Trade Policy:
Home Market Effect versus Terms-of-Trade Externality*

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Abstract

We study trade policy in a two-sector Krugman (1980) trade model, allowing for production, import and export subsidies/taxes. We consider non-cooperative and cooperative trade policy, first for each individual instrument and then for the situation where all instruments can be set simultaneously, and contrast those with the efficient allocation. While previous studies have identified the home market externality, which gives incentives to agglomerate firms in the domestic economy, as the driving force behind non-cooperative trade policy in this model, we show that this, in fact, is not the case. Instead, the incentives for a non-cooperative trade policy arise from the desire to eliminate monopolistic distortions and to improve domestic terms of trade. As a consequence, terms-of-trade externalities remain the only motive for international trade agreements in the Krugman (1980) model once a complete set of instruments is available.

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1 Introduction

The aim of this paper is to study optimal trade policy in the canonical two-sector Krugman (1980) model, where one sector is characterized by monopolistic competition, increasing returns and iceberg trade costs, while the other features perfect competition and constant returns. Within this framework we study cooperative, unilateral and strategic (Nash) production, import and export subsidies/taxes.

The common wisdom of the literature[1] (Venables (1987), Helpman and Krugman (1989), Ossa (2011)) is that in this model unilateral trade policy is set so as to agglomerate firms in the domestic economy in order to reduce the domestic price index, thereby increasing domestic welfare. An import tariff makes foreign differentiated goods more expensive relative to domestic ones so that domestic consumers shift expenditure towards domestic differentiated goods. As a consequence, domestic firms sell more thus making profits and foreign firms sell less thus making losses. This triggers entry into the domestic differentiated sector and exit out of the foreign differentiated sector, thereby reducing the domestic price index – since now less of the domestically consumed goods are subject to transport costs – and increasing the foreign one. Similarly, a production or an export subsidy also renders the domestic market a more attractive location and reduces the domestic price index at the expense of increasing the foreign one. According to the literature, this home market externality (also called production relocation externality) provides a reason for protectionist and ultimately welfare detrimental unilateral trade policy in the Krugman (1980) model and, as argued by Ossa (2011), gives an alternative theoretical justification to the neoclassical terms-of-trade externality explanation (Johnson (1953-1954), Grossman and Helpman (1985) and Bagwell and Staiger (1999)) as to why countries need to sign trade agreements. Similarly, the same mechanism also provides a theoretical justification for the World Trade Organization (WTO)'s limitation of production and export subsidies,[2] which cannot be explained within the neoclassical framework.[3]

Our contribution is twofold. First, we show that, in contrast to the previous literature, the

[1]A detailed review of the literature is provided in the next section.

[2]See, e.g., WTO (2006). GATT Article XVI and the Uruguay Round Subsidies Code prohibit the use of export subsidies, while the second also establishes that countervailing duties can be imposed on countries using production subsidies subject to an injury test.

[3]Production and export subsidies are puzzling within the neoclassical framework because they increase foreign welfare at the expense of domestic welfare.

home market externality – defined as the incentive to reduce the domestic price index via a relocation of firms to the domestic economy – is generally not a motive for unilateral or strategic trade policy in the Krugman (1980) model. Instead, non-cooperative trade policies are driven by two incentives: on the one hand, domestic policy makers try to increase production efficiency and, on the other hand, they want to improve domestic terms of trade. Second, we find that once countries are allowed to simultaneously choose all three instruments, they set production subsidies to eliminate the production inefficiency and they use trade instruments to improve their terms of trade. This implies that the only remaining international externality – and thus the only reason why countries should sign a trade agreement – is due to countries' attempt to manipulate their terms of trade.

Observe that the production inefficiency arises because there are two sectors in the model, so that monopolistic markups lead to a too low provision of variety in the monopolistically competitive sector.[4] Thus, policy makers try to improve the use of domestic resources by increasing entry into the differentiated sector. Depending on the set of policy instruments available, this attempt to increase production efficiency may impose a relocation externality on the other country. However, such relocation externality disappears once policy makers can completely eliminate the production inefficiency. Indeed, non-cooperative policy makers do not have an incentive to exploit the home market externality: They recognize that a reduction in the aggregate price index due to firm relocation cannot be welfare improving, since it is exactly offset by a fall in household's income. Finally, the use of policy instruments required to achieve greater production efficiency has negative terms-of-trade effects and non-cooperative trade policy is determined by the trade off between these two motives.

To clarify the interplay between these incentives, we start by considering production subsidies/taxes as the only available policy instrument. A production subsidy increases profits of firms in the domestic differentiated sector and triggers a relocation of firms from the foreign to the domestic economy, thereby increasing domestic production efficiency. However, this comes at the cost of a negative terms-of-trade effect because the production subsidy reduces the international price of domestically produced varieties. We show that the balance always tips in favor of the terms-of-trade effect before production efficiency is achieved: the non-cooperative

[4]In their seminal paper, Dixit and Stiglitz (1977) show that the market solution is not first-best Pareto optimal in such a model, and that subsidies on fixed costs and on marginal costs are required to implement it.

outcome is a production subsidy that is always lower than the cooperatively set one. Thus, the relocation externality – which is a consequence of policy makers' attempt to increase production efficiency – does not induce inefficiently high production subsidies. Instead, the terms-of-trade externality leads to an inefficiently low subsidy level.

The result on production subsidies makes it clear that the desire to achieve production efficiency is an important motive for non-cooperative policy choice. Keeping this in mind, we next study import subsidies/tariffs. First, we consider a situation where monopolistic distortions have been eliminated by appropriate production subsidies, so that the market allocation is first-best efficient. In this case the only remaining motive for import policy is the terms-of-trade externality. As a consequence, the optimal non-cooperative import policy entails import subsidies, which aim at relocating firms to the Foreign economy and thereby indirectly improving domestic terms of trade: by reducing the number of domestically produced varieties and increasing the foreign one, an import subsidy increases the welfare-relevant price index of exports relative to the one of imports. In contrast, when starting from the (inefficient) free trade allocation production efficiency calls for a tariff, which shifts domestic demand towards domestically produced varieties, causes entry into the differentiated sector at home, and reallocates labor to this sector. This imposes a relocation externality on the other country, where firms exit. However, such a policy comes at the cost of worsened terms of trade due to a fall in the relative price index of exportables. Overall, production efficiency effects dominate the indirect terms-of-trade motive and the Nash equilibrium outcome is tariffs.

A similar result holds for non-cooperative export policy. When monopolistic distortions have been eliminated by appropriate production subsidies, terms-of-trade effects are the only motive for non-cooperative policy makers. In this case the optimal non-cooperative export policy is an export tax, which aims at improving domestic terms of trade both directly, by increasing the international price of exported varieties, and indirectly by triggering exit of firms from the domestic economy and entry in the foreign one. Differently, when starting from the (inefficient) free trade allocation non-cooperative policy makers set export subsidies, which intend to induce entry into the domestic differentiated sector by relocating firms from the foreign economy and thus improve production efficiency. This motive dominates the negative terms-of-trade effect of export subsidies.

Finally, we analyze a situation where countries can set production, import and export policy instruments simultaneously. This is the relevant situation if one wants to address the question why countries need to sign trade agreements, given that in the absence of such agreements the set of tax instruments that can be used strategically is not limited to a single production or trade tax. In line with our above results for single instruments, we find that non-cooperative policy makers choose the level of production subsidies that exactly offsets the monopolistic distortions, and that they set import subsidies and export taxes, both of which aim at improving domestic terms of trade (directly or indirectly). This result is important for various reasons: first, it corroborates our previous claim that relocation externalities are only a consequence of production inefficiency. If policy makers dispose of sufficiently many instruments to address both incentives – namely production efficiency and terms of trade – separately, only the latter imposes an international externality; second, it clarifies that in the Krugman (1980) model the only role of international trade agreements is to solve international externalities due to terms-of-trade effects. Relocation effects only become a relevant motive for trade policy, once the set of policy instruments is restricted, so that production efficiency cannot be improved without causing other distortions.

1.1 Related Literature

Our results differ markedly from those of the previous literature on trade policy in the two-sector Krugman (1980) model (Venables (1987), Helpman and Krugman (1989) chapter 7 and Ossa (2011)). All these contributions find that in this model non-cooperative trade policy is driven by home market effects, leading to inefficiencies compared to free trade. In particular, Venables (1987) studies unilateral incentives to set, alternatively, tariffs, production or export subsidies and shows that any of those can improve domestic welfare compared to free trade due to the home market effect. However, he does not study the welfare consequences of a strategic game. Helpman and Krugman (1989) limit their discussion to unilaterally set tariffs, while Ossa (2011) considers a tariff game, where positive tariffs are set in equilibrium due to the home market effect. While we also find that non-cooperative import policy leads to tariffs, these are set to improve production efficiency and not to reduce the domestic price level.

A key difference between those contributions and our paper is the treatment of income effects generated by the policy intervention. To avoid the complications arising from the revenue consequences of tax policies, all previous contributions have assumed that tariff income is a pure waste[5] and that there are no other taxes, so that income is fixed. However, it is only in this special case, that the incentive to reduce the domestic price level via a relocation of firms is present. In the more general case, the relocation incentive is just a consequence of production inefficiency. Observe that this difference matters: once production subsidies can be set so as to achieve production efficiency, the relocation externality disappears and import subsidies are set, which aim at improving terms of trade.

Also closely related to our paper is Bagwell and Staiger (2009), who consider a two-sector Krugman (1980) model, allowing policy makers to simultaneously choose import and export taxes. They show that in this case Nash-equilibrium policy choices are explained exclusively by the terms-of-trade effects and not by the relocation externality, because import-tariff-induced relocation effects are counterbalanced by export-subsidy-induced relocation effects. While allowing for revenue effects of trade taxes, for analytical tractability they assume quasi-linear utility. This ensures that tax policies do not generate income effects on the demand for differentiated products. Compared to their work our contributions are the following ones: we provide analytical solutions for the general specification where tax revenues generate income effects on the demand of differentiated goods; we allow for production taxes in addition to import and export taxes. The advantage of this more general approach is that it underlines the crucial role played by production inefficiency and points out the absence of the home market externality. Finally, in line with Bagwell and Staiger (2009)'s result, we show that when all three policy instruments can be set strategically, the only remaining international externality is the terms-of-trade effect.

Other related work is Gros (1987), who studies an import tariff game in the one-sector variant of the Krugman (1980) model. In that version of the model relocation effects are absent and the free trade allocation is Pareto-optimal. He finds that in the Nash equilibrium policy makers set import tariffs which aim at increasing domestic wages due to terms-of-trade effects.

Finally, Flam and Helpman (1987) and Helpman and Krugman (1989), chapter 7 discuss a production efficiency effect of trade policy, which is very similar to the production efficiency effect as defined in the present paper. Since with imperfect competition prices are set above

[5]Ossa (2011) provides a numerical solution for the case when tariff revenues are redistributed. However, he does non allow for other tax instruments and does not consider the role played by the production inefficiency.

marginal costs, domestic consumption of each variety is too low. Thus, an import tariff (or a production or export subsidy), which shifts demand towards domestic varieties, can improve efficiency. However, their effect refers to changes in average cost induced by changes in firm size and not to changes in the number of domestic firms. Since firm size provided by the market is optimal in the Krugman (1980) model, there is no room for a production efficiency effect in their sense.

The paper proceeds as follows. In the next section we set up the model. In section 3 we compare the market allocation with the planner solution and discuss cooperative and non-cooperative policy makers' problems and incentives. Sections 4, 5 and 6 are dedicated to the study of individual policy instruments: production taxes/subsidies, import tariffs/subsidies and export taxes/subsidies. Section 7 considers simultaneous choice of all policy instruments and the last section presents our conclusions.

2 The Model

The setup is exactly as in Venables (1987) and Ossa (2011). The only difference is that we allow for transfers. The world economy consists of two countries: Home (H) and Foreign (F). Each country produces a homogeneous good and a continuum of differentiated goods. All goods are tradable but only the differentiated goods are subject to transport costs. The differentiated goods sector is characterized by monopolistic competition, while there is perfect competition in the homogeneous good sector. Both countries are identical in terms of preferences, production technology, market structure and size. All variables are indexed such that the first sub-index refers to the location of consumption and the second subindex to the location of production. Finally, varieties in the differentiated sector are indexed with i, while countries are indexed with j.

2.1 Households

Households' utility function in the Home country is given by:

$$U(C_H, Z_H) \equiv C_H^{\alpha} Z_H^{1-\alpha}, \tag{1}$$

where C_H aggregates over the varieties of differentiated goods, Z_H represents consumption of the homogeneous good and α is the expenditure share of the differentiated bundle in the aggregate consumption basket. While the homogeneous good is identical across countries, each country produces a different subset of differentiated goods. In particular, N_H varieties are produced in the Home country while N_F are produced by Foreign. The differentiated varieties produced in the two countries are aggregated with a CES function:[6]

$$C_H = \left[C_{HH}^{\frac{\varepsilon-1}{\varepsilon}} + C_{HF}^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{2}$$

$$C_{HH} = \left[\int_0^{N_H} c_{HH}(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} \qquad C_{HF} = \left[\int_0^{N_F} c_{HF}(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{3}$$

Here, C_{HH} is the domestic consumption bundle of varieties produced at Home, C_{HF} is the domestic consumption bundle of Foreign produced varieties, $c_{HH}(i)$ denotes domestic consumption of a domestically produced variety, $c_{HF}(i)$ is domestic consumption of a Foreign produced variety and $\varepsilon > 1$ is the elasticity of substitution between domestic and Foreign bundles and between different varieties. Analogous definitions hold for Foreign consumption bundles.

Given the Dixit-Stiglitz structure of preferences, the households' maximization problem can be solved in three stages. At the first two stages, households choose how much to consume of each Home and Foreign variety and how to allocate consumption between the domestic and the Foreign bundle. The optimality conditions imply the following domestic demand functions and domestic price indices:

$$c_{HH}(i) = \left[\frac{p_{HH}(i)}{P_{HH}} \right]^{-\varepsilon} C_{HH} \qquad C_{HH} = \left[\frac{P_{HH}}{P_H} \right]^{-\varepsilon} C_H \tag{4}$$

$$c_{HF}(i) = \left[\frac{p_{HF}(i)}{P_{HF}} \right]^{-\varepsilon} C_{HF} \qquad C_{HF} = \left[\frac{P_{HF}}{P_H} \right]^{-\varepsilon} C_H, \tag{5}$$

[6]Note that our definitions for C_{HH} and C_{HF} imply $C_H = \left[\int_0^{N_H} c_{HH}(i)^{\frac{\varepsilon-1}{\varepsilon}} di + \int_0^{N_F} c_{HF}(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}$ i.e., the model is the standard one considered in this literature. However, it is convenient to define optimal consumption indices.

$$P_H = \left[P_{HH}^{1-\varepsilon} + P_{HF}^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} \tag{6}$$

$$P_{HH} = \left[\int_0^{N_H} p_{HH}(i)^{1-\varepsilon} di\right]^{\frac{1}{1-\varepsilon}} \qquad P_{HF} = \left[\int_0^{N_F} p_{HF}(i)^{1-\varepsilon} di\right]^{\frac{1}{1-\varepsilon}}, \tag{7}$$

where P_H is the domestic price index of the differentiated bundle, P_{HH} and P_{HF} are domestic price indices of Home and Foreign produced bundles of differentiated goods respectively and $p_{HH}(i)$ ($p_{HF}(i)$) is the domestic price of variety i produced by Home (Foreign).

In the last stage, households choose how to allocate income between the homogeneous good and the differentiated bundle. Thus, they maximize (1) subject to the following budget constraint:

$$P_H C_H + p_{ZH} Z_H = I_H, \tag{8}$$

where $I_H = W_H L + T_H$, L is the total labor available in each country, W_H is the domestic wage rate, p_{ZH} is the domestic price of the homogeneous good, and T_H is a lump sum transfer which depends on the tax scheme adopted by the domestic government. The solution to the domestic consumer problem implies that the marginal rate of substitution between the homogeneous good and the differentiated bundle equals their relative price:

$$\frac{\alpha}{1-\alpha}\frac{Z_H}{C_H} = \frac{P_H}{p_{ZH}} \tag{9}$$

Foreign households solve a symmetric problem.

2.2 Firms

Firms in the differentiated sector operate under monopolistic competition. They pay a fixed cost in terms of labor, f, and then produce with linear technology:

$$y_H(i) = L_{CH}(i) - f, \tag{10}$$

where $L_{CH}(i)$ is the amount of labor allocated to the production of variety i in the differentiated sector. Goods sold in the Foreign market are subject to an iceberg transport cost $\tau > 1$. The government of each country $j \in \{H, F\}$ disposes of three fiscal instruments. A production tax/subsidy (τ_{Cj}) on firms' fixed and marginal costs,[7] a tariff/subsidy on imports (τ_{Ij}) and a tax/subsidy on exports (τ_{Xj}). Note that τ_{mj} indicates a gross tax for $m \in \{C, I, X\}$ i.e., $\tau_{mj} < 1$ indicates a subsidy and $\tau_{mj} > 1$ indicates a tax. In what follows, we will use the word tax whenever we refer to a policy instrument without specifying whether τ_{mj} is smaller or larger than one. We assume that taxes are paid directly by the firms. Given the constant price elasticity of demand, optimal prices charged by Home firms in the domestic market ($p_{HH}(i)$) are a fixed markup over their perceived marginal cost ($\tau_{CH} W_H$), and optimal prices paid by Foreign consumers for Home produced varieties ($p_{FH}(i)$) equal domestic prices augmented by transport costs and trade taxes:[8]

$$p_{HH}(i) = \tau_{CH} \frac{\varepsilon}{\varepsilon - 1} W_H \qquad\qquad p_{FH}(i) = \tau_{IF} \tau_{XH} \tau p_{HH}(i) \tag{11}$$

Foreign firms adopt symmetric optimal pricing rules:

$$p_{FF}(i) = \tau_{CF} \frac{\varepsilon}{\varepsilon - 1} W_F \qquad\qquad p_{HF}(i) = \tau_{IH} \tau_{XF} \tau p_{FF}(i) \tag{12}$$

The homogenous good is produced in both countries j with identical production technology:

$$Q_{Zj} = L_{Zj}, \tag{13}$$

where L_{Zj} is the amount of labor allocated to producing the homogeneous good. Since the good is sold in a perfectly competitive market without trade costs, price equals marginal cost and is the same in both countries. We assume that the homogeneous good is produced in both countries in equilibrium. Given the production technology, this implies factor price equalization:

$$p_{ZH} = p_{ZF} = W_H = W_F \tag{14}$$

[7]Production taxes are levied on both fixed and marginal costs. This assumption is necessary to keep firm size unaffected by production taxes, which turns out to be optimal, as we will show in section 3.1.

[8]Following the previous literature (Venables (1987), Ossa (2011)), we assume that tariffs and export taxes are charged ad valorem on the factory gate price augmented by transport costs. This implies that transport services are taxed.

For convenience, we normalize $p_{ZH} = 1$.

Using the optimal pricing rules just derived, it is possible to rewrite the domestic price index of the differentiated bundle as:

$$P_H = \left[N_H \left(\frac{\varepsilon}{\varepsilon - 1} \tau_{CH} \right)^{1-\varepsilon} + N_F \left(\frac{\varepsilon}{\varepsilon - 1} \tau_{CF} \tau_{IH} \tau_{XF} \tau \right)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}} \tag{15}$$

Note that trade policy can reduce the price index through three different channels. First, because of Dixit-Stiglitz preferences, increasing the total number of varieties reduces the price level. This is the so called *love for variety effect*. Second, by increasing N_H at the expense of N_F, the policy maker lowers the price level since Home households can now consume a larger fraction of goods for which they do not pay transport costs. This is the so called *home market externality*. Finally, trade policy can reduce the price level through the direct effect of subsidies on the prices of individual varieties.

2.3 Government

All government revenues are redistributed to consumers through a lump sum transfer T_j. The government is assumed to run a balanced budget. Hence, the domestic government's budget constraint is given by:

$$(\tau_{IH} - 1)\tau_{XF}\tau P_{FF} C_{HF} + (\tau_{XH} - 1)\tau P_{HH} C_{FH} + (\tau_{CH} - 1) \int_0^{N_H} W_H(y_H(i) + f) di = T_H \tag{16}$$

Government income consists of import tax revenues charged on imports of differentiated goods gross of transport costs and Foreign export taxes (thus, tariffs are charged on CIF values of Foreign exports); export tax revenues charged on exports gross of transport costs; and production tax revenues from taxes on marginal and fixed costs. The foreign government has a symmetric budget constraint.

2.4 Market Clearing Conditions

The market clearing condition for a differentiated variety produced at Home is given by:

$$y_H(i) = c_{HH}(i) + \tau c_{FH}(i) \tag{17}$$

A similar condition holds for Foreign varieties. Free entry in the differentiated sector implies that monopolistic producers make zero profit in equilibrium[9] and that production of each differentiated variety is fixed: $y_H(i) \equiv y = (\varepsilon - 1)f$.[10] Moreover, given that firms share the same production technology, the equilibrium is symmetric: all firms in the differentiated sector of a given country charge the same price and produce the same quantity. Hence, in equilibrium $\frac{p_{HH}(i)}{P_{HH}} = N_H^{\frac{1}{\varepsilon-1}}$ and $P_{HF} = \tau_{IH}\tau_{XF}\tau P_{FF}$. Using these price relations, the demand functions (4) and (5) and the fact that the production of each variety is equal to $(\varepsilon - 1)f$, we can rewrite the market clearing condition of domestically produced differentiated varieties (17) as:

$$(\varepsilon - 1)f = N_H^{\frac{\varepsilon}{1-\varepsilon}} P_{HH}^{-\varepsilon} \left[P_H^{\varepsilon} C_H + \tau^{1-\varepsilon}(\tau_{IF}\tau_{XH})^{-\varepsilon} P_F^{\varepsilon} C_F\right] \tag{18}$$

Using the demand functions, the market clearing condition for the homogeneous good – $Q_{ZH} + Q_{ZF} = Z_H + Z_F$ – can be written as:

$$Q_{ZH} + Q_{ZF} = \frac{(1-\alpha)}{\alpha}\left[P_H C_H + P_F C_F\right] \tag{19}$$

Equilibrium in the labor market implies that $L = L_{CH} + L_{ZH}$ with $L_{CH} = N_H L_{CH}(i)$. Making use of (10) and (13), labor market clearing can be written as:

$$Q_{ZH} = L - N_H \varepsilon f \tag{20}$$

Finally, we assume that there is no trade in financial assets, so trade is balanced. The balanced trade condition is given by:[11]

$$(Q_{ZH} - Z_H) + \tau\tau_{XH} P_{HH} C_{FH} = \tau\tau_{XF} P_{FF} C_{HF} \tag{21}$$

The left hand side of (21) is the sum of the net export value of the homogeneous goods and

[9] $\Pi_H(i) = c_{HH}(i)\left[p_{HH}(i) - \tau_{CH}\right] + c_{FH}(i)\left[\tau p_{HH}(i) - \tau\tau_{CH}\right] - f\tau_{CH} = 0$.

[10] Note that production taxes on fixed costs are necessary for this result, as can be easily verified from the free entry condition.

[11] Import taxes are collected directly by the governments at the border so they do not enter into this condition.

the value of exports of differentiated varieties (at CIF inclusive international prices), while the right hand side is the value of imports of differentiated varieties (at CIF inclusive international prices).

