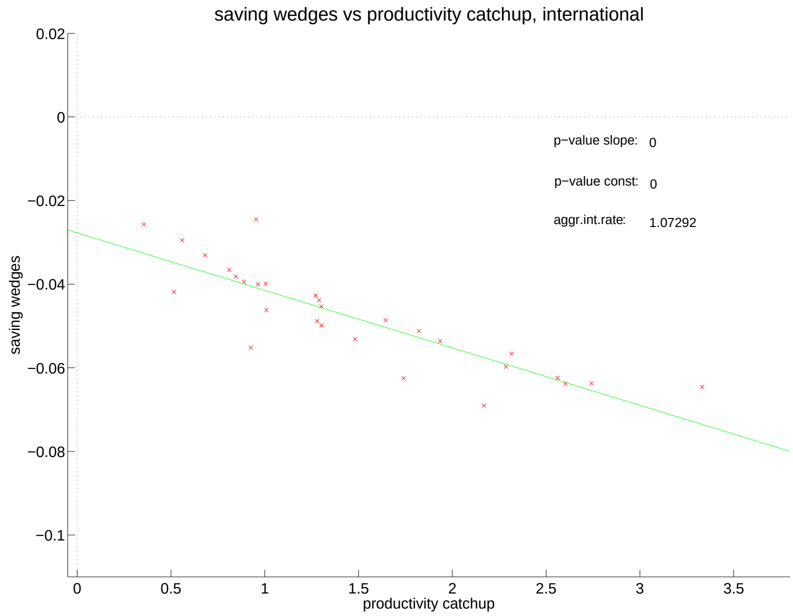


Somewhat surprisingly, compared to the net export flows, one gets a broadly similar pattern of saving wedges across provinces using international flows (correlation of 0.93). There are some differences too, again due to endowment in resources: Ningxia turns from taxing (0.96%) to subsidizing (-1.01%) savings and Xinjiang more than double the size of its negative wedge (1.27 to 2.71%). As for the saving term, Beijing goes the other way round and changed its relative position from a region encouraging savings (-0.22) to one deterring them (1.24). If we part China in bigger regions, a minor change arises: the West becomes more investment friendly than Manchuria.

The negative relationship between saving wedges and capital wedges is unchanged[55]: even if all provinces catch up relative to the rest of the world and all have negative saving wedges, the one that do more have lower wedges.

[54] Xinjiang plummets from -14.9 to -35.18, Ningxia from -3.37 to -34.35 and Qinghai from -0.58 to -19.51.

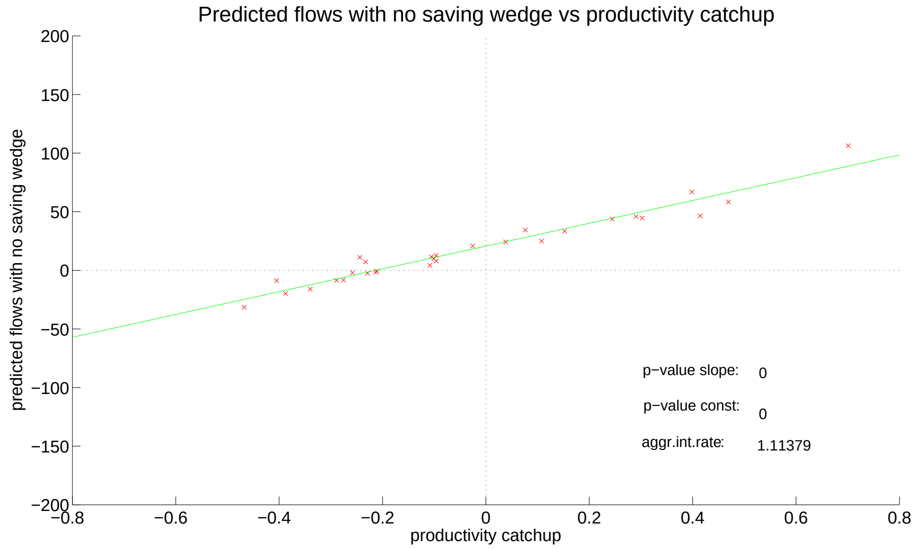
[55] We assumed a world growth rate of productivity of 3% while we kept the same value over all flow specification before. It has a negligible influence on the relative distribution.



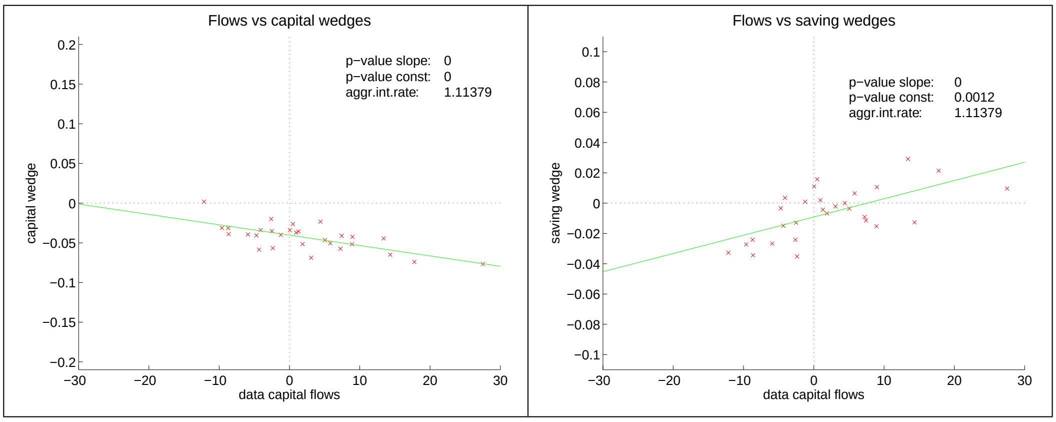
Turning to the internal flows, a new pattern emerges: regions all along the Grand Canal and Henan are attributed the lowest saving flows with values between -61.12 (Beijing) and -19.95 (Shangai). Southern and Western provinces have far less negative saving terms. Another interesting fact is that the current distribution partly disconnect from the one of the productivity catch-up (-0.41).

As for net exports at provincial boundary, the saving channel remains the main force driving the spatial distribution of saving wedges. The distribution is nearly identical to the one of our first indicator (correlation of 0.96).
The relative ranking of gross regions is similar as well and by using the same growth rate of productivity as for the first flow indicator, the negative relationship is preserved.

As in the seminal paper, the saving wedge is the driving force behind the explanation of the allocation puzzle. A Prediction of flows without saving wedges ($\tau_s = 0$) would restore the positive relationship between capital flows and rise in productivity (see appendix for derivation).



Thus, even though the distribution of capital wedges across chinese provinces is the opposite of the one in G&J and doesn't follow the classical model[56], the saving wedge is still needed to make the model correspond to reality as well. Empirically, we see provinces with relatively lower (more negative) capital wedge and higher saving wedges have higher flows as in Gourinchas and Jeanne.



5 Beyond the wedges

In this part of our study, we intend to discuss to which extent the usual suspects used in the literature are correlated with the distribution of the investment and saving wedges. As we have seen in the preceding sections the saving wedge and to a lesser extent the investment wedges are the driver of the capital allocation puzzle. We want to shed light on whether some variables have explanatory power. In a first step, the geographic distribution of variables is shortly discussed and a simple univariate regression of the wedges on them enable to identify

[56] The positive relationship between catch-up and capital wedge implies that successful provinces (high π) have lower returns (R) compared to the rest of the world (R^*). As a consequence, they experience capital outflows (positive net exports). This is a first step toward the capital allocation puzzle. In G&J, the capital wedges are in line with the theory and help to equalize marginal product of capital across countries.

possible correlation patterns. In a second step, we will try to account for the wedges variability using some selected variables.
At this point, it should be emphasized that this is only an exploratory, mainly descriptive step towards better understanding their geographic distribution.
The limitations of this approach are manifold:

- any econometric estimation suffers from the limited cross-section number of data as the model only delivers one wedge for each province over the entire 1984-2009 period. Variations over time would be needed to better identify the effects in a panel framework.
- most time series used on the right-hand-side aren't available over the entire period or change in definition/recording methodology over time. We always take the longest possible time horizon. This marked asymmetry of the variables is ubiquitous.
- our three measures of flow are approximations of current account mainly using trade in goods and services flows. It is unclear to which extent the errors generated by this approach affect provinces in the same way and don't distort totally the wedges distribution.
- wedges have no straightforward interpretation: for example, a positive value would mean there is an implicit tax on savings (capital income) and captures a distortion. A negative saving wedge is to be interpreted "as if" there was a saving subsidy but it could stand for imperfections of domestic financial markets as well.
- what makes sense on the microeconomic level doesn't automatically imply the corresponding macroeconomic patterns help to rationalize the discrepancy between the model and the reality.

In the baseline tests, the wedges of provincial net exports (first flows indicator) are used. A list of the variables and their availability is to be found in appendix.

5.1 Potential factors in existing literature

A small but increasing number of studies use sample data on household levels to try to identify the reasons for the high savings in China. Chamon and Prasad (2008) relied on annual household survey data for 1986-2005 and found that saving rates increased in all demographic groups as a consequence of the "breaking of the iron rice bowl": expenditures on education, housing and health care became more expensive during the reforms. Financial underdevelopment possibly played a role too but no impact of demographic factors was observed. In another paper, Chamon et al. (2010) established the last decades saw an increase in income uncertainty and a decline in pension replacement rate. These results contrast with studies focusing on demographic factors: Modigliani and Cao (2004) argued that a good chunk of the increase in saving is accounted by changes in demographic policies. In a related paper, Chao et al. (2011) develop a structural model of household saving behavior based on life-cycle hypothesis. It explained roughly a third of the surge in savings. Interestingly, the model does far better if one includes motivation to invest in housing. The exogenous decline in fertility in the early 1970s enabled to estimate that urban households having had daughter at the time increased savings by 27% of average income as they provide less elderly support (Banerjee et al., 2010). Horioka and Wan (2007) found weak support fot the life-cycle hypothesis over 1995-2004 using cross-provincial variation. In a more exotic stance, Du and Wei (2010) suggested intensified competition in the marriage market due to the gender gap could be held responsible for up to half the rise in saving. If household savings have been under close scrutiny, corporate and government saving have contributed to imbalances as well Ma and Yi (2011). Kuijs (2006) points to the fact that corporate and government saving are high relative to international standards.

On the macroeconomic level, differences in financial development are typically invoked to explain large imbalances. Caballero et al. (2008) highlight the central role played by heterogeneity in countries' ability to produce financial assets for global savers. Mendoza et al. (2009) introduce differences in the enforceability of financial contracts. The limited access to credit of high-productivity private firms could force them to rely heavily on selft-financing and spurs a rise in savings as in Song et al. (2011).

5.2 Related factors and usual suspects

Demographics

Investment wedges are unrelated to demographic indicators with the exception of gender imbalances: a higher male sex ratio is related to a higher wedge.

According to economic theory, prospects of an aging population should trigger off an increase in savings. In our framework, this would be interpreted as a lower saving wedge. A regression of saving wedges on the ratio of the 65 and more population over the 1997-2009 period relative to the total confirms this intuition. The assumption here is that past figures are good predictors of future values. In the same spirit, regions with high rate of youngsters are susceptible to experience low saving rates as intra familiar transfers are expected to be higher in the long-run and expenses for children have to be done on the spot. Again, results are in line with theory as the relationship is positive: provinces with higher rate of 0-14 persons have higher saving wedges.

As mentioned in the preceding section, Du and Wei (2010) have been emphasizing the effects of gender imbalances on the saving behavior of households. Regions with high men surplus would experience rising savings, more positive current account balance and capital outflow as the price of women on the love market rises. In our case, it is the opposite: provinces with high gender imbalances have very significant higher saving wedges and therefore experience lower savings.

Provinces with high urbanization rate tends to have lower saving wedges.

All demographic variables we discussed are not robust to the inclusion of the relative mean real gdp per capita over the period.

Financial development

The investment wedges are negatively correlated with the size of the property insurance premium relative to gdp. Thus, provinces with higher distortions in favor of investment pay a higher premium. Regions where real estate firms have more self-raised funds relative to the total funds are less biased toward investment.

Property and life insurance premium over gdp have no explanation power on saving wedges. Using statistics on real estate firms, we regress the wedges on the part of self-raised funds relative to total funds over 1999-2009 but no pattern emerges. No result has been obtained using owner's equity over total fund as well. Obviously, our poor indicators of financial development are not correlated with saving wedges.

Human capital

Investment wedges are unrelated to human capital indicators. The strong relationship with enrollment in higher education relative to population (EnrHigh), with the number of students in higher education (Tertiary) or with enrollment in undergraduate/specialized course in institutions of higher education (HighEduc) are driven by the Metropolises.

One would expect regions where human capital is more relevant for professional opportunities to have higher saving rates and lower wedges. Indeed, the enrollment in secondary relative to total population from 1984 to 2004 has a negative significant effect. The relative number of students in tertiary education without Beijing and senior secondary education from 2004 to 2009 are clearly negatively related to the wedges as well.
This pattern is confirmed by the use of an alternative measure: the part of total population enrolled in undergraduate or specialized courses in institutions of higher education in 1997-2009.
As a proxy for innovation capacity, we use the mean of the yearly numbers of patents granted per 100000 habitants from 1997 to 2009. Here again, the coefficient is significant and negative. Only senior secondary education would survive to the inclusion of relative gdp. Surprisingly, secondary/professional education seems to better account for the cross-sectional variability in saving wedges than tertiary/academic education.

Social security

Although their introduction is quite recent, we use data on contributors to injury insurance (2006-2009), basic medical care insurance (2000-2009) and unemployment insurance (2000-2009) relative to the labor force. Statistics on basic pension insurance are separated for rural (2006-2009) and urban areas (2005-2009) and are divided by the corresponding labor employment figures. Urban ones are used.

A better coverage of medical care or unemployment insurance goes hand in hand with higher

	Inv.wedge		Sav.wedge	
	Coeff.	*P-value*	*Coeff.*	*P-value*
Demographics				
DepRatioO	0.171	*0.313*	-0.246	*0.025*
DepRatioY	0.025	*0.600*	0.103	*0.020*
SexRatio	0.002	*0.062*	0.004	*0.000*
UrbRate	-0.017	*0.359*	-0.049	*0.021*
Financial dvpt				
PropInsPrem	-0.029	*0.003*	0.015	*0.103*
LifeInsPrem	-0.005	*0.105*	0.001	*0.687*
RE_SelfRAisedFund	-0.025	*0.398*	0.037	*0.199*
RE_OwnerEquity	-0.011	*0.116*	0.002	*0.834*
REMortLoan	0.142	*0.052*	-0.061	*0.413*
RE_DepPrep	-0.055	*0.331*	-0.015	*0.743*
Human capital				
EnrSec	0.033	*0.878*	-0.446	*0.028*
EnrHigh	-1.926	*0.000*	-1.058	*0.269*
EnrSecHigh	-0.108	*0.559*	-0.527	*0.021*
Tertiary	-0.005	*0.008*	-0.004	*0.215*
SenSec	0.000	*0.979*	-0.016	*0.004*
HighEduc	-0.096	*0.028*	-0.106	*0.122*
Patents	-0.005	*0.828*	-0.039	*0.035*
Social securitiy				
InIns	-0.016	*0.221*	-0.034	*0.054*
MatIns	-0.023	*0.223*	-0.057	*0.044*
MedIns	-0.030	*0.002*	-0.029	*0.047*
PensIns	0.009	*0.517*	-0.033	*0.025*
UnIns	-0.044	*0.005*	-0.052	*0.024*
Invst.structure				
SOInvFA	-0.104	*0.001*	0.126	*0.000*
COInvFA	0.132	*0.005*	-0.206	*0.000*
FOINvFA	0.129	*0.061*	-0.194	*0.000*
PRInvFA	0.034	*0.375*	-0.023	*0.551*
SBInvFA	-0.202	*0.011*	0.252	*0.008*
DLInvFA	-0.101	*0.027*	0.046	*0.466*
SRInvFA	0.047	*0.065*	-0.018	*0.436*
FIInvFA	0.358	*0.042*	-0.518	*0.000*
OTInvFA	-0.032	*0.552*	-0.010	*0.758*
Econ/trade structure				
PrimSec	0.022	*0.145*	0.050	*0.019*
SecSec	-0.016	*0.484*	-0.097	*0.000*
TertSec	-0.005	*0.008*	-0.030	*0.410*
Openess	0.005	*0.712*	-0.022	*0.002*
MNEShare_mean	0.029	*0.179*	-0.069	*0.000*
RealGDP_pc	-0.004	*0.367*	-0.016	*0.020*

Table 2: OLS, robust standard errors, univariate

returns on capital and higher investment rates.

Following the precautionary savings motive, provinces with underdeveloped/lower coverage insurance systems should witness higher savings. Without exception, all these variables are significantly negatively related to saving wedges. Thus, regions with higher coverage save more. Most of this counterintuitive results is driven by the very high correlation between our development level indicator and provision of social security (the correlation between injury, medical and unemployment insurance and relative gdp is of more than 0.91). Once one introduces relative gdp as new regressor, the sign of the insurance indicator becomes significantly positive for injury and medical insurance and nearly weakly significant for unemployment. At this point, one has to bear in mind that these insurances are recent and only cover basic needs.

Investment structure

We use two types of decomposition of investment in fixed assets. The first is a subdivision by status of registration or ownership. We have access to detailed data from 1997 until 2009. Unfortunately, the definition of some of the non-state private domestic channels varies over the period. As a consequence, we first focus on state-ownership (investment by state enterprises) and collective investment in fixed assets (investment in township and village enterprises by urban and rural collective units).

Investment wedges are highly related to nearly all indicators of investment structure. By using the status of registration, we observe that a higher share of state-owned investment is correlated with a more negative investment wedges. Private, collectively and foreigny owned investment have the opposite effect.

We expect provinces where the state plays a predominant role to have higher saving wedges as households or corporate savings are less needed. It is the case indeed: the relationship is very significant and positive for state-owned investment share. The pattern of collective-owned investment is the opposite and it makes sense as collective schemes rely on private funds as well. Its relative importance is negligible with a share of only 8.5% against a little more than 44% for our first indicator. Both survive to the inclusion of real gdp per capita.
The role of foreign-owned funds is obtained by taking funds belonging to Hong-Kong, Macau and Taiwan and add them to the foreign ownership. Although less important in absolute value (around 7% across provinces over the period), the indicator is highly negatively related to saving wedges and robust to the inclusion of relative gdp, even making it insignificant. The left-over term has no explanation power (it gathers all other categories and errors, for example private and joint ownership, limited liability, shareholding, others).

The second subdivision is based on a sources of fund classification: state budget, domestic loans (including state banks), foreign investment, self-raised funds (covers institutional and local government funds as well as bonds from private enterprises) and other funds (contains some private funds as well). We are confronted with a recording change as the time series abruptly change from 2004 onward. We concentrate on the 2004-2009 period.

