Enjoying Probability and Fluids in Lausanne

September 18th-22th 2023, EPFL

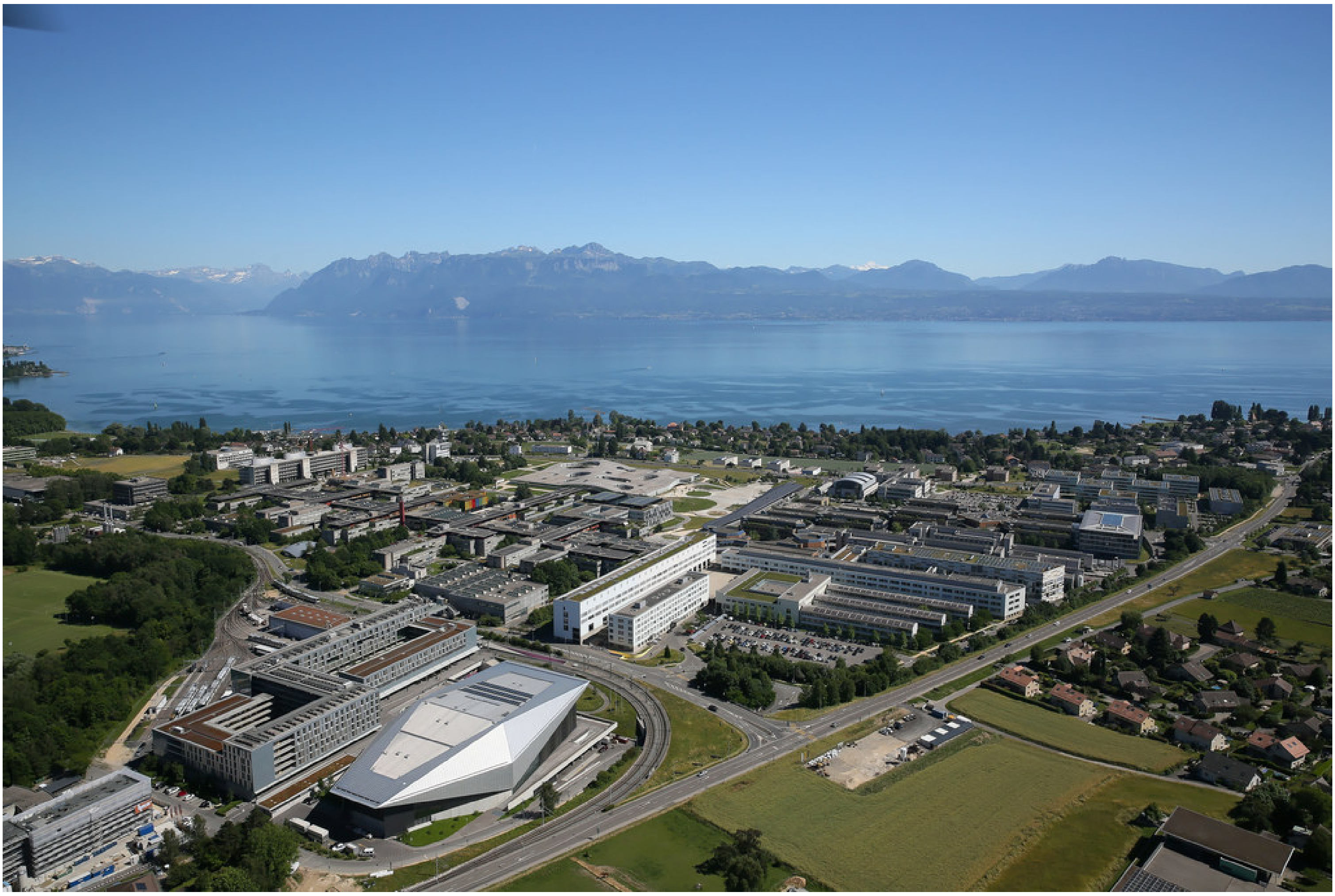

Figure 1: Courtesy of WikiCommons.

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Schedule

	Monday 18	Tuesday 19	Wednesday 20	Thursday 21	Friday 22
9:00 - 10:00		Ciampa	La		Rosati
10:00 - 11:00		Bianchini	Shi	Kwon	Carigi
11:00 - 11:30		Coffee break	Coffee break	Coffee break	Coffee break
11:30 - 12:30		Cooperman	Haziot	Bruè	Gvalani
12:30 - 14:30	Registration	Lunch	Lunch	Lunch	
14:30 - 15:30	Wu	García		Kumar	
15:30 - 16:00	Coffee break	Coffee break		Coffee break	
16:00 - 17:00	Pappalettera	Hassainia		Said	

The workshop will start on Monday 18th afternoon and end on Friday 22nd morning.

Monday 18th is bank holiday in Switzerland, therefore unfortunately the university canteens will be closed.

There will be a **social dinner on Wednesday 20th September** at 19:00 at the Lacustre (Quai Jean-Pascal Delamuraz 1, 1006 Lausanne).