As standard in the trade literature (see e.g., Helpman and Krugman (1989)), we define the *direct terms-of-trade effect* as a change of the international price of exports ($\tau_{XH} p_{HH} = \tau_{XH} \tau_{CH} \frac{\varepsilon}{\varepsilon-1}$) relative to the one of imports ($\tau_{XF} p_{FF} = \tau_{XF} \tau_{CF} \frac{\varepsilon}{\varepsilon-1}$) of individual varieties. This implies that only production and export taxes have direct terms-of-trade effects. In particular, a domestic production or export tax increases the international price of exports one to one and improves domestic terms of trade, while a foreign export tax or production tax increases the international price of imports and worsens domestic individual terms of trade. As will become clear in section 3.3, where we discuss policy makers' incentives, it is useful to define also the *consumption-based terms-of-trade effect* as a change in the international prices of the aggregate exported bundle ($\tau_{XH} P_{HH} = N_H^{\frac{-1}{\varepsilon-1}} \tau_{XH} \tau_{CH} \frac{\varepsilon}{\varepsilon-1}$) relative to the one of the aggregate imported bundle ($\tau_{XF} P_{FF} = N_F^{\frac{-1}{\varepsilon-1}} \tau_{XF} \tau_{CF} \frac{\varepsilon}{\varepsilon-1}$).[12] The main difference between the two definitions is that trade policy can influence the consumption-based terms of trade both directly, through its effect on the international prices of individual varieties, and also indirectly, through its effect on the number of varieties produced in the two countries. In particular, Home's consumption-based terms of trade improve whenever the number of Home varieties decreases or when the number of Foreign varieties increases. The intuition for this alternative definition becomes clear from the trade balance condition (21): an increase in the number of Foreign varieties implies that domestic consumers obtain a larger amount of the Foreign consumption bundle – which includes more varieties and therefore is more valuable for consumers – for each unit of the domestic consumption bundle. Note that, according to this alternative definition, import taxes do have indirect terms-of-trade effects through their impact on the number of varieties produced in each country. When studying trade policy, we will always clearly differentiate between direct and indirect terms-of-trade effects.

[12]Defining terms-of-trade effects as changes of the relative international prices of aggregate export and import bundles follows the convention of the international macroeconomics literature. See, for example, Corsetti and Pesenti (2001) or Epifani and Gancia (2009).

2.5 Equilibrium

The optimal pricing rules (11), the good market clearing condition for Home's differentiated varieties (18), the labor market clearing condition (20), the corresponding conditions for Foreign, and the balanced trade condition (21), together with the expressions for the price indices, fully characterize the equilibrium of the economy.

It is possible to solve this system explicitly for N_H and N_F as functions of the trade policy instruments:

$$N_H = \frac{L(A_{2H} - A_{1F})}{A_{2F}A_{2H} - A_{1H}A_{1F}} \qquad\qquad N_F = \frac{L(A_{2F} - A_{1H})}{A_{2F}A_{2H} - A_{1H}A_{1F}}, \tag{22}$$

where A_{1H}, A_{2H}, A_{1F} and A_{2F} are non-linear functions of Home policy instruments $\Lambda_H \equiv \{\tau_{CH}, \tau_{IH}, \tau_{XH}\}$ and Foreign policy instruments $\Lambda_F \equiv \{\tau_{CF}, \tau_{IF}, \tau_{XF}\}$. The expressions for these coefficients, as well as the derivation of the equilibrium allocation, can be found in Appendix A.

Let the superscript FT denote the market allocation in the absence of trade policies (free trade allocation). We already showed that production of each differentiated variety is fixed, thus for both countries $y^{FT} = (\varepsilon - 1)f$. Given the assumption of symmetric countries, the equilibrium allocation is symmetric too and (22) simplifies to $N^{FT} = \frac{\alpha L}{\varepsilon f}$. In the next section we compare the free trade allocation with the first-best allocation. We then lay out the general structure of the policy makers' problems and discuss the incentives that determine their trade policy choices.

3 Trade Policy

3.1 The First-Best Allocation

The first-best allocation constitutes the natural benchmark to which one can compare the equilibrium outcomes under different policy regimes. The social planner chooses an allocation that maximizes total world welfare subject to the technology constraints and full employment in each country.[13]

$$\max_{C_H, C_F, Z_H, Z_F} C_H^{\alpha} Z_H^{1-\alpha} + C_F^{\alpha} Z_F^{1-\alpha} \tag{23}$$

subject to (10), (13), (17), $Q_{ZH} + Q_{ZF} = Z_H + Z_F$, $L = L_{CH} + L_{ZH}$, the definitions of consumption indices and the corresponding constraints for Foreign.

Proposition 1 presents the solution to this problem and compares it with the free trade allocation:[14]

Proposition 1: First-Best Allocation. *The first-best allocation entails the same firm size but more varieties than the free trade allocation. Formally,*

(1) $y^{FB} = f(\varepsilon - 1) = y^{FT}$ and $N^{FB} = \frac{\alpha L}{(\varepsilon - 1 + \alpha) f} > N^{FT} = \frac{\alpha L}{\varepsilon f}$.

This result replicates Dixit and Stiglitz (1977)'s finding that the market provides optimal firm size but too little variety. Because of monopolistic competition in the differentiated sector, individual free trade prices are too high. As a consequence, there is too little demand for the differentiated goods and thus too little entry. Therefore, the free trade equilibrium is characterized by a *production inefficiency*: both countries would be better off by simultaneously shifting some of their labor force from the homogenous sector to the differentiated sector.

3.2 Optimal Policy Problems

We now turn to the description of the optimal policy problems. We consider three policy instruments: production, import and export taxes. First, we assume that policy makers choose only one policy instrument at a time and subsequently we let them choose all three policy instruments simultaneously. For each case, we study cooperative and non-cooperative policies.

Note that given Cobb-Douglas utility, Home welfare, represented by the indirect utility function, can be written as:

$$V_H(P_H(\Lambda_H, \Lambda_F), I_H(\Lambda_H, \Lambda_F)) = -\alpha \log\left(P_H(\Lambda_H, \Lambda_F)\right) + \log\left(I_H(\Lambda_H, \Lambda_F)\right) \tag{24}$$

[13]More generally, there exists a whole set of Pareto-efficient allocations such that no country can be made better off, without making the other one worse off, which can be traced out by varying the welfare weights in the planner problem. We choose the point on the frontier that corresponds to equal weights of both countries because we always study symmetric allocations, which seems natural given that both countries are identical.

[14]All proofs can be found in the Appendix.

where P_H and I_H are functions of the policy instruments Λ_H and Λ_F.

The *cooperative policy maker* chooses Home and Foreign trade policy instruments in order to maximize total world welfare, which is given by the sum of Home and Foreign indirect utility:

$$\max_{\lambda_H,\lambda_F} V_H(P_H(\Lambda_H,\Lambda_F), I_H(\Lambda_H,\Lambda_F)) + V_F(P_F(\Lambda_H,\Lambda_F), I_F(\Lambda_H,\Lambda_F)) \tag{25}$$

where $\lambda_j \in \{\tau_{Cj}, \tau_{Ij}, \tau_{Xj}, \Lambda_j\}$. Differently, the *single-country policy maker* chooses the domestic trade policy instruments Λ_H in order to maximize Home welfare, given the level of the Foreign trade policy instruments:

$$\max_{\lambda_H} V_H(P_H(\Lambda_H,\Lambda_F), I_H(\Lambda_H,\Lambda_F)) \tag{26}$$

where again $\lambda_H \in \{\tau_{CH}, \tau_{IH}, \tau_{XH}, \Lambda_H\}$.

3.3 Policy Makers' Incentives

Next, we decompose welfare changes in order to reveal policy makers' incentives to set policy instruments.[15] Remember that domestic income is given by labor income plus transfers:

$$I_H = L + T_H = L + (\tau_{IH}-1)\tau\tau_{XF}P_{FF}C_{HF} + (\tau_{XH}-1)\tau P_{HH}C_{FH} + (\tau_{CH}-1)N_H(y+f),$$

which, using the labor and goods market clearing conditions, can be rewritten as:

$$I_H = (Z_H + P_H C_H) + (\tau_{XH}\tau P_{HH}C_{FH} - \tau\tau_{XF}P_{FF}C_{HF} + Q_{ZH} - Z_H) \tag{27}$$

The terms in the first bracket equal domestic expenditure and the remaining terms are net exports at international prices (i.e., the trade balance). While welfare can in principle be decomposed in many ways, the decomposition of welfare changes exposed in Helpman and Krugman (1989), chapter one, turns out to be particularly useful for our purposes. In particular, they show that changes in indirect utility induced by changes in the trade policy instruments can be split into: terms-of-trade effects; gains from improved production composition; consumption-wedges.[16] Totally differentiating (24), and using the above expression for income, the change

[15] All derivations can be found in Appendix C.

[16] In chapter 1, page 23, they show that $\frac{dV(p_C,I)}{\frac{\partial V}{\partial I}} = -(C-X)dp^* + p^*dX + (p_c - p^*)dC$, where C is the

in domestic welfare can be expressed as:

$$
\begin{aligned}
dV_H &= -\alpha \frac{dP_H}{P_H} + \frac{dI_H}{I_H} \\
&= -\alpha \frac{dP_H}{P_H} + \alpha \frac{dP_H}{P_H} \\
&+ \frac{d(\tau_{XH} P_{HH}) \tau C_{FH} - d(\tau_{XF} P_{FF}) \tau C_{HF}}{I_H} \\
&+ \frac{(\tau_{XH} \tau_{CH} \frac{\varepsilon}{\varepsilon - 1} - 1)(y + f) dN_H}{I_H} \\
&+ \frac{(1 - \tau_{XH}) P_{HH} dC_{HH} + (\tau_{IH} - 1) \tau \tau_{XF} P_{FF} dC_{HF}}{I_H}
\end{aligned} \tag{28}
$$

Here, the first term is the change in welfare due to changes in the domestic price level, which is exactly compensated by the first term of the income derivative. Thus, the change in domestic welfare is actually function of three terms, like in Helpman and Krugman (1989). A first implication of (28) is that in general equilibrium, domestic policy makers do not try to reduce the domestic price level via the home market externality (as described in equation (15)) because they internalize that in terms of utility, any reduction in the domestic price level is always exactly compensated by a simultaneous reduction in domestic income. Note that if lump sum transfers/taxes are not allowed for, like for example when taxes are considered to be a pure waste (as is the case in Venables (1987) or Ossa (2011)[17]), income is simply constant and given by $I_H = L$. This implies that $dV_H = -\alpha \frac{dP_H}{P_H}$. In this case, a reduction in the domestic price level is not associated with a change in income, and therefore policy makers do have incentives to reduce the domestic price level by exploiting the home market externality.

The terms in the second line represent *consumption-based terms-of-trade effects*. They consist of changes in the international price of the differentiated export bundle times exports minus the change in the international price of the differentiated import bundle times imports.[18] An increase in the price of exportables raises welfare, while an increase in the price of importables reduces it. The change in the international price of the export and import bundle can be further

consumption vector, X is the production vector, p_c is the vector of consumer prices and p^* is the vector of international prices. *"The first term on the right-hand side represents the gain from improved terms of trade [...] The second term represents the gain from an improved production composition. [...] The last term represents the consumption-wedge effect."*

[17]He uses this assumption for the analytical results. He only considers the more general case in a simulation exercise.

[18]The international price of the homogeneous good does not change and thus drops from this expression.

decomposed as:

$$d(\tau_{XH}P_{HH}) = \frac{\varepsilon}{\varepsilon-1}N_H^{-\frac{1}{\varepsilon-1}}d(\tau_{XH}\tau_{CH}) - \frac{\tau_{XH}\tau_{CH}}{\varepsilon-1}\frac{\varepsilon}{\varepsilon-1}N_H^{-\frac{\varepsilon}{\varepsilon-1}}dN_H \quad (29)$$
$$d(\tau_{XF}P_{FF}) = \frac{\varepsilon}{\varepsilon-1}N_F^{-\frac{1}{\varepsilon-1}}d(\tau_{XF}\tau_{CF}) - \frac{\tau_{XF}\tau_{CF}}{\varepsilon-1}\frac{\varepsilon}{\varepsilon-1}N_F^{-\frac{\varepsilon}{\varepsilon-1}}dN_F$$

Thus, consumption-based terms of trade improve directly through increases in τ_{XH} and τ_{CH}, which make exports more expensive, and through reductions in τ_{XF} and τ_{CF}, which make imports cheaper. This is the traditional *direct terms-of-trade effect.* Moreover, consumption-based terms of trade improve indirectly through reductions in N_H and increases in N_F because they increase the international price of exports and lower the international price of imports of one unit of the respective sub-utility. In this sense, import taxes can also have *indirect terms-of-trade effects* by changing the number of varieties produced in each country. Finally, note that an increase in N_H or a reduction in N_F cannot be interpreted as a home market externality because both are welfare reducing, whereas positive welfare effects of such changes would be required to make them interpretable as such.

The term in the third line represents what Helpman and Krugman (1989) refer to as 'gain from an improved production composition'. We label this the *production efficiency effect.* It represents the trade off between producing one more variety of the differentiated good and giving up $L_H(i) = y + f$ units of Q_{ZH}, evaluated at international prices.[19] The production efficiency effect is zero when $\tau_{XH}\tau_{CH} = \frac{\varepsilon-1}{\varepsilon}$, i.e., when production and/or export subsidies are set so as to eliminate the price markup charged by domestic firms. When this is the case, there are no efficiency gains from relocating labor from one sector to the other. In contrast – as shown in section 3.1 – at the free trade allocation there is too little provision of differentiated varieties and too much production of the homogenous good. In this case, the production efficiency term equals $\frac{\varepsilon}{\varepsilon-1}fdN_H$, implying that policy makers have an incentive to induce a reallocation of labor from the homogenous to the differentiated sector. Given that a relocation of firms to the domestic economy will exactly achieve this goal, they have an incentive to use trade policy to induce this outcome. However, observe that such relocation motive is no longer present once production efficiency has been reached. Indeed, the term becomes negative if the subsidies exceed the price markup i.e., whenever there is over-subsidizing further increasing N_H

[19] Indeed, in equilibrium, $dN_H = -\varepsilon f dQ_{ZH}$.

reduces welfare. This term will be crucial to understand the policy outcomes of the different instruments.

Finally, the terms in the last line represent *consumption wedges* due to import or export taxes. By generating a differences between domestic and international prices, trade taxes induce domestic households to consume too little or too much of Foreign or domestic differentiated goods. A tariff, for instance, renders Foreign varieties too expensive for domestic households and generates an inefficiency by lowering C_{HF}. Hence, as long as $\tau_{IH} > 1$, any increase in the Home demand for Foreign differentiated goods partially corrects for this distortion and raises welfare. On the other hand, an export subsidy increases the demand of foreigners for Home varieties and reduces that of domestic households. As a result, C_{HH} is inefficiently low and as long as $\tau_{XH} < 1$, an increase in the domestic demand for Home differentiated goods is welfare improving. Observe that these terms are zero when there are no trade taxes like, for example, at the free trade allocation or when production taxes are the only instruments. In fact, in the absence of trade taxes, domestic and international prices are equal and consumption wedges are absent.[20]

4 Production Taxes

In this section we study cooperative and non-cooperative production subsidies/taxes, assuming that they are the only available policy instruments, i.e., $\tau_{IH} = \tau_{IF} = \tau_{XH} = \tau_{XF} = 1$. We first discuss cooperative production taxes and then turn to a discussion of strategic ones.

Proposition 2: Cooperative Production Subsidy. *The optimal cooperative production subsidy is set to exactly offset the price markup generated by monopolistic competition. This subsidy implements a symmetric equilibrium with the first-best number of varieties. Formally,*

(1) $\tau_C^{Coop} = \frac{\varepsilon-1}{\varepsilon}$ and $N^{Coop} = N^{FB}$.

To gain intuition for the incentives behind such policy outcome, it is useful to express the

[20]Helpman and Krugman (1989) pointed out that whenever the trade tax τ_{mj} is close to one, the consumption-wedge effect is second order in size and can even be disregarded. Nonetheless, we always account for those effects in our welfare decomposition.

cooperative welfare changes using (28) for both countries:

$$dV_H + dV_F = \frac{(\tau_{CH}\frac{\varepsilon}{\varepsilon-1} - 1)(y+f)dN_H}{I_H} + \frac{(\tau_{CF}\frac{\varepsilon}{\varepsilon-1} - 1)(y+f)dN_F}{I_F} \tag{30}$$

where $dN_H = \frac{\partial N_H}{\partial \tau_{CH}} d\tau_{CH} + \frac{\partial N_H}{\partial \tau_{CF}} d\tau_{CF}$ and $dN_F = \frac{\partial N_F}{\partial \tau_{CF}} d\tau_{CF} + \frac{\partial N_F}{\partial \tau_{CH}} d\tau_{CH}$. Under cooperation, the common authority takes into account the externality produced on the other country, so that consumption-based terms-of-trade effects exactly compensate since they are equal and of opposite signs. Moreover, when production taxes are the only instrument, consumption wedges are absent. Therefore, the production efficiency effect is the only driving incentive of the cooperative policy maker. The cooperative welfare is increasing in the two subsidies ($dN_H > 0$, $dN_F > 0$)[21] as long as $\frac{\varepsilon-1}{\varepsilon} < \tau_C^{Coop} \leq 1$, and is maximized for $\tau_C^{Coop} = \frac{\varepsilon-1}{\varepsilon}$.

Differently, unilateral setting of production taxes does not lead to the first-best outcome due to the consumption-based terms-of-trade externality, as formally stated in Lemma 1.

Lemma 1: Unilaterally Set Production Subsidies. *The optimal unilateral deviation entails a reduction in the production subsidy when starting from the efficient allocation. When starting from the free trade allocation, the optimal unilateral deviation entails a production subsidy. Formally,*

(1) If $\tau_{CH} = \tau_{CF} = \frac{\varepsilon-1}{\varepsilon}$, $\frac{\partial V_H}{\partial \tau_{CH}} > 0$.

(2) If $\tau_{CH} = \tau_{CF} = 1$, $\frac{\partial V_H}{\partial \tau_{CH}} < 0$.

Using our welfare decomposition, welfare changes induced by unilateral production taxes are given by:

$$dV_H = \frac{d(P_{HH})\tau C_{FH} - d(P_{FF})\tau C_{HF}}{I_H} + \frac{(\tau_{CH}\frac{\varepsilon}{\varepsilon-1} - 1)(y+f)dN_H}{I_H}, \tag{31}$$

where dP_{HH} and dP_{FF} can be further decomposed as in (29), and $dN_H = \frac{\partial N_H}{\partial \tau_{CH}} d\tau_{CH}$. Thus, single-country policy makers' actions are determined both by the consumption-based terms-of-trade effect (first term) and the production efficiency effect (second term). There is a trade off between these two effects: terms-of-trade effects call for a production tax which improves terms

[21] See Lemma A1 (1) in Appendix D for the proof of the incentives driving cooperative policy choice.

of trade both directly – by increasing the international price of individual varieties (direct terms-of-trade effect) – and indirectly – by reducing the number of domestically produced varieties. Instead, the production efficiency effect warrants a production subsidy that increases domestic entry and brings factory gate prices down to marginal costs.[22] Overall, this trade off leads to a production subsidy which, however, is inefficiently small. Note also that any production subsidy that is larger than the first-best level would be clearly welfare detrimental because it would make both terms-of-trade and production efficiency effects negative. This intuition carries over to strategically set production taxes, as stated in Proposition 3.

Proposition 3: Nash-Equilibrium Production Subsidies. *In the Nash equilibrium both countries set a production subsidy. However, this subsidy is smaller than the one needed to implement the first-best allocation. The equilibrium number of varieties larger than in the free trade allocation, but lower than the first-best level. Formally,*

(1) $\tau_C^{coop} < \tau_C^{Nash} < 1$ and $N^{FT} < N^{Nash} < N^{FB}$.

Thus, single-country policy makers never over-subsidize domestic production, as would be required if the home market externality were the dominating incentive for non-cooperative policy choice. Instead, the trade off between production efficiency effects and terms-of-trade effects leads policy makers to choose an inefficiently low level of production subsidies. This is an important result, because it contradicts the standard wisdom that in the two-sector Krugman model countries have an incentive to over-subsidize production in order to attract more firms (Venables (1987)).

5 Import Taxes

Here, we assume that the only strategic trade policy instrument available is an import tariff/subsidy. Given the results of the previous section, where we pointed out the importance of the production efficiency effect, we study cooperative and non-cooperative import taxes under two scenarios. In the first scenario, production subsidies have already been set in a non-strategic fashion such as to eliminate monopolistic distortions and to implement the first-best allocation

[22] See Lemmata A1 (2) and A1 (3) in Appendix D for the proofs of the incentives driving unilateral deviations.

(i.e., $\tau_{CH} = \tau_{CF} = \frac{\varepsilon-1}{\varepsilon}$ and production efficiency effects are absent), while in the second scenario monopolistic distortions are present (i.e., $\tau_{CH} = \tau_{CF} = 1$). Let us first study the cooperative policy maker's problem.

Proposition 4: Cooperative Import Subsidy. *If $\tau_{CH} = \tau_{CF} = \frac{\varepsilon-1}{\varepsilon}$, the cooperative policy maker refrains from using taxes on imports and the number of varieties equals the first-best level. If $\tau_{CH} = \tau_{CF} = 1$, the cooperative policy maker finds it optimal to subsidize imports. The number of varieties is larger than in the free trade allocation, but remains lower than the first-best level. Formally,*

(1) If $\tau_{CH} = \tau_{CF} = \frac{\varepsilon-1}{\varepsilon}$, then $\tau_I^{Coop} = 1$ and $N^{Coop} = N^{FB}$.

(2) If $\tau_{CH} = \tau_{CF} = 1$, then $\frac{\varepsilon-1}{\varepsilon} < \tau_I^{Coop} < 1$ and $N^{FT} < N^{Coop} < N^{FB}$.

Again, we can use (28) for both countries to decompose the cooperative welfare change:

$$\begin{aligned} dV_H + dV_F = &\frac{(\tau_{CH}\frac{\varepsilon}{\varepsilon-1} - 1)(y+f)dN_H}{I_H} + \frac{(\tau_{CF}\frac{\varepsilon}{\varepsilon-1} - 1)(y+f)dN_F}{I_F} \\ &+ \frac{(\tau_{IH} - 1)\tau P_{FF} dC_{HF}}{I_H} + \frac{(\tau_{IF} - 1)\tau P_{HH} dC_{FH}}{I_F}, \end{aligned} \tag{32}$$

where $dN_H = \frac{\partial N_H}{\partial \tau_{IH}} d\tau_{IH} + \frac{\partial N_H}{\partial \tau_{IF}} d\tau_{IF}$, $dN_F = \frac{\partial N_F}{\partial \tau_{IF}} d\tau_{IF} + \frac{\partial N_F}{\partial \tau_{IH}} d\tau_{IH}$, $dC_{HF} = \frac{\partial C_{HF}}{\partial \tau_{IH}} d\tau_{IH} + \frac{\partial C_{HF}}{\partial \tau_{IF}} d\tau_{IF}$ and $dC_{FH} = \frac{\partial C_{FH}}{\partial \tau_{IF}} d\tau_{IF} + \frac{\partial C_{FH}}{\partial \tau_{IH}} d\tau_{IH}$. Thus, the cooperative welfare change can be decomposed into the production efficiency effect (first line) and consumption wedges (second line). Observe that when production subsidies are set at the first-best level, $\tau_{CH} = \tau_{CF} = \frac{\varepsilon-1}{\varepsilon}$, the terms related to production efficiency are zero. In this case, setting $\tau_{IH} = \tau_{IF} = 1$, also makes consumption wedges disappear and this solves the cooperative import policy problem. Differently, when production subsidies are not available, i.e., $\tau_{CH} = \tau_{CF} = 1$, the production efficiency effect is positive and an increase in domestic and foreign varieties ($dN_H > 0$, $dN_F > 0$) increases welfare. To achieve that, the cooperative policy maker has an incentive to implement symmetric import subsidies in both countries, which increase the demand for imported varieties and trigger entry into the differentiated sectors in both countries. However, this comes at the cost of creating wedges: the domestic price of imported varieties becomes lower than its international price and this tilts the consumption allocation inefficiently towards imported varieties. This can be clearly seen from the above formula. When $\tau_{IH} = \tau_{IF} < 1$, the terms $(\tau_{Ij} - 1)$ are negative so that an increase in consumption of imported varieties induced by import subsidies ($dC_{HF} > 0$, $dC_{FH} > 0$) reduces welfare.[23] As a result, when only import taxes are available, production and consumption inefficiencies cannot be eliminated simultaneously and the cooperative import subsidy implements a second-best allocation.