The patterns for investment wedges with status of sources of funds is similar: state-budget investment and domestic loans (mostly form state banks) is related to higher investment compared to the theoretical model while the share of self-raised funds and foreign loans imply a higher investment wedge.
Turning to saving wedges, state budget relative to total sources of funds is highly significant and positively sloped. Foreign funds are strongly negatively related. Both are robust to gdp inclusion and thus corroborate our earlier findings. All three remaining categories are not strongly correlated with the distribution of wedges, probably due to their mix of public and private influences.

Economic and trade structure

A three-sectors decomposition of the provincial economies is available for the entire sample-length (1984-2009).
Investment wedges are not related with these indicators, except with the share of tertiary + secondary sector because of the Metropolises.

Not surprisingly, the primary sector is positively related to the saving wedges as poorer provinces have higher τ_s's and a more important agricultural sector. The mean value for the secondary sector is highly significant and negatively sloped. It stays significant even if one accounts for relative income. The part attributed to the tertiary sector affects wedges negatively (significant without Beijing which is an outlier). It gains in significancy once we introduce relative wealth.

Openness (exports + imports over gdp) is strongly negatively correlated with saving wedges.

In order to shed some light on the potential role of the structure of trade on saving wedges, we use Girardin and Owen's (2011) share of multinational enterprises in provincial exports for 1996, 2008 and the average of both. All three indicators have very high explaining power and are negatively correlated to the saving wedges. All remain highly significant and even make relative gdp per capita lose significancy.

5.3 Basic regression results

The preceding sections enabled us to better appreciate the relationship between macro data and the wedges. In the following tables, we pick variables that are highly correlated with the wedges in the same regression. Apart from some significant relationships mostly driven by the Metropolises, investment wedges were mainly related to the investment structure of provinces. The coefficient becomes even higher once one accounts for the per capita gdp level. The inclusion of foreign-owned investment halves the coefficient but all variables are significant. A nine percentage point increase in share of state-owned investment (ones standard deviation) goes hand in hand with a decrease in investment wedge between 0.7 and 1.26 percentage point,

Inv.wedge

	Coeff.	*P-value*	*Coeff.*	*P-value*	*Coeff.*	*P-value*
SOInvFA	-0.104	*0.001*	-0.140	*0.000*	-0.077	*0.003*
RealGDP_pc			-0.014	*0.000*	-0.023	*0.000*
FOInvFA					0.201	*0.005*
$Adj.R^2$	0.285		0.439		0.573	

Table 3: OLS, robust standard errors

depending on the specification. A higher output per capita fosters investment, the effect is comparable (-0.8 to -1.3 pp). At last, while being a small part of total investment, foreign investments in fixed assets affect investment wedges positively (1.12pp increase) and reduce distortions.

Saving wedges are highly correlated with nearly all categories of indicators. In the following table, we propose three specifications with one indicator of each "successful" category. We refrain from using demographics (they were not robust to the inclusion of real gdp per capita), financial development (no correlation) and social security (counterintuitive results). We use three alternative indicators of trade/economic structure (first variable of each regression): the share of multinational enterprises in exports, the share of the secondary sector and openness. The structure of investment is represented by state-owned or collectively owned investment in fixed assets. Indicators for human capital are senior secondary education, enrollment in higher education and number of patents relative to population.

First, all variables related to the economic and trade structures are strongly negatively correlated with the saving wedges even after controlling for the structure of investment, human capital and relative income. The coefficient somewhat vary over specification. A one standard deviation increase in MNE share makes saving wedge more negative (between 0.64 and 1.23 percentage point). The effect of a variation in the share of the secondary sector is roughly comparable: one standard deviation decreases the wedges between 0.75 and 1 percentage point. Openness has a smaller contribution of at most 1 percentage point decrease for one standard deviation.

Second, a higher weight of the state in investment in fixed assets causes saving wedges to be round half a percentage point higher. The opposite is true with collectively owned assets (-0.65 to -0.79 pp).

Third, human capital proxies are small in magnitude and two of our three variables switch sign compared to the univariate case. While higher senior secondary education brings the wedge down by half a percentage point and thus increase savings compared to the model's prediction, enrollment in higher education increases it by 0.33pp and relative patents attribution by 0.6 to 1.3pp.

At last, relatively more developed provinces save more and have a more negative saving wedge (between 0.23 and 1pp).

Sav.wedge

	Coeff.	*P-value*	*Coeff.*	*P-value*	*Coeff.*	*P-value*	*Coeff.*	*P-value*
MNE_mean	-0.069	*0.000*	-0.049	*0.001*	-0.046	*0.001*	-0.036	*0.047*
SOInvFA			0.059	*0.065*	0.048	*0.061*	0.051	*0.045*
SenSec					-0.010	*0.004*	-0.010	*0.003*
RealGDP_pc							-0.004	*0.336*
Adj.R^2	0.530		0.569		0.635		0.629	
SecSec	-0.097	*0.000*	-0.072	*0.000*	-0.099	*0.000*	-0.079	*0.047*
COInvFA			-0.158	*0.000*	-0.136	*0.000*	-0.130	*0.002*
EnrHigh					0.989	*0.070*	1.378	*0.180*
RealGDP_pc							-0.006	*0.609*
Adj.R^2	0.338		0.529		0.528		0.514	
Openness	-0.022	*0.002*	-0.012	*0.052*	-0.025	*0.016*	-0.020	*0.011*
SOInvFA			0.102	*0.001*	0.106	*0.000*	0.105	*0.000*
Patents					0.034	*0.096*	0.074	*0.033*
RealGDP_pc							-0.018	*0.030*
Adj.R^2	0.259		0.486		0.497		0.548	

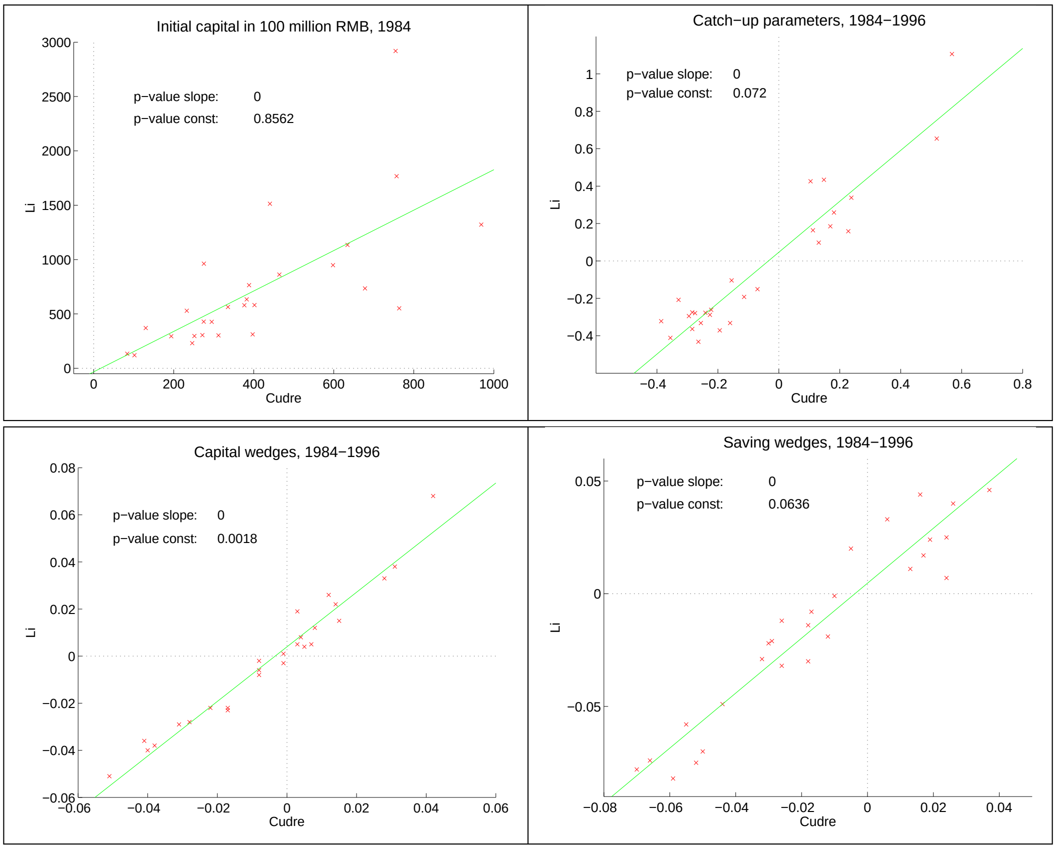
Table 4: OLS, robust standard errors

6 Data robustness checks

6.1 Capital stock

We come back to the issues related to the computation of provincial capital stock. To get the start value, we divided a smoothed value of GFCF by an assumed depreciation rate of 6% and the mean growth rate of GFCF over the sample period. Values of the last mentioned variable varied from 9.93% in Beijing to 16.30% in Fujian. Provinces with a higher value are automatically attributed a lower initial capital stock. A contradicting effect arise from the prospective smoothing of the future GFCF flows: high investment regions have high capital stock. Ideally, the perpetual inventory method is based on past data but given the limited availability for Chinese provinces, we had no choice but to have recourse to later data.
In order to partly correct for that loophole, a common value for GFCF growth is adopted. In the following, the national value of real GFCF growth over the 1952-1984 period (9.39%) is used for all provinces to compute the initial capital stock while prospective provincial values are kept in the smoothing process. The average initial capital stock is a little higher but still low (from 1.18 to 1.43). In general, although starting values can differ among provinces between this new approach and the original one, end results are nearly similar as the aggregation of new flows waters down the inital value.

Some effects of this method deserve more attention. This alternative method affects catch-up parameters. It is best understood using an example: Inner Mongolia had a high real GFCF growth (15.39%) over the period and thus started with a low initial capital relative to output (0.83). The ratio rised to 1.16 with the actual methodology. In absolute terms, we therefore observe a higher capital stock and a lower A. In the wake of the normalization by initial values of all variables of the production function, though, this smaller value causes the subsequent increases to be relatively bigger. The mean growth rate of productivity for Inner Mongolia is thus higher. In parallel to that, national productivity value (g^*) rises as well making up for a part of the regional increase. All in all, this particular region still experience an increase in π (from 0.15 to 0.21). Some other regions have lower π. This leads to small different shifts in our regression data points but doesn't influence the patterns found in the preceding parts.



The methodology we adopt is by no means the only possibility to compute a time series for provincial and national capital stocks. Li (2003) used statistics on sources of total investment in fixed assets from the Statistical Yearbook of China; à savoir state appropriation, domestic national bank loans, utilized foreign direct investment and self-raised funds. We use their data to rerun the analysis for the 1984-1996 period[57]. The author's capital stock values are clearly bigger than ours: in average, the initial one is 78% higher while the end capital stock is 93% higher. In general, the relative attribution across provinces is comparable. There are some big discrepancies though: Li attributed Liaoning the highest capital stock (6.6 relative to our 1984 real output) while the perpetual inventory method suggests a ratio of 1.7. The national mean growth rate of technology dwindles from 5.12 to 4.72%[58]. Fortunatley, following the normalization by initial capital, these differences are of little consequence for the

[57] In Li(2003), Sichuan and Chonqing were merged. We exclude these provinces from our sample.
[58] We neglected the fact that Li's national values are in 1978 RMB.

distribution of the catch-up parameters[59]. The capital and saving wedges are closely related across both specifications. Thus, our results are robust to the use of an alternative capital stock estimation method.

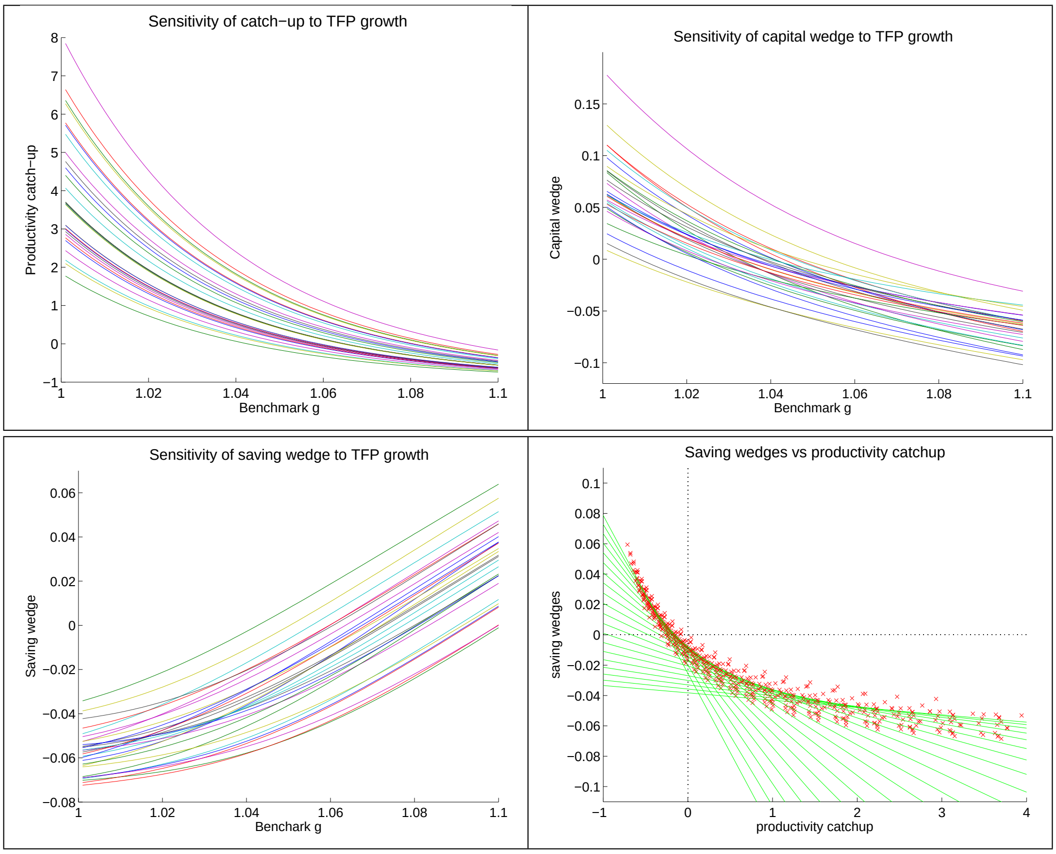
6.2 Common investment rate

As discussed in preceding sections, the empirical distribution of investment isn't in line with the findings of G&J. The clear positive correlation between capital wedge and the path of relative productivity is a first deviation from the prediction of the model. We are interested in removing the influence of this somewhat counterintuitive result on the estimation of the wedges. The strategy consist in replacing the empirical provincial average relative investment rates by the national value of 38.4% over the sample period. As a result, estimated capital and saving wedges are more compact around the regression line and the explanatory power of the regression rises considerably. The general patterns observed earlier are identical. Thus, the empirical distribution of investment caused the patterns to be more noisy but didn't invalidate our conclusions.

6.3 Productivity growth rate

The reference rate of productivity growth is an important parameter of the G&J model. It determines the benchmark against which provinces catch-up or fall behind and directly influences the national/world interest rate[60]. While the authors of the initial paper chose US average TFP growth (1.7%), we adopted our estimated values for China[61] most of the time. Obviously, these high rates only plainly make sense when the focus lays on intranational capital flows. The purely international and mixed flows should be discussed using world rates. In the past sections, we mostly restrained from that complication for comparison's sake.

[59]-0.26 vs -0.22 with our method for Liaoning.
[60]In the baseline model, the assumption that $R^* = g^{*\gamma}/\beta$ has been used.
[61]6.92% for the 1984-2009 period, 5.13% for 1984-1996 and 10.48% between 1997 and 2009.



Logically, a higher benchmark productivity value lowers the level of the catch-up parameters. Thus, as g^* increases, the relative position of the provinces shifts along the catch-up axis. More interestingly, the flow-productivity relationship stays highly significant all along and becomes even more negative. Capital wedges are quite sensitive as well: an increase brings about a drop in the τ_k's and modifies their distribution. The data points become more compact and the positive relationship weakens over g^*: significancy at the 5% confidence level is lost for growth rates above 7.4%. In our case, it doesn't endanger our conclusions in the preceding sections because the highest possible value adopted from chinese national data is less than 7%. As for saving wedges, they remain highly negatively correlated with π but here again, the slope becomes more negative with increasing benchmark growth rate.

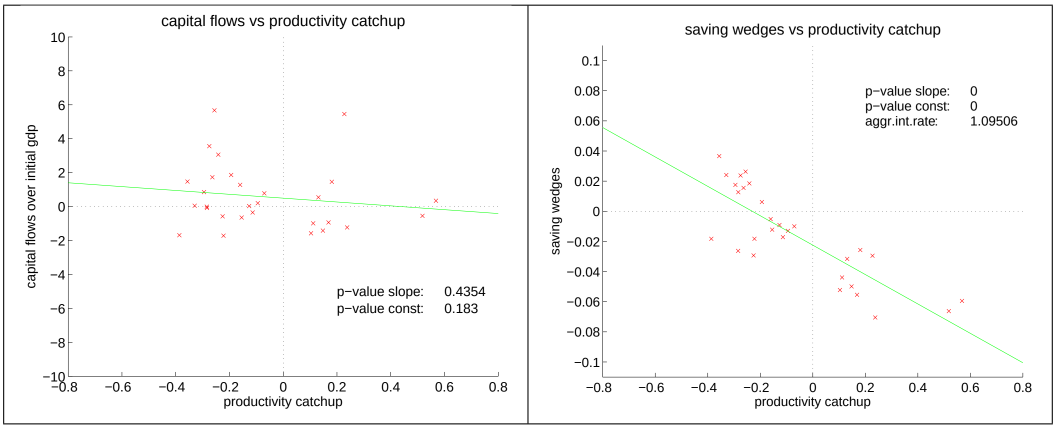
6.4 Revised data

Revisions affect nearly all variables used in this paper, particularly the net exports at provincial boundary in nearly half of provinces. One has to consult each provincial yearbook and check whether adjustments have been made (they are usually not announced). For the sample period 1984-2009, the 2011 Statistical Yearbook is needed as it provides estimates for 2010 and revised values for 2009. Unfortunately, the 2011 Yearbooks just has been published at the time of writing. This important check has to be postponed to a later version of the project.

