Exercise 11 – Low-rank approximation techniques

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Problem 1: Sum of tensors in TT decomposition

Given two tensors $\mathcal{X}$ and $\mathcal{Y}$ in TT decomposition, derive a TT decomposition for $\mathcal{X} + \mathcal{Y}$.

Problem 2: TT ranks and multilinear rank

Given a tensor in TT decomposition, derive upper bounds for its multilinear rank.

Problem 3: Implementation of TT-SVD

Implement the TT-SVD algorithm from the slides. Adjust the algorithm to work with prescribed accuracy $\varepsilon > 0$ for which it determines the TT ranks adaptively such that $\|\mathcal{X} - \mathcal{X}_{SVD}\| \le \varepsilon$. (Use the theorem from the slides.)

Problem 4: Left/right orthogonalization of TT tensors

Let $\mathcal{X}$ be a tensor in TT decomposition with TT cores $\mathcal{U}_1, \dots, \mathcal{U}_d$.

1. Show that $\|\mathcal{X}\| = \|\mathcal{U}_d\|$ holds for a left-orthogonal TT decomposition. What can you say about the singular values of $X^{<d-1>}$?
2. Show that $\|\mathcal{X}\| = \|\mathcal{U}_1\|$ holds for a right-orthogonal TT decomposition. What can you say about the singular values of $X^{<1>}$?
3. In Matlab, implement the left and right orthogonalization process as described in the slides and check the conjectures from Points 1 and 2.

Problem 5: Computing the average of a TT tensor

Using tensor contractions, develop an efficient method for computing the mean,

$$\bar{\mathcal{X}} = \frac{1}{n_1 \cdots n_d} \sum_{i_1,\dots,i_d} \mathcal{X}(i_1, \dots, i_d),$$

of a tensor in TT decomposition.