Once we move to the case of non-cooperation, trade policy outcomes crucially depend on whether production efficiency effects are present, as stated formally by Lemma 2 for the case of unilateral import taxes.

Lemma 2: Unilaterally Set Import Tariffs/Subsidies. *Let $\tau_{IH} = \tau_{IF} = 1$. The optimal unilateral deviation entails an import subsidy when starting from the first-best allocation implemented by a production subsidy, and an import tariff when starting from the free trade allocation. Formally,*

(1) If $\tau_{CH} = \tau_{CF} = \frac{\varepsilon-1}{\varepsilon}$, then $\frac{\partial V_H}{\partial \tau_{IH}} < 0$.

(2) If $\tau_{CH} = \tau_{CF} = 1$, then $\frac{\partial V_H}{\partial \tau_{IH}} > 0$.

To understand this difference in import policy choice, we use once more our welfare decomposition (28):

$$dV_H = \frac{d(P_{HH})\tau C_{FH} - d(P_{FF})\tau C_{HF}}{I_H} + \frac{(\tau_{CH}\frac{\varepsilon}{\varepsilon-1} - 1)(y+f)dN_H}{I_H} \tag{33}$$

where dP_{HH} and dP_{FF} can be further decomposed as in (29) and $dN_H = \frac{\partial N_H}{\partial \tau_{IH}} d\tau_{IH}$. Consumption wedges are absent since $\tau_{IH} = \tau_{IF} = 1$ and the change in domestic welfare induced by changes in unilateral import taxes is given by consumption-based terms-of-trade effects (first term) and production efficiency effects (second term). When $\tau_{CH} = \frac{\varepsilon-1}{\varepsilon}$, there are no production efficiency effects and import taxes have only indirect consumption-based terms-of-trade effects through their effect on the number of varieties produced in each country. The optimal unilateral policy choice is an import subsidy which shifts domestic demand towards imported varieties. This triggers exit of firms from the domestic differentiated sector and entry in Foreign, thereby indirectly improving domestic consumption-based terms of trade.

In contrast, at the free trade allocation with $\tau_{CH} = \tau_{CF} = 1$, both consumption-based terms-of-trade effects and production efficiency effects are present. While the first one calls for an

[23] See Lemma A2 (1) in Appendix E for the proof of the incentives driving cooperative policy choice.

import subsidy, which indirectly improves domestic terms of trade, the second one warrants a tariff. This is so because a unilaterally set tariff shifts domestic demand towards domestically produced varieties and triggers entry into the domestic differentiated sector, where domestic firms now make profits, at the expense of the Foreign differentiated sector, where Foreign firms now make losses. As a result, the domestic price level is reduced by entry and the Foreign one increases through exit, restoring zero profits in equilibrium. This relocation of firms from Foreign to Home allows Home to shift labor from the homogeneous to the differentiated sector, thus increasing domestic production efficiency at the expense of the other country ($dN_H > 0$ and $dN_F < 0$).[24] Overall, the production efficiency effect dominates the indirect terms-of-trade effect and the outcome is a tariff. Thus, while production efficiency is the reason for the tariff, a relocation externality is the means by which production efficiency gains are achieved.

Our finding for the case $\tau_{CH} = \tau_{CF} = 1$ is in line with Venables (1987)'s and Ossa (2011)'s results that in absence of retaliation import tariffs can increase domestic welfare compared to a situation with free trade. What is different is the interpretation. According to their interpretation, the home market externality – defined as the desire to reduce the domestic price level via reduced transport costs that are achieved by relocating firms from the Foreign to the domestic economy – is the only incentive driving unilateral import policy choices. Both Venables (1987) and Ossa (2011) consider tariff income as a pure waste. In this special case, the motive to reduce the price index is indeed the only incentive for policy makers because effects of tariffs on income – and thus terms-of-trade effects – are absent. Instead, we have shown that when allowing for transfers, it is the desire to increase production efficiency which drives incentives to relocate firms to the domestic economy. Thus, it is the production efficiency effect which implies a relocation externality. Observe that this is not a purely semantic difference. The relocation externality is in general only present when the initial allocation is inefficient. Just in the special case when income effects are not allowed for, the relocation externality is always present. The next Proposition shows that the results for unilateral case extend to a setting with strategic choice of import taxes.

Proposition 5: Nash-Equilibrium Import Tariffs/Subsidies. *When starting from the first-best allocation, the Nash-equilibrium policy consists of an import subsidy, implying more*

[24]See Lemmata A2 (2) and A2 (3) in Appendix E for the proofs of the incentives driving unilateral deviations.

varieties than the first-best allocation. In contrast, when starting from the free trade allocation, the Nash-equilibrium entails a tariff, implying less varieties than the free trade allocation. Formally,

(1) If $\tau_{CH} = \tau_{CF} = \frac{\varepsilon-1}{\varepsilon}$, then $\tau_I^{Nash} < 1$ and $N^{FT} < N^{FB} < N^{Nash}$.

(2) If $\tau_{CH} = \tau_{CF} = 1$, then there exists a $\tau_I^{Nash} > 1$ such that $N^{Nash} < N^{FT} < N^{FB}$.

First, note that on top of the production efficiency effect and terms-of-trade effect described in Lemma 2, in the Nash equilibrium import taxes also generate consumption wedges. However, the Nash outcomes are exactly the ones we would have expected from the incentives for unilateral deviations. When starting from the first-best allocation, the optimal Nash policy is an import subsidy. The intuition follows from the incentives for unilateral policies: the import subsidy aims at improving consumption-based domestic terms of trade indirectly by reducing the number of domestic firms. Yet, in equilibrium no country reaches this aim and entry increases beyond efficiency. According to the second part of Proposition 5, when there is no correction of the monopolistic distortion, non-cooperative trade policy brings about a positive tariff. From Lemma 2 (2) we know that the production efficiency effect is behind the choice of setting a unilateral import tariff. However, in the Nash equilibrium no country manages to relocate firms to its domestic market thereby failing to increase production efficiency. Instead, tariffs reduce the world equilibrium number of varieties.

6 Export Taxes

In this section, we consider export subsidies/taxes as the only strategic trade policy instrument available. In line with the previous analysis, we study cooperative and Nash policies under two scenarios. In the first one production subsidies have been set such as to implement the first-best allocation (i.e., $\tau_{CH} = \tau_{CF} = \frac{\varepsilon-1}{\varepsilon}$), while in the second scenario monopolistic distortions have not been corrected (i.e., $\tau_{CH} = \tau_{CF} = 1$).

Proposition 6: Cooperative Export Subsidy. *If $\tau_{CH} = \tau_{CF} = \frac{\varepsilon-1}{\varepsilon}$, the cooperative policy maker refrains from using taxes on exports and the number of varieties equals the first-best level. If $\tau_{CH} = \tau_{CF} = 1$, the cooperative policy maker finds it optimal to subsidize exports.*

The number of varieties is larger than in the free trade allocation, but remains lower than the first-best level. Formally,

(1) If $\tau_{CH} = \tau_{CF} = \frac{\varepsilon-1}{\varepsilon}$, then $\tau_X^{Coop} = 1$ and $N^{Coop} = N^{FB}$.

(2) If $\tau_{CH} = \tau_{CF} = 1$, then $\tau_X^{Coop} < 1$ and $N^{FT} < N^{Coop} < N^{FB}$.

Incentives to set export subsidies can again be best understood using the welfare decomposition (28), which now becomes

$$dV_H + dV_F = \frac{(\tau_{XH}\tau_{CH}\frac{\varepsilon}{\varepsilon-1} - 1)(y+f)dN_H}{I_H} + \frac{(\tau_{XF}\tau_{CF}\frac{\varepsilon}{\varepsilon-1} - 1)(y+f)dN_F}{I_F} \\ + \frac{(1-\tau_{XH})P_{HH}dC_{HH}}{I_H} + \frac{(1-\tau_{XF})P_{FF}dC_{FF}}{I_F}, \tag{34}$$

where $dN_H = \frac{\partial N_H}{\partial \tau_{XH}} d\tau_{XH} + \frac{\partial N_H}{\partial \tau_{XF}} d\tau_{XF}$, $dN_F = \frac{\partial N_F}{\partial \tau_{XH}} d\tau_{XH} + \frac{\partial N_F}{\partial \tau_{XF}} d\tau_{XF}$, $dC_{HH} = \frac{\partial C_{HH}}{\partial \tau_{XH}} d\tau_{XH} + \frac{\partial C_{HH}}{\partial \tau_{XF}} d\tau_{XF}$ and $dC_{FF} = \frac{\partial C_{FF}}{\partial \tau_{XH}} d\tau_{XH} + \frac{\partial C_{FF}}{\partial \tau_{XF}} d\tau_{XF}$. Terms in the first line of (34) are production efficiency effects while terms in the second line are consumption wedges. When production subsidies are set at their first-best level, i.e., $\tau_{CH} = \tau_{CF} = \frac{\varepsilon-1}{\varepsilon}$, it is easy to see that both production efficiency effects and consumption wedges are zero if $\tau_{XH} = \tau_{XF} = 1$. In contrast, when no production subsidies are available i.e., $\tau_{CH} = \tau_{CF} = 1$, production efficiency can be improved by setting export subsidies in both countries, which increase demand for differentiated varieties and trigger entry into the differentiated sectors ($dN_H > 0$, $dN_F > 0$).[25] However, this comes at the cost of creating consumption wedges. As for the case of import subsidies, this trade off leads to a second-best outcome, which improves upon the free trade allocation but does not eliminate all distortions. We now turn to a discussion of non-cooperative export taxes.

Lemma 3: Unilaterally Set Export Taxes/Subsidies. *Let $\tau_{XH} = \tau_{XF} = 1$. The optimal unilateral deviation entails an export tax when starting from the first-best allocation implemented by a production subsidy, and an export subsidy when starting from the free trade allocation. Formally,*

(1) If $\tau_{CH} = \tau_{CF} = \frac{\varepsilon-1}{\varepsilon}$, then $\frac{\partial V_H}{\partial \tau_{XH}} > 0$.

(2) If $\tau_{CH} = \tau_{CF} = 1$, then $\frac{\partial V_H}{\partial \tau_{XH}} < 0$.

[25]See Lemma A3 (1) in Appendix F for the proof of the incentives driving cooperative policy choices.

Once again, we use our welfare decomposition (28):

$$dV_H = \frac{d(\tau_{XH}P_{HH})\tau C_{FH} - d(\tau_{XF}P_{FF})\tau C_{HF}}{I_H} + \frac{(\tau_{XH}\tau_{CH}\frac{\varepsilon}{\varepsilon-1} - 1)(y+f)dN_H}{I_H} \tag{35}$$

where $d(\tau_{XH}P_{HH})$ and $d(\tau_{XF}P_{FF})$ can be further decomposed as in (29) and $dN_H = \frac{\partial N_H}{\partial \tau_{XH}} d\tau_{XH}$. The first term in (35) is the consumption based terms-of-trade effect, while the second term is the production efficiency effect. When $\tau_{CH} = \frac{\varepsilon-1}{\varepsilon}$, production efficiency effects are absent as long as $\tau_{XH} = 1$. However, policy makers have incentives to unilaterally deviate by setting a small export tax. Such an export tax indeed improves domestic terms of trade both directly, through an increase in the international price of domestically produced varieties, and indirectly, via a reduction in the number of domestic firms and an increase in the number of foreign ones. Differently, at the free trade allocation with $\tau_{CH} = \tau_{CF} = 1$ production efficiency effects are present and call for an export subsidy.[26] Overall, negative terms-of-trade effects of an export subsidy are out-weighted by production efficiency gains. A small export subsidy triggers entry into the domestic differentiated sector, thereby improving domestic production efficiency. This creates a relocation externality, since it induces exit of firms from Foreign. However, the relocation effect is just the means to increase domestic production efficiency. The next Proposition shows that the results on unilateral changes extend to a setup with strategic choice of the export policy instrument.

Proposition 7: Nash-Equilibrium Export Taxes/Subsidies. *When starting from the first-best allocation, the Nash-equilibrium policy consists of an export tax, implying less varieties than the first-best allocation. In contrast, when starting from the free trade allocation, the Nash equilibrium entails an export subsidy, implying more varieties than the free trade allocation. Formally,*

(1) If $\tau_{CH} = \tau_{CF} = \frac{\varepsilon-1}{\varepsilon}$, then $\tau_X^{Nash} > 1$ and $N^{Nash} < N^{FB}$.

(2) If $\tau_{CH} = \tau_{CF} = 1$, then $\tau_X^{Nash} < 1$ and $N^{FT} < N^{Nash} < N^{FB}$.

Proposition 7 makes it clear that in this case too, the outcome of the policy game depends crucially on whether the initial allocation is (in)efficient. When starting from the free trade

[26] See Lemmata A3 (2) and A3 (3) in Appendix F for the proofs of the incentives driving unilateral deviations.

allocation the optimal Nash policy is an export subsidy, whereas when starting from the first-best allocation the optimal non-cooperative policy is an export tax. Again, even though the export taxes/subsidies will induce consumption wedges in the Nash equilibrium, the intuition is the one provided for the unilateral policy choice. If the initial allocation is efficient consumption-based terms-of-trade effects call for an export tax, but in the Nash outcome both countries fail to improve their terms of trade. If instead the initial allocation is inefficient, the production efficiency effect prevails and the policy makers choose an export subsidy.

7 Simultaneous Policy Choice

Finally, in this section we allow for simultaneous choice of all three policy instruments.

Proposition 8: Cooperative Policy Instruments. *The cooperative policy maker sets the first-best level of production subsidies and chooses the trade taxes such that* $\tau_I^{Coop} \cdot \tau_X^{Coop} = 1$. *The number of varieties equals the first-best level.*

(1) $\tau_C^{Coop} = \frac{\varepsilon-1}{\varepsilon}$, $\tau_I^{Coop} \cdot \tau_X^{Coop} = 1$ and $N^{Coop} = N^{FB}$.

This result is straightforward: the cooperative policy maker uses the production subsidy to reach the first-best allocation and either refrains from using trade instruments ($\tau_I^{Coop} = \tau_X^{Coop} = 1$), or uses them in a way that does not create consumption wedges ($\tau_I^{Coop} \cdot \tau_X^{Coop} = 1$). Differently, non-cooperative policy makers intend to manipulate international prices in their favor.

Proposition 9: Nash-Equilibrium Policy Instruments. *The Nash-equilibrium policy consists of the first-best level of production subsidies, and inefficient import subsidies and export taxes. Formally,*

(1) $\tau_C^{Nash} = \tau_C^{Coop} = \frac{\varepsilon-1}{\varepsilon}$, $\tau_I^{Nash} < 1$ and $\tau_X^{Nash} > 1$.

The result that production subsidies are set so as to completely offset monopolistic distortions is an application of the principle of targeting in public economics (Dixit (1985)). It states that an externality or distortion is best countered with a tax instrument that acts directly on the appropriate margin. Only when such an instrument is not available, trade policy can be used as a second-best policy. This confirms that the inefficiency of the market allocation crucially affects policy makers' incentives to set import tariffs or export subsidies. Once uncoordinated policy makers have all the necessary instruments to eliminate these distortions, the only motive to set trade policy is the incentive to improve domestic terms of trade. Export taxes achieve this directly by increasing the international price of domestic varieties and indirectly by reducing the number of domestically produced varieties, while import tariffs only impact on terms of trade through this indirect channel.[27] Moreover, this finding also strengthens the results from the previous sections, where first-best production subsidies were set in a non-strategic fashion and confirms our approach to isolate efficiency considerations from other motives to set trade policy.

Finally, note that our finding that terms-of-trade effects are the dominating motive for trade policy in the Krugman model is closely related to Bagwell and Staiger (2009) who derive a very similar result for the case where countries can set import and export taxes simultaneously but do not have access to production taxes. They find that countries' best response to an import tariff would be to set an offsetting export subsidy, and thus the relocation motive is not present in the incentives that determine Nash-equilibrium policy choice. Instead, only terms-of-trade effects survive. This is in line with our result that when additionally production subsidies are available, they will be set to the first-best level while the trade instruments are driven by terms-of-trade effects.

8 Conclusions

In this paper we have studied cooperative, unilateral and strategic trade policies in a two-sector Krugman (1980) model of intra-industry trade, considering production, import and export taxes as trade policy instruments. It is common wisdom that in this model non-cooperative trade policies are set in order to try to agglomerate firms in the domestic economy, which reduces transport costs for domestic consumers and thus the domestic price level (*home market effect*).

Contrary to the results of the previous literature, we show that in this model the home market effect is not a motive for non-cooperative trade policy choices. Instead, they are driven by *pro-*

[27] It is easy to show that using both trade instruments unilaterally improves terms of trade by more than when relying only on a single one.

duction efficiency considerations, on the one hand, and by *consumption-based terms-of-trade effects* on the other. Indeed, due to monopolistic competition, in the free trade equilibrium there are too few firms in the differentiated sector and this affects policy makers' incentives in a crucial way. Thus, when production taxes are available, non-cooperative policy makers increase production efficiency by setting production subsidies. However, due to terms-of-trade effects these subsidies are lower than the cooperatively set ones. When only import (export) tax instruments are available, non-cooperative policy makers use tariffs (export subsidies) to increase production efficiency, thereby imposing a relocation externality on the other country. However, once monopolistic distortions have been offset by appropriate production subsidies, results turn around: policy makers set import subsidies (export taxes), which improve domestic consumption-based terms of trade. Finally, when policy makers can set all three policy instruments simultaneously, they choose to set production subsidies, which exactly offset monopolistic distortions. Moreover, they set import subsidies and export taxes, both of which aim at improving domestic terms of trade. The implications of our findings are important: also in the Krugman (1980) model, terms-of-trade externalities remain the only reason why countries need to sign trade agreements.

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APPENDIX

A Equilibrium

A.1 Equilibrium Allocation and Prices

Substituting the optimal pricing rules (11) and (12) into the definition of Home (6) (and Foreign) aggregate price indices we obtain:

$$P_H = \frac{\varepsilon}{\varepsilon - 1}\left[N_H \tau_{CH}^{1-\varepsilon} + N_F\left(\tau_{IH}\tau_{XF}\tau\tau_{CF}\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} \quad P_F = \frac{\varepsilon}{\varepsilon - 1}\left[N_F \tau_{CF}^{1-\varepsilon} + N_H\left(\tau_{IF}\tau_{XH}\tau\tau_{CH}\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} \tag{36}$$

Combining the market clearing condition (18) with the analogous one for Foreign and substituting out the expressions for the prices (36), gives:

$$C_H = \frac{fP_H^{-\varepsilon}(\varepsilon-1)\left(\frac{\varepsilon}{\varepsilon-1}\right)^{\varepsilon}\tau^{\varepsilon}[-\tau\tau_{CF}^{\varepsilon} + (\tau\tau_{CH}\tau_{IF}\tau_{XH})^{\varepsilon}](\tau_{IH}\tau_{XF})^{\varepsilon}}{\tau^{2\varepsilon}(\tau_{IF}\tau_{XH}\tau_{IH}\tau_{XF})^{\varepsilon} - \tau^2} \tag{37}$$

$$C_F = \frac{fP_F^{-\varepsilon}(\varepsilon-1)\left(\frac{\varepsilon}{\varepsilon-1}\right)^{\varepsilon}\tau^{\varepsilon}[-\tau\tau_{CH}^{\varepsilon} + (\tau\tau_{CF}\tau_{IH}\tau_{XF})^{\varepsilon}](\tau_{IF}\tau_{XH})^{\varepsilon}}{\tau^{2\varepsilon}(\tau_{IF}\tau_{XH}\tau_{IF}\tau_{XF})^{\varepsilon} - \tau^2} \tag{38}$$

Using the trade balance condition (21), the labor market clearing condition (20), the equivalent equations for Foreign, and the expressions for C_H, C_F, P_H and P_F just derived, we obtain the following system of equations in N_H and N_F:

$$A_{1H}N_H + A_{2H}N_F - L = 0 \tag{39}$$

$$A_{2F}N_H + A_{1F}N_F - L = 0 \tag{40}$$

The solution to this system is:

$$N_H = \frac{L(A_{2H} - A_{1F})}{A_{2F}A_{2H} - A_{1H}A_{1F}} \qquad N_F = \frac{L(A_{2F} - A_{1H})}{A_{2F}A_{2H} - A_{1H}A_{1F}} \tag{41}$$

where:

$$A_{1H} = \frac{f\varepsilon\tau_{CH}^{-\varepsilon}\tau^{2\varepsilon}(\tau_{CH}\tau_{IH}\tau_{IF}\tau_{XH}\tau_{XF})^{\varepsilon}(\alpha + (1-\alpha)\tau_{CH})}{\alpha(\tau^{2\varepsilon}(\tau_{IH}\tau_{IF}\tau_{XH}\tau_{XF})^{\varepsilon} - \tau^2)} + \frac{f\varepsilon\tau_{CH}^{-\varepsilon}\tau[\alpha\tau\tau_{CH}^{\varepsilon}(\tau_{CH}\tau_{XH} - 1) - \tau_{CH}(\tau\tau_{CF}\tau_{IH}\tau_{XF})^{\varepsilon}(1-\alpha+\alpha\tau_{XH})]}{\alpha(\tau^{2\varepsilon}(\tau_{IH}\tau_{IF}\tau_{XH}\tau_{XF})^{\varepsilon} - \tau^2)} \tag{42}$$

$$A_{2H} = \frac{f\varepsilon\tau\tau_{XF}\tau_{CF}^{1-\varepsilon}(-\alpha - (1-\alpha)\tau_{IH})[\tau\tau_{CF}^{\varepsilon} - (\tau\tau_{CH}\tau_{IF}\tau_{XH})^{\varepsilon}]}{\alpha(\tau^{2\varepsilon}(\tau_{IH}\tau_{IF}\tau_{XH}\tau_{XF})^{\varepsilon} - \tau^2)} \tag{43}$$

$$A_{1F} = \frac{f\varepsilon\tau_{CF}^{-\varepsilon}\tau^{2\varepsilon}(\tau_{CF}\tau_{IH}\tau_{IF}\tau_X\tau_{XF})^{\varepsilon}(\alpha+(1-\alpha)\tau_{CF})}{\alpha(\tau^{2\varepsilon}(\tau_{IH}\tau_{IF}\tau_{XH}\tau_{XF})^{\varepsilon}-\tau^2)} + \frac{f\varepsilon\tau_{CF}^{-\varepsilon}\tau[\alpha\tau\tau_{CF}^{\varepsilon}(\tau_{CF}\tau_{XF}-1)-\tau_{CF}(\tau\tau_{CH}\tau_{IF}\tau_{XH})^{\varepsilon}(1-\alpha+\alpha\tau_{XF})]}{\alpha(\tau^{2\varepsilon}(\tau_{IH}\tau_{IF}\tau_{XH}\tau_{XF})^{\varepsilon}-\tau^2)} \tag{44}$$

$$A_{2F} = \frac{f\varepsilon\tau\tau_{XH}\tau_{CH}^{1-\varepsilon}(-\alpha-(1-\alpha)\tau_{IF})[\tau\tau_{CH}^{\varepsilon}-(\tau\tau_{CF}\tau_{IH}\tau_{XF})^{\varepsilon}]}{\alpha(\tau^{2\varepsilon}(\tau_{IH}\tau_{IF}\tau_{XH}\tau_{XF})^{\varepsilon}-\tau^2)} \tag{45}$$

A.2 Free Trade Allocation

Let $\tau_{CH} = \tau_{CF} = \tau_{IH} = \tau_{IF} = \tau_{XH} = \tau_{XF} = 1$. Then (41) simplifies to:

$$N_H = N_F = \frac{\alpha L}{\varepsilon f} \equiv N^{FT} \tag{46}$$

B The Planner's Problem

Proposition 1: First-Best Allocation. *The first-best allocation entails the same firm size but more varieties than the free trade allocation. Formally,*

(1) $y^{FB} = f(\varepsilon-1) = y^{FT}$ and $N^{FB} = \frac{\alpha L}{(\varepsilon-1+\alpha)f} > N^{FT} = \frac{\alpha L}{\varepsilon f}$.

