ÉCOLE POLYTECHNIQUE
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► Setting

The goal of this project is to understand the need for recompression of tensors in low-rank formats, in particular the TT format, and to develop an efficient algorithm for such procedure.

Let $\mathcal{X}$ and $\mathcal{Y}$ be two tensors given in TT decompositions

$$\mathcal{X}(i_1, i_2, \ldots, i_d) = U_1(i_1)U_2(i_2)\cdots U_d(i_d),$$
$$\mathcal{Y}(i_1, i_2, \ldots, i_d) = V_1(i_1)V_2(i_2)\cdots V_d(i_d),$$

then their Hadamard product $\mathcal{Z} = \mathcal{X} * \mathcal{Y}$ has the following TT decomposition

$$\mathcal{Z}(i_1, i_2, \ldots, i_d) = \left(U_1(i_1) \otimes V_1(i_1)\right)\left(U_2(i_2) \otimes V_2(i_2)\right)\cdots\left(U_d(i_d) \otimes V_d(i_d)\right). \quad (1)$$

► Tasks

1. Implement the right-orthogonalization procedure of the cores of a TT decomposition.
2. Implement the algorithm for recompression of TT decomposition from Slide 18 (Lecture 7). Adjust the algorithm to work with given rank and with given precision.
3. Create $n \times \cdots \times n$ tensor $\mathcal{X}$ of order $d = 10$ with rank $(r, \ldots, r)$ the following way. Generate d random $n \times r$ matrices with orthogonal columns U_μ, $\mu = 1, \ldots, d$ and define a vector σ as $\sigma_i = e^{1-i}$, for $i = 1, \ldots, r$. Create the CP decomposition of $\mathcal{X}$ as

$$\mathcal{X}(i_1, \ldots, i_d) = \sum_{\alpha=1}^{r} \left(\sigma_\alpha U_1(i_1, \alpha)\right) U_2(i_2, \alpha) \cdots U_d(i_d, \alpha),$$

   and use it to get the TT decomposition of $\mathcal{X}$. Apply recompression algorithm from Point 2 to tensor $\mathcal{X}$ such that the norm of the error is smaller that some ε. Try different combinations of parameters.
4. Prove formula (1).
5. Create tensors $\mathcal{X}$ and $\mathcal{Y}$ following the same procedure as in Point 3 for $n = 30$ and $r = 10$. Create the Hadamard product in TT format $\mathcal{Z} = \mathcal{X} * \mathcal{Y}$ using representation (1) and apply the algorithm from Point 2 to get its TT format with rank r.