ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

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▶ Setting

The goal of this project is to develop a modification of the HOSVD algorithm which is based on the randomized algorithm.

The modified HOSVD should be of the following form:

```
1: procedure MHOSVD(𝒳)
2:     for μ = 1,2,3 do
3:         Ũ_μ ← orthonormal basis for approximation of range of X^(μ)
4:     end for
5:     C̃ ← Ũ_1^T ∘_1 Ũ_2^T ∘_2 Ũ_3^T ∘_3 𝒳
6:     [𝒞, V_1, V_2, V_3] = HOSVD(C̃)
7:     for μ = 1,2,3 do
8:         U_μ ← Ũ_μ V_μ
9:     end for
10: end procedure
```

Lines 6-9 are only necessary if the basis matrices from Line 3 and the core tensor from Line 5 are unnecessary large, for example when there is an oversampling parameter involved in their calculation. If their size is optimal, Lines 6-9 can be skipped.

▶ Tasks

1. Implement the randomized range finder, both for given rank (use oversampling parameter) and for given precision – Algorithms 4.1 and 4.2 from [Halko/Martinsson/Tropp'2010: Finding Structure with Randomness: Probabilistic algorithms for constructing approximate matrix decompositions].

2. Implement the modified HOSVD algorithm such that the Line 3 is obtained from the randomized range finder from Point 1. Adjust the algorithm to work with given rank and with given precision. Test the algorithm on a function related tensor – e.g. evaluate the function

$$f(x,y,z) = \frac{1}{\sqrt{x+y+z}},$$

on a grid $\{0.1, 0.2, \ldots, n/10\}^3$ for $n = 50$.

3. Create a random tensor of size $200 \times 200 \times 200$ and approximate it by Tucker decomposition with multilinear ranks (R, R, R) for $R = 5, 10, \ldots, 50$. Report the times needed for the modified HOSVD and the HOSVD to obtain such approximations and the norms of the errors of the resulting tensors.

4. Derive an error bound for the modified HOSVD algorithm – if each basis matrix is obtained such that

$$\|(I - U_\mu U_\mu^T) X^{(\mu)}\|_F \leq \varepsilon,$$

find a bound for

$$\|\mathcal{X} - U_1 \circ_1 U_2 \circ_3 U_3 \circ_3 \mathcal{C}\|_F.$$