6.5 Subsamples

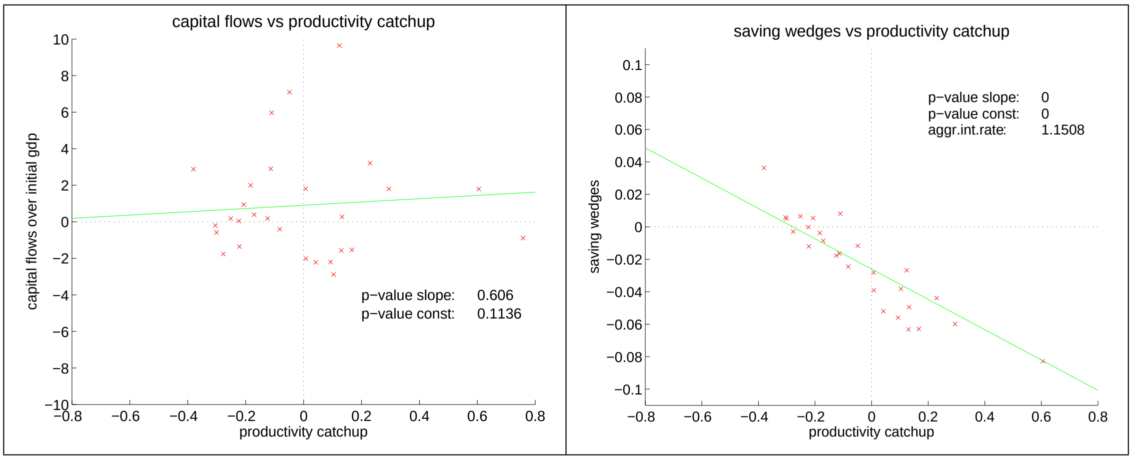
In this section, we aim at testing wether the capital allocation puzzle survives over different subsamples and methodologies. An intuitively obvious check is to let the entire estimation method run for two periods of equal length. The 1984-1996 period includes the second and part of the third wave of reforms. It ends at the onset of the Asian crisis. As for 1997-2009, it characterizes the emergence of China as a international trade hub and the backlash in domestic market regulations of the mid 00s. We will focus on net exports at provincial boundary (our first imbalances indicator).

In the first half of our dataset, national productivity grew slower than in the entire sample (5.13% versus 6.92%). As in the preceding parts, large differences in capital flows and technological progress among provinces are observed. Surprisingly, the negative relationship between capital flows and productivity catchup isn't significant anymore. More than a third of regions are in the "normal" quadrant where falling behind implies a capital outflow and catching-up a capital inflow. Some poorer parts of the country like Xinjiang and to a lesser extent Guangxi and Yunnan have been making headway and imported capital. On the opposite, Shangai and Liaoning have been falling back and lost capital. These provinces greatly influcence the general pattern: without Xinjiang, the slope becomes nearly significant at the 10% level. By excluding Guangxi and Shangai as well, one gets the strong negative relationship already observed for the entire sample (significant at 5% level). The positive correlation between capital wedges and productivity growth is less strong than for the entire sample but always significant. Most importantly, saving wedges and technology catchup are still very negatively related.



The second half of the dataset is characterized by a skyrocketing of national productivity (10.48% yearly). In the same time, the total range and discrepancies between provinces relative to productivy have been on the rise. The range of registered flows relative to initial output (here 1997) nearly doubled. As for the first sub-sample, the negative π-$\Delta D/Y$ relationship becomes nearly flat. Again, roughly a third of observations are in line with classical theory. Inner Mongolia and Shaanxi are the driving force: they turned from a clearly negative catch-up of round a quarter in 1984-1996 to respectively 1.05 and 0.61 in 1997-2009. Ningxia reversed its position (-0.26 to 0.12) as well at the cost of a near doubling of its capital imports

(9.64 vs 5.67). Another impressing shift happened in Tianjin (-0.23 to 0.76) but without consequences for its flow balance[62]. Striking is the observation that nearly half of provinces experienced a reversal of catch-up parameter between the first and the second period. There is no change in the correlation of relative productivity path with the capital and saving wedges. Relative cumulated flows are stable over both periods: there has been no big shift in patterns of surplus and deficit regions.



A similar subsample analysis conducted with international and intranational flows would delivers no significant relationship between flows and productivity.
Clearly, we have never registered a positive significant relationship between relative productivity path and capital flows yet. Furthermore, the highly significant negative correlation of π with the saving wedges is robust over sample specifications. However, the exact mirror pattern of the standard theory hasn't been detected in either subsamples.

In order to better appreciate its evolution over time, we recompute the national growth rate of productivity (g) for each additional year including past values. Thus, year-specific catch-up are obtained for the entire time series with the last one corresponding to the steady-state value used in the preceding sections. Relative flows are updated in the same way for each time period. The rolling capital allocation regressions enable us to shed light on the evolution of the puzzle over the years: the slope started flat and evolved toward an increasing negative value all along. The relationship became significantly negative at 5% in 1997 and at 1% in 2001. The allocation puzzle has been steadily reinforcing.

An non negligible loophole of our approach is that for fast growing economies, late flows are overweighted relative to initial output. We repeat the process using a rolling average of negative net exports over output in the corresponding year instead of the preceding flows indicator. Results are comparable[63]. Patterns for both sub-samples are roughly identical as well by using the alternative flow indicator.

[62]Unfortunately, no saving wedge has been found to reasonably mimick the observed flows for Tianjin, hence its value of 0.1 (upper bound).
[63]Significancy at the 5% level is reached in 1998. The correlation is less significant as the p-value of the slope coefficient never drops under 1%.

7 Model robustness checks

7.1 Exogenous interest rate

In the seminal paper, G&J assumed that $R^* = g^{*\gamma}/\beta$. It implicitly means the rest of the world is composed of steady-state economies that have no saving wedge and preferences similar to the domestic small economy. This assumption is by no way as innocuous as it initially seems. It is used in the second step of our detailed derivation of their model. It enables to somewhat simplify the maths and get an easily estimable flow decomposition (stable long-run distribution). We crunch through an alternative version refraining from any restriction on the functional form of world interest rates (details available in appendix). The convergence and investment channels are identical. A new variable is defined for convienience's sake.

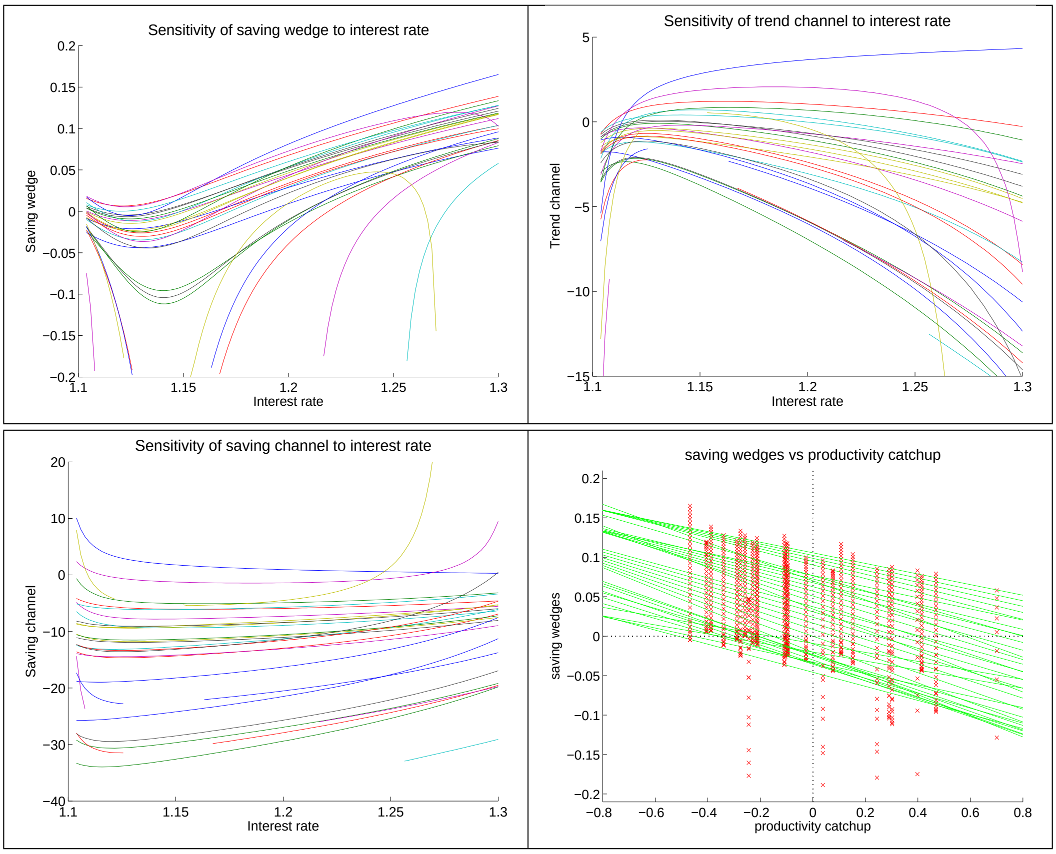
$$\Omega = \frac{(R^* - n(\beta R^*)^{\frac{1}{\gamma}})(\beta R^*)^{\frac{T}{\gamma}}}{R^* - ng^*}$$

The new trend and saving components are:

$$\frac{\Delta D^t}{Y_0} = \frac{\tilde{k}_0 (ng^*)^T - \tilde{d}_0}{\tilde{y}_0} + \Omega\psi(\tau_s)\left[n\phi(\tau_s)\right]^T \frac{\tilde{d}_0 - \tilde{k}_0}{\tilde{y}_0}$$

$$\frac{\Delta D^s}{Y_0} = \frac{\tilde{w} + \tilde{z}_k}{\tilde{y}_0} n^T \left[\frac{g^{*T}(1+\pi)}{R^* - ng^*} - \Omega\frac{\psi(\tau_s)\phi(\tau_s)^T}{R^*}\sum_{t=0}^{\infty}\left(\frac{ng^*}{R^*}\right)^t (1+\pi_t)\right]$$

Unfortunately, we cannot use Ψ to get rid of the infinite summation term anymore. Clearly, the saving channel will only converge if $R^* > ng^*$. For our dataset, it means aggregate interest rates have to be considerably high: ng^* varies between 7.46 (Tianjin) and 10.38% (Beijing). This last figure will constitute the lower boundary of our tests. Even for sufficient rates, any estimation of the infinite sum is only precise for big t. The computation time of the wedges estimation increases. We use a one hundred periods approximation of the infinite. A time series of different interest rates is used to shed light on wether the main findings are comparable.



No precise estimation of capital flows is obtained for Tianjin and it is therefore excluded from the sample. Across units, a general pattern emerges: saving wedges initially plummet and then begin to rise mostly between 13 and 14%. Regrettably, some regions have highly non-linear saving wedge patterns over R^*[64]. We restrict ourself to a -20 +20% interval to keep computation time low and appreciate graphical properties more easily. Not surprisingly, the capital wedges linearly shift down as the world interest rate increases. Although the location of some provinces in the catch-up vs saving wedge representation is highly sensitive to the interest rate, the negative significant relationship is maintained over the entire time series even by including outliers. As the convergence and investment channels are not affected, we focus on the two channels left. Compared to the baseline case, the slope of the trend channel relative to the catch-up parameters is far less negative for initial interest rates and begin to become more pronounced as R^* increases. The saving component is strongly negatively correlated all along and is less affected. Thus, in spite of some encountered difficulties relative to the estimation procedure, we have been able to show that the results are not hinging on the simplifying assumption made by G&J. Although more cumbersome, a model with variable world interest rates would lead to similar conclusions.

[64]These are Zhejiang, Fujian, Henan, Guangdong and Ningxia.

7.2 Capital adjustment costs

In the preceding sections and according to the G&J paper, we abstracted from capital adjustment costs. The extension of the existing model to include them could potentially mitigate the problem discussed before where some of the poorest regions have among the highest initial capital to output ratio and then accumulated huge relative GFCF. We introduce one adjustment cost taking effect in the dynamics of investment over time and a second one influencing the initial jump in capital to reach the steady-state level. The parameters κ_1 and κ_2 enable us to test the effects of various costs of adjusting capital stock (see appendix for derivation):

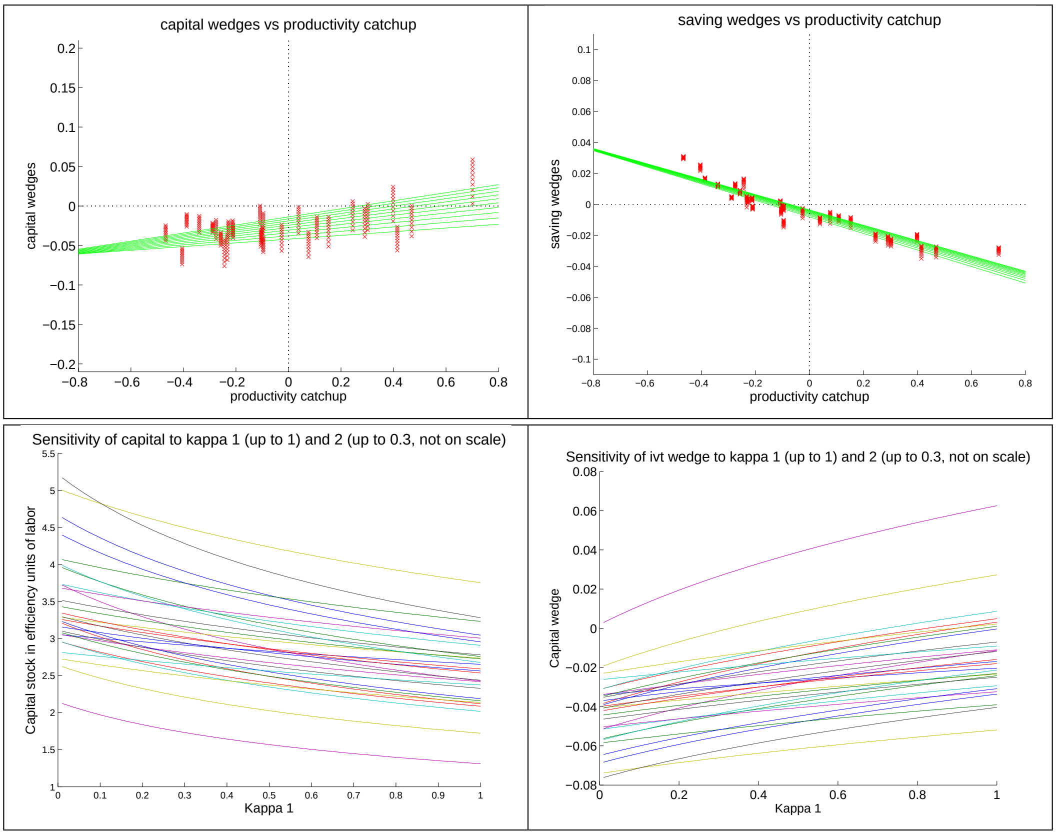
$$
\begin{aligned}
i &= \frac{1}{T}\sum_{t=0}^{T-1}\kappa_1\tilde{k}^{*(2\text{-}\alpha)}\left(\frac{A_{t+1}^2}{A_t}\frac{N_{t+1}^2}{N_t}-2A_{t+1}N_{t+1}+A_tN_t\right)\\
&+\frac{1}{T}\frac{\tilde{k}^*-\tilde{k}_0}{\tilde{k}_0^\alpha}+\frac{1}{T}\frac{\tilde{k}^*-\tilde{k}_0}{\tilde{k}_0^\alpha}\kappa_2+g^*\frac{\pi}{T}n\tilde{k}^{*(1-\alpha)}+(g^*n+\delta-1)\tilde{k}^{*(1-\alpha)}
\end{aligned}
$$

The implementation of the initial jump cost driven by κ_2 presents no difficulty. The estimation procedure of the κ_1 part is somewhat more complicated as the terms in bracket don't simplify. A first possibility would be to use data for normalized A and N. This would violate the assumption of constant population growth rate used to simplify the expression. Alternatively, time series of both variables using the corresponding mean growth rates over the period are generated and used to compute the new channel. For labor force, the growth rate used corresponds to the provincial ns. Technology's case is less obvious. Remember it is assumed that catch-up parameters converge linearly ($\pi_t = f(t)\pi$ with $f(t) = min(\frac{t}{T}, 1) \leq 1$). Using the expression for π_t, one can find the A_ts implied by this rule[65]. In this case, the dynamics of technology de facto corresponds to an assumed constant growth rate as differences between the implied time series and constant growth time series are negligible. In the following, each channel is implemented separately to better appreciate their effect on our results.

The κ_1 channel implies that provinces have to use a supplementary part of ther investment flows to maintain capital in efficiency unit of labor at steady-state level. In terms of the grid search, it means provinces with high average investment rate are attributed a lower $\tilde{k}^*$and a higher adjustment cost channel relative to their average investment. As an example, κ_1 is set to 0.2. For Beijing, it corresponds to a cost relative to the corresponding capital between 1.31 and 1.67%. This adjustment cost is substantial and represents 17.77% of the capital increment in average between 1984 and 2009. High investment provinces like Ningxia and Beijing are attributed high adjustment channels relative to average relative investment of respectively 11.3 and 10.1%. There are some exceptions though: Guangdong and Fujian have among the lowest i but have relative values of more than 10% as well due to their high labor force and productivity growth. Over all regions, the new channel has an average of 3.3%. Compared to the baseline version, the productivity and trend channels both register a similar 6% decrease. The convergence component is lower by 11.9% and capital stock dwindle by 8.3%.

[65] $A_t = (1 + \pi_t)A_0 g^{*t}$

The κ_2 channel makes the initial jump more expensive in terms of required investment. By fixing κ_2 to 7.15%, we get initial adjustment costs as high as a quarter of the initial increase for Beijing. Contrary to the preceding modification where average investment, labor force and productivity growth implied a higher channel relative to average investment, this channel is only influenced by the attributed initial capital. Thus, its effect relative to investment is neatly positively related to investment rate and to capital stock while the former channel didn't particularly penalize regions with high capital. The new channel has an average of 1.7%. It causes the productivity and trend components to drop by 2.9%. Again, the convergence component absorbs most of the decrease (5.5%) and capital dwindles by 4.1%.



By introducing both channels, we attribute relatively higher relative adjustment costs to provinces with high investment and high growth rate of productivity/labor force. Thus, regions with relatively high capital stock are mostly more affected. The positive pattern of capital/saving wedges relative to flows is broadly unchanged and shifts upward as both adjustment parameters increase. As adjustment costs gain in importance, the correlation between average relative investment and attributed capital gets lower but stays high for reasonable κ_1 and κ_2 values[66]. Interestingly, the negative relationship between capital wedges and capital becomes more pronounced as the former variable increases and the last one decreases. In spite of our extensions, all patterns of the preceding sections are preserved. This is to be explained by the monotonic and limited non-linearity of the capital wedges reaction to the increase in adjustment costs.

[66]It amounts to 0.91 in the baseline version and 0.84 with $\kappa_1 = 0.2$ and $\kappa_2 = 0.066$.

