Deterministic and random features in fluids

July 3rd-7th 2023, EPFL

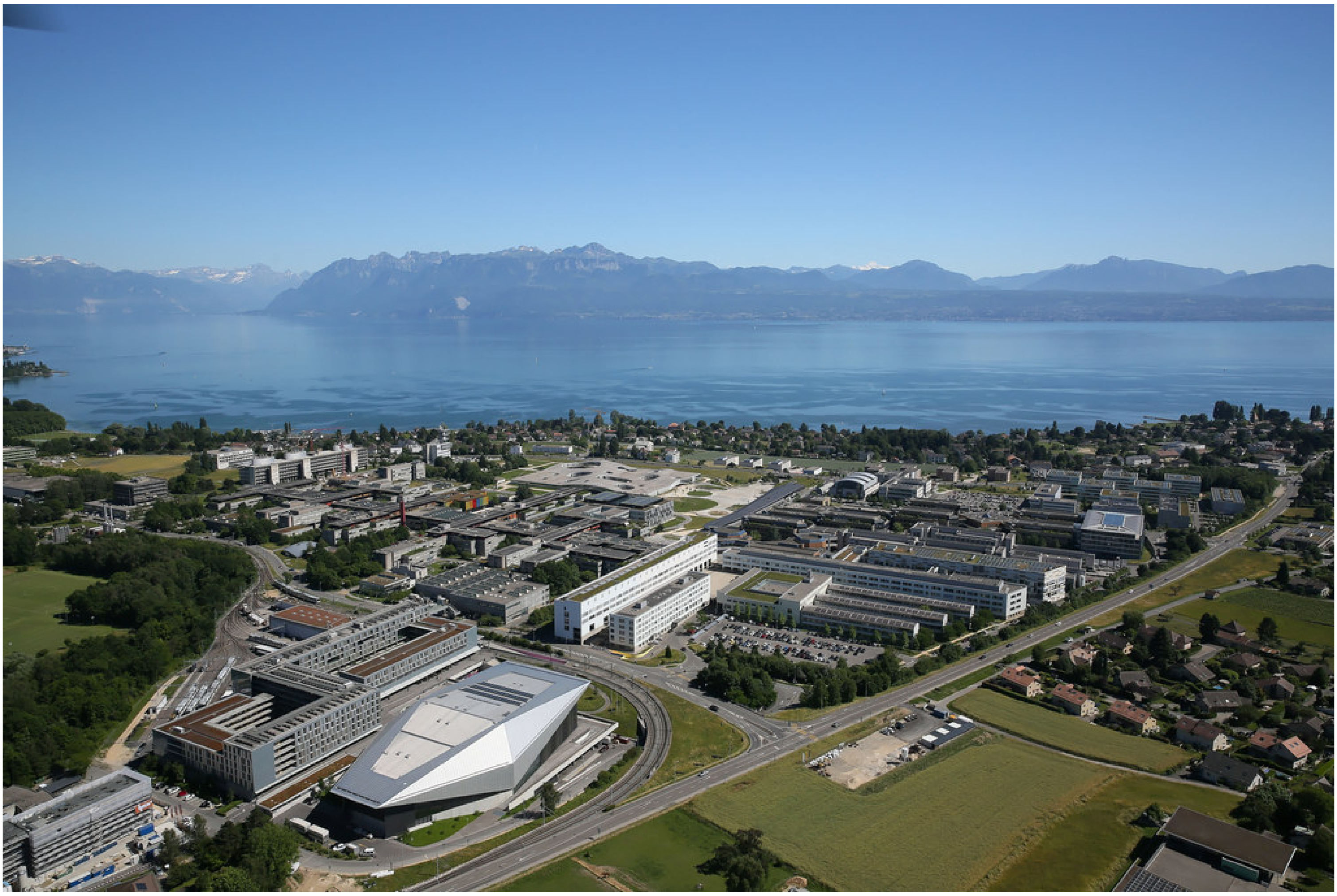

Figure 1: Courtesy of WikiCommons.

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Schedule

	Monday 3	Tuesday 4	Wednesday 5	Thursday 6	Friday 7
9:00 - 10:00	Registration	Bedrossian	Bedrossian	Gallay	Bedrossian
10:00 - 11:00	Gallay	Bedrossian	Bedrossian	Gallay	Bedrossian
11:00 - 11:30	Coffee break	Coffee break	Coffee break	Coffee break	Coffee break
11:30 - 12:30	Gallay	Gallay	Gallay	Armstrong	Albritton
12:30 - 14:00	Lunch	Lunch	Lunch	Lunch	Lunch
14:00 - 15:00	Zhu	Jeong		Punshon-Smith	
15:00 - 15:30	Liss	Park		Zizza	
15:30 - 16:00	Coffee break	Coffee break		Coffee break	
16:00 - 17:00	Yao	Novack		Lange/Lim	

There will be a **social dinner on Thursday 6th July** at 20:00 at the Petit Port (Chem. du Petit-Port 11, 1025 Saint-Sulpice).