Proof of Proposition 1.

The Lagrangian for the planner's problem is:

$$\begin{aligned}\mathcal{L} = &\left[\int_0^{N_H} c_{HH}(i)^{\frac{\varepsilon-1}{\varepsilon}}di + \int_0^{N_F} c_{HF}(i)^{\frac{\varepsilon-1}{\varepsilon}}di\right]^{\frac{\varepsilon\alpha}{\varepsilon-1}} Z_H^{1-\alpha} + \left[\int_0^{N_F} c_{FH}(i)^{\frac{\varepsilon-1}{\varepsilon}}di + \int_0^{N_F} c_{FF}(i)^{\frac{\varepsilon-1}{\varepsilon}}di\right]^{\frac{\varepsilon\alpha}{\varepsilon-1}} Z_F^{1-\alpha} \\ &+ \int_0^{N_H} \lambda_1(i)[L_{CH}(i) - f - c_{HH}(i) - \tau c_{FH}(i)]di + \int_0^{N_F} \lambda_2(i)[L_{CF}(i) - f - c_{FF}(i) - \tau c_{HF}(i)]di \\ &+ \lambda_3[L_H + L_F - \int_0^{N_H} L_{CH}(i)di - \int_0^{N_F} L_{CF}(i)di - Z_H - Z_F]\end{aligned}$$

The first-order conditions are:

$$\frac{\partial \mathcal{L}}{\partial c_{HH}(i)} = 0 : \alpha C_H^{\alpha}\left[\int_0^{N_H} c_{HH}(i)^{\frac{\varepsilon-1}{\varepsilon}}di + \int_0^{N_F} c_{HF}(i)^{\frac{\varepsilon-1}{\varepsilon}}di\right]^{-1} Z_H^{1-\alpha}c_{HH}(i)^{\frac{-1}{\varepsilon}} = \lambda_1(i) \tag{47}$$

$$\frac{\partial \mathcal{L}}{\partial c_{HF}(i)} = 0 : \alpha C_H^{\alpha}\left[\int_0^{N_H} c_{HH}(i)^{\frac{\varepsilon-1}{\varepsilon}}di + \int_0^{N_F} c_{HF}(i)^{\frac{\varepsilon-1}{\varepsilon}}di\right]^{-1} Z_H^{1-\alpha}c_{HF}(i)^{\frac{-1}{\varepsilon}} = \tau\lambda_2(i) \tag{48}$$

$$\frac{\partial \mathcal{L}}{\partial Z_H} = 0 : (1-\alpha)C_H^{\alpha}Z_H^{-\alpha} = \lambda_3 \tag{49}$$

$$\frac{\partial \mathcal{L}}{\partial L_{CH}(i)} = 0 : \lambda_1(i) = \lambda_3 \tag{50}$$

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial N_H} = 0 :& \alpha \frac{\varepsilon}{\varepsilon - 1} \left\{ C_H^{\alpha} Z_H^{1-\alpha} \left[\int_0^{N_H} c_{HH}(i)^{\frac{\varepsilon-1}{\varepsilon}} di + \int_0^{N_F} c_{HF}(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{-1} c_{HH}(N_H)^{\frac{\varepsilon-1}{\varepsilon}} + \right. \\ & \left. C_F^{\alpha} Z_F^{1-\alpha} \left[\int_0^{N_H} c_{FH}(i)^{\frac{\varepsilon-1}{\varepsilon}} di + \int_0^{N_F} c_{FF}(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{-1} c_{FH}(N_H)^{\frac{\varepsilon-1}{\varepsilon}} \right\} = \lambda_3 L_{CH}(N_H),\end{aligned} \tag{51}$$

where in the last condition we have already used the fact that $\lambda_1(N_H)[L_{CH}(N_H) - f - c_{HH}(N_H) - \tau c_{FH}(N_H)] = 0$.
The first-order conditions with respect to Foreign variables are completely symmetric and are thus omitted for the sake of space. By imposing symmetry we find $\lambda_1(i) = \lambda_2(i)$. Combining (47) and (48) we obtain:

$$c_{HF}(i) = c_{HH}(i)\tau^{-\varepsilon} \tag{52}$$

Combining (47), (50) and (51) we get that:

$$\frac{\varepsilon}{\varepsilon - 1}[c_{HH}(i)^{\frac{\varepsilon-1}{\varepsilon}} + c_{HF}(i)^{\frac{\varepsilon-1}{\varepsilon}}] = L_{CH}(i) c_{HH}(i)^{\frac{1}{\varepsilon}} \tag{53}$$

Combining (52) and (53), we obtain:

$$c_{HH}(i) = \frac{\varepsilon}{\varepsilon - 1}[1 + \tau^{1-\varepsilon}]^{-1} \tag{54}$$

Substituting the expression for $c_{HH}(i)$ and $c_{HF}(i)$ into the resource condition for domestic varieties $L_{CH}(i) = f + c_{HH}(i) + \tau c_{FH}(i)$, we get $L_{CH}(i) = \varepsilon f$ and using the production function $y_H(i) = L_{CH}(i) - f$ we obtain $y^{FB} = (\varepsilon - 1)f$. Moreover, $c_{HH}^{FB}(i) = (\varepsilon - 1)f[1 + \tau^{1-\varepsilon}]^{-1}$ and $c_{HF}^{FB}(i) = (\varepsilon - 1)f\tau^{-\varepsilon}[1 + \tau^{1-\varepsilon}]^{-1}$.
Using the resource condition for Z_H, we get $Z_H = L - N_H \varepsilon f$. Finally, combining (47), (49) and (50):

$$(1 - \alpha)C_H^{\frac{\varepsilon-1}{\varepsilon}} = \alpha Z_H c_{HH}(i)^{-\frac{1}{\varepsilon}} \tag{55}$$

Substituting the expressions for Z_H, C_H, $c_{HH}^{FB}(i)$ and $c_{HF}^{FB}(i)$ into (55), we can solve for $N_H = N_F \equiv N^{FB} = \frac{\alpha L}{f(\varepsilon + \alpha - 1)}$. ■

C Derivation of the Welfare Decomposition

In this section we decompose the welfare change due to a change in the policy instruments following Helpman and Krugman (1989). Totally differentiating indirect utility, we have:

$$dV_H = -\alpha \frac{dP_H}{P_H} + \frac{dI_H}{I_H} \tag{56}$$

Domestic income is given by labor income plus transfers:

$$I_H = L_H + T_H = L_H + (\tau_{IH}-1)\tau\tau_{XF}P_{FF}C_{HF} + (\tau_{XH}-1)\tau P_{HH}C_{FH} + (\tau_{CH}-1)N_H(y+f) \tag{57}$$

Note that:

(i) From labor market clearing it follows that $L_H = Q_{ZH} + N_H(y+f)$;

(ii) From optimal pricing we have $\tau_{IH}\tau\tau_{XF}P_{FF} = P_{HF}$;

(iii) $\tau_{CH}N_H(y+f) - \tau P_{HH}C_{FH} = P_{HH}C_{HH}$ given that:

- From $p_{HH}(i) = \tau_{CH}\frac{\varepsilon}{\varepsilon-1}$ and $y = (\varepsilon-1)f$ we have $\tau_{CH}N_H(y+f) = p_{HH}(i)N_Hy$;
- Market clearing (17) implies that $p_{HH}(i)N_Hy = N_Hp_{HH}(i)c_{HH}(i) + \tau N_Hp_{HH}(i)c_{FH}(i)$;
- From the pricing definitions and the optimal demand equations we have: $N_Hp_{HH}(i)c_{HH}(i) + \tau N_Hp_{HH}(i)c_{FH}(i) = P_{HH}C_{HH} + \tau P_{HH}C_{FH}$

(iv) Finally, remember that $P_{HF}C_{HF} + P_{HH}C_{HH} = P_HC_H$.

Therefore, we can rewrite (57) as follows:

$$I_H = [Z_H + P_HC_H] + \tau_{XH}\tau P_{HH}C_{FH} - \tau\tau_{XF}P_{FF}C_{HF} + (Q_{ZH} - Z_H) \tag{58}$$

Totally differentiating this expression, we obtain:

$$\begin{aligned} dI_H &= dZ_H + P_HdC_H + C_HdP_H \\ &\quad + d(\tau_{XH}P_{HH})\tau C_{FH} + (\tau\tau_{XH}P_{HH})dC_{FH} \\ &\quad - d(\tau_{XF}P_{FF})\tau C_{HF} - (\tau\tau_{XF}P_{FF})dC_{HF} + dQ_{ZH} - dZ_H \end{aligned} \tag{59}$$

Note that:

- From (2) it follows that $dC_H = \left(\frac{C_H}{C_{HH}}\right)^{\frac{1}{\varepsilon}} dC_{HH} + \left(\frac{C_H}{C_{HF}}\right)^{\frac{1}{\varepsilon}} dC_{HF}$. Using the optimal demand equations (4) and (5) we have: $\left(\frac{C_H}{C_{HH}}\right)^{\frac{1}{\varepsilon}} = \frac{P_{HH}}{P_H}$ and $\left(\frac{C_H}{C_{HF}}\right)^{\frac{1}{\varepsilon}} = \frac{P_{HF}}{P_H}$. Therefore,

$$P_HdC_H = P_{HH}dC_{HH} + P_{HF}dC_{HF}$$

- From (iii) we have $\tau P_{HH}C_{FH} = N_Hp_{HH}(i)y - P_{HH}C_{HH}$. Therefore:

$$\begin{aligned} &d(\tau_{XH}P_{HH})\tau C_{FH} + \tau\tau_{XH}P_{HH}dC_{FH} = \\ &yN_Hd(\tau_{XH}p_{HH}(i)) - C_{HH}d(\tau_{XH}P_{HH}) + \tau_{XH}p_{HH}(i)ydN_H - \tau_{XH}P_{HH}dC_{HH} \end{aligned}$$

Note also that from (8) and (9) we have $-\alpha\frac{dP_H}{P_H} = -\frac{C_H}{I_H}dP_H$. Therefore, indirect utility is given by:

$$dV_H = -\frac{C_H}{I_H}dP_H + \frac{C_H}{I_H}dP_H + \frac{yN_H d(\tau_{XH}p_{HH}(i)) - d(\tau_{XH}P_{HH})C_{HH} - d(\tau_{XF}P_{FF})\tau C_{HF}}{I_H} + \frac{dQ_{ZH} + \tau_{XH}p_{HH}(i)ydN_H}{I_H} + \frac{(1-\tau_{XH})P_{HH}dC_{HH} + (\tau_{IH}-1)\tau\tau_{XF}P_{FF}dC_{HF}}{I_H} \tag{60}$$

From (iii) and the fact that $p_{HH}(i) = P_{HH}N_H^{\frac{1}{\varepsilon-1}}$ we have $C_{HH} = yN_H^{\frac{\varepsilon}{\varepsilon-1}} - \tau C_{FH}$. Using this and observing that $N_H^{\frac{\varepsilon}{\varepsilon-1}}d(\tau_{XH}P_{HH}) = N_H d(\tau_{XH}p_{HH}(i)) + \frac{1}{1-\varepsilon}p_{HH}(i)\tau_{XH}dN_H$, we can write (60) as:

$$\begin{aligned} dV_H = & -\frac{C_H}{I_H}dP_H + \frac{C_H}{I_H}dP_H \\ & + \frac{d(\tau_{XH}P_{HH})\tau C_{FH} - d(\tau_{XF}P_{FF})\tau C_{HF}}{I_H} \\ & + \frac{dQ_{ZH} + \tau_{XH}p_{HH}(i)ydN_H + \frac{\tau_{XH}p_{HH}(i)y}{(\varepsilon-1)}dN_H}{I_H} \\ & + \frac{(1-\tau_{XH})P_{HH}dC_{HH} + (\tau_{IH}-1)\tau\tau_{XF}P_{FF}dC_{HF}}{I_H} \end{aligned} \tag{61}$$

Equation (28) follows immediately from this expression by observing that $Q_{ZH} = L - N_H\varepsilon f$, $y = (\varepsilon-1)f$ and $p_{HH}(i) = \tau_{CH}\frac{\varepsilon}{\varepsilon-1}$. Thus, the production efficiency effect can be rewritten as:

$$dQ_{ZH} + \tau_{XH}p_{HH}(i)y\frac{\varepsilon}{\varepsilon-1}dN_H = \left(\tau_{XH}\tau_{CH}\frac{\varepsilon}{\varepsilon-1} - 1\right)(y+f)dN_H \tag{62}$$

D Production Taxes

In this section we set $\tau_{IH} = \tau_{IF} = \tau_{XH} = \tau_{XF} = 1$ and we prove the propositions and the lemmata of section 4.

Lemma A1: Policy Incentives for Production Subsidies. *Assume* $\frac{\varepsilon-1}{\varepsilon} \leq \tau_{CH} = \tau_{CF} \leq 1$. *Then, when there is a symmetric increase in the production subsidies in Home and Foreign, production efficiency effects are positive. At the same time, when there is an unilateral increase in the production subsidy in Home, the production efficiency effect is positive in the domestic economy and negative in the foreign one, while domestic terms-of-trade effects are negative. Formally,*

(1) Let $d\tau_{CH} = d\tau_{CF} < 0$, then $\frac{(\tau_{CH}\frac{\varepsilon}{\varepsilon-1}-1)(y+f)dN_H}{I_H} = \frac{(\tau_{CF}\frac{\varepsilon}{\varepsilon-1}-1)(y+f)dN_F}{I_F} > 0$.

(2) Let $d\tau_{CH} < 0$, $\frac{(\tau_{CH}\frac{\varepsilon}{\varepsilon-1}-1)(y+f)dN_H}{I_H} > 0$, $\frac{(\tau_{CF}\frac{\varepsilon}{\varepsilon-1}-1)(y+f)dN_F}{I_F} < 0$ and $\frac{dP_{HH}\tau C_{FH} - dP_{FF}\tau C_{HF}}{I_H} < 0$.

Proof of Lemma A1.

(1) By imposing symmetry we obtain:

$$dN_H = dN_F = \frac{\partial N_H}{\partial \tau_{CH}} d\tau_{CH} + \frac{\partial N_H}{\partial \tau_{CF}} d\tau_{CF} = \left(\frac{\partial N_H}{\partial \tau_{CH}} + \frac{\partial N_F}{\partial \tau_{CH}}\right) d\tau_{CH} = \left(\frac{\partial N_H}{\partial \tau_{CF}} + \frac{\partial N_F}{\partial \tau_{CF}}\right) d\tau_{CF}$$

Then, (1) follows from the fact that if $\tau_{CH} = \tau_{CF} = \tau_C \leq 1$:

$$\frac{\partial N_H}{\partial \tau_{CH}} + \frac{\partial N_F}{\partial \tau_{CH}} = \frac{\partial N_H}{\partial \tau_{CF}} + \frac{\partial N_F}{\partial \tau_{CF}} = -\frac{L(1-\alpha)\alpha}{f\varepsilon[\alpha - (\alpha-1)\tau_C]^2} < 0 \tag{63}$$

which implies that $dN_H > 0$ and $dN_F > 0$ as long as $d\tau_{CH} < 0$ and $d\tau_{CF} < 0$.

(2) Observe that for $\frac{\varepsilon-1}{\varepsilon} \leq \tau_{CH} = \tau_{CF} \leq 1$, we have that $(\tau_{CH}\frac{\varepsilon}{\varepsilon-1} - 1)(y + f) > 0$. Hence the domestic (Foreign) production efficiency effect has the same sign as dN_H (dN_F). Moreover, once we impose symmetry, terms of trade effects in (31) can be written as:

$$\frac{\left(\frac{\varepsilon}{\varepsilon-1} N_H^{-\frac{1}{\varepsilon-1}} d\tau_{CH} - \frac{\tau_{CH}}{\varepsilon-1}\frac{\varepsilon}{\varepsilon-1} N_H^{-\frac{\varepsilon}{\varepsilon-1}} dN_H + \frac{\tau_{CF}}{\varepsilon-1}\frac{\varepsilon}{\varepsilon-1} N_F^{-\frac{\varepsilon}{\varepsilon-1}} dN_F\right)\tau C_{HF}}{I_H} \tag{64}$$

Thus, to prove (2) it suffices to show that as long as $d\tau_C < 0$, $dN_H = \frac{\partial N_H}{\partial \tau_{CH}} d\tau_{CH} > 0$ and $dN_F = \frac{\partial N_F}{\partial \tau_{CH}} d\tau_{CH} < 0$. This follows from the fact that if $\tau_{CH} = \tau_{CF} = \tau_C \leq 1$, then:

$$\frac{\partial N_H}{\partial \tau_{CH}} = \frac{L\alpha\left[\tau^2\left(\alpha^2 + (1-\alpha^2)\tau_C\right) + \tau^{\varepsilon+1}\left(2(1-\alpha)(\varepsilon-1)\tau_C + \alpha(2\varepsilon-1)\right) + (1-\alpha)\tau^{2\varepsilon}\left((1-\alpha)\tau_C + \alpha\right)\right]}{f\varepsilon\left(\tau^\varepsilon - \tau\right)\left[\alpha - (\alpha-1)\tau_C\right]^2\left[\alpha(\tau+\tau^\varepsilon)(\tau_C - 1) - \tau_C(\tau^\varepsilon - \tau)\right]} < 0$$

$$\frac{\partial N_F}{\partial \tau_{CH}} = \frac{L\alpha\tau\left[\alpha(\tau^\varepsilon - \tau) + \tau^\varepsilon\left(2(\alpha-1)\varepsilon\tau_C - 2\alpha\varepsilon\right)\right]}{f\varepsilon\left(\tau^\varepsilon - \tau\right)\left[\alpha - (\alpha-1)\tau_C\right]^2\left[\alpha(\tau+\tau^\varepsilon)(\tau_C - 1) - \tau_C(\tau^\varepsilon - \tau)\right]} > 0$$

In fact, the denominator of both expressions is negative whenever $\tau_C \leq 1$. The numerator of the first expression is always positive being the sum of only positive terms. For the numerator of the second expression to be positive we would need $\tau_C < \frac{\alpha(1-\tau^{1-\varepsilon}-2\varepsilon)}{2(1-\alpha)\varepsilon}$, which is not possible given that $\tau_C \geq 0$ by definition.

■

Proposition 2: Cooperative Production Subsidy. *The optimal cooperative production subsidy is set to exactly offset the price markup generated by monopolistic competition. This subsidy implements a symmetric equilibrium with the first-best number of varieties. Formally,*

(1) $\tau_C^{Coop} = \frac{\varepsilon-1}{\varepsilon}$ and $N^{Coop} = N^{FB}$.

Proof of Proposition 2. By setting $\tau_C^{Coop} = \frac{\varepsilon-1}{\varepsilon}$ in both countries, the cooperative policy maker exactly eliminates the price markup charged by the monopolistic firms in the differentiated sector. Indeed, from equation (11) we see that individual domestic varieties are now priced at their marginal costs i.e., $p_{HH}(i) = 1$ and $p_{FH}(i) = \tau$ and the same holds for the foreign country. Substituting $\tau_{CH} = \tau_{CF} = \frac{\varepsilon-1}{\varepsilon}$ into equation (41), we obtain $N_H = N_F = \frac{\alpha L}{f(\varepsilon-1+\alpha)} \equiv N^{Coop}$. This coincides with N^{FB} of Proposition 1. ■

Lemma 1: Unilaterally Set Production Subsidies. *The optimal unilateral deviation entails a reduction in the production subsidy when starting from the efficient allocation. When starting from the free trade allocation, the optimal unilateral deviation entails a production subsidy. Formally,*

(1) If $\tau_{CH} = \tau_{CF} = \frac{\varepsilon-1}{\varepsilon}$, $\frac{\partial V_H}{\partial \tau_{CH}} > 0$.

(2) If $\tau_{CH} = \tau_{CF} = 1$, $\frac{\partial V_H}{\partial \tau_{CH}} < 0$.

Proof of Lemma 1.

(1) If $\tau_{CH} = \tau_{CF} = \frac{\varepsilon-1}{\varepsilon}$, then:

$$\frac{\partial V_H}{\partial \tau_{CH}} = \frac{\alpha\varepsilon^2\tau\left(\tau^\varepsilon + \tau\right)}{(\varepsilon-1)\left(\tau^\varepsilon - \tau\right)\left(\alpha(\tau+\tau^\varepsilon) + (\varepsilon-1)(\tau^\varepsilon - \tau)\right)} > 0$$

(2) If $\tau_{CH} = \tau_{CF} = 1$, then

$$\frac{\partial V_H}{\partial \tau_{CH}} = -\frac{\alpha\left((1-\alpha)\tau^\varepsilon + \tau(\alpha+\varepsilon-1)\right)}{(\varepsilon-1)\left(\tau^\varepsilon - \tau\right)} < 0 \tag{65}$$

■

Proposition 3: Nash-Equilibrium Production Subsidies. *In the Nash equilibrium both countries set a production subsidy. However, this subsidy is smaller than the one needed to implement the first-best allocation. The equilibrium number of varieties larger than in the free trade allocation, but lower than the first-best level. Formally,*

(1) $\tau_C^{coop} < \tau_C^{Nash} < 1$ and $N^{FT} < N^{Nash} < N^{FB}$.

Proof of Proposition 3. First, we prove that $\tau_C^{coop} < \tau_C^{Nash} < 1$. The Nash solution of this game will be symmetric due to the symmetry assumption for the two countries. Therefore, to derive τ_C^{Nash} it is enough to compute the best reply of Home, $\frac{\partial V_H(P_H(\tau_{CH},\tau_{CF}), I_H(\tau_{CH},\tau_{CF}))}{\partial \tau_{CH}} = 0$, and then impose symmetry, i.e., $\tau_{CH} = \tau_{CF} = \tau_C$. Here, $P_H(\tau_{CH}, \tau_{CF})$ is given by equation (36), which is implied by the equilibrium expressions for $N_H(\tau_{CH}, \tau_{CF})$ and $N_F(\tau_{CH}, \tau_{CF})$, equation (41). Moreover, $I_H(\tau_{CH}, \tau_{CF})$ is given by $L + (\tau_{CH} - 1)\varepsilon f N_H(\tau_{CH}, \tau_{CF})$. When doing so, we obtain a quadratic expression in τ_C^{Nash}:

$$a(\tau_C^{Nash})^2 + b\tau_C^{Nash} + c = 0 \tag{66}$$

where $a \equiv \alpha(1-\alpha)\varepsilon\tau^{\varepsilon}[(3-2\varepsilon-\alpha)\tau-(1-\alpha)\tau^{\varepsilon}]$, $b \equiv \alpha[(\varepsilon-1+\alpha)\tau^2+(1-\alpha)(\varepsilon-1-\alpha(2\varepsilon-1))\tau^{2\varepsilon}+(2\varepsilon-2+\alpha)(\varepsilon-1-\alpha(2\varepsilon-1))\tau^{1+\varepsilon}]$ and $c \equiv \alpha^2(\varepsilon-1)\tau^{\varepsilon}((2\varepsilon-1+\alpha)\tau+(1-\alpha)\tau^{\varepsilon})$. Note that $a<0$ and $c>0$. To prove that $a<0$ it suffices to see that:

(i) $\tau^{\varepsilon} > \tau \ \forall\varepsilon > 1$ and $\forall\tau > 1$;

(ii) $1-\alpha > 3-2\varepsilon-\alpha \ \forall\varepsilon > 1$.