7.3 Sensitivity to main parameters

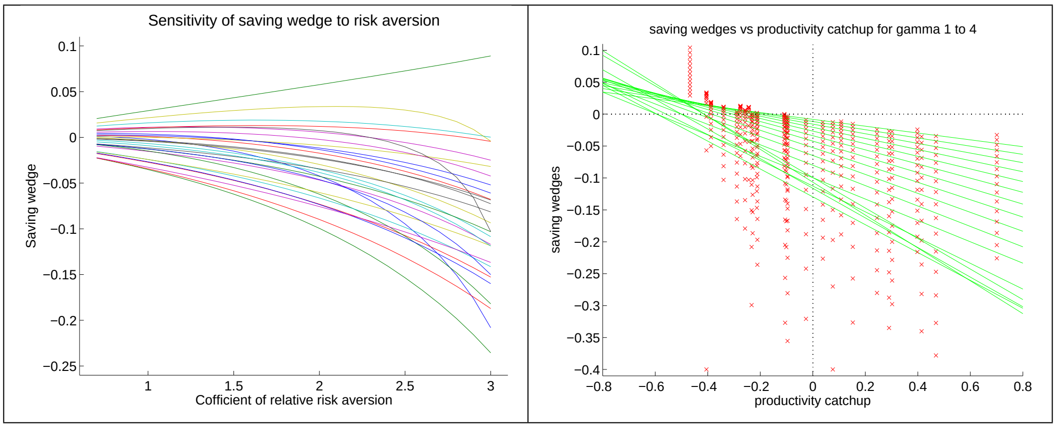
In the baseline version, a value of 6% has been assumed for capital depreciation. Alternative values have near inexistent influence on the relationship between relative flows and productivity catch-up as long as δ is common across provinces. The distribution of capital wedges is pretty similar and is shifted down as depreciation rises. Put in relationship with capital flows, the τ_s are shifted upwards but keep the same pattern[67].

We assumed the initial value of relative foreign assets/debts to be 0. An alternative value potentially influences the estimated saving wedges. By starting from a negative value (initial foreign assets) and evolving toward positive values (foreign debt position), one observes a downward shift of the relationship. Here again, the distribution is stable.

The initial capital share had been set to 0.3. Changes in α provoke changes in the rank of catch-up parameters for many provinces. The negative relationship between flows and catch-up looses significancy at the 1% level for capital share higher than 0.5 and becomes increasingly flat. The distribution of capital wedges is shifted up and the positive slope flattens out as α increases. Concerning saving wedges, estimates are wandering toward the North-West quadrant but the usual pattern remains.

Variations of the discount factor β brings about nearly linear changes in τ_w. It is less the case for the τ_s's: although the distribution shifts up, the estimates gather more compactly around the regression line as the parameter converges against 1. The discrepancies across provinces dwindles.

Up to this point, the entire procedure has been run with log utility. Alternative γs influence capital wedges in a near linear way. Small increases in the relative risk aversion greatly affect saving wedges: the relationship becomes more negative and changes in ranking occur.



All in all, our results are very robust to variations in parameters. If at all, a more China specific calibration compared to G&J would rather reinforce the patterns we have found, for example by using a discount factor higher than 0.96 (more patient households) and a higher

[67] For a δ of a little more than 0.2, the regression line goes through the origin as in the seminal paper.

gamma (more risk-averse). In this case, catch-up parameters would remain unaffected, capital wedges would only shift in level but would keep the same distribution and the saving wedges would have smaller sum of square residuals and would be even more negatively related to relative flows.

8 Conclusion

We shed light on issues related to data quality and aggregation properties of Chinese provincial macroeconomic figures. We found large discrepancies between the regional and national official components of the national accounts. While the pre-reform patterns are greatly affected by the lack of some economic heavyweights, the capitalist era started in 1978 has been far from able to alleviate persistent doubts about the way macroeconomics figures are computed. Investment errors are the key driver of differences between provincial and national output: phantom local investments have largely pushed aggregate figures up as they have exceeded China's statistics since 1990. This clear trend toward higher investment has been a constant statistical companion of emerging China over the 1990s and 2000s. Provincial net exports are found to be highly sensitive to it and logically end up being too low. Errors in savings seem more stationary and have been making for a part of investment errors in the 2000s as regional figures exceeded national ones. Still, by comparing the ratio of aggregate local net exports to aggregate local output with national values, one can observe the match was reasonable from the mid 1980s to the mid 2000s. Since then, national figures have gone through the roof while provincial ones have staid stable.

International trade data have been improving. Provincial imports were largely underestimated but exports seem to match national data pretty well. Errors in imports dwindled in the 1990s and and no large discrepancy has been observed since then. The fact that national data are better than provincial one is far from established and our findings could be interpreted the other way round since national data have been taken as reference point. All in all data quality certainly is an issue and there is little doubt the next economic Census is going to send ripples through the statistical system.

Large differences in output per capita were already striking in the 1950s although the pattern was different: a clear North-South divide was observable and morphed into an East-West divide as coastal regions were first integrated in the world economy. This pattern is to be taken cautiously as it remains unclear to which extent this shift is only a byproduct of the transition from the Material Product System to the System of National Accounts. We observe a large decrease in terms of output per capita disparities among regions during the initial reform period, although refraining from including the Metropolises in the sample would level the effect down. In accordance with the literature, we find that the cross-regional variability has been on the rise since the 1990s and is slowly heading toward pre-reform level.

Even more than well-known differences in development among regions, the historical distribution of net exports surpluses and deficits is striking and has never been discussed in the literature. The prominent coastal regions have indeed accumulated surpluses since the opening-up of China but they already had large ones before reforms. Interestingly, some other regions have even larger saving-investment gaps. In the stern Communist era, the Metropolises and Manchuria accumulated huge surpluses and converged to neutral position during the reforms. Over the last decades, Central China has had roughly balanced net exports. Western and Southern China have been running huge deficits independently of the adopted system.

From the 1950s to today, positive net exports have been related to high output per capita, albeit with varying intensity: the initial domestic reforms lowered the relationship but it then increased following the uneven integration of regions in the world economy in the 1990s and 2000s.

The cross-provincial variability in relative net exports dwindled considerably from 1970 to the mid 1990s but has been on the rise since then.

Imbalances in international trade are less dramatic: with the exception of the large deterioration of the Metropolises and Manchuria since the 1990s and the steadily increasing export surpluses of the East Coast, most provinces have been running small surpluses. Thus, some large discrepancies in net exports are driven by interprovincial flows. Disparities in international relativ flows have only started to increase massively in the 2000s.

We imposed some structure on the discussion of internal imbalances by resorting to the methodology of Gourinchas and Jeanne. By using net exports, international trade and interprovincial trade as flow indicators, we showed there is a capital allocation puzzle in China over the 1984-2009 period. Provinces which catched up more relative to China experienced capital outflows while provinces which fell behind experienced massive inflows. This negative relationship holds for international flows as well and turns flat with interprovincial flows. In contrast with Gourinchas and Jeanne, investment wedges necessary to mimick empirical average investment rates are positively related to productivity catch-up parameters. It is a first discrepancy between the model and the patterns observed in Chinese regions: provinces which fell behind in terms of productivity subsidized investment more. Western, Center North China and the Metropolises have high negative values while the Center and East Coast ones are less negative.

As in the original paper, the saving wedges enabling to reproduce the patterns of relative cumulated flows are strongly negatively related to productivity. It is the second systematic discrepancy between the prediction of the model and the reality as regions which taxed savings more experienced lower productivity gains and capital inflows. The East Coast, the Metropolises and to a lesser extent the Center North have negative saving wedges while Western and Southern China exhibit positive values. Central China and Manchuria have a near neutral bias. As in Gourinchas and Jeanne, the saving wedges are the key driver of the capital allocation puzzle pattern.

By trying to account for the cross-sectional variability in investment wedges, we find that variables capturing the investment structure account for a great deal of discrepancies among provinces and are robust to controlling for the level of development. A higher share of state-

owned investment is related to lower investment wedges (higher investment compared to the prediction of the neoclassical model) while foreign-owned investments push wedges toward a more neutral level.
The distribution of saving wedges is related to indicators of the trade and economic structure, investment structure and education. The share of multinational enterprises in provincial exports, the importance of the secondary sector or to a lesser extent openness have a large negative impact on the saving wedges and therefore push saving up compared to the standard neoclassical model. The share of state-owned investment in fixed assets has a big positive effect (lower savings) while the one of collectively owned investment influences them positively (higher savings). Surprisingly, greater innovation capacity captured by the number of patents or higher education level implies a higher saving wedge. In general, more developed regions save more than the model predicts and thus have more negative distortion.

The fact other categories of variables were less successful or not robust to alternative specifications doesn't mean they don't potentially play a role in explaining wedges. We have to come up with better indicators for financial development. Demographic parameters are highly correlated with the level of development and lose significancy once we control for it. Still, our finding that regions with large gender imbalances are exactly the ones that have massive net exports deficit is not speaking in favor of Du and Wei. Social security is highly related to saving wedges in a counterintuitve way in the sense that regions benefiting from better coverage are the ones which subsidize savings. At this point, one has to bear in mind that most of these insurances are recent and only cover basic needs.

9 Literature

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Table 5:

	n	g	π	nx	tb	intra
Beijing	1.032	1.058	-0.232	3.08	34.25	-31.17
Tianjin	1.005	1.084	0.415	-2.36	-3.63	1.26
Hebei	1.017	1.080	0.302	-9.61	-5.32	-4.3
Shanxi	1.014	1.064	-0.105	1.87	-2.53	4.4
In.Mongolia	1.013	1.075	0.152	8.91	0.53	8.38
Liaoning	1.010	1.059	-0.211	-4.7	-5.95	1.25
Jilin	1.012	1.064	-0.101	5.05	1.38	3.67
Heilongjiang	1.012	1.054	-0.289	-4.09	-1.94	-2.15
Shangai	1.007	1.064	-0.096	-4.33	-1.03	-3.3
Jiangsu	1.014	1.085	0.469	-8.65	-9.08	0.43
Zhejiang	1.021	1.078	0.245	-8.71	-23.97	15.25
Anhui	1.018	1.051	-0.340	0.05	-1.5	1.54
Fujian	1.027	1.083	0.398	-2.61	-17	14.4
Jiangxi	1.015	1.064	-0.096	1.29	-2.47	3.77
Shandong	1.017	1.080	0.290	-5.91	-7.5	1.59
Henan	1.023	1.070	0.038	-2.52	-1.8	-0.72
Hubei	1.012	1.058	-0.229	-1.22	-1.43	0.21
Hunan	1.015	1.048	-0.388	0.48	-1.84	2.32
Guangdong	1.030	1.092	0.701	-12.15	-21.78	9.62
Guanxi	1.019	1.064	-0.108	4.41	-2.62	7.02
Chongqing	1.011	1.073	0.108	7.44	-0.96	8.38
Sichuan	1.012	1.059	-0.213	0.95	-1.14	2.09
Guizhou	1.024	1.042	-0.468	13.39	-0.95	14.34
Yunnan	1.021	1.055	-0.275	8.97	-1.57	10.54
Shaanxi	1.014	1.068	-0.026	7.24	-2.48	9.6
Gansu	1.011	1.056	-0.258	5.81	0.08	5.73
Qinghai	1.019	1.047	-0.405	17.76	-2.05	19.81
Ningxia	1.026	1.057	-0.244	27.51	-4.32	31.83
Xinjiang	1.015	1.072	0.076	14.33	-6.61	20.94

10 Appendix 1: results

10.1 Tables

Table 6:

	τ_k	i^{conv}	i^{prod}	i^{trend}	i
Beijing	-6.89	14.67	-3.02	48.16	59.73
Tianjin	-5.67	11.93	4.69	35.42	51.95
Hebei	-3.12	7.87	2.82	31.66	42.25
Shanxi	-5.17	10.97	-1.15	36.46	46.20
In.Mongolia	-5.19	11.01	1.67	36.12	48.70
Liaoning	-4.07	9.18	-2.10	32.39	39.37
Jilin	-4.66	10.10	-1.05	34.47	43.42
Heilongjiang	-3.39	8.22	-2.73	31.08	36.47
Shangai	-5.86	12.31	-1.11	36.73	47.86
Jiangsu	-3.90	8.92	4.63	32.92	46.37
Zhejiang	-3.13	7.89	2.29	32.63	42.72
Anhui	-3.41	8.24	-3.24	32.70	37.61
Fujian	-1.99	6.56	3.45	31.30	41.23
Jiangxi	-3.56	8.45	-0.92	32.22	39.65
Shandong	-3.95	9.01	2.88	33.68	45.48
Henan	-3.51	8.38	0.37	34.00	42.65
Hubei	-3.99	9.06	-2.27	32.73	39.43
Hunan	-2.63	7.27	-3.48	30.07	33.77
Guangdong	0.19	4.58	5.27	27.68	37.44
Guanxi	-2.33	6.93	-0.95	30.29	36.17
Chongqing	-4.11	9.24	1.08	32.68	42.89
Sichuan	-3.71	8.66	-2.06	31.89	38.39
Guizhou	-4.45	9.75	-4.86	36.84	41.65
Yunnan	-4.23	9.42	-2.81	35.42	41.95
Shaanxi	-5.73	12.06	-0.30	38.32	50.00
Gansu	-5.04	10.75	-2.78	35.39	43.27
Qinghai	-7.41	16.09	-5.46	46.36	56.90
Ningxia	-7.67	16.86	-3.40	50.16	63.53
Xinjiang	-6.49	13.71	0.94	41.45	56.01

Table 7: Flows decomposition using net exports

	$\frac{\Delta D^c}{Y_0}$	$\frac{\Delta D^i}{Y_0}$	$\frac{\Delta D^t}{Y_0}$	$\frac{\Delta D^s}{Y_0}$	τ_s
Beijing	43.33	-12.84	-0.07	-27.42	-0.22
Tianjin	18.04	9.99	-1.77	-28.71	-3.52
Hebei	16.15	7.36	-1.29	-31.92	-2.73
Shanxi	20.96	-3.01	-0.35	-15.82	-0.68
In.Mongolia	20.26	4.21	-0.79	-14.85	-1.53
Liaoning	15.94	-4.83	-0.18	-15.7	-0.34
Jilin	18.4	-2.59	-0.19	-10.65	-0.36
Heilongjiang	14.81	-6.37	0.19	-12.83	0.35
Shangai	19.81	-2.52	-0.78	-20.93	-1.49
Jiangsu	17.08	11.61	-1.64	-35.76	-3.44
Zhejiang	17.89	6.59	-1.07	-32.18	-2.42
Anhui	17.54	-8.85	0.58	-9.31	1.10
Fujian	17.22	11.04	-0.89	-30.07	-2.42
Jiangxi	16.44	-2.33	-0.23	-12.69	-0.44
Shandong	18.36	7.7	-1.27	-30.77	-2.67
Henan	19.87	1.13	-0.58	-23	-1.29
Hubei	16.6	-5.48	-0.05	-12.47	0.09
Hunan	14.17	-8.51	0.87	-6.11	1.58
Guangdong	13.06	17.15	-0.95	-41.49	-3.27
Guanxi	14.89	-2.54	0	-8.02	0
Chongqing	16.33	2.52	-0.6	-10.92	-1.15
Sichuan	15.66	-4.87	0.11	-10.05	0.20
Guizhou	23.68	-15.61	1.52	3.71	2.92
Yunnan	21.17	-8.3	0.55	-4.53	1.06
Shaanxi	23.08	-0.79	-0.47	-14.67	-0.91
Gansu	19.26	-6.82	0.35	-7.06	0.64
Qinghai	34.52	-17.45	1.17	-0.58	2.14
Ningxia	43.5	-13.13	0.43	-3.37	0.96
Xinjiang	27.13	2.67	-0.64	-14.9	-1.27

Table 8: Flows decomposition using international trade

	$\frac{\Delta D^c}{Y_0}$	$\frac{\Delta D^i}{Y_0}$	$\frac{\Delta D^t}{Y_0}$	$\frac{\Delta D^s}{Y_0}$	τ_s
Beijing	43.33	-12.84	0.43	3.25	1.24
Tianjin	18.04	9.99	-1.82	-29.92	-3.62
Hebei	16.15	7.36	-1.19	-27.73	-2.49
Shanxi	20.96	-3.01	-0.51	-20.05	-0.99
In.Mongolia	20.26	4.21	-1.06	-22.96	-2.09
Liaoning	15.94	-4.83	-0.24	-16.92	-0.44
Jilin	18.4	-2.59	-0.33	-14.19	-0.63
Heilongjiang	14.81	-6.37	0.29	-10.75	0.53
Shangai	19.81	-2.52	-0.63	-17.79	-1.20
Jiangsu	17.08	11.61	-1.65	-36.2	-3.46
Zhejiang	17.89	6.59	-1.39	-47.13	-3.23
Anhui	17.54	-8.85	0.53	-10.81	1
Fujian	17.22	11.04	-1.07	-44.29	-3.01
Jiangxi	16.44	-2.33	-0.35	-16.3	-0.68
Shandong	18.36	7.7	-1.31	-32.34	-2.76
Henan	19.87	1.13	-0.56	-22.31	-1.25
Hubei	16.6	-5.48	0.04	-12.67	0.08
Hunan	14.17	-8.51	0.77	-8.35	1.40
Guangdong	13.06	17.15	-1.03	-51.03	-3.61
Guanxi	14.89	-2.54	-0.2	-14.83	-0.40
Chongqing	16.33	2.52	-0.88	-19.02	-1.71
Sichuan	15.66	-4.87	0.02	-12.03	0.04
Guizhou	23.68	-15.61	1.04	-10.13	2.05
Yunnan	21.17	-8.3	0.21	-14.73	0.42
Shaanxi	23.08	-0.79	-0.81	-24.05	-1.60
Gansu	19.26	-6.82	0.09	-12.55	0.17
Qinghai	34.52	-17.45	0.29	-19.51	0.56
Ningxia	43.5	-13.13	-0.41	-34.35	-1.01
Xinjiang	27.13	2.67	-1.31	-35.18	-2.71

Table 9: Flows decomposition using intranational trade

	$\frac{\Delta D^c}{Y_0}$	$\frac{\Delta D^i}{Y_0}$	$\frac{\Delta D^t}{Y_0}$	$\frac{\Delta D^s}{Y_0}$	τ_s
Beijing	43.33	-12.84	-0.61	-61.12	-2.21
Tianjin	18.04	9.99	-1.64	-25.22	-3.24
Hebei	16.15	7.36	-1.16	-26.73	-2.43
Shanxi	20.96	-3.01	-0.26	-13.36	-0.50
In.Mongolia	20.26	4.21	-0.8	-15.38	-1.57
Liaoning	15.94	-4.83	0.08	-10.01	-0.14
Jilin	18.4	-2.59	-0.24	-11.98	-0.46
Heilongjiang	14.81	-6.37	0.28	-10.97	0.51
Shangai	19.81	-2.52	-0.73	-19.95	-1.40
Jiangsu	17.08	11.61	-1.42	-26.9	-2.92
Zhejiang	17.89	6.59	-0.59	-8.74	-1.25
Anhui	17.54	-8.85	0.64	-7.86	1.20
Fujian	17.22	11.04	-0.67	-13.29	-1.76
Jiangxi	16.44	-2.33	-0.14	-10.29	-0.27
Shandong	18.36	7.7	-1.09	-23.47	-2.25
Henan	19.87	1.13	-0.54	-21.27	-1.20
Hubei	16.6	-5.48	0.11	-11.12	0.20
Hunan	14.17	-8.51	0.95	-4.37	1.72
Guangdong	13.06	17.15	-0.77	-19.86	-2.56
Guanxi	14.89	-2.54	0.08	-5.48	0.15
Chongqing	16.33	2.52	-0.57	-9.99	-1.08
Sichuan	15.66	-4.87	0.15	-8.94	0.29
Guizhou	23.68	-15.61	1.56	4.63	2.97
Yunnan	21.17	-8.3	0.6	-2.99	1.16
Shaanxi	23.08	-0.79	-0.39	-12.39	-0.75
Gansu	19.26	-6.82	0.34	-7.13	0.64
Qinghai	34.52	-17.45	1.26	1.39	2.30
Ningxia	43.5	-13.13	0.55	0.83	1.20
Xinjiang	27.13	2.67	-0.43	-8.51	-0.84

10.2 List of variables used in econometrics part

Demographics:

DepRatioYO: people aged 0-14 and 65 and more over population. Have 2000 census data or mean over 1997-2009. Have values for only young or only old people as well relative to total population.