Minicourses

Jacob Bedrossian, *Lagrangian chaos and almost-sure mixing in stochastic fluid mechanics.*

This course will introduce students to the basics of the random dynamical systems methods in order to study "chaos" of the Lagrangian flow map and almost-sure exponential mixing of passive scalars by stochastically forced fluid equations and similar random flows. First we discuss Lyapunov exponents, formulas for them, and how to estimate them. Time permitting, we will then discuss a general scheme for how to upgrade positive Lyapunov exponents to almost-sure exponential mixing estimates of passive scalars. While we will focus on finite-dimensional examples (i.e. Galerkin truncations of the Navier-Stokes equations rather than the full NSE themselves) we will discuss what ingredients need to be added and enhanced in order to extend the results to the infinite dimensional Navier-Stokes equations.

Thierry Gallay, *Stability of Vortex Rings at High Reynolds Number.*

The goal of these lectures is to present in some detail a recent result in collaboration with V. Sverak, which is devoted to the vanishing viscosity limit for vortex rings originating from circular filaments. The whole analysis is carried out in the framework of axisymmetric flows without swirl, which is both mathematically convenient and physically relevant for the phenomena we want to study. We first review a few standard properties of the axisymmetric solutions without swirl of the incompressible Navier–Stokes equations. We then explain how to construct an approximate solution of our problem, which is accurate enough in the high Reynolds number regime. Finally, we establish the stability of our approximation using carefully designed energy estimates, which partially rely on Arnold's geometric approach to the stability of stationary flows for the two-dimensional Euler equations. Open questions and perspectives will also be discussed.

Useful material for this minicourse, including handwritten notes, bibliography and related papers, can be found at this link.

Talks

Dallas Albritton, *Kinetic shock profiles for the Landau equation.*

Compressible Euler solutions develop jump discontinuities known as shocks. However, physical shocks are not, strictly speaking, discontinuous. Rather, they exhibit an internal structure which, in certain regimes, can be represented by a smooth function, the shock profile. We demonstrate the existence of weak shock profiles to the kinetic Landau equation. Joint work with Jacob Bedrossian (UCLA) and Matthew Novack (Purdue University).

Scott Armstrong, *Anomalous diffusion by fractal homogenization.*

I will try to explain the ideas underlying our recent work with Vlad Vicol in which we give an example of divergence-free vector field, which is nearly $C^{1/3}$, such that the corresponding advection-diffusion equations admits anomalous diffusion, for all initial data, along a subsequence of diffusivities tending to zero. This work is based on the (very old) idea that anomalous diffusion for such a scalar equation may be understood as a consequence of an enhancement of diffusivity, due to homogenization, iterated across many scales.

In-Jee Jeong, *Stability and instability of vortex patches.*

Vortex patches are solutions to 2D incompressible Euler equations in which the vorticity is the characteristic function of a set moving with the fluid. There are many known uniformly rotating patch solutions, the most famous example being the Kirchhoff ellipses (1876). We discuss how the nonlinear stability of some rotating patches can be obtained from a variational framework. Then we show how stability can be used to prove various unstable behavior for certain perturbations.

Matthew Novack, *The strong Onsager conjecture.*

The phenomenon of anomalous dissipation in turbulence predicts the existence of solutions to the incompressible Euler equations that enjoy regularity consistent with Kolmogorov's 4/5 law and satisfy a local energy inequality. The "strong Onsager conjecture" asserts that such solutions do indeed exist. In this talk, we will discuss the background and motivation behind the strong Onsager conjecture. In addition, we outline a construction of solutions with regularity (nearly) consistent with the 4/5 law, thereby proving the conjecture in the natural L^3 scale of Besov spaces. This is based on joint work with Hyunju Kwon and Vikram Giri.

Sam Punshon-Smith, *Using regularity to estimate Lyapunov exponents for SDE.*

I will present a general method, developed with Jacob Bedrossian and Alex Blumenthal, for estimating the top Lyapunov exponent for finite dimensional hypoelliptic diffusions from below using local regularity of stationary measures on the projective bundle. For damped-driven SDE in fluctuation dissipation scaling, this provides a robust strategy for proving positivity of the top Lyapunov exponent as the damping parameter is taken to zero by verifying certain rigidity and hypoellipticity conditions. These conditions can be verified explicitly for arbitrary Galerkin truncations of the stochastic Navier Stokes equations on $\mathbb{T}^2$ using techniques from computational algebraic geometry to verify hypoellipticity. Time permitting, I will discuss recent ongoing work estimating the sum of the first k Lyapunov exponents and its connection to lower bounds on attractor dimension.

