Exercise 8 – Low-rank approximation techniques

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Problem 1. Given two tensors $\mathcal{X}$ and $\mathcal{Y}$ in TT decomposition, derive a TT decomposition for $\mathcal{X} + \mathcal{Y}$.

Problem 2. Show that a tensor of tensor rank R has TT rank at most $(R, \ldots, R)$ by converting its CP decomposition into a TT decomposition.

Problem 3. Given a tensor in TT decomposition, derive upper bounds for its multilinear rank.

Problem 4. In Matlab, implement the TT-SVD algorithm from slide 10. Adjust the algorithm to work with prescribed accuracy $\varepsilon > 0$ for which it determines the TT ranks adaptively such that $\|\mathcal{X} - \mathcal{X}_{SVD}\| \leq \varepsilon$. (Use theorem from slide 11.)

Problem 5. Let $\mathcal{X}$ be a tensor in TT decomposition with TT cores $\mathcal{U}_1, \ldots, \mathcal{U}_d$.

1. Show that $\|\mathcal{X}\| = \|\mathcal{U}_d\|$ holds for a left-orthogonal TT decomposition. What can you say about the singular values of $X^{<\mu-1>}$?
2. Show that $\|\mathcal{X}\| = \|\mathcal{U}_1\|$ holds for a right-orthogonal TT decomposition. What can you say about the singular values of $X^{<1>}$?
3. In Matlab, implement the left and right orthogonalization process as described on slide 16 and check the conjectures from Points 1 and 2.

Problem 6. Using tensor contractions, develop an efficient method for computing the mean,

$$\bar{\mathcal{X}} = \frac{1}{n_1 \cdots n_d} \sum_{i_1, \ldots, i_d} \mathcal{X}(i_1, \ldots, i_d),$$

of a tensor in TT decomposition.