SexRatio: nb of men for 100 women. Have mean over 1997-2009 period (without 2000 and 2001) and 2000 census results. Excluded census cause results quite different than the entire time series.

UrbRate: urbanization rate, part of total population living in urban area. Mean of 2006-09 or 2000 census.

Financial development:

PropInsPrem: property insurance premium over gdp, 2007-09.

LifeInsPrem: life insurance premium over gdp, 2007-09.

RE_SelfRaisedFund: Self-raised funds over total funds in real estate development enterprises, 1999-2009. They are sources of funds measures.

RE_OwnerEquity: owners' equity relative to total funds, 2005 census.

RE_MortLoan: personal mortgage loans of real estate dvpt enterprises relative to total funds, 2005 census.

RE_DepPrep: deposits and pre payment of enterprises in real estate dvpt, 2005 census..

Human capital:

EnrSec: enrollment in secondary education relative to total population, 1984-2004.

EnrHigh: enrollment in tertiary education relative to total population, 1984-2004.

EnrSecHigh: enrollment in secondary and higher educ relative to total population, 1984-2004.

Tertiary: higher education, nb of students relative to 100 persons, 2004-09.

SenSec: senior secondary, 2004-09.

HighEduc: enrollment in undergraduate or specialized course in institutions of higher education relative to total population, 1997-2009.

Patents: nb of patents granted per 10000 habitants, 1997-2009.

Social security:

InjIns: work injury insurance, contributor over labor force (employed people), 2006-2009.

MatIns: maternity insurance, contributors over population, 2006-09.

MedIns: basic medical care insurance, contributors over labor force, 2000-09.

PensIns_rural: rural basic pension insurance, contributors over rural employment, 2006-2009 (unbalanced). PensIns_urban: same, 2005-2009 (balanced).

UnIns: unemployment insurance, contributors over labor force, 2000-2009.

Investment structure:

SOInvFA_own: state-owned investment in fixed assets over total investment in fixed assets, 1997-2009. By status of registration.

COInvFA_own: collectively-owned investment in fixed assets, 1997-2009.
FOInvFA_own: foreign-owned (HK, T and M included), 1997-2009.
PRInvFA_own: privately-owned, sum of joint-ownership, limited liability, shareholding, private, self-employed, other ownership, individuals... definitions vary over time, roughly corresponds to total funds minus three first categories.
SBInvFA_source: state-budget investment in fixed assets, 2004-2009, by status of sources of funds over total of sources of funds.
DLInvFA_source: domestic loans, 2004-2009.
SRInvFA_source: self-raised loans, 2004-2009. It is a mix of self-raised funds of enterprises AND government.
FIInvFA_source: foreign, 2004-2009.
OTInvFA_source: other categories, include private capital from bonds and individuals.

Economic and trade structure:
PrimSec: primary sector, 1984-2009.
SecSec: secondary sector, 1984-2009.
TertSec: tertiary sector, 1984-2009.
Openness: exports + imports over gdp, mean over 1997-2009 period.
MNEShare_1996 or 2008: share of multinational enterprises in provincial exports
MNEShare_mean: mean of 1996 and 2008.
Wealth: RealGDP_pc: real gdp per capita in 84 RMB, relative to national value, 1984-2009 average.

11 Appendix 2: math

11.1 Decomposition of average investment over gdp

Dynamics of capital stock:

$$I_t = K_{t+1} - (1-\delta)K_t$$

Divide by gdp in period t:

$$i_t = \frac{K_{t+1} - (1-\delta)K_t}{Y_t}$$

Find an alternative expression for Y_t assuming that labor supply is equal to the entire population ($L_t = N_t$):

$$\begin{aligned} Y_t &= K_t^\alpha (A_t L_t)^{1-\alpha} \\ &= K_t^\alpha (A_t N_t)^{1-\alpha} \\ &= \left(\frac{K_t}{A_t N_t}\right)^\alpha A_t N_t \\ &= \tilde{k}_t^\alpha A_t N_t \end{aligned}$$

For $t \geq 1$we have a constant steady-state capital in efficiency units. Thus:

$$\begin{aligned} Y_t &= A_t N_t \tilde{k}^{*\alpha} \\ K_{t+1} &= A_{t+1} N_{t+1} \tilde{k}^* \\ K_t &= A_t N_t \tilde{k}^* \end{aligned}$$

Using $g_{t+1} = \frac{A_{t+1}}{A_t}$ and the assumption that population grows at a constant rate $n_{t+1} = n = \frac{N_{t+1}}{N_t}$, we can rewrite i_t as:

$$\begin{aligned} i_t &= \frac{A_{t+1} N_{t+1} \tilde{k}^* - (1-\delta) A_t N_t \tilde{k}^*}{A_t N_t \tilde{k}^{*\alpha}} \\ &= \frac{A_{t+1} N_{t+1} \tilde{k}^*}{A_t N_t \tilde{k}^{*\alpha}} - (1-\delta)\frac{\tilde{k}^*}{\tilde{k}^{*\alpha}} \\ &= g_{t+1} n \tilde{k}^{*(1-\alpha)} - (1-\delta)\tilde{k}^{*(1-\alpha)} \\ &= \tilde{k}^{*(1-\alpha)} \left[g_{t+1} n - 1 + \delta\right] \end{aligned}$$

In $t = 0$, need a term reflecting the initial jump from $\tilde{k}_0$to $\tilde{k}^*$. First note that:

$$\begin{aligned} K_0^* &= A_0 N_0 \tilde{k}^* \\ K_0 &= A_0 N_0 \tilde{k}_0 \\ Y_0 &= A_0 N_0 \tilde{k}_0^{*\alpha} \end{aligned}$$

Thus one gets the following jump term:

$$\begin{aligned}\frac{K_0^* - K_0}{Y_0} &= \frac{A_0 N_0 \tilde{k}^* - A_0 N_0 \tilde{k}_0}{A_0 N_0 \tilde{k}_0^{*\alpha}} \\ &= \frac{\tilde{k}^* - \tilde{k}_0}{\tilde{k}_0^{*\alpha}}\end{aligned}$$

Add the jump term to the expression from before to get i_0:

$$i_0 = \tilde{k}^{*(1-\alpha)}(g_1 n + \delta - 1) + \frac{\tilde{k}^* - \tilde{k}_0}{\tilde{k}_0^{*\alpha}}$$

Time frame: start at $t = 0$ until $t = T$ which is the last observation considered as steady-state. Thus, the last data is ignored. For example, for the $1984 - 2009$ period we have $\frac{1}{25}\sum_{t=0}^{t=24} i_t$ although we have a total of 26 periods.

The average investment rate between t and $T - 1$ is:

$$\begin{aligned} i &= \frac{1}{T}\sum_{t=0}^{T-1} i_t \\ &= \frac{1}{T}\frac{\tilde{k}^* - \tilde{k}_0}{\tilde{k}_0^{\alpha}} + \frac{1}{T}\sum_{t=0}^{T-1}(g_{t+1} n + \delta - 1)\tilde{k}^{*(1-\alpha)}\end{aligned}$$

The first part is the initial jump term in $i_0$whereas the second one gather the standard elements from 0 to $T - 1$. Note that sum from 0 to $T - 1$ divided by T has no effect on constants. Define average productivity growth rate as $\frac{1}{T}\sum_{t=0}^{T-1} g_{t+1} = \bar{g}$.
Rewrite the second term of last expression and expand with $g^* n \tilde{k}^{*(1-\alpha)}$:

$$\begin{aligned}(\bar{g}n + \delta - 1)\tilde{k}^{*(1-\alpha)} &+ g^* n \tilde{k}^{*(1-\alpha)} - g^* n \tilde{k}^{*(1-\alpha)} \\ (\bar{g}n - g^* n)(\tilde{k}^{*(1-\alpha)}) &+ (\delta - 1 + g^* n)\tilde{k}^{*(1-\alpha)} \\ (\bar{g} - g^*) n \tilde{k}^{*(1-\alpha)} &+ (g^* n + \delta - 1)\tilde{k}^{*(1-\alpha)}\end{aligned}$$

We want to express g_{t+1}relative to the catchup parameters $\pi_t$and π_{t+1}. First note that:

$$\begin{aligned} A_t \le A_t^* &= A_0^* g^{*t} \\ \pi_t &= \frac{A_t}{A_0 g^{*t}} - 1 \\ \pi_{t+1} &= \frac{A_{t+1}}{A_0 g^{*t+1}} - 1\end{aligned}$$

Start with definition of technology growth rate and use $A_t$and $A_{t+1}$from former equations:

$$
\begin{aligned}
g_{t+1} &= \frac{A_{t+1}}{A_t} \\
&= \frac{1+\pi_{t+1}}{1+\pi_t}\frac{A_0 g^{*t+1}}{A_0 g^{*t}} \\
&= \frac{1+\pi_{t+1}}{1+\pi_t} g^*
\end{aligned}
$$

One can approximate $\frac{1+\pi_{t+1}}{1+\pi_t}$ with $1+\pi_{+1}-\pi_t$.
Can rewrite $\bar{g}$:

$$
\begin{aligned}
\bar{g} &= \frac{1}{T}\sum_{t=0}^{T-1} g_{t+1} \\
&= \frac{1}{T}\sum_{t=0}^{T-1} \frac{1+\pi_{t+1}}{1+\pi_t} g^* \\
&\approx \frac{1}{T}\sum_{t=0}^{T-1} (1+\pi_{t+1}-\pi_t) g^*
\end{aligned}
$$

Focus on the concrete form of the summation, taking into account that $\pi_0 = 0$ and $\pi_T = \pi$ (steady-state at $T = 25$, in the 26th period):

$$
\begin{aligned}
\frac{1}{T}[(1+\pi_1-\pi_0 \quad)+ \quad & (1+\pi_2-\pi_1)+(1+\pi_3-\pi_2)+\ldots+(1+\pi_T-\pi_{T-1})] \\
& \frac{1}{T}[1+1+1+...+\pi] \\
& \frac{25}{25}+\frac{\pi}{25} \\
& 1+\frac{\pi}{T}
\end{aligned}
$$

Thus we have that $\bar{g} = (1+\frac{\pi}{T})g^*$which we use in i:

$$
\begin{aligned}
i &= \frac{1}{T}\frac{\tilde{k}^* - \tilde{k}_0}{\tilde{k}_0^\alpha} + \frac{1}{T}\sum_{t=0}^{T-1}(g_{t+1}n+\delta-1)\tilde{k}^{*(1-\alpha)} \\
&= \frac{1}{T}\frac{\tilde{k}^* - \tilde{k}_0}{\tilde{k}_0^\alpha} + (\bar{g}-g^*)n\tilde{k}^{*(1-\alpha)} + (g^*n+\delta-1)\tilde{k}^{*(1-\alpha)} \\
&= \frac{1}{T}\frac{\tilde{k}^* - \tilde{k}_0}{\tilde{k}_0^\alpha} + \left[g^*(1+\frac{\pi}{T})-g^*\right] n\tilde{k}^{*(1-\alpha)} + (g^*n+\delta-1)\tilde{k}^{*(1-\alpha)} \\
&= \underbrace{\frac{1}{T}\frac{\tilde{k}^* - \tilde{k}_0}{\tilde{k}_0^\alpha}}_{convergence} + \underbrace{g^*\frac{\pi}{T}n\tilde{k}^{*(1-\alpha)}}_{catch-up} + \underbrace{(g^*n+\delta-1)\tilde{k}^{*(1-\alpha)}}_{depreciation}
\end{aligned}
$$

Convergence component: initial investment at $t = 0$ required to put capital at its steady-state level. Then $\tilde{k} = \tilde{k}^*$is constant.

Catch-up component: additional investment required by the productivity catch-up.
Depreciation component: investment required to offset capital depreciation taking into account productivity and population (labor force) growth.

11.2 Closed-form expression for relative cumulated capital flows

11.2.1 Step 1: debt ratio

Noticing that $\triangle D = D_T - D_0$, $\tilde{d}_T = \frac{D_T}{A_T N_T}$ and $\tilde{y}_0 = \frac{Y_0}{A_0 N_0}$ we can rewrite relative change in debt as:

$$\begin{aligned}
\frac{\triangle D}{Y_0} = \frac{D_T - D_0}{Y_0} &= \frac{\tilde{d}_T A_T N_T - \tilde{d}_0 A_0 N_0}{A_0 N_0 \tilde{y}} \\
&= \frac{\tilde{d}_T A_T N_T / A_0 N_0 - \tilde{d}_0}{\tilde{y}_0} \\
&= \frac{\tilde{d}_T g^{*T}(1+\pi) n^T - \tilde{d}_0}{\tilde{y}_0} \\
&= \frac{\tilde{d}_T (g^* n)^T (1+\pi) - \tilde{d}_0}{\tilde{y}_0} \qquad (1)
\end{aligned}$$

where we used:

$$\begin{aligned}
\frac{A_t}{A_0 g^{*t}} &= 1 + \pi_t \\
\pi_T &= \pi \\
\frac{A_T}{A_0} &= g^{*T}(1+\pi) \\
\frac{N_{t+1}}{N_t} &= n_{t+1} = n \\
\frac{N_T}{N_0} &= n^T
\end{aligned}$$

At $t = 0$ there is a jump in external debt to $\tilde{d}_0^+ = \tilde{d}_0 + \tilde{k}^* - \tilde{k}_0$ to finance the initial increase in capital. Normalization by initial output level occurs before capital jump.
Now we look for $\tilde{d}_T$ using the BC of the representative HH:

$$N_t w_t + N_t z_t = C_t + K_{t+1} - (1-\tau_s) R^* K_t + (1-\tau_s) R^* D_t - D_{t+1}$$

HH resources: work income, transfers, net returns from capital, higher debt.
HH expenditures: consumption, investment in capital for next period, debt repayment.
The marginal influence of the saving wedge is $\frac{\partial .}{\partial \tau_s} = R^*(K_t - D_t)$. Thus, if $K_t > D_t$ we have a positive effect of an increase in τ_s on the expenditures (less resources). If $K_t < D_t$, HH have more resources at disposal. There are two opposite effects given higher τ_s: HH get less returns on capital but pay less on debt. Which effects dominates depends on size of $K_t$and D_t. The first situation will be considered as the standard case.
Divide BC by N_t and use $c_t = \frac{C_T}{N_T}$ and $N_t = \frac{N_{t+1}}{n}$:

$$\begin{aligned}
w_t + z_t &= \frac{C_t}{N_t} + \frac{K_{t+1} - D_{t+1}}{N_t} + (1-\tau_s)\frac{R^*(D_t - K_t)}{N_t} \\
&= c_t + n(k_{t+1} - d_{t+1}) + R^*(d_t - k_t) - \tau_s R^*(d_t - k_t)
\end{aligned}$$

In a next step, the terms involving the saving wedge are consolidated.
The total revenue from the wedges is, in level and per capita unit:

$$\begin{aligned}
Z_t &= \tau_k R_t K_t + \tau_s R^*(K_t - D_t) \\
z_t &= \tau_k R_t k_t + \tau_s R^*(k_t - d_t)
\end{aligned}$$

In the BC, focus on the terms involving τ_s and use z_t:

$$\begin{aligned}
z_t &+ \tau_s R^*(d_t - k_t) \\
\tau_k R_t k_t + \tau_s R^*(k_t - d_t) &+ \tau_s R^*(d_t - k_t) \\
\tau_k R_t k_t &
\end{aligned}$$

Using $R_t = \frac{R^*}{1-\tau_k}$ we get z_{kt}, the lump-sum transfers financed by capital wedge:

$$z_{kt} = \frac{\tau_k}{1-\tau_k} R^* k_t$$

The BC is:

$$z_{kt} + w_t = c_t + n(k_{t+1} - d_{t+1}) + R^*(d_t - k_t)$$

Divide BC by A_t and use $A_t = \frac{A_{t+1}}{g_{t+1}}$:

$$\begin{aligned}
\frac{z_{kt}}{A_t} + \frac{w_t}{A_t} &= \frac{c_t}{A_t} + n(\frac{k_{t+1}}{A_t} - \frac{d_{t+1}}{A_t}) + R^*(\frac{d_t - k_t}{A_t}) \\
\tilde{z}_{kt} + \tilde{w}_t &= \tilde{c}_t + n g_{t+1}(\tilde{k}_{t+1} - \tilde{d}_{t+1}) + R^*(\tilde{d}_t - \tilde{k}_t)
\end{aligned}$$

In steady-state, $\tilde{k}_{t+1} = \tilde{k}_t = \tilde{k}^*$and $\tilde{z}_k = \frac{\tau_k}{1-\tau_k} R^* \tilde{k}^*$. The normalized BC is:

$$\tilde{z}_k + \tilde{w} = \tilde{c}_t + n g_{t+1}(\tilde{k}^* - \tilde{d}_{t+1}) + R^*(\tilde{d}_t - \tilde{k}^*)$$

After (rather at) time T, the economy is in steady-state and the saving wedge disappears. Using $g_{t+1} = g^*$, $\tilde{d}_t = \tilde{d}_T$ and $\tilde{c}_t = c_T$, we get the SS debt:

$$\begin{aligned}
\tilde{z}_k + \tilde{w} &= \tilde{c}_T + n g^*(\tilde{k}^* - \tilde{d}_T) + R^*(\tilde{d}_T - \tilde{k}^*) \\
&= \tilde{c}_T - \tilde{k}^*(R^* - n g^*) + \tilde{d}_T(R - n g^*) \\
\tilde{d}_T &= \frac{\tilde{z}_k + \tilde{w} - \tilde{c}_T}{R^* - n g^*} + \tilde{k}^* \qquad (2)
\end{aligned}$$