Yao Yao, *Small scale formation for the 2D Boussinesq equation.*

In this talk, we consider the 2D incompressible Boussinesq equation without thermal diffusion, and aim to construct rigorous examples of small scale formations as time goes to infinity. In the viscous case, we construct examples of global-in-time smooth solutions where the H^1 norm of density grows to infinity algebraically in time. For the inviscid equation in the strip, we construct examples whose vorticity grows at least like t^3 and gradient of density grows at least like t^2 during the existence of a smooth solution. These growth results work for a broad class of initial data, where we only require certain symmetry and sign conditions. As an application, we also construct solutions to the 3D axisymmetric Euler equation whose velocity has infinite-in-time growth. (joint work with Alexander Kiselev and Jaemin Park)

Rongchan Zhu, *Non-unique ergodicity for* 3D *Navier–Stokes and Euler equations.*

We establish existence of infinitely many stationary solutions as well as ergodic stationary solutions to the three dimensional Navier–Stokes and Euler equations in the deterministic as well as stochastic setting, driven by an additive noise. The solutions belong to the regularity class $C(\mathbb{R}; H^\vartheta) \cap C^\vartheta(\mathbb{R}; L^2)$ for some $\vartheta > 0$ and satisfy the equations in an analytically weak sense. Moreover, we are able to make conclusions regarding the vanishing viscosity limit. The result is based on a new stochastic version of the convex integration method which provides uniform moment bounds locally in the aforementioned function spaces.

Mini talks

Theresa Lange, *Global existence and non-uniqueness-in-law for 3D Euler equations with transport noise.*

Consider the three-dimensional Euler equations perturbed by a multiplicative noise of transport type. With the help of a flow transformation of the resulting SPDE and a convex integration scheme inspired by De Lellis-Szekelyhidi-2014, we show existence of Hölder continuous, global-in-time solutions. Our analysis allows to prescribe the kinetic energy up to a stopping time which gives non-uniqueness in law. The results presented in this talk are based on joint work with Martina Hofmanová (Bielefeld University) and Umberto Pappalettera (Bielefeld University).

Deokwoo Lim, *Stability–instability of vortex solutions in incompressible Euler equations.*

In this talk, we consider the vorticity form of the 2D incompressible Euler equations either in the whole plane $\mathbb{R}^2$ or in the half cylinder $S_+ := \mathbb{R}_+ \times \mathbb{T}$. It is well-known that in $\mathbb{R}^2$, any radial vorticity is a steady solution of the vorticity form, such as circular vortex patches. We will discuss about ideas which shows stability of circular patches in some norm involving the angular impulse. Then we adapt these ideas to S_+, and construct a vortex patch in this domain that shows infinite perimeter growth for infinite time. This is a joint work with Kyudong Choi(UNIST) and In-Jee Jeong(SNU).

Kyle Liss, *Enhanced diffusion for alternating shear flows.*

We consider the long-time behavior of solutions to the advection-diffusion equation with an incompressible velocity field. An intriguing feature of this equation is that the formation of small scales due to advection can cause solutions to approach equilibrium much faster than on the standard diffusive timescale in the regime of weak diffusion. This phenomenon is often referred to as enhanced diffusion. In this talk, I will discuss recent joint work with Tarek Elgindi and Jonathan Mattingly in which we study enhanced diffusion on the periodic box for time-periodic, alternating piecewise linear shear flows. Our main result is an estimate on the convergence rate of solutions to equilibrium which is optimal with respect to scaling in the small diffusive parameter. The proof is based on the probabilistic representation formula of the advection-diffusion equation and ideas from dynamical systems.

Jaemin Park, *Construction of quasi-periodic solutions to active scalar equations.*

Tautologically, a smooth, steady fluid remains smooth for all time since it does not evolve non-trivially in time. A natural question is whether a small perturbation of such a smooth steady state can also remain smooth for all time. Especially, when the well-posedness of the governing equation is in question, the investigation of initial data near stable steady states can give insight into potential global-in-time solutions. In this talk, we will discuss the construction of global solutions to the generalized surface quasi-geostrophic equations (gSQG) by means of the KAM theory. This is a joint work with Javier Gomez-Serrano and Alexandru Ionescu.

Martina Zizza, *The role of Permutations in mixing Phenomena.*

In this talk I will present an approximation result of divergence-free vector fields in the unit square by permutation vector fields: namely, vector fields whose flow permutes the subsquares of a grid of $[0,1]^2$. I will describe a way to obtain mixing properties of the vector fields starting from permutations, focusing both on weak and strong mixing.