Talks

Roberta Bianchini, *Mathematical Analysis of Stratified Fluids.*

We will be interested in the analysis of a system of PDEs modeling continuously stratified fluids under the influence of gravity. I will present some mathematical results related to (in)stability and long-time dynamics.

Elia Brué, *Transport equation and Cauchy problem for Sobolev velocity fields: old and new.*

We are focusing on the question of well-posedness for the linear transport equation involving incompressible Sobolev vector fields. We'll revisit the foundational ideas of the Ambrosio-DiPerna-Lions theory, the quantitative technique by Crippa-De Lellis, and the convex integration approach to show ill-posedness. Finally, we'll present new sharp examples of non-uniqueness. These examples steam on a novel approach inspired by a cantor-like construction recently introduced by Kumar.

Giulia Carigi, *Response theory for dissipative SPDEs.*

An easy to apply framework to establish response theory is presented which is tailored for nonlinear dissipative SPDEs of Navier-Stokes type. With response theory we mean in this context the following: one considers a dynamical system whose dynamical law depends on a parameter (here given by an SPDE where the parameter is in the forcing) and we say that one has a response theory if one can show a regularity in the dependence of the invariant measure on the parameter (here differentiability in weak topology or Holder continuity in a Wasserstein metric). The results are applied to the 2D stochastic Navier-Stokes equation and the stochastic two-layer quasi-geostrophic model, an intermediate complexity model popular in the geosciences to study atmosphere and ocean dynamics. The physical motivation for studying the response to perturbations in the forcings for models in geophysical fluid dynamics comes from climate change and relate to the question as to whether statistical properties of the dynamics derived under current conditions will be valid under different forcing scenarios. This work is jointly with Jochen Bröcker (University of Reading) and Tobias Kuna (Università dell'Aquila).

Gennaro Ciampa, *On the topology of the magnetic lines of solutions of the MHD equations.*

The goal of this talk is to provide examples of periodic smooth solutions of the Magnetohydrodynamics equations (MHD) for which the topology of the magnetic field lines changes under the evolution without any loss of regularity. This is known to be impossible in the non-resistive case by Alfvén's theorem. The reconnection of the magnetic field lines occurs instead in the resistive case, being responsible for many dynamic phenomena in astrophysics such as solar flares and solar winds. Although numerical and experimental evidences exist, analytical examples of magnetic reconnection were not known.

William Cooperman, *Homogenization for the random G-equation.*

How can a fish swim from point A to point B if the ocean current flows faster than the fish can swim? If the current is mean-zero and random in space, then the fish has a chance. I will discuss some results on stochastic homogenization of the G-equation, a convex but non-coercive Hamilton-Jacobi equation. Along the way, I will present a simplified proof of a quantitative first-passage percolation shape theorem of Kesten.

Claudia García, *Self-similar spirals for the generalized surface quasi-geostrophic equations.*

In this talk, we will construct a large class of non-trivial (non radial) self-similar solutions of the generalized surface quasi-geostrophic equation. To the best of our knowledge, this is the first rigorous construction of any self-similar solution for these equations. Moreover, the solutions are of spiral type, locally integrable and may have a change of sign. This is a joint work with Javier Gómez-Serrano.

Rishab Gvalani, *The thin-film equation with thermal noise.*

In this talk, we will study the lubrication approximation of the well-known fluctuating hydrodynamics model introduced by Landau and Lifschitz. The corresponding system is a fourth-order, degenerate, quasilinear singular PDE commonly referred to as the thin-film equation with thermal noise. We start by presenting an alternative derivation of the equation from thermodynamic principles using as inputs the correct invariant measure for the dynamics (the 1D Gaussian free field restricted to positive functions) and the correct dissipation mechanism (a weighted version of the H^{-1} inner product). Next, we propose a natural structure-preserving discretisation which preserves the strict positivity of the film height for large enough mobilities. Finally, we study the equation in the framework of the theory of regularity structures: we estimate (uniformly in the regularisation parameter) the appropriately renormalised centered model associated to the equation completing the first step in obtaining a solution theory for the equation. This talk is based on two separate works with Benjamin Gess (MPI-MiS/Bielefeld), Florian Kunick (MPI-MiS), and Felix Otto (MPI-MiS) and with Markus Tempelmayr (Uni. Muenster), respectively.

Zineb Hassainia, *On the desingularization of pairs of point vortices in the presence of background shear.*

In this talk, I will discuss the existence of periodic pairs of simply connected patches for Euler equations with a background shear flow. In particular, using a Nash-Moser implicit function iterative scheme, we show the existence of a curve of periodic pair of patches when their separation distance belongs to a Cantor set with almost full Lebesgue measure. This is a joint work with Taoufik Hmidi and Nader Masmoudi.

Susanna Haziot, *Desingularization and long-term dynamics of solutions to fluid models.*

The Muskat equation models the interaction of two incompressible fluids with equal viscosity propagating in porous medium, governed by Darcy's law. The Peskin problem describes the flow of a Stokes fluid through the heart valves. In this talk, we investigate the small data critical regularity theory for these two models, and in particular, the desingularization of interfaces with small corners. The first part is joint work with E. Garcia-Juarez, J. Gomez-Serrano and B. Pausader. The second part is joint work with E. Garcia-Juarez.

