

# New Challenges in Hyperbolic Problems

Bernoulli Workshop

September 10 – 12, 2025

## Schedule

	Wednesday 10 <sup>th</sup>	Thursday 11 <sup>th</sup>	Friday 12 <sup>th</sup>
9:30 - 10:30		Kang	Del Zotto
10:30 - 11:00	Registration	Coffee break	Coffee break
11:00 - 12:00	Pacherie	Cristian	Krupa
12:00 - 14:00	Lunch	Lunch	Lunch
14:00 - 15:00	Nobili	Huysmans	
15:00 - 15:30	Coffee break	Coffee break	
15:30 - 16:30	Meyer	Coiculescu	

## Booklet of abstracts

### Non-Unique Smooth Solutions of the Navier-Stokes Equations from Critical Data

**Matei Coiculescu**

*Princeton University*

We consider the Cauchy problem for the incompressible Navier-Stokes equations in dimension three and construct initial data in the critical space  $BMO^{-1}$  from which there exist two distinct global solutions that are smooth after initial time. One consequence of this construction is the sharpness of the small data global well-posedness result of Koch and Tataru. This is joint work with Stan Palasek.

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### Coagulation models with hyperbolic flavors

**Iulia Cristian**

*Sorbonne Université*

Coagulation equations describe the evolution in time of a system of particles characterized by their volume. In this talk, we introduce the standard coagulation model. Depending on the interaction rate of the particles, solutions may lose mass instantaneously. Our aim is to identify coagulation models that admit mass-conserving solutions in cases where the standard model fails to conserve mass.

For this, we present various coagulation models with transport terms. We then compare these models and discover clear conditions for which mass-conserving solutions exist. Our main focus is a spatially inhomogeneous coagulation model that contains a transport term in the spatial variable. It arises in applications in order to describe rain initiation times or the behavior of air bubbles in water. For this model, we show that mass-conserving solutions exist, in contrast to the standard coagulation model.

This is based on some works with B. Niethammer and J. J. L. Velázquez.

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### On the stability of the rotating Couette flow

**Augusto Del Zotto**

*University of Zurich*

In this talk, we present an overview of our recent work (joint with M. Coti Zelati and K. Widmayer) on the linear and nonlinear stability of rotating Couette flow. We examine the behaviour of perturbations to this steady state under the Navier–Stokes equations, highlighting how the strength of rotation gives rise to distinct regimes of (in)stability at the linear level. Furthermore, we show how these stabilising effects can be exploited to address the corresponding nonlinear stability problem.

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## Mixing estimates for BV vector fields

**Lucas Huysmans**

*Max Planck Institute*

We show the first quantitative mixing estimate for the transport problem along a divergence-free vector field with bounded variation. The growth of the solution in a negative Sobolev space is bounded by repeated exponentiation in the time variable. The proof relies on a careful quantification of Ambrosio's original well-posedness result for BV vector fields, together with a compactness argument which we show can be quantified by a careful use of the pigeonhole principle.

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## Stability of small BV solutions to compressible Euler in a class of vanishing physical viscosity limits

**Moon-Jin Kang**

*KAIST*

The stability of a Riemann shock, in the absence of any technical conditions for perturbations, is a major challenging problem even within a mono-dimensional framework. A physically natural approach to justify the stability of such a singularity involves considering a class of vanishing physical dissipation limits (or viscosity limits) of physical viscous flows with evanescent viscosities. I will present the recent result for the well-posedness theory of entropy solutions to the 1D isentropic Euler system evolving from small BV initial data in the class of inviscid limits from the associated Navier-Stokes system. More precisely, small BV entropy solutions to the isentropic Euler can be constructed by inviscid limits from Navier-Stokes, and those are unique and stable among inviscid limits from Navier-Stokes. The proof is based on the three main methodologies: the modified front tracking algorithm; the  $\alpha$ -contraction method; the method of compensated compactness.

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## Are $L^\infty$ solutions to hyperbolic systems of conservation laws unique?

**Sam G. Krupa**

*ENS Paris*

For hyperbolic systems of conservation laws in 1D, fundamental questions about uniqueness and blow up of weak solutions still remain even for the apparently "simple" systems of two conserved quantities such as isentropic Euler and the p-system. Similarly, in the multi-dimensional case, a longstanding open question has been the uniqueness of weak solutions with initial data corresponding to the compressible vortex sheet. We address all of these questions by using the lens of

convex integration, a general method of constructing highly irregular and non-unique solutions to PDEs. Our proofs involve computer-assistance. This talk is based on joint work with László Székelyhidi, Jr.

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## Desingularization of vortex sheets

**David Meyer**

*ICMAT*

We show how to regularize vortex sheets by means of smooth, compactly supported vorticities that asymptotically evolve according to the Birkhoff–Rott vortex sheet dynamics. More precisely, consider a vortex sheet initial datum  $\omega_{\text{sing}}^0$ , which is a signed Radon measure supported on a closed curve. We construct a family of initial vorticities  $\omega_\epsilon^0 \in C_c^\infty(\mathbb{R}^2)$  converging to  $\omega_{\text{sing}}^0$  distributionally as  $\epsilon \rightarrow 0^+$ , and show that the corresponding solutions  $\omega_\epsilon(x, t)$  to the 2D incompressible Euler equations converge to the measure defined by the Birkhoff–Rott system with initial datum  $\omega_{\text{sing}}^0$ . Based on joint work with Alberto Enciso and Antonio Fernandez.

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## Buoyancy-Driven Flow in 2D Boussinesq Systems with Navier Slip: Scaling Laws and Long-Time Behavior

**Camilla Nobili**

*University of Surrey*

In this talk, we consider the two-dimensional Boussinesq equations on a bounded domain with Navier-slip boundary conditions, both in the presence and absence of thermal diffusion. The first part of the talk focuses on the derivation and interpretation of scaling laws for the Nusselt number in the context of Rayleigh–Bénard convection. In particular, we examine how the nature of the boundary conditions influences heat transport. In the second part, we turn to the large-time behaviour of solutions in absence of thermal diffusion. We show that, in appropriate norms, solutions converge to the hydrostatic equilibrium, and we establish its linear stability when the background temperature profile is an increasing affine function of height—that is, under stable vertical stratification.

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## Asymptotic profiles for the viscous Burgers equation with infinite mass

**Eliot Pacherie**

*CY Cergy*

We consider the viscous Burgers equation on the real line with initial data that decay slowly, in particular with infinite mass. We show that for some of them, the solution converges, after

rescaling to a limit profile, with two unexpected properties: 1) The solution of the viscous Burgers equation converges to 0 faster than the solution of the heat equation for the same initial data. That is, the nonlinear transport term enhanced the dissipation 2) The profile has a discontinuity that can be seen as a boundary layer. In other words, there are two scales to the profile.

We will explain how to prove these two properties, as well as particular cases where there are more than two scales near the boundary layer.

This talk is based on joint works with Tej-Eddine Ghoul and Nader Masmoudi, as well as with Nicola De Nitti.