11.2.2 Step 2: consumption

Start with the Euler equation and use $R^* = \frac{g^{*\gamma}}{\beta}$ to get rid of β with $\beta = \frac{g^{*\gamma}}{R^*}$:

$$\begin{aligned}
c_t^{-\gamma} &= \beta R^*(1-\tau_s)c_{t+1}^{-\gamma} \\
c_{T-1}^{\gamma} &= \frac{g^{*\gamma}}{R^*} R^*(1-\tau_s)c_T^{-\gamma} \\
c_T^{-\gamma} &= \frac{c_{T-1}^{-\gamma}}{g^{*\gamma}(1-\tau_s)} \\
c_T &= \frac{c_{T-1}}{g^{*-1}(1-\tau_s)^{-\frac{1}{\gamma}}} \\
c_T &= c_{T-1} g^*(1-\tau_s)^{\frac{1}{\gamma}}
\end{aligned}$$

This result is used to get c_T relative to $c_0$with $\phi(\tau_s) = (1-\tau_s)^{\frac{1}{\gamma}}$:

$$\begin{aligned}
c_T &= c_0 \left[g^*(1-\tau_s)^{\frac{1}{\gamma}} \right]^T \\
&= c_0 \left[g^* \phi(\tau_s) \right]^T
\end{aligned}$$

Using $A_T = (1+\pi)A_0 g^{*T}$, consumption in efficiency units can be expressed as:

$$\begin{aligned}
\tilde{c}_T &= \frac{c_0 \left[g^* \phi(\tau_s) \right]^T}{(1+\pi)A_0 g^{*T}} \\
&= \frac{\tilde{c}_0 \phi(\tau_s)^T}{1+\pi} \qquad (3)
\end{aligned}$$

Now one needs to rewrite the per capita BC of the households:

$$\begin{aligned}
t=0: \quad z_{k0} + w_0 &= c_0 + n(k_1 - d_1) + R^*(d_0 - k_0) \\
t=1: \quad z_{k1} + w_1 &= c_1 + n(k_2 - d_2) + R^*(d_1 - k_1) \\
t=2: \quad z_{k2} + w_2 &= c_2 + n(k_3 - d_3) + R^*(d_2 - k_2)
\end{aligned}$$

From $t = 1$ get:

$$k_1 - d_1 = \frac{c_1 + n(k_2 - d_2) - z_{k1} - w_1}{R^*}$$

and plug it in $t = 0$:

$$z_{k0} + w_0 = c_0 + n\frac{c_1}{R^*} + \frac{n^2(k_2 - d_2)}{R^*} - \frac{n z_{k1}}{R^*} - \frac{n w_1}{R^*} + R^*(d_0 - k_0)$$

Rearrange:

$$z_{k0}+\frac{nz_{k1}}{R^*}+w_0+\frac{nw_1}{R^*}=c_0+\frac{nc_1}{R^*}+R^*(d_0-k_0)+\frac{n^2(k_2-d_2)}{R^*}$$

From $t=2$ get:

$$k_2-d_2=\frac{c_2+n(k_3-d_3)-z_{k2}-w_2}{R^*}$$

and plug it in the result from before:

$$z_{k0}+\frac{nz_{k1}}{R^*}+\frac{n^2z_{k2}}{R^{*2}}+w_0+\frac{nw_1}{R^*}+\frac{n^2w_2}{R^{*2}}=c_0+\frac{n^2c_1}{R^{*2}}+\frac{n^2c_2}{R^{*2}}+R^*(d_0-k_0)+\frac{n^3}{R^{*2}}(k_3-d_3)$$

By repeating the process until ∞, the $k-d$ term on the right disappears if $R^*>n$ and we are left with the intertemporal BC:

$$\sum_0^\infty\left(\frac{n}{R^*}\right)^t(z_{kt}+w_t)=\sum_0^\infty\left(\frac{n}{R^*}\right)^t c_t+R^*(d_0-k_0)$$

From before, we know that consumption per capita grows at rate $g^*\phi(\tau_s)$ until T and at g^*afterwards:

$$c_t=A_0\phi^{min(t,T)}g^{*t}\tilde{c}_0$$

Rewrite the consumption part of the intertemporal BC:

$$\begin{aligned}
\sum_{t=0}^\infty\left(\frac{n}{R^*}\right)^t c_t &= \sum_{t=0}^\infty\left(\frac{n}{R^*}\right)A_0\phi^{min(t,T)}g^{*t}\tilde{c}_0\\
&= A_0\tilde{c}_0\sum_{t=0}^T\left(\frac{n}{R^*}\right)^t\phi^t g^{*t}+A_0\tilde{c}_0\phi^T\sum_{t=T+1}^\infty\left(\frac{n}{R^*}\right)^t g^{*t}\\
&= A_0\tilde{c}_0\left[\sum_{t=0}^T\left(\frac{\phi ng^*}{R^*}\right)^t+\phi^T\sum_{t=T+1}^\infty\left(\frac{ng^*}{R^*}\right)^t\right]\\
&= \frac{A_0\tilde{c}_0}{(1-\frac{ng^*}{R^*})\psi(\tau_s)} \qquad (4)
\end{aligned}$$

All elements influenced by τ_s are gathered in $\psi(\tau_s)$:

$$\psi(\tau_s)=\left(1-\frac{ng^*}{R^*}\right)^{-1}\left[\sum_{t=0}^T\left(\frac{\phi ng^*}{R^*}\right)^t+\phi^T\sum_{t=T+1}^\infty\left(\frac{ng^*}{R^*}\right)^t\right]^{-1} \qquad (5)$$

Remember that:

$$\sum_{t=0}^{n-1} x^t = \frac{1-x^n}{1-x}$$
$$\sum_{t=0}^{\infty} x^t = \frac{1}{1-x}$$
$$\sum_{t=1}^{\infty} x^t = \sum_{t=0}^{\infty} x^t - 1 = \frac{x}{1-x}$$

Using them we get:

$$\left(1-\frac{ng^*}{R^*}\right)^{-1}\left[\frac{1-(\frac{\phi ng^*}{R^*})^{T+1}}{1-\frac{\phi ng^*}{R^*}} + \phi^T\frac{1}{1-\frac{ng^*}{R^*}} - \phi^T\frac{1-\frac{ng^*}{R^*}^{T+1}}{1-\frac{ng^*}{R^*}}\right]$$

$$\left(1-\frac{ng^*}{R^*}\right)^{-1}\left[\frac{1-(\frac{\phi ng^*}{R^*})^{T+1}}{1-\frac{\phi ng^*}{R^*}} + \phi^T\frac{\frac{ng^*}{R^*}^{T+1}}{1-\frac{ng^*}{R^*}}\right]^{-1}$$

$$\frac{R^*}{R^*-ng^*}\left[\frac{1-(\frac{\phi ng^*}{R^*})^{T+1}}{\frac{R^*-\phi ng^*}{R^*}} + \phi^T\frac{\frac{ng^*}{R^*}^{T+1}}{\frac{R^*-ng^*}{R^*}}\right]^{-1}$$

$$\frac{R^*}{R^*-ng^*}\left[R^*\frac{1-(\frac{\phi ng^*}{R^*})^{T+1}}{R^*-\phi ng^*} + R^*\phi^T\frac{\frac{ng^*}{R^*}^{T+1}}{R^*-ng^*}\right]^{-1}$$

$$\frac{1}{R^*-ng^*}\left[\frac{(1-(\frac{\phi ng^*}{R^*})^{T+1})(R^*-ng^*)+\phi^T\frac{ng^*}{R^*}^{T+1}(R^*-\phi ng^*)}{(R^*-\phi ng^*)(R^*-ng^*)}\right]^{-1}$$

Focus on the numerator, first gather terms containing ng^*:

$$-ng^* + \left[\frac{\phi ng^*}{R^*}\right]^{T+1} ng^* - \phi^T\left[\frac{ng^*}{R^*}\right]^{T+1}\phi ng^*$$

$$-ng^*$$

then, concentrate on R^*:

$$R^* - \frac{\phi ng^*}{R^*}^{T+1} R^* + \phi^T\frac{ng^*}{R^*}^{T+1} R^*$$

$$R^* - R^*\left[\frac{\phi ng^*}{R^*}\right]^T\frac{\phi ng^*}{R^*} + R^*\left[\frac{\phi ng^*}{R^*}\right]^T\frac{ng^*}{R^*}$$

$$R^* - \left[\frac{\phi ng^*}{R^*}\right]^T\phi ng^* + \left[\frac{\phi ng}{R^*}\right]^T ng^*$$

$$R^* + \left[\frac{\phi ng^*}{R^*}\right]^T ng^*(1-\phi)$$

Thus, we get:

$$\frac{1}{R^*-ng^*}\left[\frac{R^*-ng^*+\left[\frac{\phi ng^*}{R^*}\right]^T ng^*(1-\phi)}{(R^*-\phi ng^*)(R^*-ng^*)}\right]^{-1}$$

$$\psi(\tau_s)=\frac{R^*-\phi ng^*}{R^*-ng^*+\left[\frac{ng^*\phi(\tau_s)}{R^*}\right]^T ng^*(1-\phi(\tau_s))} \tag{6}$$

The consumption in period 0 in efficiency units can now be computed. First note that:

$$\begin{aligned}
(1+\pi_T)A_0 g^{*T} &= A_{T,}\ \pi_T=\pi \\
(1+\pi_t)A_0 g^{*t} &= A_t \\
\tilde{w}=\frac{w_t}{A_t} &= \frac{w_t}{A_0(1+\pi_t)g^{*t}} \\
\tilde{z}_{kt}=\frac{z_{kt}}{A_t} &= \frac{z_{kt}}{A_0(1+\pi_t)g^{*t}} \\
w_t+z_{kt} &= (\tilde{w}+\tilde{z}_k)A_0(1+\pi_t)g^{*t} \qquad (7)
\end{aligned}$$

Plug 4 and 7 in the intertemporal version of the budget constraint:

$$\begin{aligned}
\sum_0^\infty\left(\frac{n}{R^*}\right)^t c_t &= \sum_0^\infty\left(\frac{n}{R^*}\right)^t (z_{kt}+w_t)+R^*(k_0-d_0) \\
\frac{A_0\tilde{c}_0}{(1-\frac{ng^*}{R^*})\psi(\tau_s)} &= \sum_{t=0}^\infty\left(\frac{n}{R^*}\right)^t(\tilde{w}+\tilde{z}_k)A_0(1+\pi_t)g^{*t}+R^*(k_0-d_0) \\
\tilde{c}_0 &= \frac{R^*-ng^*}{R^*}\psi(\tau_s)\frac{1}{A_0}(\tilde{w}+\tilde{z}_k)A_0\sum_{t=0}^\infty\left(\frac{n}{R^*}\right)^t(1+\pi_t)g^{*t} \\
&\quad +\frac{R^*-ng^*}{R^*}\psi(\tau_s)\frac{1}{A_0}R^*(k_0-d_0) \\
&= \frac{R^*-ng^*}{R^*}\psi(\tau_s)(\tilde{w}+\tilde{z}_k)\sum_{t=0}^\infty\left(\frac{ng^*}{R^*}\right)^t(1+\pi_t) \\
&\quad +(R^*-ng^*)\psi(\tau_s)\frac{k_0-d_0}{A_0} \\
&= \ldots+(R^*-ng^*)\psi(\tau_s)(\tilde{k}_0-\tilde{d}_0) \\
\tilde{c}_0 &= (R^*-ng^*)\psi(\tau_s)\left[\frac{\tilde{w}+\tilde{z}_k}{R^*}\sum_{t=0}^\infty\left(\frac{ng^*}{R^*}\right)^t(1+\pi_t)+\tilde{k}_0-\tilde{d}_0\right]
\end{aligned}$$

The saving wedge $\tau_s$enters consumption choice through the marginal propensity to consume $(R^*-ng^*)\psi(\tau_s)$ out of wealth [...]. If τ_s is higher, then $\phi(\tau_s)$is lower, $\psi(\pi_s)$is higher and the marginal propensity to consume out of wealth rises.
Note that using $\tilde{c}_0$ we can now find expressions for $\tilde{c}_t$ and $\tilde{c}_T$ as well:

$$
\begin{aligned}
\tilde{c}_t = \frac{c_t}{A_t} &= \frac{1}{(1+\pi_t)A_0 g^{*t}} \frac{A_0 \tilde{c}_0}{(1-\frac{ng^*}{R^*})\psi(\tau_s)} \frac{1}{\sum_{t=0}^{\infty}\left(\frac{n}{R^*}\right)^t} \\
&= \frac{1}{(1+\pi_t)g^{*t}} \frac{\tilde{c}_0}{(1-\frac{ng^*}{R^*})\psi(\tau_s)} \frac{1}{\sum_{t=0}^{\infty}\left(\frac{n}{R^*}\right)^t} \\
\tilde{c}_T = \frac{c_T}{A_T} &= \frac{\tilde{c}_0 \phi(\tau_s)^T}{1+\pi_T}
\end{aligned}
$$

11.2.3 Step 3: closed-form expression for the relative flows

The found expressions for $\tilde{d}_T$, $\tilde{c}_T$ and $\tilde{c}_0$ can be successively plugged in the initial $\frac{\triangle D}{Y_0}$:

$$
\begin{aligned}
\frac{\triangle D}{Y_0} &= \frac{\tilde{d}_T (g^* n)^T (1+\pi) - \tilde{d}_0}{\tilde{y}_0} \\
&= \left(\frac{\tilde{z}_k + \tilde{w} - \tilde{c}_T}{R^* - ng^*} + \tilde{k}^*\right)\left(\frac{(g^* n)^T (1+\pi)}{\tilde{y}_0}\right) - \frac{\tilde{d}_0}{\tilde{y}_0} \\
&= \frac{\tilde{k}^* (g^* n)^T (1+\pi)}{\tilde{y}_0} + \frac{\tilde{w} + \tilde{z}_k}{R^* - ng^*} \frac{(g^* n)^T (1+\pi)}{\tilde{y}_0} - \frac{\tilde{c}_T}{R^* - ng^*} \frac{(g^* n)^T (1+\pi)}{\tilde{y}_0} - \frac{\tilde{d}_0}{\tilde{y}_0}
\end{aligned}
$$

Focus on the last term containing $\tilde{c}_T$:

$$
-\frac{\tilde{c}_0 \phi(\tau_s)^T}{1+\pi} \frac{1}{(R^* - ng^*)} \frac{(g^* n)^T (1+\pi)}{\tilde{y}_0}
$$

$$
-(R^* - ng^*)\psi(\tau_s)\left[\frac{\tilde{w} + \tilde{z}_k}{R^*} \sum_{t=0}^{\infty}\left(\frac{ng^*}{R^*}\right)^t (1+\pi_t) + (\tilde{k}_0 - \tilde{d}_0)\right] \frac{\phi(\tau_s)^T}{1+\pi} \frac{1}{(R^* - ng^*)} \frac{(g^* n)^T (1+\pi)}{\tilde{y}_0}
$$

$$
-\frac{\psi(\tau_s)\phi(\tau_s)^T}{\tilde{y}_0}(g^* n)^T \left[\frac{\tilde{w} + \tilde{z}_k}{R^*} \sum_{t=0}^{\infty}\left(\frac{ng^*}{R^*}\right)^t (1+\pi_t) + (\tilde{k}_0 - \tilde{d}_0)\right]
$$

The entire expression is:

$$
\begin{aligned}
\frac{\triangle D}{Y_0} = & \frac{\tilde{k}^* (g^* n)^T (1+\pi)}{\tilde{y}_0} + \frac{\tilde{w} + \tilde{z}_k}{R^* - ng^*} \frac{(g^* n)^T (1+\pi)}{\tilde{y}_0} \\
& - \frac{\psi(\tau_s)\phi(\tau_s)^T}{\tilde{y}_0}(g^* n)^T \left[\frac{\tilde{w} + \tilde{z}_k}{R^*} \sum_{t=0}^{\infty}\left(\frac{ng^*}{R^*}\right)^t (1+\pi_t) + \tilde{k}_0 - \tilde{d}_0\right] - \frac{\tilde{d}_0}{\tilde{y}_0}
\end{aligned}
$$

The part of the equation gathering $\tilde{k}^*$ is the first response term:

$$
\frac{\tilde{k}^*}{\tilde{y}_0}(ng^*)^T (1+\pi)
$$

Focusing on terms containing $\tilde{k}_0$, one gets the second element of the answer:

$$-\frac{\psi(\tau_s)\phi(\tau_s)^T}{\tilde{y}_0}(g^*n)^T\tilde{k}_0$$

$$-\frac{\tilde{k}_0}{\tilde{y}_0}\psi(\tau_s)[ng^*\phi(\tau_s)]^T$$

As for the third one, take the terms containing $\tilde{d}_0$

$$\frac{\psi(\tau_s)\phi(\tau_s)^T}{\tilde{y}_0}(g^*n)^T\tilde{d}_0-\frac{\tilde{d}_0}{\tilde{y}_0}$$

$$\frac{\tilde{d}_0}{\tilde{y}_0}\left(\psi(\tau_s)[ng^*\phi(\tau_s)]^T-1\right)$$

Collecting the left-over terms of the initial expression, we have:

$$\frac{\tilde{w}+\tilde{z_k}}{R^*-ng^*}\frac{(g^*n)^T(1+\pi)}{\tilde{y}_0}-\frac{\psi(\tau_s)\phi(\tau_s)^T(g^*n)^T}{\tilde{y}_0}\frac{\tilde{w}+\tilde{z}_k}{R^*}\sum_{t=0}^{\infty}\left(\frac{ng^*}{R^*}\right)^t(1+\pi_t)$$

$$\frac{\tilde{w}+\tilde{z_k}}{\tilde{y}_0}(g^*n)^T\left[\frac{1}{R^*-ng^*}(1+\pi)-\frac{\psi(\tau_s)\phi(\tau_s)^T}{R^*}\sum_{t=0}^{\infty}\left(\frac{ng^*}{R^*}\right)^t(1+\pi_t)\right]$$

$$\ldots\left[\frac{1}{R^*-ng^*}(1+\pi)-\frac{\psi(\tau_s)\phi(\tau_s)^T}{R^*}\sum_{t=T}^{\infty}\left(\frac{ng^*}{R^*}\right)^t(1+\pi)-\frac{\psi(\tau_s)\phi(\tau_s)^T}{R^*}\sum_{t=0}^{T-1}\left(\frac{ng^*}{R^*}\right)^t(1+\pi_t)\right]$$

where the assumption that from T on, $\pi_t=\pi$ has been used in the second part.
From now on, one needs to proceed sequentially. First, keep the third term in the fridge for the moment:

$$-\frac{\tilde{w}+\tilde{z_k}}{\tilde{y}_0}(g^*n)^T\frac{\psi(\tau_s)\phi(\tau_s)^T}{R^*}\sum_{t=0}^{T-1}\left(\frac{ng^*}{R^*}\right)^t(1+\pi_t) \tag{8}$$