Hence, (66) has two real solutions, one positive and one negative $\forall\varepsilon > 1$, $\alpha \in (0,1)$ and $\tau > 1$. Then, since $\tau_C^{Nash} \in [0,\infty)$, (66) implies that the Nash solution always exists and is unique. As a consequence:

(i) At $\tau_C = 1$ we have: $a\tau_C^2 + b\tau_C + c = -\alpha(\tau^{\varepsilon}-\tau)[(\varepsilon+\alpha-1)\tau+(1-\alpha)\tau^{\varepsilon}] < 0$, implying that $\tau_C^{Nash} < 1$ since $a < 0$.

(ii) At $\tau_C^{FB} = \frac{\varepsilon-1}{\varepsilon}$ we have: $a\tau_C^2 + b\tau_C + c = \frac{\alpha(\varepsilon-1)(\varepsilon+\alpha-1)\tau(\tau+\tau^{\varepsilon})}{\varepsilon} > 0$, implying $\tau_C^{FB} < \tau_C^{Nash}$.

Second, we show that $N^{FT} < N^{Nash} < N^{FB}$. This follows from $\tau_C^{Coop} < \tau_C^{Nash} < 1$ and $dN_H > 0$ since $\frac{\partial N_H}{\partial\tau_{CH}} + \frac{\partial N_F}{\partial\tau_{CH}} < 0 \ \forall\tau_{CH}$ such that $\frac{\varepsilon-1}{\varepsilon} \leq \tau_{CH} = \tau_{CF} \leq 1$ (see the proof of point (1) of Lemma A1 and condition (63)). ■

E Import Taxes

In this section, while retaining the assumption $\tau_{XH} = \tau_{XF} = 1$, we prove the propositions and the lemmata of section 5 where we allow for the use of an import tariff as the policy instrument.

Lemma A2: Policy Incentives for Import Subsidies/Tariffs. *If $\tau_{CH} = \tau_{CF} = 1$, then when there is a symmetric increase in the import subsidies in Home and in Foreign, production efficiency effects are positive and consumption wedges are negative. At the same time, if $\tau_{CH} = \tau_{CF} = 1$ and $\tau_{IH} = \tau_{IF} = 1$ when there is a unilateral increase in the import tariff at Home, the production efficiency effect is positive in the domestic economy and negative in the foreign one, while domestic terms-of-trade effects are negative. Instead, if $\tau_{CH} = \tau_{CF} = \frac{\varepsilon-1}{\varepsilon}$ and $\tau_{IH} = \tau_{IF} = 1$, a unilateral increase in the import subsidy at Home generates positive domestic terms-of-trade effects. Formally,*

(1) Let $\tau_{CH} = \tau_{CF} = 1$, $\tau_{IH} = \tau_{IF} \leq 1$ and $d\tau_{IH} = d\tau_{IF} < 0$, then $\frac{(\frac{\varepsilon}{\varepsilon-1}-1)(y+f)dN_H}{I_H} = \frac{(\frac{\varepsilon}{\varepsilon-1}-1)(y+f)dN_F}{I_F} > 0$, $\frac{(\tau_{IH}-1)\tau P_{FF}dC_{HF}}{I_H} < 0$ and $\frac{(\tau_{IF}-1)\tau P_{HH}dC_{FH}}{I_F} < 0$.

(2) Let $\tau_{CH} = \tau_{CF} = 1$, $\tau_{IH} = \tau_{IF} = 1$ and $d\tau_{IH} > 0$, then $\frac{(\frac{\varepsilon}{\varepsilon-1}-1)(y+f)dN_H}{I_H} > 0$, $\frac{(\frac{\varepsilon}{\varepsilon-1}-1)(y+f)dN_F}{I_F} < 0$ and $\frac{dP_{HH}\tau C_{FH}-dP_{FF}\tau C_{HF}}{I_H} < 0$.

(3) Let $\tau_{CH} = \tau_{CF} = \frac{\varepsilon-1}{\varepsilon}$, $\tau_{IH} = \tau_{IF} = 1$ and $d\tau_{IH} < 0$, then $\frac{dP_{HH}\tau C_{FH}-dP_{FF}\tau C_{HF}}{I_H} < 0$.

Proof of Lemma A2.

(1) By imposing symmetry we obtain:

$$dN_H = dN_F = \frac{\partial N_H}{\partial \tau_{IH}} d\tau_{IH} + \frac{\partial N_H}{\partial \tau_{IF}} d\tau_{IF} = \left(\frac{\partial N_H}{\partial \tau_{IH}} + \frac{\partial N_F}{\partial \tau_{IH}}\right) d\tau_{IH} = \left(\frac{\partial N_H}{\partial \tau_{IF}} + \frac{\partial N_F}{\partial \tau_{IF}}\right) d\tau_{IF}$$

$$dC_{HF} = dC_{FH} = \frac{\partial C_{HF}}{\partial \tau_{IH}} d\tau_{IH} + \frac{\partial C_{HF}}{\partial \tau_{IF}} d\tau_{IF} = \left(\frac{\partial C_{HF}}{\partial \tau_{IH}} + \frac{\partial C_{FH}}{\partial \tau_{IH}}\right) d\tau_{IH} = \left(\frac{\partial C_{HF}}{\partial \tau_{IF}} + \frac{\partial C_{FH}}{\partial \tau_{IF}}\right) d\tau_{IF}$$

Then, production efficiency effects are positive since if $\tau_{IH} = \tau_{IF} = \tau_I \leq 1$ and $\tau_{CH} = \tau_{CF} = 1$:

$$\frac{\partial N_H}{\partial \tau_{IH}} + \frac{\partial N_F}{\partial \tau_{IH}} = \frac{\partial N_H}{\partial \tau_{IF}} + \frac{\partial N_F}{\partial \tau_{IF}} = -\frac{L(1-\alpha)\alpha\tau\left((\varepsilon(1-\tau_I)+\tau_I)\,\tau^{\varepsilon}\tau_I^{\varepsilon} + \tau\tau_I\right)}{f\varepsilon\tau_I\left(\tau_I\tau(1-\alpha) + \tau^{\varepsilon}\tau_I^{\varepsilon} + \alpha\tau\right)^2} < 0 \quad (67)$$

At the same time, consumption wedges are negative since if $\tau_{IH} = \tau_{IF} = \tau_I \leq 1$ and $\tau_{CH} = \tau_{CF} = 1$:

$$\frac{\partial C_{HF}}{\partial \tau_{IH}} + \frac{\partial C_{FH}}{\partial \tau_{IH}} = \frac{\partial C_{HF}}{\partial \tau_{IF}} + \frac{\partial C_{FH}}{\partial \tau_{IF}}$$
$$= -\frac{f\varepsilon\left((1-\alpha)\tau^2\tau_I + \tau^{\varepsilon}\tau_I^{\varepsilon}\left((\varepsilon-\alpha)\tau + (\varepsilon-1)\tau^{\varepsilon}\tau_I^{\varepsilon}\right)\right)\left(\frac{P_{HH}}{\tau\tau_I}\frac{\varepsilon-1}{\varepsilon}\right)^{-\varepsilon}}{\tau_I\left(\tau^{\varepsilon}\tau_I^{\varepsilon} + \tau\right)^2\left(\alpha\tau + (1-\alpha)\tau\tau_I + \tau^{\varepsilon}\tau_I^{\varepsilon}\right)} < 0$$

which imply that $dC_H > 0$ and $dC_F > 0$ as long as $d\tau_{CH} < 0$ and $d\tau_{CF} < 0$.

(2) First, note that domestic (Foreign) production efficiency effects have the same sign as dN_H (dN_F), given that $(\frac{\varepsilon}{\varepsilon-1} - 1)(y + f) > 0$. Moreover, once we impose symmetry, terms of trade effects in (33) can be written as:

$$\frac{\left(-\frac{1}{\varepsilon-1}\frac{\varepsilon}{\varepsilon-1}N_H^{-\frac{\varepsilon}{\varepsilon-1}}dN_H + \frac{1}{\varepsilon-1}\frac{\varepsilon}{\varepsilon-1}N_F^{-\frac{\varepsilon}{\varepsilon-1}}dN_F\right)\tau C_{HF}}{I_H}$$

Thus, to prove (2) it suffices to show that if $\tau_{IH} = \tau_{IF} = 1$ and $\tau_{CH} = \tau_{CF} = 1$, then $dN_H = \frac{\partial N_H}{\partial \tau_{IH}} d\tau_{IH} > 0$ and $dN_F = \frac{\partial N_F}{\partial \tau_{IH}} < 0$ as long as $d\tau_{IH} > 0$. Indeed, in this case:

$$\frac{\partial N_H}{\partial \tau_{IH}} = \frac{L\alpha\tau^{\varepsilon+1}\left[(1+\varepsilon-\alpha)\tau + (\alpha+\varepsilon-1)\tau^{\varepsilon}\right]}{f\varepsilon(\tau-\tau^{\varepsilon})^2(\tau+\tau^{\varepsilon})} > 0$$

$$\frac{\partial N_F}{\partial \tau_{IH}} = -\frac{L\alpha\tau\left[(1-\alpha)\tau^2 + \varepsilon\tau^{2\varepsilon} + (\alpha+\varepsilon-1)\tau^{\varepsilon+1}\right]}{f\varepsilon(\tau-\tau^{\varepsilon})^2(\tau+\tau^{\varepsilon})} < 0$$

which means that domestic production efficiency effects are positive, Foreign ones are negative and domestic terms-of-trade effects are negative, since $dN_H = \frac{\partial N_H}{\partial \tau_{IH}} d\tau_{IH} > 0$ and $dN_F = \frac{\partial N_F}{\partial \tau_{IH}} d\tau_{IH} < 0$ if $d\tau_{IH} > 0$.

(3) Similarly to point (2), to prove (3) we need to show that if $\tau_{IH} = \tau_{IF} = 1$ and $\tau_{CH} =$

$\tau_{CF} = \frac{\varepsilon-1}{\varepsilon}$, then:

$$\frac{\partial N_H}{\partial \tau_{IH}} = \frac{L\alpha\tau(\varepsilon-1)\left[(1-\alpha)\alpha\tau^2 + (\alpha+\varepsilon-1)^2\tau^{2\varepsilon} + (\varepsilon^2+\alpha-1)\tau^{1+\varepsilon}\right]}{f(\alpha+\varepsilon-1)^2\left[\alpha(\tau+\tau^\varepsilon) + (\varepsilon-1)(\tau^\varepsilon-\tau)\right](\tau^{2\varepsilon}-\tau^2)} > 0$$

$$\frac{\partial N_F}{\partial \tau_{IH}} = -\frac{L\alpha\tau(\varepsilon-1)\left[(\varepsilon-1)(1-\alpha)\tau^2 + \varepsilon(\alpha+\varepsilon-1)\tau^{2\varepsilon} + ((\varepsilon-1)^2+\alpha(2\varepsilon-1))\tau^{\varepsilon+1}\right]}{f(\alpha+\varepsilon-1)^2\left[\alpha(\tau+\tau^\varepsilon) + (\varepsilon-1)(\tau^\varepsilon-\tau)\right](\tau^{2\varepsilon}-\tau^2)} < 0$$

■

Proposition 4: Cooperative Import Subsidy. *If $\tau_{CH} = \tau_{CF} = \frac{\varepsilon-1}{\varepsilon}$, the cooperative policy maker refrains from using taxes on imports and the number of varieties equals the first-best level. If $\tau_{CH} = \tau_{CF} = 1$, the cooperative policy maker finds it optimal to subsidize imports. The number of varieties is larger than in the free trade allocation, but remains lower than the first-best level. Formally,*

(1) If $\tau_{CH} = \tau_{CF} = \frac{\varepsilon-1}{\varepsilon}$, then $\tau_I^{Coop} = 1$ and $N^{Coop} = N^{FB}$.

(2) If $\tau_{CH} = \tau_{CF} = 1$, then $\frac{\varepsilon-1}{\varepsilon} < \tau_I^{Coop} < 1$ and $N^{FT} < N^{Coop} < N^{FB}$.

Proof of Proposition 4. In the case of tariffs, the cooperative policy maker maximizes:

$$\max_{\tau_{IH},\tau_{IF}} V_H(P_H(\tau_{IH},\tau_{IF}), I_H(\tau_{IH},\tau_{IF})) + V_F(P_F(\tau_{IH},\tau_{IF}), I_F(\tau_{IH},\tau_{IF})) \tag{68}$$

where $P_H(\tau_{IH},\tau_{IF})$ is given by equation (36) once we substitute in $N_H(\tau_{IH},\tau_{IF})$ and $N_H(\tau_{IH},\tau_{IF})$ as implicitly determined by equation (22). $I_H(\tau_{IH},\tau_{IF})$ is equal to $L+(\tau_{CH}-1)N_H(\tau_{IH},\tau_{IF})\varepsilon f+(\tau_{IH}-1)\tau P_{FF}(\tau_{IH},\tau_{IF})C_{HF}(\tau_{IH},\tau_{IF})$ where $P_{FF}(\tau_{IH},\tau_{IF}) = \frac{\varepsilon}{\varepsilon-1}\tau_{CF}N_F(\tau_{IH},\tau_{IF})^{\frac{1}{1-\varepsilon}}$, $C_{HF}(\tau_{IH},\tau_{IF}) = P_{HF}(\tau_{IH},\tau_{IF})^{-\varepsilon}P_H(\tau_{IH},\tau_{IF})^{\varepsilon}C_H(\tau_{IH},\tau_{IF})$, $P_{HF}(\tau_{IH},\tau_{IF}) = \frac{\varepsilon}{\varepsilon-1}\tau\tau_{IH}\tau_{CF}N_F(\tau_{IH},\tau_{IF})^{\frac{1}{1-\varepsilon}}$ and finally $C_H(\tau_{IH},\tau_{IF})$ is given by its equilibrium value in equation (37). Symmetric conditions apply to foreign variables.

(1) To prove the first part of the proposition it suffices to show that if $\tau_{CH} = \tau_{CF} = \frac{\varepsilon-1}{\varepsilon}$, the cooperative policy maker finds it optimal to set $\tau_{IH} = \tau_{IF} = 1$.

By differentiating (68) with respect to τ_{IH} and imposing symmetry, i.e. $\tau_{IH} = \tau_{IF} = \tau_I$, the first-order condition can be written as:

$$-\frac{\alpha\varepsilon\tau\,(\tau_I-1)\left((\alpha+\varepsilon-1)\tau^{2\varepsilon}\tau_I^{2\varepsilon} + (1-\alpha)\tau^2\tau_I + \varepsilon\tau^{\varepsilon+1}\tau_I^{\varepsilon}\right)}{\tau_I\left(\tau^\varepsilon\tau_I^\varepsilon+\tau\right)\left(\tau^\varepsilon\tau_I^\varepsilon+\tau\tau_I\right)\left((\alpha+\varepsilon-1)\tau^\varepsilon\tau_I^\varepsilon + (1-\alpha)(\varepsilon-1)\tau\tau_I + \alpha\varepsilon\tau\right)} = 0$$

Then, it is easy to verify that this condition is satisfied iff $\tau_I = 1$.

(2) To prove the second part of the proposition we follow three steps: (i) first, we show that if $\tau_{CH} = \tau_{CF} = 1$, no cooperative solution exists for $\tau_I > 1$; (ii) second, we show that if $\tau_{CH} = \tau_{CF} = 1$, there exists a solution for $\tau_I < 1$; (iii) third, we prove that if $\tau_{CH} = \tau_{CF} = 1$ $\tau_I^{Coop} < 1$ and $N^{FT} < N_I^{Coop} < N^{FB}$. If $\tau_{CH} = \tau_{CF} = 1$, then by taking the derivative of (68) with respect to τ_{IH} and imposing symmetry, the first-order

condition can be written as:

$$\frac{A_I^{Coop}(\tau_I)}{B_I^{Coop}(\tau_I)} = 0$$

where:

$$\begin{aligned}
A_I^{Coop}(\tau_I) &\equiv \alpha\tau(\tau^{\varepsilon+1}\tau_I^{\varepsilon}\left(\tau_I\left(2\alpha-\varepsilon^2+\varepsilon-2\right)+(\varepsilon-1)\varepsilon\right) \\
&\quad + \tau^{2\varepsilon}\tau_I^{2\varepsilon}\left(\varepsilon(\alpha+\varepsilon-2)-(\varepsilon-1)\tau_I(\alpha+\varepsilon-1)\right)+(\alpha-1)\tau^2\tau_I\left(\varepsilon\tau_I-\varepsilon+1\right)) \\
B_I^{Coop}(\tau_I) &\equiv (\varepsilon-1)\tau_I\left(\tau^{\varepsilon}\tau_I^{\varepsilon}+\tau\right)\left(\tau^{\varepsilon}\tau_I^{\varepsilon}+\tau\tau_I\right)\left(\tau_I(\tau-\alpha\tau)+\tau^{\varepsilon}\tau_I^{\varepsilon}+\alpha\tau\right)
\end{aligned}$$

(i) At the optimum, it must be that $A_I^{Coop}(\tau_I) = A_I^{Coop}(\tau_I^{Coop}) = 0$. To prove that no cooperative solution exists for $\tau_I > 1$ it suffices to notice that all terms in $A_I^{Coop}(\tau_I)$ are strictly negative for $\tau_I > 1$ and thus $A_I^{Coop}(\tau_I)$ has no zeros for $\tau_I > 1$.

(ii) In order to prove that a cooperative solution exists for $\tau_I < 1$, consider that: (a) $A_I^{Coop}(\tau_I)$ is a continuous function in τ_I; (b) $A_I^{Coop}(1) = -(1-\alpha)\alpha\tau\left(\tau^{\varepsilon}+\tau\right)^2 < 0$; (c) $A_I^{Coop}(0) = 0$ and $\partial A_I^{Coop}(0)/\partial\tau_I = (1-\alpha)\alpha(\varepsilon-1)\tau^3 > 0$. Then, by the intermediate value theorem there exists a value $\tau_I \in (0,1)$ such that $A_I^{Coop}(\tau_I) = 0$.

(iii) Finally, we need to prove that if $\tau_{CH} = \tau_{CF} = 1$, it holds that $\tau_I^{Coop} < 1$, $N^{FT} < N^{Coop} < N^{FB}$. We do this in several steps.

   (a) If $0 < \tau_I < 1$, by symmetrically increasing the subsidy on imports in both countries the cooperative policy maker increases the total number of varieties. Indeed, by (67) at the symmetric equilibrium $dN_H = dN_F > 0$ since $\frac{\partial N_H}{\partial\tau_{IH}} + \frac{\partial N_H}{\partial\tau_{IF}}) < 0$ $d\tau_{IH} = d\tau_{IF} < 0$. Thus, $dN_H > 0$ together with point (ii) – proving that the cooperative solution entails $\tau_I < 1$ – imply that $N^{FT} < N^{Coop}$. Then, we are left with the comparison between the cooperative solution and the first-best. After imposing $\tau_{CH} = \tau_{CF} = \tau_{XH} = \tau_{XF} = 1$ and $\tau_{IH} = \tau_{IF}$ in (41), we compute the number of varieties produced in each country in the symmetric equilibrium: $N_H(\tau_I) = \frac{L\alpha(\tau+(\tau\tau_I)^{\varepsilon})}{f\varepsilon(\alpha\tau+\tau(1-\alpha)\tau_I+(\tau\tau_I)^{\varepsilon})}$ and we observe that $\lim_{\tau_I\to 0} N_H(\tau_I) = \frac{L}{f\varepsilon}$ where $\frac{L}{f\varepsilon} > \frac{L\alpha}{f(\varepsilon+\alpha-1)} = N^{FB}$ since $\varepsilon > 1$ and $0 < \alpha < 1$. As a consequence, there exists a τ_I small enough so that the cooperative policy maker can implement the first-best number of varieties. The question is whether he wants to do so.

   (b) Let $\tau_I^{Coop} = f(\alpha,\varepsilon,\tau)$ and $\tau_I^{FB} = g(\alpha,\varepsilon,\tau)$ be, respectively, the solution to the cooperative problem and the subsidy implementing the first-best level of number of varieties. Thus, τ_I^{Coop} is such that $A_I^{Coop}(\tau_I^{Coop}) = 0$, while τ_I^{FB} is such that $N(\tau_I^{FB}) = \frac{L\alpha(\tau+(\tau\tau_I^{FB})^{\varepsilon})}{f\varepsilon(\alpha\tau+\tau(1-\alpha)\tau_I^{FB}+(\tau\tau_I^{FB})^{\varepsilon})} = \frac{L\alpha}{f(\varepsilon+\alpha-1)} = N^{FB}$. Though it is not possible to find an explicit solution for τ_I^{FB}, the condition $N_H(\tau_I^{FB}) = N^{FB}$ simplifies to $(\tau\tau_I^{FB})^{\varepsilon} = -\varepsilon\tau\tau_I^{FB} + \tau(\varepsilon-1)$. If we substitute this condition into $A_I^{Coop}(\tau_I) = 0$ we obtain a cubic expression in τ_I. The solutions are $\tau_I = \{\frac{\varepsilon-1}{\varepsilon}, 1, 1\}$. However, none of these solves $(\tau\tau_I)^{\varepsilon} = -\varepsilon\tau\tau_I + \tau(\varepsilon-1)$. More precisely, they all imply a level of subsidy on imports smaller than what needed to implement the first-best level of varieties. Thus, we conclude that there is no intersection between the set of τ_I^{Coop} and the set of τ_I^{FB}.

   (c) The last step is to prove that $\tau_I^{FB} < \tau_I^{Coop}$ always. From (67) it will then follow that $N_I^{Coop} < N_{FB}$. To this purpose, note that f and g are two continuous functions in the space $\{0 < \alpha < 1, \tau > 1, \varepsilon > 1\}$. This is so since, by the implicit function theorem, we can compute the derivatives of τ_I^{FB} and τ_I^{Coop} with respect to the three parameters and the derivative always exists in such parametric space. In point (b) we proved that there is no intersection between g and f. It must then be that one always lies on top of the other, i.e., it either always holds that $\tau_I^{FB} < \tau_I^{Coop}$ or the other way around. We evaluate both functions at $\{\alpha = 0.5, \varepsilon = 2, \tau = 1.5\}$ and find $\tau_I^{FB} = 0.39 < 0.63 = \tau_I^{Coop}$.[28] Thus, the cooperative import subsidy is always smaller than the one needed to implement the first-best number of varieties.

■

Lemma 2: Unilaterally Set Import Tariffs/Subsidies. *Let $\tau_{IH} = \tau_{IF} = 1$. The optimal unilateral deviation entails an import subsidy when starting from the first-best allocation implemented by a production subsidy, and an import tariff when starting from the free trade allocation. Formally,*

(1) If $\tau_{CH} = \tau_{CF} = \frac{\varepsilon-1}{\varepsilon}$, then $\frac{\partial V_H}{\partial \tau_{IH}} < 0$.

(2) If $\tau_{CH} = \tau_{CF} = 1$, then $\frac{\partial V_H}{\partial \tau_{IH}} > 0$.

Proof of Lemma 2.