Anuj Kumar, *Construction of nonunique solutions of the transport and continuity equation for Sobolev vector fields.*

In this talk, we are concerned with DiPerna–Lions' theory for the transport equation. In the first part of the talk, I will discuss a few results regarding the nonuniqueness of trajectories of the associated ODE. Alberti '12 asked the following question: are there continuous Sobolev vector fields with bounded divergence such that the set of initial conditions for which the trajectories are not unique is of full measure? We construct an explicit example of divergence-free Hölder continuous Sobolev vector field for which trajectories are not unique on a set of full measure, which then answers the question of Alberti. The construction is based on building an appropriate Cantor set and a "blob flow" vector field to translate cubes in space. The vector field constructed also implies nonuniqueness in the class of measure solutions. The second part to talk is a more recent work jointly with E. Bruè and M. Colombo. We construct nonunique solutions of the continuity equation in the class L^∞ in time and L^r in space. We achieve this by introducing two novel ideas: (1) In the construction, we interweave the scaled copies of the vector field itself. (2) Asynchronous translation of cubes, which makes the construction heterogeneous in space. These new ideas allow us to prove nonuniqueness in the range of exponents beyond what is available using the method of convex integration and sharply match with the range of uniqueness of solutions from Bruè, Colombo, De Lellis '21.

Hyunju Kwon, *The Strong Onsager conjecture.*

Smooth solutions to the incompressible 3D Euler equations conserve kinetic energy in every local region of a periodic spatial domain. In particular, the total kinetic energy remains conserved. When the regularity of an Euler flow falls below a certain threshold, a violation of total kinetic energy conservation has been predicted due to anomalous dissipation in turbulence, leading to Onsager's theorem. Subsequently, the L^3-based strong Onsager conjecture has been proposed to reflect the intermittent nature of turbulence and the local evolution of kinetic energy. This conjecture states the existence of Euler flows with regularity below the threshold of $B^{1/3}_{3,\infty}$ which not only dissipate total kinetic energy but also exhibit intermittency and satisfy the local energy inequality. In this talk, I will discuss the resolution of this conjecture based on recent collaboration with Matthew Novack and Vikram Giri.

Joonhyun La, *Hydrodynamic limit of incompressible Euler equation.*

In this talk, we discuss recent progress in the hydrodynamic limit of incompressible Euler equation from the Boltzmann equation.

Umberto Pappalettera, *Anomalous and total dissipation due to advection by solutions of randomly forced Navier-Stokes equations.*

We show the existence of a velocity field v, solution of (randomly) forced Navier-Stokes equations, which produces total dissipation of kinetic energy in finite time when advecting a passive scalar ρ. The total dissipation holds true uniformly in the viscosity parameter and the initial conditions ρ_0, in particular the dissipation is anomalous. Dissipation induced by single realizations of v is also discussed. Our results extend to the case when ρ is replaced by a solution to the two or three dimensional (deterministic) Navier-Stokes equations advected by v. Based on a joint work with M. Hofmanová, R. Zhu and X. Zhu.

Tommaso Rosati, *Lower bounds to Lyapunov exponents for stochastic PDEs.*

We introduce an approach to control the projective dynamic of dissipative linear stochastic PDEs with adapted initial data. Our results apply to vector valued stochastic heat equations and hyperviscous equations. Our proof relies on the introduction of a novel Lyapunov functional for the projective process associated to the equation, based on the study of dynamics of the energy median and on a notion of non-degeneracy of the noise that leads to high-frequency stochastic instability. This technique is applied to obtain - for the first time in a setting without order preservation - lower bounds to Lyapunov exponents of the equation with adapted initial data, and - under more stringent conditions on the model - their uniqueness via Furstenberg-Khasminskii formulas. Joint work with Martin Hairer.

Ayman Rimah Said, *Local well-posedness and singularity formation beyond the Yudovich class.*

In this talk, I will present recent results obtained in collaboration with Tarek M. Elgindi and Ryan M. Murray. We give a new supercritical class of data for the 2D Euler equation that allows for unbounded vorticities well beyond the Yudovich class. Within this class, we can demonstrate local existence and uniqueness of the solutions. Furthermore, we construct data for which a finite-time blow-up occurs.

Jia Shi, *On the analyticity of the Muskat equation.*

The Muskat equation describes the interface of two liquids in a porous medium. We will show that if a solution to the Muskat problem in the case of same viscosity and different densities is sufficiently smooth, then it must be analytic except at the points where a turnover of the fluids happens. We will also show analyticity in a region that degenerates at the turnover points provided some additional conditions are satisfied.

Bian Wu, *Scale invariant bounds for mixing in the Rayleigh-Taylor instability.*

I will talk about Rayleigh-Taylor instability for two miscible, incompressible, inviscid fluids. Scale-invariant estimates for the size of the mixing zone and coarsening of internal structures in the fully nonlinear regime are established. These bounds provide optimal scaling laws and reveal the strong role of dissipation in slowing down mixing. This is a joint work with Konstantin Kalinin, Govind Menon.