Focus on the first and second part:

$$\frac{\tilde{w}+\tilde{z_k}}{\tilde{y}_0}(g^*n)^T\left[\frac{1}{R^*-ng^*}(1+\pi)-\frac{\psi(\tau_s)\phi(\tau_s)^T}{R^*}\sum_{t=T}^{\infty}\left(\frac{ng^*}{R^*}\right)^t(1+\pi)\right]$$

$$\frac{\tilde{w}+\tilde{z_k}}{\tilde{y}_0}(g^*n)^T\left[(1+\pi)\frac{1}{R^*}\sum_{t=0}^{\infty}\left(\frac{ng^*}{R^*}\right)^t-(1+\pi)\frac{\psi(\tau_s)\phi(\tau_s)^T}{R^*}\sum_{t=T}^{\infty}\left(\frac{ng^*}{R^*}\right)^t\right]$$

$$\frac{\tilde{w}+\tilde{z_k}}{\tilde{y}_0}(g^*n)^T\frac{(1+\pi)}{R^*}\left[\sum_{t=0}^{\infty}\left(\frac{ng^*}{R^*}\right)^t-\psi(\tau_s)\phi(\tau_s)^T\sum_{t=T}^{\infty}\left(\frac{ng^*}{R^*}\right)^t\right] \tag{9}$$

Before, we had defined:

$$\psi(\tau_s) = \left(1 - \frac{ng^*}{R^*}\right)^{-1} \left[\sum_{t=0}^{T} \left(\frac{\phi ng^*}{R^*}\right)^t + \phi(\tau_s)^T \sum_{t=T+1}^{\infty} \left(\frac{ng^*}{R^*}\right)^t\right]^{-1}$$

Use this definition to get an expression for $\sum_{t=0}^{\infty} \left(\frac{ng^*}{R^*}\right)^t = \left(1 - \frac{ng^*}{R^*}\right)^{-1}$:

$$\sum_{t=0}^{T} \left(\frac{ng^*}{R^*}\right)^t = \psi(\tau_s) \left[\sum_{t=0}^{T} \left(\frac{\phi ng^*}{R^*}\right)^t + \phi(\tau_s)^T \sum_{t=T+1}^{\infty} \left(\frac{ng^*}{R^*}\right)^t\right]$$

Plug it in 9 and focus on the interior of the bracket:

$$\ldots \left[\psi(\tau_s) \sum_{t=0}^{T} \left(\frac{\phi ng^*}{R^*}\right)^t + \psi(\tau_s)\phi(\tau_s)^T \sum_{t=T+1}^{\infty} \left(\frac{ng^*}{R^*}\right)^t - \psi(\tau_s)\phi(\tau_s)^T \sum_{t=T}^{\infty} \left(\frac{ng^*}{R^*}\right)^t\right]$$

$$\begin{aligned} \psi(\tau_s) \sum_{t=0}^{T-1} \left(\frac{\phi ng^*}{R^*}\right)^t + \psi(\tau_s) \sum_{t=T}^{T} \left(\frac{\phi ng^*}{R^*}\right)^t + \psi(\tau_s)\phi(\tau_s)^T \sum_{t=T}^{\infty} \left(\frac{ng^*}{R^*}\right)^t \\ -\psi(\tau_s)\phi^T \sum_{t=T}^{T} \left(\frac{ng^*}{R^*}\right)^t - \psi(\tau_s)\phi(\tau_s)^T \sum_{t=T}^{\infty} \left(\frac{ng^*}{R^*}\right)^t \end{aligned}$$

$$\begin{aligned} \psi(\tau_s) \sum_{t=0}^{T-1} \left(\frac{\phi ng^*}{R^*}\right)^t + \psi(\tau_s) \left(\frac{\phi ng^*}{R^*}\right)^T + \psi(\tau_s)\phi(\tau_s)^T \sum_{t=T}^{\infty} \left(\frac{ng^*}{R^*}\right)^t \\ -\psi(\tau_s)\phi^T \left(\frac{ng^*}{R^*}\right)^T - \psi(\tau_s)\phi(\tau_s)^T \sum_{t=T}^{\infty} \left(\frac{ng^*}{R^*}\right)^t \end{aligned}$$

$$\frac{\tilde{w} + \tilde{z}_k}{\tilde{y}_0} (g^* n)^T \frac{(1+\pi)}{R^*} \left[\psi(\tau_s) \sum_{t=0}^{T-1} \left(\frac{\phi ng^*}{R^*}\right)^t\right]$$

Now take the last expression and 8 together to get the fourth part:

$$\frac{\tilde{w} + \tilde{z}_k}{\tilde{y}_0} (g^* n)^T \frac{(1+\pi)}{R^*} \left[\psi(\tau_s) \sum_{t=0}^{T-1} \left(\frac{\phi ng^*}{R^*}\right)^t\right] - \frac{\tilde{w} + \tilde{z}_k}{\tilde{y}_0} (g^* n)^T \frac{\psi(\tau_s)\phi(\tau_s)^T}{R^*} \sum_{t=0}^{T-1} \left(\frac{ng^*}{R^*}\right)^t (1+\pi_t)$$

$$\frac{\tilde{w} + \tilde{z}_k}{\tilde{y}_0} \frac{\psi(\tau_s)}{R^*} [ng^* \phi(\tau_s)]^T \sum_{t=0}^{T-1} \left(\frac{ng^*}{R^*}\right)^t \left[\phi(\tau_s)^{t-T}(1+\pi) - (1+\pi_t)\right]$$

The equation of the flows decomposition is constituted of the four expressions we have just computed:

$$\begin{aligned} \frac{\triangle D}{Y_0} &= \frac{\tilde{k}^*}{\tilde{y}_0}(ng^*)^T(1+\pi) - \frac{\tilde{k}_0}{\tilde{y}_0}\psi(\tau_s)[ng^*\phi(\tau_s)]^T + \frac{\tilde{d}_0}{\tilde{y}_0}\left(\psi(\tau_s)[ng^*\phi(\tau_s)]^T - 1\right) \\ &\quad + \frac{\tilde{w} + \tilde{z}_k}{\tilde{y}_0} \frac{\psi(\tau_s)}{R^*} [ng^* \phi(\tau_s)]^T \sum_{t=0}^{T-1} \left(\frac{ng^*}{R^*}\right)^t \left[\phi(\tau_s)^{t-T}(1+\pi) - (1+\pi_t)\right] \qquad (10) \end{aligned}$$

From *proposition 1*, we know that under certain conditions:

$$\frac{\triangle D}{Y_0} = D\left(\underset{+}{\tilde{d}_0}, \underset{-}{\tilde{k}_0}, \underset{+}{\pi}, \underset{-}{\tau_k}, \underset{+}{\tau_s}\right)$$

Cumulated relative capital inflows increase with $\tilde{d}_0$ if $\psi(\tau_s)[ng^*\phi(\tau_s)]^T > 1$ which is the case if τ_s is small enough. They clearly decrease with higher $\tilde{k}_0$. An increase in π causes the first term to be higher. In the fourth one, concentrate on the bracket: as $\pi_t = f(t)\pi$ with $f(t) = min(\frac{t}{T}, 1) \leq 1$ and $\phi(\tau_s)^{t-T} = (1-\tau_s)^{\frac{1}{\gamma}(t-T)} > 1$ but only if $\tau_s > 0$. If not, $\phi(\tau_s)^{t-T} < 1$ and the fourth term could even overcompensate for the positive effect of the first term. Countries with a relatively higher τ_k have lower $\tilde{k}^*$and thus less inflows. At last, a relatively higher τ_s implies higher inflows if $\tilde{k}_0 \geq \tilde{d}_0$.

11.2.4 Step 4: channel decomposition

Focus on the first three terms and distribute:

$$\frac{\tilde{k}^*}{\tilde{y}_0}(ng^*)^T + \frac{\tilde{k}^*}{\tilde{y}_0}(ng^*)^T\pi - \frac{\tilde{k}_0}{\tilde{y}_0}\psi(\tau_s)\left[ng^*\phi(\tau_s)\right]^T + \frac{\tilde{d}_0}{\tilde{y}_0}\left(\psi(\tau_s)[ng^*\phi(\tau_s)]^T - 1\right)$$

Expand with $\frac{\tilde{k}_0}{\tilde{y}_0}(ng^*)^T$:

$$\underbrace{\frac{\tilde{k}^* - \tilde{k}_0}{\tilde{y}_0}(ng^*)^T}_{convergence} + \underbrace{\frac{\tilde{k}^*}{\tilde{y}_0}(ng^*)^T\pi}_{investment} + \frac{\tilde{k}_0}{\tilde{y}_0}(ng^*)^T - \frac{\tilde{k}_0}{\tilde{y}_0}\psi(\tau_s)\left[ng^*\phi(\tau_s)\right]^T + \frac{\tilde{d}_0}{\tilde{y}_0}\left(\psi(\tau_s)[ng^*\phi(\tau_s)]^T - 1\right)$$

Focus on the left-over terms:

$$\frac{\tilde{k}_0}{\tilde{y}_0}(ng^*)^T - \frac{\tilde{k}_0}{\tilde{y}_0}\psi(\tau_s)\left[ng^*\phi(\tau_s)\right]^T + \frac{\tilde{d}_0}{\tilde{y}_0}\psi(\tau_s)[ng^*\phi(\tau_s)]^T - \frac{\tilde{d}_0}{\tilde{y}_0}$$

$$\underbrace{\frac{\tilde{k}_0(ng^*)^T - \tilde{d}_0}{\tilde{y}_0} + \psi(\tau_s)\left[ng^*\phi(\tau_s)\right]^T\frac{\tilde{d}_0 - \tilde{k}_0}{\tilde{y}_0}}_{trend}$$

The saving channel is the fourth part of (10):

$$\underbrace{\frac{\tilde{w} + \tilde{z}_k}{\tilde{y}_0}\frac{\psi(\tau_s)}{R^*}[ng^*\phi(\tau_s)]^T\sum_{t=0}^{T-1}\left(\frac{ng^*}{R^*}\right)^t\left[\phi(\tau_s)^{t-T}(1+\pi) - (1+\pi_t)\right]}_{saving}$$

11.3 Relative flows with no saving wedge

With $\tau_s = 0$, we have:

$$\begin{aligned}\phi(\tau_s) &= (1-\tau_s)^{\frac{1}{\gamma}} = 1 \\ \psi(\tau_s) &= \frac{R^* - ng^*}{R^* - ng^* + \left(\frac{ng^*}{R^*}\right)^T ng^*(1-1)} = 1\end{aligned}$$

The convergence and investment channels are the same:

$$\frac{\tilde{k}^* - \tilde{k}_0}{\tilde{y}_0}(ng^*)^T$$

$$\frac{\tilde{k}^*}{\tilde{y}_0}(ng^*)^T\pi$$

The trend channel becomes:

$$\frac{\tilde{k}_0(ng^*)^T - \tilde{d}_0}{\tilde{y}_0} + (ng^*)^T\frac{\tilde{d}_0 - \tilde{k}_0}{\tilde{y}_0}$$

$$\frac{\tilde{d}_0(ng^*)^T}{\tilde{y}_0} - \frac{\tilde{k}_0}{\tilde{y}_0}(ng^*)^T + \frac{\tilde{k}_0}{\tilde{y}_0}(ng^*)^T - \frac{\tilde{d}_0}{\tilde{y}_0}$$

$$\frac{\tilde{d}_0}{\tilde{y}_0}\left[(ng^*)^T - 1\right]$$

The saving channel is:

$$\frac{\tilde{w} + \tilde{z}_k}{\tilde{y}_0}\frac{1}{R^*}(ng^*)^T\sum_{t=0}^{T-1}\left(\frac{ng^*}{R^*}\right)^t[(1+\pi) - (1+\pi_t)]$$

Remember that $\pi_t = f(t)\pi$:

$$\frac{\tilde{w} + \tilde{z}_k}{\tilde{y}_0}\frac{1}{R^*}(ng^*)^T\sum_{t=0}^{T-1}\left(\frac{ng^*}{R^*}\right)\pi\,[1 - f(t)]$$

11.4 Extension 1: exogenous interest rate

The debt ratio is not influenced, à savoir:

$$\tilde{d}_T = \frac{\tilde{z}_k + \tilde{w} - \tilde{c}_T}{R^* - ng^*} + \tilde{k}^*$$

Start with the Euler equation and don't use $R^* = \frac{g^{*\gamma}}{\beta}$:

$$\begin{aligned}
c_t^{-\gamma} &= \beta R^*(1-\tau_s)c_{t+1}^{-\gamma} \\
c_T^{-\gamma} &= \frac{c_{T-1}^{-\gamma}}{\beta R^*(1-\tau_s)} \\
c_T &= \frac{c_{T-1}}{[\beta R^*(1-\tau_s)]^{-\frac{1}{\gamma}}} \\
c_T &= c_{T-1}[\beta R^*(1-\tau_s)]^{\frac{1}{\gamma}}
\end{aligned}$$

This result is used to get c_T relative to $c_0$with $\phi(\tau_s) = (1-\tau_s)^{\frac{1}{\gamma}}$:

$$
\begin{aligned}
c_T &= c_0 \left[(\beta R^*)^{\frac{1}{\gamma}} (1-\tau_s)^{\frac{1}{\gamma}} \right]^T \\
&= c_0 \left[(\beta R^*)^{\frac{1}{\gamma}} \phi(\tau_s) \right]^T
\end{aligned}
$$

Using $A_T = (1+\pi)A_0 g^{*T}$, consumption in efficiency units can be expressed as:

$$
\begin{aligned}
\tilde{c}_T &= \frac{c_0 \left[(\beta R^*)^{\frac{1}{\gamma}} \phi(\tau_s) \right]^T}{(1+\pi)A_0 g^{*T}} \\
&= \frac{\tilde{c}_0 \phi(\tau_s)^T (\beta R^*)^{\frac{T}{\gamma}}}{(1+\pi) g^{*T}}
\end{aligned}
$$

The intertemporal BC is the same as in the baseline case:

$$
\sum_0^\infty \left(\frac{n}{R^*} \right)^t (z_{kt} + w_t) = \sum_0^\infty \left(\frac{n}{R^*} \right)^t c_t + R^*(d_0 - k_0)
$$

From before, we know that consumption per capita grows at rate $(\beta R^*)^{\frac{1}{\gamma}} \phi(\tau_s)$ until T and at $(\beta R^*)^{\frac{1}{\gamma}}$ afterwards:

$$
c_t = A_0 \phi^{min(t,T)} (\beta R^*)^{\frac{t}{\gamma}} \tilde{c}_0
$$

Rewrite the consumption part of the intertemporal BC:

$$
\begin{aligned}
\sum_{t=0}^\infty \left(\frac{n}{R^*} \right)^t c_t &= \sum_{t=0}^\infty \left(\frac{n}{R^*} \right)^t A_0 \phi^{min(t,T)} (\beta R^*)^{\frac{t}{\gamma}} \tilde{c}_0 \\
&= A_0 \tilde{c}_0 \sum_{t=0}^T \left(\frac{n}{R^*} \right)^t \phi^t (\beta R^*)^{\frac{t}{\gamma}} + A_0 \tilde{c}_0 \phi^T \sum_{t=T+1}^\infty \left(\frac{n}{R^*} \right)^t (\beta R^*)^{\frac{t}{\gamma}} \\
&= A_0 \tilde{c}_0 \left[\sum_{t=0}^T \left(\frac{\phi n}{R^*} \right)^t (\beta R^*)^{\frac{t}{\gamma}} + \phi^T \sum_{t=T+1}^\infty \left(\frac{n}{R^*} \right)^t (\beta R^*)^{\frac{t}{\gamma}} \right] \\
&= \frac{A_0 \tilde{c}_0}{(1 - \frac{n(\beta R^*)^{\frac{1}{\gamma}}}{R^*}) \psi(\tau_s)}
\end{aligned}
$$

All elements influenced by τ_s are gathered in $\psi(\tau_s)$:

$$
\psi(\tau_s) = \left(1 - \frac{n(\beta R^*)^{\frac{1}{\gamma}}}{R^*} \right)^{-1} \left[\sum_{t=0}^T \left(\frac{\phi n (\beta R^*)^{\frac{1}{\gamma}}}{R^*} \right)^t + \phi^T \sum_{t=T+1}^\infty \left(\frac{n(\beta R^*)^{\frac{1}{\gamma}}}{R^*} \right)^t \right]^{-1}
$$

Remember that:

$$\begin{aligned}\sum_{t=0}^{n-1} x^t &= \frac{1-x^n}{1-x} \\ \sum_{t=0}^{\infty} x^t &= \frac{1}{1-x} \\ \sum_{t=1}^{\infty} x^t &= \sum_{t=0}^{\infty} x^t - 1 = \frac{x}{1-x}\end{aligned}$$

Using them we get:

$$\left(1-\frac{n(\beta R^*)^{\frac{1}{\gamma}}}{R^*}\right)^{-1}\left[\frac{1-(\frac{\phi n(\beta R^*)^{\frac{1}{\gamma}}}{R^*})^{T+1}}{1-\frac{\phi n(\beta R^*)^{\frac{1}{\gamma}}}{R^*}}+\phi^T\frac{1}{1-\frac{n(\beta R^*)^{\frac{1}{\gamma}}}{R^*}}-\phi^T\frac{1-\frac{n(\beta R^*)^{\frac{1}{\gamma}}}{R^*}^{T+1}}{1-\frac{n(\beta R^*)^{\frac{1}{\gamma}}}{R^*}}\right]$$

$$\left(1-\frac{n(\beta R^*)^{\frac{1}{\gamma}}}{R^*}\right)^{-1}\left[\frac{1-(\frac{\phi n(\beta R^*)^{\frac{1}{\gamma}}}{R^*})^{T+1}}{1-\frac{\phi n(\beta R^*)^{\frac{1}{\gamma}}}{R^*}}+\phi^T\frac{\frac{n(\beta R^*)^{\frac{1}{\gamma}}}{R^*}^{T+1}}{1-\frac{n(\beta R^*)^{\frac{1}{\gamma}}}{R^*}}\right]^{-1}$$

$$\frac{R^*}{R^*-n(\beta R^*)^{\frac{1}{\gamma}}}\left[\frac{1-(\frac{\phi n(\beta R^*)^{\frac{1}{\gamma}}}{R^*})^{T+1}}{\frac{R^*-\phi n(\beta R^*)^{\frac{1}{\gamma}}}{R^*}}+\phi^T\frac{\frac{n(\beta R^*)^{\frac{1}{\gamma}}}{R^*}^{T+1}}{\frac{R^*-n(\beta R^*)^{\frac{1}{\gamma}}}{R^*}}\right]^{-1}$$

$$\frac{R^*}{R^*-n(\beta R^*)^{\frac{1}{\gamma}}}\left[R^*\frac{1-(\frac{\phi n(\beta R^*)^{\frac{1}{\gamma}}}{R^*})^{T+1}}{R^*-\phi n(\beta R^*)^{\frac{1}{\gamma}}}+R^*\phi^T\frac{\frac{n(\beta R^*)^{\frac{1}{\gamma}}}{R^*}^{T+1}}{R^*-n(\beta R^*)^{\frac{1}{\gamma}}}\right]^{-1}$$

$$\frac{1}{R^*-n(\beta R^*)^{\frac{1}{\gamma}}}\left[\frac{(1-(\frac{\phi n(\beta R^*)^{\frac{1}{\gamma}}}{R^*})^{T+1})(R^*-n(\beta R^*)^{\frac{1}{\gamma}})+\phi^T\frac{n(\beta R^*)^{\frac{1}{\gamma}}}{R^*}^{T+1}(R^*-\phi n(\beta R^*)^{\frac{1}{\gamma}})}{(R^*-\phi n(\beta R^*)^{\frac{1}{\gamma}})(R^*-n(\beta R^*)^{\frac{1}{\gamma}})}\right]^{-1}$$