(1) If $\tau_{IH} = \tau_{IF} = 1$ and $\tau_{CH} = \tau_{CF} = \frac{\varepsilon-1}{\varepsilon}$, it is easy to show that:

$$\frac{\partial V_H}{\partial \tau_{IH}} = -\frac{\alpha\tau^2\left((\alpha+2\varepsilon-1)\tau^\varepsilon + (1-\alpha)\tau\right)}{\left((\alpha(\tau^\varepsilon+\tau) + (\varepsilon-1)(\tau^\varepsilon-\tau)\right)\left(\tau^{2\varepsilon}-\tau^2\right)} < 0.$$

(2) If $\tau_{IH} = \tau_{IF} = 1$ and $\tau_{CH} = \tau_{CF} = 1$, it is easy to show that:

$$\frac{\partial V_H}{\partial \tau_{IH}} = \frac{\alpha\tau\left((\alpha+\varepsilon-1)\tau^\varepsilon + (1-\alpha)\tau\right)}{(\varepsilon-1)\left(\tau^{2\varepsilon}-\tau^2\right)} > 0.$$

■

Proposition 5: Nash-Equilibrium Import Tariffs/Subsidies. *When starting from the first-best allocation, the Nash-equilibrium policy consists of an import subsidy, implying more varieties than the first-best allocation. In contrast, when starting from the free trade allocation, the Nash-equilibrium entails a tariff, implying less varieties than the free trade allocation. Formally,*

(1) If $\tau_{CH} = \tau_{CF} = \frac{\varepsilon-1}{\varepsilon}$, then $\tau_I^{Nash} < 1$ and $N^{FT} < N^{FB} < N^{Nash}$.

(2) If $\tau_{CH} = \tau_{CF} = 1$, then there exists a $\tau_I^{Nash} > 1$ such that $N^{Nash} < N^{FT} < N^{FB}$.

[28] The other solutions are either negative or zero, thus we exclude them since $\tau_I > 0$.

Proof of Proposition 5. In the case of tariffs, the non-cooperative policy maker maximizes:

$$\max_{\tau_{IH}} V_H(P_H(\tau_{IH}, \tau_{IF}), I_H(\tau_{IH}, \tau_{IF})) \tag{69}$$

where $P_H(\tau_{IH}, \tau_{IF})$ is given by equation (36) once we substitute in $N_H(\tau_{IH}, \tau_{IF})$ and $N_F(\tau_{IH}, \tau_{IF})$ as implicitly determined by equation (22). $I_H(\tau_{IH}, \tau_{IF})$ is equal to $L+(\tau_{CH}-1)N_H(\tau_{IH}, \tau_{IF})\varepsilon f+$ $(\tau_{IH}-1)\tau P_{FF}(\tau_{IH}, \tau_{IF})C_{HF}(\tau_{IH}, \tau_{IF})$ where $P_{FF}(\tau_{IH}, \tau_{IF}) = \frac{\varepsilon}{\varepsilon-1}\tau_{CF}(N_F(\tau_{IH}, \tau_{IF}))^{\frac{1}{1-\varepsilon}}$, $C_{HF}(\tau_{IH}, \tau_{IF})$ $= P_{HF}(\tau_{IH}, \tau_{IF})^{-\varepsilon}P_H(\tau_{IH}, \tau_{IF})^{\varepsilon}C_H(\tau_{IH}, \tau_{IF})$, $P_{HF}(\tau_{IH}, \tau_{IF}) = \frac{\varepsilon}{\varepsilon-1}\tau\tau_{IH}\tau_{CF}(N_F(\tau_{IH}, \tau_{IF}))^{\frac{1}{1-\varepsilon}}$ and finally $C_H(\tau_{IH}, \tau_{IF})$ is given by its equilibrium value in equation (37).

(1) If $\tau_{CH} = \tau_{CF} = \frac{\varepsilon-1}{\varepsilon}$ and $\tau_{IH} = \tau_{IF}$, the first-order condition of (69) with respect to τ_{IH} can be written as:

$$\frac{A_I^{Nash}(\tau_I)}{B_I^{Nash}(\tau_I)} = 0$$

where

$$\begin{aligned}
A_I^{Nash}(\tau_I) \equiv\ & \alpha(\varepsilon-2)\varepsilon^2\tau^{\varepsilon+3}\tau_I^{\varepsilon} + (\alpha-1)\varepsilon\left(\alpha+\varepsilon^2-1\right)\tau^{\varepsilon+3}\tau_I^{\varepsilon+2} \\
& + (\alpha-1)\left(\varepsilon^2+\varepsilon-1\right)(\alpha+\varepsilon-1)\tau^{2\varepsilon+2}\tau_I^{2\varepsilon+2} \\
& - (\alpha+\varepsilon-1)\left(\alpha\varepsilon+\alpha+\varepsilon^2+\varepsilon-1\right)\tau^{3\varepsilon+1}\tau_I^{3\varepsilon+1} \\
& + \left((1-2\alpha)\varepsilon^3+2(\alpha-1)\varepsilon^2-(\alpha-1)\alpha\varepsilon+(\alpha-1)^2\right)\tau^{\varepsilon+3}\tau_I^{\varepsilon+1} \\
& + \varepsilon(\alpha(\varepsilon-1)-1)(\alpha+\varepsilon-1)\tau^{2\varepsilon+2}\tau_I^{2\varepsilon} \\
& + \varepsilon(\alpha+\varepsilon-1)^2\tau^{3\varepsilon+1}\tau_I^{3\varepsilon} - \varepsilon(\alpha+\varepsilon-1)((2\alpha-1)\varepsilon+2)\tau^{2\varepsilon+2}\tau_I^{2\varepsilon+1} \\
& + \varepsilon(\alpha+\varepsilon-1)^2\tau^{4\varepsilon}\tau_I^{4\varepsilon} - \varepsilon(\alpha+\varepsilon-1)^2\tau^{4\varepsilon}\tau_I^{4\varepsilon+1} - (\alpha-1)^2(\varepsilon-1)\varepsilon\tau^4\tau_I^3 \\
& + (\alpha-1)(\varepsilon-1)\tau^4\tau_I^2(\alpha(2\varepsilon-1)-\varepsilon+1) + (1-\alpha)\alpha(\varepsilon-2)\varepsilon\tau^4\tau_I \\
B_I^{Nash}(\tau_I) \equiv\ & \tau_I\left(\tau^{\varepsilon}\tau_I^{\varepsilon}+\tau\tau_I\right)\left(\tau^{2\varepsilon}\tau_I^{2\varepsilon}-\tau^2\right)((\alpha+\varepsilon-1)\tau^{\varepsilon}\tau_I^{\varepsilon}+(\alpha-1)(\varepsilon-1)\tau\tau_I-\alpha(\varepsilon-2)\tau) \\
& ((\alpha+\varepsilon-1)\tau^{\varepsilon}\tau_I^{\varepsilon}+\tau\tau_I(-\alpha\varepsilon+\alpha+\varepsilon-1)+\alpha\varepsilon\tau)
\end{aligned}$$

The first part of Proposition 5 is proved by showing that if $\tau_{CH} = \tau_{CF} = \frac{\varepsilon-1}{\varepsilon}$: (i) there is no solution of the Nash equilibrium of the non-cooperative policy game for $\tau_I > 1$; (ii) if $\varepsilon > 2$, there exists a solution of the non-cooperative policy game for $\tau_I < 1$; (iii) $N^{Nash} > N^{FB} > N^{FT}$.

(i) In order to show that no Nash equilibrium exists, we need to prove that there are no zeros of $A_I^{Nash}(\tau_I)$ for $\tau_I > 1$. This is so because: (a) A_I^{Nash} is a second-order polynomial in α; (b) if $\alpha = 0$ or $\alpha = 1$, $A_I^{Nash}(\tau_I) < 0$; (c) $\frac{\partial A_I^{Nash}(\tau_I)}{\partial \tau_I}|_{\alpha=0} < 0$.

(a) It is straightforward to see that A_I^{Nash} is quadratic in α.

(b) If $\alpha = 0$ and $\tau_I > 1$

$$\begin{aligned}
A_I^{Nash}(\tau_I) = & -(\varepsilon-1)(\tau^{\varepsilon+3}\tau_I^{\varepsilon+1}\left((\varepsilon+1)\varepsilon\tau_I-\varepsilon(\varepsilon-1)+1\right) \\
& + \tau^{2\varepsilon+2}\tau_I^{2\varepsilon}\left(\left(\varepsilon^2+\varepsilon-1\right)\tau_I^2-(\varepsilon-2)\varepsilon\tau_I+\varepsilon\right) \\
& + \tau^{3\varepsilon+1}\tau_I^{3\varepsilon}\left(\left(\varepsilon^2+\varepsilon-1\right)\tau_I-(\varepsilon-1)\varepsilon\right) \\
& + \tau^{4\varepsilon}\tau_I^{4\varepsilon}(\varepsilon-1)\varepsilon\left(\tau_I-1\right) \\
& + \tau^4\tau_I^2\left(\varepsilon\left(\tau_I-1\right)+1\right)) < 0
\end{aligned}$$

If $\alpha = 1$ and $\tau_I > 1$

$$A_I^{Nash}(\tau_I) = -\varepsilon^2 \tau^\varepsilon \tau_I^\varepsilon \left(\tau^\varepsilon \tau_I^\varepsilon + \tau\right) \left(2\tau^{\varepsilon+1}\tau_I^{\varepsilon+1} + \varepsilon\left(\tau_I - 1\right)\tau^{2\varepsilon}\tau_I^{2\varepsilon} + \tau^2\left(\varepsilon\tau_I - \varepsilon + 2\right)\right) < 0$$

(c) To see why $\partial A_I^{Nash}(\tau_I)/\partial\tau_I < 0$, first consider that if $\alpha = 0$:

$$\frac{\partial A_I^{Nash}(\tau_I)}{\partial \tau_I} = \tau_I^{4\varepsilon}\tau^{4\varepsilon}\kappa_1 + \tau^{3\varepsilon+1}\tau_I^{3\varepsilon}\kappa_2 + \tau^{2\varepsilon+2}\tau_I^{2\varepsilon}\kappa_3 + \tau^{\varepsilon+3}\tau_I^{\varepsilon}\kappa_4 + \tau^4\tau_I\kappa_5$$

where:

$$\begin{aligned}
\kappa_1 &\equiv -2(\varepsilon-1)\varepsilon\left(\tau_I - 1\right) \\
\kappa_2 &\equiv -\left(\left(2\varepsilon^2 + \varepsilon - 2\right)\tau_I - 2(\varepsilon-1)\varepsilon\right) \\
\kappa_3 &\equiv (\varepsilon-2)\left(\varepsilon^2+\varepsilon-1\right)\tau_I^2 + ((3-2\varepsilon)\varepsilon - 2)\varepsilon\tau_I + (\varepsilon-2)\varepsilon^2 \\
\kappa_4 &\equiv \tau_I\left[\left(\varepsilon^2-2\right)\varepsilon\tau_I - 2(\varepsilon-1)\varepsilon^2 + \varepsilon - 2\right] + (\varepsilon-2)\varepsilon^2 \\
\kappa_5 &\equiv (\varepsilon-1)\tau_I\left(2\varepsilon\tau_I - 3\varepsilon + 2\right) + (\varepsilon-2)\varepsilon
\end{aligned}$$

First, we show that $\partial A_I^{Nash}(\tau_I)/\partial\tau_I < 0$ for $\varepsilon < 2$. Under this assumption $\kappa_1 < 0$, $\kappa_2 < 0$, $\kappa_3 < 0$ and $\kappa_3 - \kappa_4 < 0$. In this case it is sufficient to show that $\tau_I^{4\varepsilon}\tau^{4\varepsilon}\kappa_1 + \tau^{3\varepsilon+1}\tau_I^{3\varepsilon}\kappa_2 + \tau^4\tau_I\kappa_5 < 0$. Note that $\tau_I^{4\varepsilon}\tau^{4\varepsilon}\kappa_1 + \tau^{3\varepsilon+1}\tau_I^{3\varepsilon}\kappa_2 + \tau^4\tau_I\kappa_5 < \delta(\tau_I)$ where $\delta(\tau_I) \equiv (\kappa_1+\kappa_2)\tau_I^{2\varepsilon} + \kappa_5$. It can be shown that $\delta'(\tau_I) < 0$. It follows then from $\delta(\tau_I) = -2\varepsilon$ at $\tau_I = 1$ that $\delta(\tau_I) < 0$.
Second, we show that $\partial A_I^{Nash}(\tau_I)/\partial\tau_I < 0$ for $\varepsilon > 2$. Under this assumption $\kappa_1 < 0$, $\kappa_2 < 0$ and $\kappa_5 < 0$. Therefore, in this case it suffices to show that $\tau_I^{4\varepsilon}\tau^{4\varepsilon}\kappa_1 + \tau^{2\varepsilon+2}\tau_I^{2\varepsilon}\kappa_3 < 0$ and $\tau^{3\varepsilon+1}\tau_I^{3\varepsilon}\kappa_2 + \tau^{\varepsilon+3}\tau_I^{\varepsilon}\kappa_4 + \tau^4\tau_I\kappa_5 < 0$ or alternatively that $\delta_1(\tau_I) \equiv \kappa_1\tau_I^{2\varepsilon} + \kappa_3 < 0$ and $\delta_2(\tau_I) \equiv \kappa_2\tau_I^{2\varepsilon} + \kappa_4 + \kappa_5 < 0$. These last conditions are always satisfied because at $\tau_I = 1$, $\delta_1(\tau_I) = 2 - 5\varepsilon$ and $\delta_2(\tau_I) = -2 - 3\varepsilon$ and it can be proved that $\delta_2^{'}(\tau_I) < 0$ and $\delta_1^{'}(\tau_I) < 0$.

(ii) This is equivalent to show that there is at least one zero of $A_I^{Nash}(\tau_I)$ for $\tau_I < 1$. A sufficient condition for the existence of a Nash solution is $\varepsilon > 2$. To see why this is the case, consider that: a) $A_I^{Nash}(\tau_I)$ is a continuous function in τ_I; b) $A_I^{Nash}(1) = -\tau(\alpha+\varepsilon-1)\left(\tau^\varepsilon+\tau\right)^2\left((1-\alpha)\tau + (\alpha+2\varepsilon-1)\tau^\varepsilon\right) < 0$; c) $A_I^{Nash}(0) = 0$ and $\partial A_I^{Nash}(0)/\partial\tau_I = (1-\alpha)\alpha(\varepsilon-2)\varepsilon\tau^4 > 0$ for $\varepsilon > 2$. Then, by the intermediate value theorem there exists a value $\tau_I \in (0,1)$ such that $A_I^{Nash}(\tau_I) = 0$.

(iii) To prove this statement recall that if $\tau_{CH} = \tau_{CF} = \frac{\varepsilon-1}{\varepsilon}$, then

$$\frac{\partial N_H}{\partial \tau_{IH}} + \frac{\partial N_H}{\partial \tau_{IF}} = -\frac{L(1-\alpha)\alpha(\varepsilon-1)\tau\left(\tau^\varepsilon\left(\varepsilon(1-\tau_I)+\tau_I\right)\tau_I^\varepsilon + \tau\tau_I\right)}{f\tau_I\left((\alpha+\varepsilon-1)\tau^\varepsilon\tau_I^\varepsilon + \tau\tau_I(1-\alpha)(\varepsilon-1) + \alpha\varepsilon\tau\right)^2} < 0$$

for all $\tau_I \leq 1$. We have already proven at point (i) and (ii) that when $\tau_{CH} = \tau_{CF} = \frac{\varepsilon-1}{\varepsilon}$, $\tau_I^{Nash} < 1$. As a consequence, $N^{Nash} > N^{FB} > N^{FT}$ since at the symmetric equilibrium $dN_H = \frac{\partial N_H}{\partial \tau_{IH}}d\tau_{IH} + \frac{\partial N_H}{\partial \tau_{IF}}d\tau_{IF} = (\frac{\partial N_H}{\partial \tau_{IH}} + \frac{\partial N_H}{\partial \tau_{IF}})d\tau_{IH} > 0$ for all $\tau_I \leq 1$ and $d\tau_{IH} = d\tau_{IF} < 0$.

(2) By taking the derivative of (69) with respect to τ_{IH} and imposing symmetry i.e., $\tau_{IH} =$

$\tau_{IF} = \tau_I$, the first-order condition evaluated at $\tau_{CH} = \tau_{CF} = 1$ can be written as:

$$\frac{A_I^{Nash}(\tau_I)}{B_I^{Nash}(\tau_I)} = 0$$

where

$$\begin{aligned}
A_I^{Nash}(\tau_I) \equiv\ & \alpha(\tau^{2\varepsilon+3}\tau_I^{2\varepsilon}(\tau_I\left((\alpha-1)(\varepsilon+1)\tau_I(\alpha+\varepsilon-1)-\alpha^2(2\varepsilon+1)-2\alpha(\varepsilon-1)\varepsilon+(\varepsilon-1)\varepsilon+1\right) \\
& +\alpha\varepsilon(\alpha+\varepsilon-1))+\varepsilon\tau^{\varepsilon+4}\left((\alpha-1)\tau_I-\alpha\right)\left(\varepsilon\tau_I-\varepsilon+1\right)\tau_I^{\varepsilon} \\
& -\varepsilon\tau^{3\varepsilon+2}\tau_I^{3\varepsilon}\left(\tau_I(\alpha+\varepsilon-1)-\alpha-\varepsilon\right) \\
& +(-\alpha-\varepsilon+1)\tau^{4\varepsilon+1}\left((\varepsilon-1)\tau_I-\varepsilon\right)\tau_I^{4\varepsilon}-(\alpha-1)\tau^5\tau_I\left((\alpha-1)\tau_I-\alpha\right)\left(\varepsilon\tau_I-\varepsilon+1\right)) \\
B_I^{Nash}(\tau_I) \equiv\ & (\varepsilon-1)\tau_I\left(\tau^{\varepsilon}\tau_I^{\varepsilon}+\tau\tau_I\right)\left(\tau^{2\varepsilon}\tau_I^{2\varepsilon}-\tau^2\right)\left((\alpha-1)\tau\tau_I+\tau^{\varepsilon}\tau_I^{\varepsilon}-\alpha\tau\right)\left(\tau_I(\tau-\alpha\tau)+\tau^{\varepsilon}\tau_I^{\varepsilon}+\alpha\tau\right)
\end{aligned}$$

To prove the second part of Proposition 5 we need to show that: (i) there exist at least one Nash equilibrium of the policy game for which $\tau_I^{Nash} > 1$; (ii) for such a $\tau_I^{Nash} > 1$, we have $N^{Nash} < N^{FT} < N^{FB}$

(i) To show this point consider that:
   (a) $A_I^{Nash}(\tau_I)$ is a continuous function of τ_I;
   (b) If $\tau_I = 1$ $A_I^{Nash} = \tau\left(\tau^{\varepsilon}-\tau\right)\left(\tau^{\varepsilon}+\tau\right)^2\left[(\alpha+\varepsilon-1)\tau^{\varepsilon}-\alpha\tau+\tau\right] > 0$;
   (c) If $\tau_I = \frac{\varepsilon}{\varepsilon-1}$:

$$\begin{aligned}
A_I^{Nash}\left(\frac{\varepsilon}{\varepsilon-1}\right) = & \\
& -\frac{\varepsilon\tau^2}{(\varepsilon-1)^3}\Big[(\varepsilon-1)\tau\left(\alpha(\varepsilon-\alpha)+\alpha(1-\alpha)+(\varepsilon-1)(2\varepsilon-1)\right)\left(\frac{\varepsilon\tau}{\varepsilon-1}\right)^{2\varepsilon} \\
& +(1-\alpha)(2\varepsilon-1)\tau^3(\varepsilon-\alpha)+(2\varepsilon-1)(\varepsilon-1)\tau^2(\varepsilon-\alpha)\left(\frac{\varepsilon\tau}{\varepsilon-1}\right)^{\varepsilon} \\
& +\alpha(\varepsilon-1)^{2-3\varepsilon}(\varepsilon\tau)^{3\varepsilon}\Big] < 0,
\end{aligned}$$

   Therefore, by the intermediate value theorem there exists a $\tau_I^{Nash} \in \{1, \frac{\varepsilon}{\varepsilon-1}\}$ such that $A_I^{Nash}(\tau_I^{Nash}) = 0$.

(ii) To prove this statement recall that by (67), if $\tau_I < \frac{\varepsilon}{\varepsilon-1}$, then at the symmetric equilibrium (i.e., $\tau_{IH} = \tau_{IF} = \tau_I$) $\frac{\partial N_H}{\partial \tau_{IH}} d\tau_{IH} + \frac{\partial N_H}{\partial \tau_{IF}} d\tau_{IF} < 0$. Hence, from (i) and (67) we can be sure that there exists a solution $\tau_I^{Nash} \in \{1, \frac{\varepsilon}{\varepsilon-1}\}$ such that $N^{Nash} < N^{FT} < N^{FB}$.

■

F Export Taxes

In this section while retaining the assumption $\tau_{IH} = \tau_{IF} = 1$, we prove the propositions and the lemmata of section 6, where we allow for the use of export taxes/subsidies as the policy instrument.

Lemma A3: Policy Incentives for Export Subsidies/Taxes *If* $\tau_{CH} = \tau_{CF} = 1$*, then when there is a symmetric increase in the export subsidies in Home and in Foreign, production efficiency effects are positive. At the same time, if* $\tau_{XH} = \tau_{XF} = \tau_{CH} = \tau_{CF} = 1$*, when there is a unilateral increase in export subsidies at Home, the production efficiency effect is positive in the domestic economy and negative in the foreign one, while domestic terms-of-trade effects are negative. If* $\tau_{CH} = \tau_{CF} = \frac{\varepsilon-1}{\varepsilon}$ *and* $\tau_{XH} = \tau_{XF} = 1$*, a unilateral increase in the export tax at Home generates positive terms-of-trade effects. Formally,*

(1) Let $\tau_{CH} = \tau_{CF} = 1$ and $\tau_{XH} = \tau_{XF} \leq 1$. If $d\tau_{XH} = d\tau_{XF} < 0$, then $\frac{(\frac{\varepsilon}{\varepsilon-1}-1)(y+f)dN_H}{I_H} = \frac{(\frac{\varepsilon}{\varepsilon-1}-1)(y+f)dN_F}{I_F} > 0$.

(2) Let $\tau_{XH} = \tau_{XF} = \tau_{CH} = \tau_{CF} = 1$. If $d\tau_{XH} < 0$, then $\frac{(\frac{\varepsilon}{\varepsilon-1}-1)(y+f)dN_H}{I_H} > 0$, $\frac{(\frac{\varepsilon}{\varepsilon-1}-1)(y+f)dN_F}{I_F} < 0$ and $\frac{d(\tau_{XH}P_{HH})\tau C_{FH} - d(\tau_{XF}P_{FF})\tau C_{HF}}{I_H} < 0$.

(3) Let $\tau_{XH} = \tau_{XF} = 1$ and $\tau_{CH} = \tau_{CF} = \frac{\varepsilon-1}{\varepsilon}$. If $d\tau_{XH} > 0$, then $\frac{d(\tau_{XH}P_{HH})\tau C_{FH} - d(\tau_{XF}P_{FF})\tau C_{HF}}{I_H} > 0$.

Proof of Lemma A3.