Focus on the numerator, first gather terms containing $n(\beta R^*)^{\frac{1}{\gamma}}$:

$$-n(\beta R^*)^{\frac{1}{\gamma}}+\left[\frac{\phi n(\beta R^*)^{\frac{1}{\gamma}}}{R^*}\right]^{T+1}n(\beta R^*)^{\frac{1}{\gamma}}-\phi^T\left[\frac{n(\beta R^*)^{\frac{1}{\gamma}}}{R^*}\right]^{T+1}\phi n(\beta R^*)^{\frac{1}{\gamma}}$$

$$-n(\beta R^*)^{\frac{1}{\gamma}}$$

then, concentrate on R^*:

$$R^*-\frac{\phi n(\beta R^*)^{\frac{1}{\gamma}}}{R^*}^{T+1}R^*+\phi^T\left[\frac{n(\beta R^*)^{\frac{1}{\gamma}}}{R^*}\right]^{T+1}R^*$$

$$R^*-R^*\left[\frac{\phi n(\beta R^*)^{\frac{1}{\gamma}}}{R^*}\right]^T\frac{\phi n(\beta R^*)^{\frac{1}{\gamma}}}{R^*}+R^*\left[\frac{\phi n(\beta R^*)^{\frac{1}{\gamma}}}{R^*}\right]^T\frac{n(\beta R^*)^{\frac{1}{\gamma}}}{R^*}$$

$$R^* - \left[\frac{\phi n(\beta R^*)^{\frac{1}{\gamma}}}{R^*}\right]^T \phi n(\beta R^*)^{\frac{1}{\gamma}} + \left[\frac{\phi n(\beta R^*)^{\frac{1}{\gamma}}}{R^*}\right]^T n(\beta R^*)^{\frac{1}{\gamma}}$$

$$R^* + \left[\frac{\phi n(\beta R^*)^{\frac{1}{\gamma}}}{R^*}\right]^T n(\beta R^*)^{\frac{1}{\gamma}}(1-\phi)$$

Thus, we get:

$$\frac{1}{R^* - n(\beta R^*)^{\frac{1}{\gamma}}} \left[\frac{R^* - n(\beta R^*)^{\frac{1}{\gamma}} + \left[\frac{\phi n(\beta R^*)^{\frac{1}{\gamma}}}{R^*}\right]^T n(\beta R^*)^{\frac{1}{\gamma}}(1-\phi)}{(R^* - \phi n(\beta R^*)^{\frac{1}{\gamma}})(R^* - n(\beta R^*)^{\frac{1}{\gamma}})}\right]^{-1}$$

$$\psi(\tau_s) = \frac{R^* - \phi n(\beta R^*)^{\frac{1}{\gamma}}}{R^* - n(\beta R^*)^{\frac{1}{\gamma}} + \left[\frac{n(\beta R^*)^{\frac{1}{\gamma}}\phi(\tau_s)}{R^*}\right]^T n(\beta R^*)^{\frac{1}{\gamma}}(1-\phi(\tau_s))}$$

Thus, we have a new expression for consumption with $(\beta R^*)^{\frac{1}{\gamma}}$ instead of g^*. The consumption in period 0 in efficiency units can now be computed similarly as in the baseline case using again:

$$w_t + z_{kt} = (\tilde{w} + \tilde{z}_k)A_0(1+\pi_t)g^{*t}$$

Plug the rewritten consumption part and the former equation in the intertemporal version of the budget constraint:

$$\begin{aligned}
\sum_0^\infty \left(\frac{n}{R^*}\right)^t c_t &= \sum_0^\infty \left(\frac{n}{R^*}\right)^t (z_{kt} + w_t) + R^*(k_0 - d_0) \\
\frac{A_0 \tilde{c}_0}{(1 - \frac{n(\beta R^*)^{\frac{1}{\gamma}}}{R^*})\psi(\tau_s)} &= \sum_{t=0}^\infty \left(\frac{n}{R^*}\right)^t (\tilde{w} + \tilde{z}_k)A_0(1+\pi_t)g^{*t} + R^*(k_0 - d_0) \\
\tilde{c}_0 &= \frac{R^* - n(\beta R^*)^{\frac{1}{\gamma}}}{R^*}\psi(\tau_s)\frac{1}{A_0}(\tilde{w} + \tilde{z}_k)A_0 \sum_{t=0}^\infty \left(\frac{n}{R^*}\right)^t (1+\pi_t)g^{*t} \\
&\quad + \frac{R^* - n(\beta R^*)^{\frac{1}{\gamma}}}{R^*}\psi(\tau_s)\frac{1}{A_0}R^*(k_0 - d_0) \\
&= \frac{R^* - n(\beta R^*)^{\frac{1}{\gamma}}}{R^*}\psi(\tau_s)(\tilde{w} + \tilde{z}_k)\sum_{t=0}^\infty \left(\frac{ng^*}{R^*}\right)^t (1+\pi_t) \\
&\quad + (R^* - n(\beta R^*)^{\frac{1}{\gamma}})\psi(\tau_s)\frac{k_0 - d_0}{A_0} \\
&= \ldots + (R^* - n(\beta R^*)^{\frac{1}{\gamma}})\psi(\tau_s)(\tilde{k}_0 - \tilde{d}_0) \\
\tilde{c}_0 &= (R^* - n(\beta R^*)^{\frac{1}{\gamma}})\psi(\tau_s)\left[\frac{\tilde{w} + \tilde{z}_k}{R^*}\sum_{t=0}^\infty \left(\frac{ng^*}{R^*}\right)^t (1+\pi_t) + (\tilde{k}_0 - \tilde{d}_0)\right]
\end{aligned}$$

The found expressions for $\tilde{d}_T$, $\tilde{c}_T$ and $\tilde{c}_0$ can be successively plugged in the initial (unchanged) $\frac{\triangle D}{Y_0}$. As $\tilde{d}_T$ is similar to the baseline case, focus on the third part of $\frac{\triangle D}{Y_0}$ containing $\tilde{c}_T$ and then plug in $\tilde{c}_T$ and $\tilde{c}_0$ from before:

$$-\frac{\tilde{c}_T}{R^* - ng^*}\frac{(g^*n)^T(1+\pi)}{\tilde{y}_0}$$

$$-\frac{\tilde{c}_0\phi(\tau_s)^T(\beta R^*)^{\frac{T}{\gamma}}}{(1+\pi)g^{*T}}\frac{1}{(R^* - ng^*)}\frac{(g^*n)^T(1+\pi)}{\tilde{y}_0}$$

$$-(R^* - n(\beta R^*)^{\frac{1}{\gamma}})\psi(\tau_s)\left[\frac{\tilde{w}+\tilde{z}_k}{R^*}\sum_{t=0}^{\infty}\left(\frac{ng^*}{R^*}\right)^t(1+\pi_t)+(\tilde{k}_0-\tilde{d}_0)\right]\frac{\phi(\tau_s)^T(\beta R^*)^{\frac{T}{\gamma}}}{(1+\pi)g^{*T}}\frac{1}{(R^* - ng^*)}\frac{[g^*n]^T(1+\pi)}{\tilde{y}_0}$$

$$-\frac{(R^* - n(\beta R^*)^{\frac{1}{\gamma}})(\beta R^*)^{\frac{T}{\gamma}}}{R^* - ng^*}\phi(\tau_s)^T n^T\frac{\psi(\tau_s)}{\tilde{y}_0}\left[\frac{\tilde{w}+\tilde{z}_k}{R^*}\sum_{t=0}^{\infty}\left(\frac{ng^*}{R^*}\right)^t(1+\pi_t)+(\tilde{k}_0-\tilde{d}_0)\right]$$

The entire expression is:

$$\begin{aligned}\frac{\triangle D}{Y_0} \quad = \quad & \frac{\tilde{k}^*(g^*n)^T(1+\pi)}{\tilde{y}_0}+\frac{\tilde{w}+\tilde{z}_k}{R^* - ng^*}\frac{(g^*n)^T(1+\pi)}{\tilde{y}_0} \\ & -\frac{(R^* - n(\beta R^*)^{\frac{1}{\gamma}})(\beta R^*)^{\frac{T}{\gamma}}}{R^* - ng^*}\phi(\tau_s)^T n^T\frac{\psi(\tau_s)}{\tilde{y}_0}\left[\frac{\tilde{w}+\tilde{z}_k}{R^*}\sum_{t=0}^{\infty}\left(\frac{ng^*}{R^*}\right)^t(1+\pi_t)+\tilde{k}_0-\tilde{d}_0\right] \\ & -\frac{\tilde{d}_0}{\tilde{y}_0}\end{aligned}$$

Note that only the third part differ compared to the baseline case.
For convenience, define a new variable:

$$\Omega = \frac{(R^* - n(\beta R^*)^{\frac{1}{\gamma}})(\beta R^*)^{\frac{T}{\gamma}}}{R^* - ng^*}$$

The part of the equation gathering $\tilde{k}^*$ is the first response term as in the baseline case:

$$\frac{\tilde{k}^*}{\tilde{y}_0}(ng^*)^T(1+\pi)$$

Focusing on terms containing $\tilde{k}_0$, one gets the second element of the answer:

$$-\frac{(R^* - n(\beta R^*)^{\frac{1}{\gamma}})(\beta R^*)^{\frac{T}{\gamma}}}{R^* - ng^*}\phi(\tau_s)^T n^T\frac{\psi(\tau_s)}{\tilde{y}_0}\tilde{k}_0$$

$$-\Omega\frac{\tilde{k}_0}{\tilde{y}_0}\psi(\tau_s)[n\phi(\tau_s)]^T$$

As for the third one, take the terms containing $\tilde{d}_0$

$$\Omega n^T \frac{\psi(\tau_s)\phi(\tau_s)^T}{\tilde{y}_0}\tilde{d}_0 - \frac{\tilde{d}_0}{\tilde{y}_0}$$

$$\frac{\tilde{d}_0}{\tilde{y}_0}\left(\Omega\psi(\tau_s)[n\phi(\tau_s)]^T - 1\right)$$

Collecting the left-over terms of the initial expression, we have:

$$\frac{\tilde{w}+\tilde{z}_k}{R^* - ng^*}\frac{(g^*n)^T(1+\pi)}{\tilde{y}_0} - \Omega[\phi(\tau_s)n]^T\frac{\psi(\tau_s)}{\tilde{y}_0}\frac{\tilde{w}+\tilde{z}_k}{R^*}\sum_{t=0}^{\infty}\left(\frac{ng^*}{R^*}\right)^t(1+\pi_t)$$

$$\frac{\tilde{w}+\tilde{z}_k}{\tilde{y}_0}n^T\left[\frac{g^{*T}(1+\pi)}{R^*-ng^*} - \Omega\frac{\psi(\tau_s)\phi(\tau_s)^T}{R^*}\sum_{t=0}^{\infty}\left(\frac{ng^*}{R^*}\right)^t(1+\pi_t)\right]$$

Unfortunately, in this case, we cannot use $\psi(\tau_s)$to rewrite the first term of in the brackets and get rid of the $\sum^{\infty}$. A numerical approximation will be implemented.
The equation of the flows decomposition is constituted of the four expressions we have just computed:

$$\begin{aligned}\frac{\triangle D}{Y_0} &= \frac{\tilde{k}^*}{\tilde{y}_0}(ng^*)^T(1+\pi) - \frac{\tilde{k}_0}{\tilde{y}_0}\psi(\tau_s)[n\phi(\tau_s)]^T\Omega + \frac{\tilde{d}_0}{\tilde{y}_0}\left[\Omega\psi(\tau_s)[n\phi(\tau_s)]^T - 1\right] \\ &+\frac{\tilde{w}+\tilde{z}_k}{\tilde{y}_0}n^T\left[\frac{g^{*T}(1+\pi)}{R^*-ng^*} - \Omega\frac{\psi(\tau_s)\phi(\tau_s)^T}{R^*}\sum_{t=0}^{\infty}\left(\frac{ng^*}{R^*}\right)^t(1+\pi_t)\right]\end{aligned}$$

By focusing on the first three terms, one sees the convergence and investment channels are identical to the baseline case. Focus on the left-over terms:

$$\frac{\tilde{k}_0}{\tilde{y}_0}(ng^*)^T - \frac{\tilde{k}_0}{\tilde{y}_0}\psi(\tau_s)\left[n\phi(\tau_s)\right]^T\Omega + \frac{\tilde{d}_0}{\tilde{y}_0}\left[\Omega\psi(\tau_s)[n\phi(\tau_s)]^T - 1\right]$$

$$\underbrace{\frac{\tilde{k}_0(ng^*)^T - \tilde{d}_0}{\tilde{y}_0} + \Omega\psi(\tau_s)\left[n\phi(\tau_s)\right]^T\frac{\tilde{d}_0 - \tilde{k}_0}{\tilde{y}_0}}_{trend}$$

The saving channel is the fourth part of flows decomposition:

$$\underbrace{\frac{\tilde{w}+\tilde{z}_k}{\tilde{y}_0}n^T\left[\frac{g^{*T}(1+\pi)}{R^*-ng^*} - \Omega\frac{\psi(\tau_s)\phi(\tau_s)^T}{R^*}\sum_{t=0}^{\infty}\left(\frac{ng^*}{R^*}\right)^t(1+\pi_t)\right]}_{saving}$$

Compared to the following form in the baseline case:

$$\frac{\tilde{w}+\tilde{z}_k}{\tilde{y}_0}(g^*n)^T\left[\frac{(1+\pi)}{R^*-ng^*} - \frac{\psi(\tau_s)\phi(\tau_s)^T}{R^*}\sum_{t=0}^{\infty}\left(\frac{ng^*}{R^*}\right)^t(1+\pi_t)\right]$$

which they were able to rewrite as:

$$\frac{\tilde{w}+\tilde{z}_k}{\tilde{y}_0}\frac{\psi(\tau_s)}{R^*}[ng^*\phi(\tau_s)]^T\sum_{t=0}^{T-1}\left(\frac{ng^*}{R^*}\right)^t\left[\phi(\tau_s)^{t-T}(1+\pi)-(1+\pi_t)\right]$$

11.5 Extension 2: capital adjustment costs

The first capital adjustment cost is introduced in the equation for the dynamics of capital stock:

$$I_t = K_{t+1}-(1-\delta)K_t+\kappa_1(K_{t+1}-K_t)^2$$

Divide by gdp in period t:

$$i_t=\frac{K_{t+1}-(1-\delta)K_t}{Y_t}+\kappa_1\frac{(K_{t+1}-K_t)^2}{Y_t}$$

We can rewrite it as:

$$i_t = \tilde{k}^{*(1-\alpha)}\left[g_{t+1}n-1+\delta\right]+\kappa_1\frac{(A_{t+1}N_{t+1}\tilde{k}^*-A_tN_t\tilde{k}^*)^2}{A_tN_t\tilde{k}^{*\alpha}}$$

A second (add-hoc) capital adjustment cost enters in the initial jump in capital stock:

$$\frac{K_0^*-K_0}{Y_0} = \frac{(1+\kappa_2)(\tilde{k}^*-\tilde{k}_0)}{\tilde{k}_0^{*\alpha}}$$

Add the jump term to the expression from before to get a new i_0:

$$i_0 = \tilde{k}^{*(1-\alpha)}(g_1n+\delta-1)+\kappa_1\frac{(A_{t+1}N_{t+1}\tilde{k}^*-A_tN_t\tilde{k}^*)^2}{A_tN_t\tilde{k}^{*\alpha}}+\frac{(1+\kappa_2)(\tilde{k}^*-\tilde{k}_0)}{\tilde{k}_0^{*\alpha}}$$

The average investment rate between t and $T-1$ is:

$$\begin{aligned} i &= \frac{1}{T}\sum_{t=0}^{T-1}i_t \\ &= \frac{1}{T}\frac{(1+\kappa_2)(\tilde{k}^*-\tilde{k}_0)}{\tilde{k}_0^{*\alpha}}+\frac{1}{T}\sum_{t=0}^{T-1}(g_{t+1}n+\delta-1)\tilde{k}^{*(1-\alpha)}+\frac{1}{T}\sum_{t=0}^{T-1}\kappa_1\frac{(A_{t+1}N_{t+1}\tilde{k}^*-A_tN_t\tilde{k}^*)^2}{A_tN_t\tilde{k}^{*\alpha}} \end{aligned}$$

Focus on the last part:

$$\frac{1}{T}\sum_{t=0}^{T-1}\kappa_1\frac{(A_{t+1}N_{t+1}\tilde{k}^*)^2-2A_{t+1}N_{t+1}\tilde{k}^*A_tN_t\tilde{k}^*+(A_tN_t\tilde{k}^*)^2}{A_tN_t\tilde{k}^{*\alpha}}$$

$$\frac{1}{T}\sum_{t=0}^{T-1}\kappa_1\tilde{k}^{*(2\text{-}\alpha)}\left(\frac{A_{t+1}^2}{A_t}\frac{N_{t+1}^2}{N_t}-2A_{t+1}N_{t+1}+A_tN_t\right)$$

We get the following decomposition of investment rate:

$$\begin{aligned} i &= \frac{1}{T}\frac{\tilde{k}^{*}-\tilde{k}_0}{\tilde{k}_0^{\alpha}} + \frac{1}{T}\frac{\tilde{k}^{*}-\tilde{k}_0}{\tilde{k}_0^{\alpha}}\kappa_2 + g^{*}\frac{\pi}{T}n\tilde{k}^{*(1-\alpha)} + (g^{*}n+\delta-1)\tilde{k}^{*(1-\alpha)} \\ &+\frac{1}{T}\sum_{t=0}^{T-1}\kappa_1\tilde{k}^{*(2\text{-}\alpha)}\left(\frac{A_{t+1}^2}{A_t}\frac{N_{t+1}^2}{N_t} - 2A_{t+1}N_{t+1} + A_tN_t\right) \end{aligned}$$

Two new channels appeared. The second term is the initial capital adjustment cost and the fifth term represents the standard capital adjustment costs.