(1) Let $\tau_{CH} = \tau_{CF} = 1$. By imposing symmetry, we obtain:

$$dN_H = dN_F = \frac{\partial N_H}{\partial \tau_{XH}} d\tau_{XH} + \frac{\partial N_H}{\partial \tau_{XF}} d\tau_{XF} = \left(\frac{\partial N_H}{\partial \tau_{XH}} + \frac{\partial N_F}{\partial \tau_{XH}}\right) d\tau_{XH}$$

When $0 < \tau_{XH} = \tau_{XF} \leq 1$ the following derivative is negative:

$$\frac{\partial N_H}{\partial \tau_{XH}} + \frac{\partial N_F}{\partial \tau_{XH}} = -\frac{L(1-\alpha)\alpha\tau(\tau_X\tau + (\tau_X\tau)^{\varepsilon}(\tau_X + \varepsilon(1-\tau_X)))}{f\tau_X\varepsilon(\alpha\tau + (\tau_X\tau)^{\varepsilon} + \tau_X\tau(1-\alpha))^2} < 0$$

which implies that $dN_H = dN_F > 0 \iff d\tau_{HX} = d\tau_{XF} < 0$ i.e., symmetric production subsidies generate positive production efficiency effects for both Home and Foreign.

(2) Let $\tau_{XH} = \tau_{XF} = \tau_{CH} = \tau_{CF} = 1$. The change in the number of Home and Foreign varieties when τ_{XH} moves unilaterally is given by:

$$dN_H = \frac{\partial N_H}{\partial \tau_{XH}} d\tau_{XH} = -\frac{L\alpha\tau(\tau^2 + (\varepsilon-\alpha)\tau^{2\varepsilon} + (\varepsilon-1+\alpha)\tau^{1+\varepsilon}}{f\varepsilon(\tau-\tau^{\varepsilon})^2(\tau+\tau^{\varepsilon})} d\tau_{XH}$$

$$dN_F = \frac{\partial N_F}{\partial \tau_{XH}} d\tau_{XH} = \frac{L\alpha\tau(\alpha\tau^2 + (\varepsilon-1)\tau^{2\varepsilon} + (\varepsilon-1+\alpha)\tau^{1+\varepsilon}}{f\varepsilon(\tau-\tau^{\varepsilon})^2(\tau+\tau^{\varepsilon})} d\tau_{XH}$$

Therefore, $dN_H > 0$ and $dN_F < 0 \iff d\tau_{XH} < 0$. Thus, unilateral increase in the export subsidy generates production efficiency effects that are positive for Home and negative for Foreign.

Once we impose symmetry and we assume that $\tau_{XH} = \tau_{XF} = 1$, terms-of-trade effects in (35) can be written as:

$$\frac{\left(\frac{\varepsilon}{\varepsilon-1} N_H^{-\frac{1}{\varepsilon-1}} d\tau_{XH} - \frac{\tau_{CH}}{\varepsilon-1}\frac{\varepsilon}{\varepsilon-1} N_H^{-\frac{\varepsilon}{\varepsilon-1}} dN_H + \frac{\tau_{CF}}{\varepsilon-1}\frac{\varepsilon}{\varepsilon-1} N_F^{-\frac{\varepsilon}{\varepsilon-1}} dN_F\right)\tau C_{HF}}{I_H} \tag{70}$$

We just showed that $d\tau_{XH} < 0$ implies $dN_H > 0$ and $dN_F < 0$, i.e., unilaterally set export subsidies generate negative consumption based terms-of-trade effects both directly, through their impact on the price of exported varieties, and indirectly, through their impact on the number of Home and Foreign varieties.

(3) It remains to show is that when $\tau_{CH} = \tau_{CF} = \frac{\varepsilon-1}{\varepsilon}$ and $d\tau_{XH} > 0$ then $dN_H < 0$ and $dN_F > 0$. This follows from the fact that:

$$\frac{\partial N_H}{\partial \tau_{XH}} = -\frac{L\alpha(\varepsilon-1)\tau\left(\tau^2\left(\alpha^2+\varepsilon-1\right)+\left(\alpha(2\varepsilon-1)+(\varepsilon-1)^2\right)\tau^{\varepsilon+1}+(\varepsilon-\alpha)(\alpha+\varepsilon-1)\tau^{2\varepsilon}\right)}{f(\alpha+\varepsilon-1)^2\left(\tau^{2\varepsilon}-\tau^2\right)\left(\alpha(\tau^\varepsilon+\tau)+(\varepsilon-1)(\tau^\varepsilon-\tau)\right)} < 0$$

$$\frac{\partial N_F}{\partial \tau_{XH}} = \frac{L\alpha(\varepsilon-1)\tau\left(\left(\alpha+\varepsilon^2-1\right)\tau^{\varepsilon+1}+(\varepsilon-1)(\alpha+\varepsilon-1)\tau^{2\varepsilon}+\alpha\varepsilon\tau^2\right)}{f(\alpha+\varepsilon-1)^2\left(\tau^{2\varepsilon}-\tau^2\right)\left(\alpha(\tau^\varepsilon+\tau)+(\varepsilon-1)(\tau^\varepsilon-\tau)\right)} > 0$$

■

Proposition 6: Cooperative Export Subsidy. *If $\tau_{CH} = \tau_{CF} = \frac{\varepsilon-1}{\varepsilon}$, the cooperative policy maker refrains from using taxes on exports and the number of varieties equals the first-best level. If $\tau_{CH} = \tau_{CF} = 1$, the cooperative policy maker finds it optimal to subsidize exports. The number of varieties is larger than in the free trade allocation, but remains lower than the first-best level. Formally,*

(1) If $\tau_{CH} = \tau_{CF} = \frac{\varepsilon-1}{\varepsilon}$, then $\tau_X^{Coop} = 1$ and $N^{Coop} = N^{FB}$.

(2) If $\tau_{CH} = \tau_{CF} = 1$, then $\tau_X^{Coop} < 1$ and $N^{FT} < N^{Coop} < N^{FB}$.

Proof of Proposition 6.

(1) If $\tau_{CH} = \tau_{CF} = \frac{\varepsilon-1}{\varepsilon}$, the cooperative policy maker solves:

$$\max_{\tau_{XH},\tau_{XF}} V_H(P_H(\tau_{XH},\tau_{XF}), I_H(\tau_{XH},\tau_{XF})) + V_F(P_F(\tau_{XH},\tau_{XF}), I_F(\tau_{XH},\tau_{XF})) \tag{71}$$

Here, $P_H(\tau_{XH},\tau_{XF})$ is given by equation (36), which is implied by the equilibrium expressions for $N_H(\tau_{XH},\tau_{XF})$ and $N_F(\tau_{XH},\tau_{XF})$, equation (41). $I_H(\tau_{XH},\tau_{XF})$, is given by $L_H + (\tau_{XH}-1)\tau P_{HH}(\tau_{XH},\tau_{XF})C_{FH}(\tau_{XH},\tau_{XF}) + (\tau_{CH}-1)N_H(\tau_{XH},\tau_{XF})\varepsilon f$ where $P_{HH}(\tau_{XH},\tau_{XF}) = \frac{\varepsilon}{\varepsilon-1}\tau_{CH}N_H(\tau_{XH},\tau_{XF})^{\frac{1}{1-\varepsilon}}$, $C_{FH}(\tau_{XH},\tau_{XF}) = P_{FH}(\tau_{XH},\tau_{XF})^{-\varepsilon}P_F(\tau_{XH},\tau_{XF})^{\varepsilon}$ $C_F(\tau_{XH},\tau_{XF})$ and $P_{FH}(\tau_{XH},\tau_{XF}) = \frac{\varepsilon}{\varepsilon-1}\tau\tau_{XH}\tau_{CH}N_H(\tau_{XH},\tau_{XF})^{\frac{1}{1-\varepsilon}}$. Finally, $C_H(\tau_{XH},\tau_{XF})$ is given by its equilibrium value in equation (37). Symmetric conditions apply to foreign variables.

Taking derivatives with respect to τ_{XH} and τ_{XF} and imposing symmetry, the first-order condition can be written as:

$$\frac{\alpha\varepsilon\tau(\tau_X - 1)[(\alpha+\varepsilon-1)\tau^{2\varepsilon}\tau_X^{2\varepsilon} + (1-\alpha)\tau^2\tau_X + \varepsilon\tau^{\varepsilon+1}\tau_X^{\varepsilon}]}{\tau_X(\tau^\varepsilon\tau_X^\varepsilon + \tau)(\tau^\varepsilon\tau_X^\varepsilon + \tau\tau_X)[-(\alpha+\varepsilon-1)\tau^\varepsilon\tau_X^\varepsilon + (\alpha-1)(\varepsilon-1)\tau\tau_X - \alpha\varepsilon\tau]} = 0$$

It is straightforward to see that $\tau_X = 1$ is the unique solution to this equation. We have already shown in Proposition 2 that $\tau_{CH} = \tau_{CF} = \frac{\varepsilon-1}{\varepsilon}$, $\tau_{XH} = \tau_{XF} = 1$ implements the first-best allocation. Thus, $N^{Coop} = N^{FB}$.

(2) If $\tau_{CH} = \tau_{CF} = 1$, the cooperative policy maker solves the same problem as in (1) but income is now given by $I_H(\tau_{XH}, \tau_{XF}) = L + (\tau_{XH} - 1)\tau P_{HH} C_{FH}$ and similarly for Foreign. Taking derivatives with respect to τ_{XH} and τ_{XF} and imposing symmetry, the first-order condition can now be written as

$$\frac{A_X^{Coop}(\tau_X)}{B_X^{Coop}(\tau_X)} = 0,$$

where

$$\begin{aligned} A_X^{Coop}(\tau_X) \equiv\ & \alpha\tau[\tau^{\varepsilon+1}\tau_X^\varepsilon[\tau_X(2\alpha - \varepsilon^2 + \varepsilon - 2) + (\varepsilon-1)\varepsilon] \\ & + \tau^{2\varepsilon}\tau_X^{2\varepsilon}[\varepsilon(\alpha+\varepsilon-2) - (\varepsilon-1)\tau_X(\alpha+\varepsilon-1)] + (\alpha-1)\tau^2\tau_X(\varepsilon\tau_X - \varepsilon + 1)] \\ B_X^{Coop}(\tau_X) \equiv\ & (\varepsilon-1)\tau_X(\tau^\varepsilon\tau_X^\varepsilon + \tau)(\tau^\varepsilon\tau_X^\varepsilon + \tau\tau_X)(\tau_X(\tau - \alpha\tau) + \tau^\varepsilon\tau_X^\varepsilon + \alpha\tau) \end{aligned}$$

Note that $A_X^{Coop}(\tau_X) = A_I^{Coop}(\tau_I)$ and $B_X^{Coop}(\tau_X) = B_I^{Coop}(\tau_I)$ with the only difference that they are functions of τ_X instead of τ_I. Thus, the proof is the same as the one for the cooperative import subsidy (Proposition 4).

■

Lemma 3: Unilaterally Set Export Taxes/Subsidies. *Let $\tau_{XH} = \tau_{XF} = 1$. The optimal unilateral deviation entails an export tax when starting from the first-best allocation implemented by a production subsidy, and an export subsidy when starting from the free trade allocation. Formally,*

(1) If $\tau_{CH} = \tau_{CF} = \frac{\varepsilon-1}{\varepsilon}$, then $\frac{\partial V_H}{\partial \tau_{XH}} > 0$.

(2) If $\tau_{CH} = \tau_{CF} = 1$, then $\frac{\partial V_H}{\partial \tau_{XH}} < 0$.

Proof of Lemma 3:

(1) If $\tau_{CH} = \tau_{CF} = \frac{\varepsilon-1}{\varepsilon}$, then:

$$\frac{\partial V_H}{\partial \tau_X} = \frac{\alpha\tau\left((1-\alpha)\tau^{\varepsilon+1} + (\alpha+\varepsilon-1)\tau^{2\varepsilon} + \varepsilon\tau^2\right)}{(\tau^{2\varepsilon} - \tau^2)\left(\alpha(\tau^\varepsilon + \tau) + (\varepsilon-1)(\tau^\varepsilon - \tau)\right)} > 0$$

(2) If $\tau_{CH} = \tau_{CF} = 1$, then:

$$\frac{\partial V_H}{\partial \tau_{XH}} = -\frac{\alpha\tau\left(\tau(\alpha+\varepsilon-1)+(1-\alpha)\tau^{\varepsilon}\right)}{(\varepsilon-1)\left(\tau^{2\varepsilon}-\tau^2\right)} < 0$$

■

Proposition 7: Nash-Equilibrium Export Taxes/Subsidies. *When starting from the first-best allocation, the Nash-equilibrium policy consists of an export tax, implying less varieties than the first-best allocation. In contrast, when starting from the free trade allocation, the Nash-equilibrium entails an export subsidy, implying more varieties than the free trade allocation. Formally,*

(1) If $\tau_{CH} = \tau_{CF} = \frac{\varepsilon-1}{\varepsilon}$, then $\tau_X^{Nash} > 1$ and $N^{Nash} < N^{FB}$.

(2) If $\tau_{CH} = \tau_{CF} = 1$, then $\tau_X^{Nash} < 1$ and $N^{FT} < N^{Nash} < N^{FB}$.

Proof of Proposition 7.

(1) If $\tau_{CH} = \tau_{CF} = \frac{\varepsilon-1}{\varepsilon}$, the Nash policy maker solves:

$$\max_{\tau_{XH}} V_H(P_H(\tau_{XH},\tau_{XF}), I_H(\tau_{XH},\tau_{XF})) \tag{72}$$

Here, $P_H(\tau_{XH},\tau_{XF})$ is given by equation (36), which is implied by the equilibrium expressions for $N_H(\tau_{XH},\tau_{XF})$ and $N_F(\tau_{XH},\tau_{XH})$, equation (41). Moreover, $I(\tau_{XH},\tau_{XF}) = L + (\tau_{XH}-1)\tau P_{HH}(\tau_{XH},\tau_{XF})C_{FH}(\tau_{XH},\tau_{XF})+(\tau_{CH}-1)N_H(\tau_{XH},\tau_{XF})\varepsilon f$, where $P_{HH}(\tau_{XH},\tau_{XF}) = \frac{\varepsilon}{\varepsilon-1}\tau_{CH}N_H(\tau_{XH},\tau_{XF})^{\frac{1}{1-\varepsilon}}$, $C_{FH} = P_{FH}(\tau_{XH},\tau_{XF})^{-\varepsilon}P_F(\tau_{XH},\tau_{XF})^{\varepsilon}C_F(\tau_{XH},\tau_{XF})$, $P_{FH}(\tau_{XH},\tau_{XF}) = \frac{\varepsilon}{\varepsilon-1}\tau\tau_{XH}\tau_{CH}N_H(\tau_{XH},\tau_{XF})^{\frac{1}{1-\varepsilon}}$ and finally $C_F(\tau_{XH},\tau_{XF})$ is given by its equilibrium value in equation (37).

Taking derivatives with respect to τ_{XH} and τ_{XF} and then imposing symmetry i.e., $\tau_{XH} = \tau_{XF} = \tau_X$, the first-order conditions at the symmetric Nash equilibrium can be written as

$$\frac{A_X^{Nash}(\tau_X)}{B_X^{Nash}(\tau_X)} = 0 \tag{73}$$

where

$$\begin{aligned}
A_X^{Nash}(\tau_X) \equiv\ & \alpha\tau\{-\tau^{\varepsilon+3}\tau_X^{\varepsilon+1}[\tau_X((\alpha^2-1)(\varepsilon-1)\varepsilon\tau_X+(-2\alpha^2+\alpha-2)\varepsilon^2+(\alpha-1)^2+\varepsilon^3)\\
&+\varepsilon((\alpha-1)\alpha\varepsilon+(\alpha-1)\alpha-\varepsilon^2+\varepsilon)]\\
&+\tau^{2\varepsilon+2}\tau_X^{2\varepsilon}[\tau_X(\varepsilon\tau_X(\alpha^2+\alpha(\varepsilon-1)^2-(\varepsilon-2)(\varepsilon-1))\\
&-\alpha^2(\varepsilon+1)+2\alpha(-\varepsilon^3+\varepsilon^2+1)+(\varepsilon-1)(\varepsilon^2+1))+\alpha\varepsilon^3]\\
&+\tau^{3\varepsilon+1}\tau_X^{3\varepsilon}(\alpha+\varepsilon-1)[(\varepsilon-1)\tau_X\left(\tau_X(\alpha(\varepsilon-1)+1)-(2\alpha+1)\varepsilon\right)+(\alpha+1)\varepsilon^2]\\
&-\tau_X^{4\varepsilon}\tau^{4\varepsilon}(\alpha+\varepsilon-1)^2[(\varepsilon-1)\tau_X-\varepsilon]+\tau^4\tau_X^2\varepsilon(\alpha+\varepsilon-1)\}
\end{aligned}$$

$$B_X^{Nash}(\tau_X) \equiv \tau_X(\tau^\varepsilon\tau_X^\varepsilon + \tau\tau_X)(\tau^{2\varepsilon}\tau_X^{2\varepsilon} - \tau^2)[(\alpha+\varepsilon-1)\tau^\varepsilon\tau_X^\varepsilon - (\alpha+1)(\varepsilon-1)\tau\tau_X + \alpha\varepsilon\tau]$$
$$[(\alpha+\varepsilon-1)\tau^\varepsilon\tau_X^\varepsilon + \tau\tau_X(-\alpha\varepsilon+\alpha+\varepsilon-1)+\alpha\varepsilon\tau]$$

(i) We first show that no solution with $\tau_X < 1$ exists. Focusing on the numerator of the first-order condition, this is so since all terms of $A_X^{Nash}(\tau_X)$ are positive for $\tau_X < 1$.

(ii) Next, we show that there exists at least one solution with $\tau_X > 1$.

(a) For $\tau_X = 1$, $A_X^{Nash}(1) = (\varepsilon+\alpha-1)\left(\tau^\varepsilon+\tau\right)^2\left[(1-\alpha)\tau^\varepsilon\tau+\varepsilon\tau^2+\tau^{2\varepsilon}(\varepsilon-1+\alpha)\right] > 0$;

(b) $\lim_{\tau_X\to\infty} A_X^{Nash}(\tau_X) = -\infty$;

(c) Thus, by continuity of $A_X^{Nash}(\tau_X)$, there exists a $\tau_X^{Nash} > 1$ such that $A_X^{Nash}(\tau_X^{Nash}) = 0$.

(iii) It remains to show that if $\tau_X^{Nash} > 1$, then $N^{Nash} < N^{FB}$. When $\tau_{CH} = \tau_{CF} = \frac{\varepsilon-1}{\varepsilon}$ and after imposing symmetry i.e., $\tau_{XH} = \tau_{XF} = \tau_X$ and $d\tau_{XH} = d\tau_{XF}$:

$$\begin{aligned} dN_H &= \frac{\partial N_H}{\partial \tau_{XH}} d\tau_{XH} + \frac{\partial N_H}{\partial \tau_{XF}} d\tau_{XF} = \left(\frac{\partial N_H}{\partial \tau_{XH}} + \frac{\partial N_H}{\partial \tau_{XF}}\right) d\tau_{XH} \\ &= \frac{L(1-\alpha)\alpha(\varepsilon-1)\tau\left[\tau^\varepsilon\tau_X^\varepsilon\left((\varepsilon-1)\tau_X-\varepsilon\right)-\tau\tau_X\right]}{f\tau_X\left[(\alpha+\varepsilon-1)\tau^\varepsilon\tau_X^\varepsilon+\tau\tau_X(1-\alpha)(\varepsilon-1)+\alpha\varepsilon\tau\right]^2} d\tau_{XH} \end{aligned}$$

Note that $d\tau_{XH} > 0 \Rightarrow dN_H \geq 0 \iff \tau^\varepsilon\tau_X^\varepsilon\left[(\varepsilon-1)\tau_X-\varepsilon\right]-\tau\tau_X \geq 0$. Let us define the following two continuous and monotonic functions $f(\tau_X) \equiv (\varepsilon-1)\tau^\varepsilon\tau_X^{\varepsilon+1}$ and $g(\tau_X) \equiv \varepsilon\tau^\varepsilon\tau_X^\varepsilon + \tau\tau_X$ with $f'(\tau_X) > 0$, $f''(\tau_X) > 0$, $g'(\tau_X) > 0$ and $g''(\tau_X) > 0$. Note that $f(1) - g(1) < 0$ implying that $dN_H < 0$ when $\tau_X = 1$. By continuity and monotonicity of the two functions, only two cases are possible. They either never cross, in which case $dN_H < 0 \forall\tau_X \in [1,\infty)$ and consequently $N^{Nash} < N^{FB}$. Or, they cross only once. That implies that $\exists\bar{\tau}_X > 1$ such that $f(\tau_X) \geq g(\tau_X)$, $\forall\tau_X \geq \bar{\tau}_X$ implying $dN_H > 0 \iff \tau_X \in (\bar{\tau}_X,\infty)$. However note that:

$$\lim_{\tau_X->\infty} N_H = \lim_{\tau_X->\infty} \frac{L\alpha\left(\tau^\varepsilon\tau_X^\varepsilon+\tau\right)}{f\left((\alpha+\varepsilon-1)\tau^\varepsilon\tau_X^\varepsilon+\tau\tau_X(\alpha(-\varepsilon)+\alpha+\varepsilon-1)+\alpha\varepsilon\tau\right)} = N^{FB}$$

implying that also in this case $N^{Nash} < N_X^{FB}$.

(2) If $\tau_{CH} = \tau_{CF} = 1$, the Nash policy maker solves the same problem as in (1) but income is now given by $I_H(\tau_{XH},\tau_{XF}) = L + (\tau_{XH}-1)\tau P_{HH}(\tau_{XH},\tau_{XF})C_{FH}(\tau_{XH},\tau_{XF})$. Taking derivatives with respect to τ_{XH} and τ_{XF} and then imposing symmetry i.e., $\tau_{XH} = \tau_{XF} = \tau_X$, the first-order conditions at the symmetric Nash equilibrium can be written as:

$$\frac{A_X^{Nash}(\tau_X)}{B_X^{Nash}(\tau_X)} = 0 \tag{74}$$

with

$$\begin{aligned}A_X^{Nash}(\tau_X) \equiv{}& \alpha\{\tau^{\varepsilon+4}\tau_X^{\varepsilon+1}[\tau_X(\tau_X(\varepsilon-\alpha^2\varepsilon)+2\alpha^2\varepsilon+(\alpha-1)\alpha-\varepsilon^2+\varepsilon)-\alpha^2(\varepsilon+1)+\alpha+(\varepsilon-1)\varepsilon] \\ &+\tau^{2\varepsilon+3}\tau_X^{2\varepsilon}[\tau_X(\tau_X((\alpha-1)\varepsilon^2-\alpha+\varepsilon+1)+\alpha(-2\varepsilon^2+\varepsilon-1)+(\varepsilon-1)^2)+\alpha(\varepsilon-1)\varepsilon] \\ &+\tau^{3\varepsilon+2}\tau_X^{3\varepsilon}[\tau_X(\alpha(\varepsilon-1)\tau_X(\alpha+\varepsilon-1)-(2\alpha+1)\varepsilon^2-2(\alpha-2)\alpha\varepsilon+(\alpha-1)\alpha) \\ &+\varepsilon(\alpha(\alpha+\varepsilon-2)+\varepsilon-1)] \\ &+\tau^{4\varepsilon+1}\tau_X^{4\varepsilon}(\varepsilon(\alpha+\varepsilon-2)-(\varepsilon-1)\tau_X(\alpha+\varepsilon-1))+\tau^5\tau_X^2(\alpha+\varepsilon-1)\} \\ B_X^{Nash}(\tau_X) \equiv{}& (\varepsilon-1)\tau_X(\tau^\varepsilon\tau_X^\varepsilon+\tau\tau_X)(\tau^{2\varepsilon}\tau_X^{2\varepsilon}-\tau^2)(-(\alpha+1)\tau\tau_X+\tau^\varepsilon\tau_X^\varepsilon+\alpha\tau) \\ &(\tau_X(\tau-\alpha\tau)+\tau^\varepsilon\tau_X^\varepsilon+\alpha\tau)\end{aligned}$$

(i) In order to show that there exists a solution with $\tau_X < 1$, we first show that when $\tau_X = 1$, $A_X^{Nash}(1)$ is negative. This is so given that $A_X^{Nash}(1) = \tau(\tau^\varepsilon - \tau)(\tau^\varepsilon + \tau)^2[(\alpha - 1)\tau^\varepsilon + \tau(-\alpha-\varepsilon+1)] < 0$.

(ii) Next, we show that for $\varepsilon > 2$ there exists a $\tau_X \in \{0, 1\}$ with $A_X^{Nash}(\tau_X) > 0$. By continuity of $A_X^{Nash}(\tau_X)$ this is enough to guarantee the existence of a solution. Consider $\tau_X = \frac{\varepsilon-2}{\varepsilon}$. Then, $A_X^{Nash}(\frac{\varepsilon-2}{\varepsilon}) = \frac{\tau}{\varepsilon^2}[(\varepsilon-2)^2\tau^4(\alpha+\varepsilon-1)+(\varepsilon-2)^{4\varepsilon}\varepsilon^{1-4\varepsilon}(2+2\varepsilon^2-5\varepsilon+3\alpha\varepsilon-2\alpha)\tau^{4\varepsilon} + (\varepsilon-2)^{1+\varepsilon}\varepsilon^{-\varepsilon}(4+2\alpha-6\alpha^2+3(\varepsilon-2)\varepsilon)\tau^{3+\varepsilon} + \left(\frac{\varepsilon-2}{\varepsilon}\right)^{2\varepsilon}(\alpha(6\varepsilon-4)+(\varepsilon-2)(\varepsilon^2-2))\tau^{2+2\varepsilon} + \tau\left(\alpha^2(6\varepsilon-4)+2\alpha(\varepsilon-2)(2\varepsilon-1)+\varepsilon^3\right)\left(\frac{(\varepsilon-2)\tau}{\varepsilon}\right)^{3\varepsilon}] > 0$ since each of the coefficients is positive for $\varepsilon > 2$. This proves that a solution with $\tau_X < 1$ exists.

(iii) Finally, we show that $N^{Nash} < N^{FB}$.

(a) Let $\tau_X^{Nash} = f(\alpha, \varepsilon, \tau)$ and $\tau_X^{FB} = g(\alpha, \varepsilon, \tau)$ be, respectively, the Nash equilibrium export subsidy and the export subsidy that implements the first-best number of varieties . First we show that there is no intersection between the set of τ_X^{Nash} and the set of τ_X^{FB} in the interval $[0, 1]$. If $\tau_X = \tau_X^{Nash}$, $A_X^{Nash}(\tau_X^{Nash}) = 0$. At the same time τ_X^{FB} is such $N = \frac{L\alpha(\tau+(\tau\tau_X^{FB})^\varepsilon)}{f\varepsilon(\alpha\tau+\tau(1-\alpha)\tau_X^{FB}+(\tau\tau_X^{FB})^\varepsilon)} = \frac{L\alpha}{f(\varepsilon+\alpha-1)} = N^{FB}$. This last condition can be rewritten as $(\tau\tau_X)^\varepsilon = -\varepsilon\tau\tau_X + \tau(\varepsilon-1)$. Note that when combined, this two conditions are a system of two equations in τ_X. We now investigate if there exists a τ_X such that both conditions are satisfied simultaneously. Once we substitute the above condition into A_X^{Nash} we obtain a fifth-order polynomial in τ_X which can be factorized into two polynomials. The first polynomial is $-\varepsilon\tau^5(\tau_X-1)^2(\alpha+\varepsilon-1)$, with solutions $\tau_X = \{1, 1\}$. None of these solutions solves $(\tau\tau_X)^\varepsilon = -\varepsilon\tau\tau_X + \tau(\varepsilon-1)$. The second polynomial is cubic and we call it ${A_X}_{mod}^{Nash}$. It can be shown that there exist at most one real solution of ${A_X}_{mod}^{Nash}$. However, evaluating ${A_X}_{mod}^{Nash}$ at $\tau_X = 1$ and $\tau_X = 0$ we find that both ${A_X}_{mod}^{Nash}(1) < 0$ and ${A_X}_{mod}^{Nash}(0) < 0$. Thus, by continuity of ${A_X}_{mod}^{Nash}$, either there exists no real solution or there are at least two zeros of ${A_X}_{mod}^{Nash} = 0$ that are real. Since there exists at most one real solution of ${A_X}_{mod}^{Nash} = 0$ in $[0, 1]$, we can conclude that there is no intersection between the set of τ_X^{Nash} and the set of τ_X^{FB} in the interval $[0, 1]$.

(b) The second step is to show that $\tau_X^{FB} < \tau_X^{Nash}$ in the interval $[0, 1]$. To this end, recall that f and g are two continuous functions in the space $\{0 < \alpha < 1, \tau > 1, \varepsilon > 1\}$, given that the derivatives of τ_X^{FB} and τ_X^{Nash} with respect to the three parameters always exists in the permitted parameter space. In point (a) we proved that there is no intersection between g and f. As a consequence, we either have $\tau_X^{FB} < \tau_X^{Nash}$ or the other way around. We evaluate both functions at $\{\alpha = 0.5, \varepsilon = 2, \tau = 1.5\}$ and find $\tau_X^{FB} = 0.39 < 0.82 = \tau_X^{Nash}$. Thus, the non-cooperative export subsidy is always smaller than the one needed to implement the first-best number of varieties.

(c) Finally, from Lemma A3 (1) we know that $0 < \tau_{XH} = \tau_{XF} \leq 1 \Rightarrow \frac{\partial N_H}{\partial \tau_{XH}} + \frac{\partial N_F}{\partial \tau_{XH}} < 0$ which implies that $dN_H = dN_F > 0 \iff d\tau_{HX} = d\tau_{XF} < 0$ i.e, by symmetrically increasing the export subsidy in both countries policy makers increase the number of varieties. It then follows that $\tau_X^{FB} < \tau_X^{Nash} \Rightarrow N^{Nash} < N^{FB}$.

■

G Simultaneous Policy Choice

Proposition 8: Cooperative Policy Instruments. *The cooperative policy maker sets the first-best level of production subsidies and chooses the trade taxes such that* $\tau_I^{Coop} \cdot \tau_X^{Coop} = 1$. *The number of varieties equals the first-best level.*

(1) $\tau_C^{Coop} = \frac{\varepsilon-1}{\varepsilon}$, $\tau_I^{Coop} \cdot \tau_X^{Coop} = 1$ and $N^{Coop} = N^{FB}$.

Proof of Proposition 8. We have already proven in Propositions 2, 4 and 6 that the first-best allocation can be implemented by eliminating the monopolistic distortion and by refraining from using import or export taxes (i.e., by setting $\tau_C^{Coop} = \frac{\varepsilon-1}{\varepsilon}$ and $\tau_I^{Coop} = \tau_X^{Coop} = 1$). What remains to show is that the first-best allocation can be implement even when $\tau_C^{Coop} = \frac{\varepsilon-1}{\varepsilon}$ and $\tau_I^{Coop}\tau_X^{Coop} = 1$. To see why this is the case, notice that if $\tau_C^{Coop} = \frac{\varepsilon-1}{\varepsilon}$ and $\tau_I^{Coop}\tau_X^{Coop} = 1$ $N_H = N_F = N^{FB}$, i.e., this policy implements the first-best allocation. Intuitively, if $\tau_C^{Coop} = \frac{\varepsilon-1}{\varepsilon}$ any policy such that $\tau_I^{Coop}\tau_X^{Coop} = 1$ allows to reach the social optimum since the effects of import tariffs/subsidies are exactly offset by those of export subsidies/taxes. ■

Proposition 9: Nash-Equilibrium Policy Instruments. *The Nash-equilibrium policy consists of the first-best level of production subsidies, and inefficient import subsidies and export taxes. Formally,*

(1) $\tau_C^{Nash} = \tau_C^{Coop} = \frac{\varepsilon-1}{\varepsilon}$, $\tau_I^{Nash} < 1$, $\tau_X^{Nash} > 1$.

Proof of Proposition 9. In order to prove Proposition 9 we follow Dixit (1985) and we use the primal approach to optimal policy instead of the indirect utility function i.e., we directly maximize utility subject to the equilibrium conditions. The non-cooperative policy maker maximizes domestic utility subject to the good market clearing conditions, the trade balance and the demand functions of the domestic and foreign economy. The Lagrangian associated with the optimal policy problem of the non-cooperative policy maker can be formulated as:

$$
\begin{aligned}
\mathcal{L} &= P_H^{-\alpha}\left(\tau\tau_{IH}\tau_{XF}P_{FF}C_{HF} + P_{HH}C_{HH}\right) \\
&\quad + \lambda_1\left[f(\varepsilon-1)N_H^{\frac{\varepsilon}{\varepsilon-1}} - C_{HH} - \tau C_{FH}\right] + \lambda_2\left[f(\varepsilon-1)N_F^{\frac{\varepsilon}{\varepsilon-1}} - C_{FF} - \tau C_{HF}\right] \\
&\quad - \lambda_3\left[\frac{(1-\alpha)}{\alpha}\left(P_{HH}C_{HH} + \tau\tau_{IH}\tau_{XF}P_{FF}C_{HF}\right) + \tau\tau_{XF}P_{FF}C_{HF} - \tau\tau_{XH}P_{HH}C_{FH} - Q_H\right] \\
&\quad - \lambda_4\left[\frac{(1-\alpha)}{\alpha}\left(P_{FF}C_{FF} + \tau\tau_{IF}\tau_{XH}P_{HH}C_{FH}\right) + \tau\tau_{XH}P_{HH}C_{FH} - \tau\tau_{XF}P_{FF}C_{HF} - Q_F\right] \\
&\quad - \lambda_5\left[P_{HH}^{\varepsilon}C_{HH} - \left(\tau\tau_{IH}\tau_{XF}\right)^{\varepsilon}P_{FF}^{\varepsilon}C_{HF}\right] - \lambda_6\left[P_{FF}^{\varepsilon}C_{FF} - \left(\tau\tau_{IF}\tau_{XH}\right)^{\varepsilon}P_{HH}^{\varepsilon}C_{FH}\right]
\end{aligned}
$$

where P_H, P_{HH} and P_{FF} are defined consistently with (6), (7) and (11). Making use of the constraints and rearranging the first-order conditions of $\mathcal{L}$ with respect to C_{HH}, C_{HF}, C_{FF}, C_{FH}, N_H, N_F, τ_{CH}, τ_{IH} and τ_{XH}, which we evaluate at the symmetric equilibrium, we obtain, respectively:

$$
\begin{aligned}
P_{HH}^{1-\alpha}\left[1+\left(\tau\tau_I\tau_X\right)^{1-\varepsilon}\right]^{-\frac{\alpha}{1-\varepsilon}} &= \lambda_1 + \frac{1-\alpha}{\alpha}\lambda_3 + \gamma_5 \\
\tau\tau_I\tau_X P_{HH}^{1-\alpha}\left[1+\left(\tau\tau_I\tau_X\right)^{1-\varepsilon}\right]^{-\frac{\alpha}{1-\varepsilon}} &= \lambda_2\tau + \lambda_3\left(\tau\tau_X + \frac{1-\alpha}{\alpha}\tau\tau_I\tau_X\right) - \gamma_4\tau\tau_X - \gamma_5\left(\tau\tau_I\tau_X\right)^{\varepsilon} \\
0 &= \lambda_2 + \frac{1-\alpha}{\alpha}\gamma_4 + \gamma_6 \\
0 &= \lambda_1\tau - \lambda_3\tau\tau_X + \gamma_4\left(\tau\tau_X + \frac{1-\alpha}{\alpha}\tau\tau_I\tau_X\right) - \gamma_6\left(\tau\tau_I\tau_X\right)^{\varepsilon} \\
0 &= \lambda_1 - \lambda_3\frac{\varepsilon-1}{\varepsilon\tau_C} \\
(1-\alpha)\left(\tau\tau_I\tau_X\right)^{1-\varepsilon}P_{HH}^{1-\alpha}\left[1+\left(\tau\tau_I\tau_X\right)^{1-\varepsilon}\right]^{-\frac{\alpha}{1-\varepsilon}} &= \lambda_2\varepsilon\left[1+\tau^{1-\varepsilon}\left(\tau_I\tau_X\right)^{-\varepsilon}\right] + \lambda_3\left(\frac{1-\alpha}{\alpha} + \frac{1}{\tau_I}\right)\left(\tau\tau_I\tau_X\right)^{1-\varepsilon} \\
&\quad + \gamma_4\left\{\frac{1-\alpha}{\alpha} - \frac{\varepsilon-1}{\tau_C}\left[1+\tau^{1-\varepsilon}\left(\tau_I\tau_X\right)^{-\varepsilon}\right] - \tau_I^{-\varepsilon}\left(\tau\tau_X\right)^{1-\varepsilon}\right\} \\
&\quad - \gamma_5\varepsilon + \gamma_6\varepsilon \\
(1-\alpha)P_{HH}^{1-\alpha}\left[1+\left(\tau\tau_I\tau_X\right)^{1-\varepsilon}\right]^{-\frac{\alpha}{1-\varepsilon}} &= \lambda_3\left[\frac{1-\alpha}{\alpha} - \tau_I^{-\varepsilon}\left(\tau\tau_X\right)^{1-\varepsilon}\right] + \gamma_4\left(\frac{1-\alpha}{\alpha} + \frac{1}{\tau_I}\right)\left(\tau\tau_I\tau_X\right)^{1-\varepsilon} \\
&\quad + \gamma_5\varepsilon - \gamma_6\varepsilon \\
(1-\alpha)P_{HH}^{1-\alpha}\left[1+\left(\tau\tau_I\tau_X\right)^{1-\varepsilon}\right]^{-\frac{\alpha}{1-\varepsilon}} &= \lambda_3\frac{1-\alpha}{\alpha} + \gamma_5\varepsilon\left(\tau\tau_I\tau_X\right)^{\varepsilon-1} \\
0 &= \lambda_3\tau_I^{-\varepsilon}\left(\tau\tau_X\right)^{1-\varepsilon} - \gamma_4\left[\frac{1-\alpha}{\alpha}\left(\tau\tau_I\tau_X\right)^{1-\varepsilon} + \tau_I^{-\varepsilon}\left(\tau\tau_X\right)^{1-\varepsilon}\right] \\
&\quad + \gamma_6\varepsilon
\end{aligned} \tag{75}
$$

where $\gamma_3 \equiv \lambda_3 P_{HH}$, $\gamma_4 \equiv \lambda_4 P_{HH}$, $\gamma_5 \equiv \lambda_5 P_{HH}^{\varepsilon}$ and $\gamma_6 \equiv \lambda_6 P_{HH}^{\varepsilon}$.
Combining the previous equations, we can solve for τ_C and the multipliers:

$$
\begin{aligned}
\tau_C &= \frac{\varepsilon-1}{\varepsilon} \\
\lambda_1 &= \gamma_3 = P_{HH}^{1-\alpha}\alpha\left[1+(\tau\tau_I\tau_X)^{1-\varepsilon}\right]^{-\frac{\alpha}{1-\varepsilon}} \\
\gamma_4 &= \frac{P_{HH}^{1-\alpha}\alpha^2(1-\alpha)(\varepsilon(\tau_X-1)-\tau_X)\left[1+(\tau\tau_I\tau_X)^{1-\varepsilon}\right]^{-\frac{\alpha}{1-\varepsilon}}}{(\varepsilon-1)(\alpha+(1-\alpha)\tau_I)\tau_X} \\
\gamma_5 &= 0 \\
\gamma_6 &= -\frac{P_{HH}^{1-\alpha}\alpha\tau^{1-\varepsilon}(\tau_I\tau_X)^{-\varepsilon}\left[1+(\tau\tau_I\tau_X)^{1-\varepsilon}\right]^{-\frac{\alpha}{1-\varepsilon}}}{\varepsilon-1} \\
\lambda_2 &= -\gamma_6-\frac{\gamma_4}{\alpha}
\end{aligned}
\tag{76}
$$

The first condition in (76) already states that the Nash equilibrium production subsidy completely offsets the monopolistic distortion. What remains to show is that $\tau_I^{Nash} < 1$ and $\tau_X^{Nash} > 1$. Substituting the expressions for the multipliers and the solution for τ_C in the first-order conditions and simplifying, we are left with two equations, the derivative with respect to C_{HF} and the one with respect to N_F. The derivative with respect to C_{HF} is given by:

$$
A_1(\tau_I,\tau_X)+A_2(\tau_I,\tau_X)+A_3(\tau_I,\tau_X)=0 \tag{77}
$$

where:

$$
\begin{aligned}
A_1(\tau_I,\tau_X) &\equiv -(\varepsilon-1)(1-\tau_I)\tau_X^2(\alpha+(1-\alpha)\tau_I) \\
A_2(\tau_I,\tau_X) &\equiv -(\varepsilon-(\varepsilon-1)\tau_X)(\alpha\tau_X+1-\alpha) \\
A_3(\tau_I,\tau_X) &\equiv -(\alpha+(1-\alpha)\tau_I)(\tau\tau_X)^{1-\varepsilon}\tau_I^{-\varepsilon}
\end{aligned}
$$

Note that:

(i) $A_3(\tau_I,\tau_X) < 0$ always;

(ii) $A_1(\tau_I,\tau_X) < 0 \iff \tau_I < 1$;

(iii) $A_2(\tau_I,\tau_X) < 0 \iff \tau_X < \frac{\varepsilon}{\varepsilon-1}$.

Thus, a necessary condition for τ_I and τ_X to solve equation (77) is that if $\tau_X < \frac{\varepsilon}{\varepsilon-1}$ then $\tau_I > 1$. By combining (77) with the first-order condition with respect to N_F we obtain a second condition:

$$
B_1(\tau_I,\tau_X)+B_2(\tau_I,\tau_X)+B_3(\tau_I,\tau_X)=0 \tag{78}
$$

where

$$
\begin{aligned}
B_1(\tau_I,\tau_X) &\equiv -\tau_X^2(\varepsilon-1)(\alpha+(1-\alpha)\tau_I)(1-\varepsilon(1-\tau_I)) \\
B_2(\tau_I,\tau_X) &\equiv (-\varepsilon+\tau_X(\varepsilon-1))(\varepsilon-(1-\alpha))\tau^{\varepsilon-1}(\tau_I\tau_X)^{\varepsilon} \\
B_2(\tau_I,\tau_X) &\equiv -\alpha(-\varepsilon+\tau_X(\varepsilon-1))^2
\end{aligned}
$$

Note that:

(i) $B_3(\tau_I, \tau_X) < 0$ always;

(ii) $B_1(\tau_I, \tau_X) < 0 \iff \tau_I > \frac{\varepsilon-1}{\varepsilon}$;

(iii) $B_2(\tau_I, \tau_X) < 0 \iff \tau_X < \frac{\varepsilon}{\varepsilon-1}$.

Thus, a necessary condition for τ_I and τ_X to solve equation (78) is that if $\tau_X < \frac{\varepsilon}{\varepsilon-1}$ then $\tau_I < \frac{\varepsilon-1}{\varepsilon}$. Note that this condition contradicts the one needed for (77). Therefore, the only possible solution is $\tau_X^{Nash} > \frac{\varepsilon}{\varepsilon-1}$ i.e., this proves that $\tau_X^{Nash} > 1$.

We now have to show that $\tau_I^{Nash} < 1$. We will prove this by contradiction. First, we show that a necessary condition for $\tau_I^{Nash} > 1$ is that $\tau_I^{Nash} \cdot \tau_X^{Nash} < 1$. Second, we show that if $\tau_I^{Nash} > 1$ it must be that $\tau_X^{Nash} < 1$, which contradicts the fact that $\tau_X^{Nash} > 1$. In order to show the first point, it is useful to rewrite equation (77) as follows:

$$-(\varepsilon-1)(1-\tau_I)\tau_X(\alpha\tau_X+(1-\alpha)\tau_{IX})+((\varepsilon-1)\tau_X-\varepsilon)(\alpha\tau_X+1-\alpha)-(\alpha\tau_X+(1-\alpha)\tau_{IX})(\tau^{1-\varepsilon}\tau_{IX}^{-\varepsilon})=0$$

where $\tau_{IX} \equiv \tau_I\tau_X$. If we solve the previous equation for τ_I we obtain:

$$\tau_I^{Nash} = \frac{C_1(\tau_X, \tau_{IX})}{C_2(\tau_X, \tau_{IX})} \tag{79}$$

where:

$$\begin{aligned} C_1(\tau_X, \tau_{IX}) \equiv\ & (\tau_{IX}\tau)^{-\varepsilon}(\tau_{IX}\tau(1-\alpha) + (\tau_{IX}\tau)^{\varepsilon}\varepsilon(1-\alpha-\tau_X+2\alpha\tau_X) \\ & + \alpha\tau_X\tau + (\tau_{IX}\tau)^{\varepsilon}\tau_X(1-\alpha) + \tau_{IX}^{1+\varepsilon}\tau^{\varepsilon}\tau_X(\varepsilon-1)(1-\alpha)) \\ C_2(\tau_X, \tau_{IX}) \equiv\ & (\varepsilon-1)\tau_X(\tau_{IX}(1-\alpha)+\alpha\tau_X) \end{aligned}$$

Now suppose that $\tau_I > 0$ and $C_2(\tau_X, \tau_{IX}) > 0$, it has to be the case that $C_1(\tau_X, \tau_{IX}) > 0$ too. Moreover, for τ_I^{Nash} to be greater than 1, $C_1(\tau_X, \tau_{IX}) - C_2(\tau_X, \tau_{IX})$ should be greater than 0:

$$\begin{aligned} & C_3(\tau_I, \tau_X) \equiv C_1(\tau_X, \tau_{IX}) - C_2(\tau_X, \tau_{IX}) = \\ & (\tau_{IX}\tau)^{-\varepsilon}(\tau_{IX}\tau(1-\alpha) + \alpha\tau\tau_X + (\tau_{IX}\tau)^{\varepsilon}((1-\alpha)\varepsilon + (1-\alpha+2\tau_{IX}(1-\alpha)(\varepsilon-1) - \varepsilon + 2\alpha\varepsilon)\tau_X + \alpha(\varepsilon-1)\tau_X^2) \end{aligned}$$

Note that:

(i) $C_3(\tau_I, \tau_X)$ is linear in α;

(ii) $\alpha = 0$ and $\tau_{IX} > 1$ implies $C_3 = -(\tau_{IX}\tau)^{-\varepsilon}(\tau_{IX}\tau + (\tau_{IX}\tau)^{\varepsilon}(\varepsilon + (\varepsilon-1)(\tau_{IX}-1) + \tau_{IX}\tau_X(\varepsilon-1))) < 0$;

(iii) $\alpha = 1$ and $\tau_{IX} > 1$ implies $C_3 = -(\tau_{IX}\tau)^{-\varepsilon}(\tau\tau_X + (\tau\tau_{IX})^{\varepsilon}(\varepsilon\tau_X + (\varepsilon-1)\tau_X^2)) < 0$;

(iv) By continuity, $\forall\alpha \in (0,1)$ $\tau_{IX} > 1 \Rightarrow C_3 < 0 \Rightarrow \tau_I^{Nash} < 1$;

(v) Thus, a necessary condition for $\tau_I^{Nash} > 1$ is $\tau_{IX} < 1$.

However, we have already proven that $\tau_X^{Nash} > 1$ thus, it cannot be that $\tau_I^{Nash} > 1$ and $\tau_{IX}^{Nash} \equiv \tau_I^{Nash}\tau_X^{Nash} < 1$. Therefore, it has to be that $\tau_I^{Nash} < 1$. ■