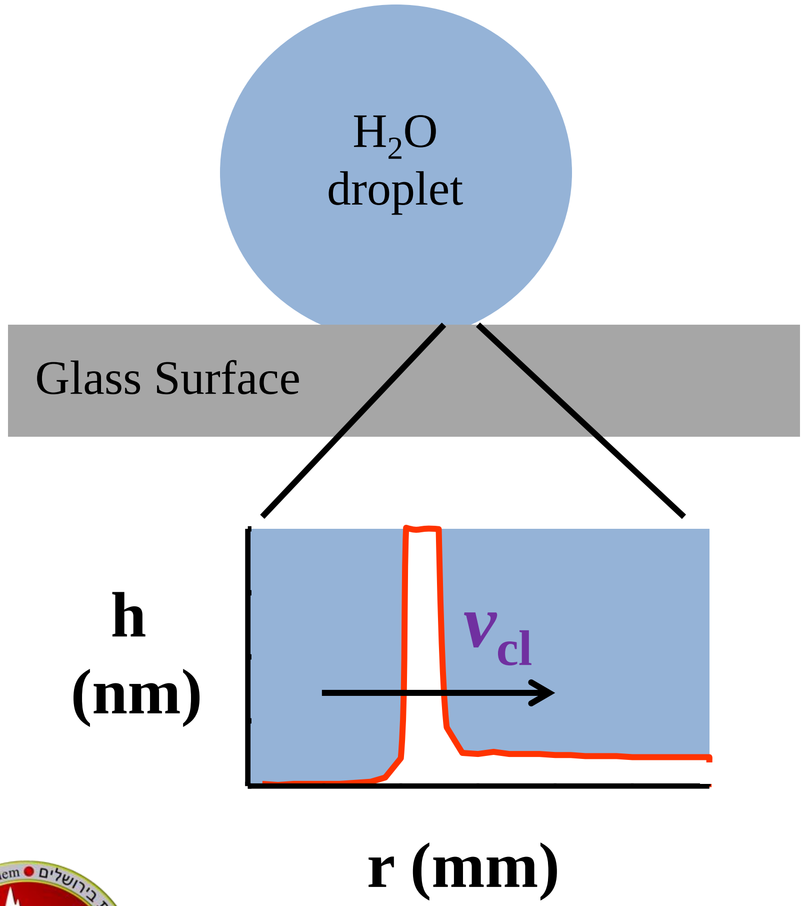
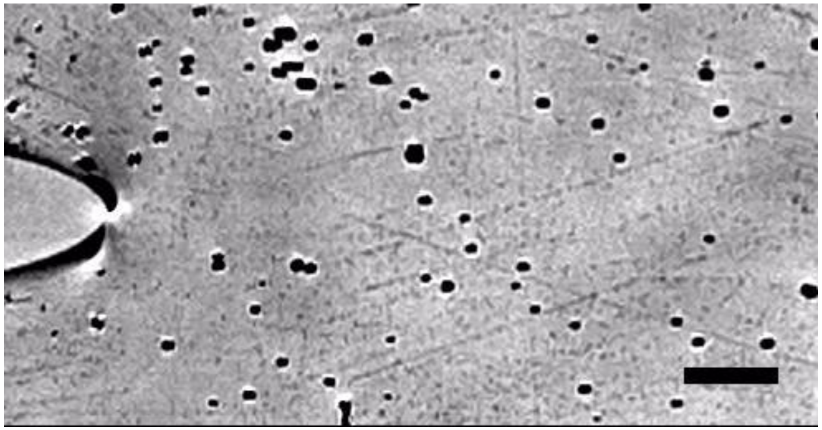
Dynamic Instabilities at Interfaces

propagating contact lines and *dynamic fracture mechanics*



Dynamic crack in a particle-laden gel:



Total time: 440 msec

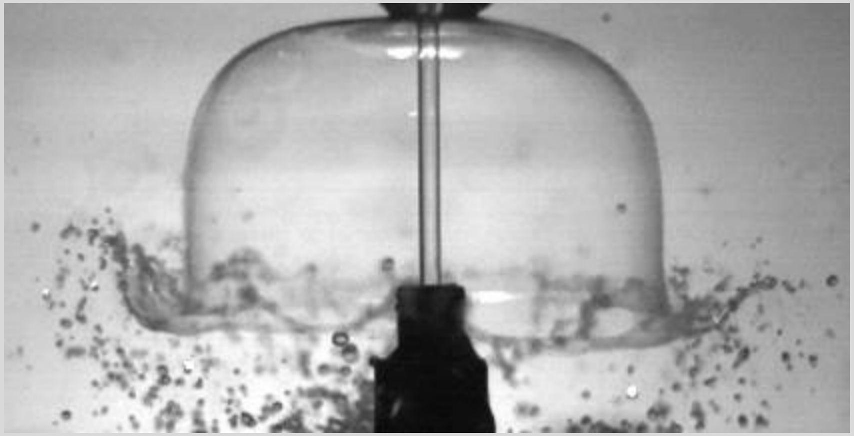
John Kolinski[1], Shmuel Rubinstein[2], Lital Levy[1], Jay Fineberg[1]

1. Hebrew University of Jerusalem
2. Harvard University

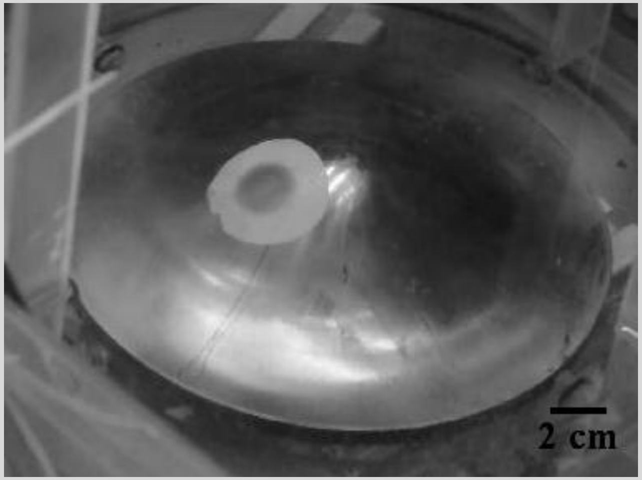
Supported by: Fulbright foundation, NSF GRFP, ERC and Israeli Science Foundation

Dynamic Instabilities at Interfaces

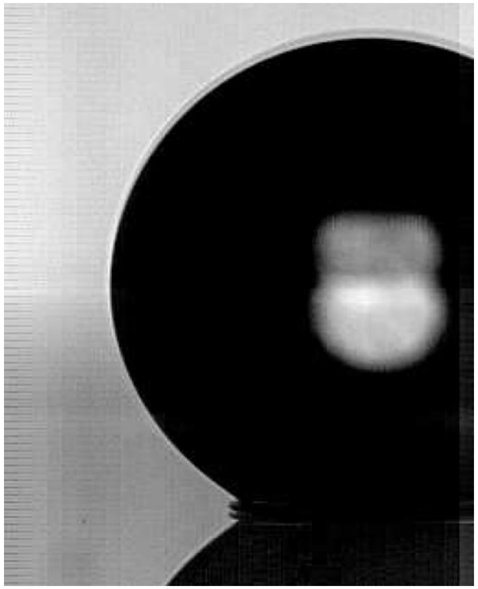
Water bells & dynamics of fluid sheets



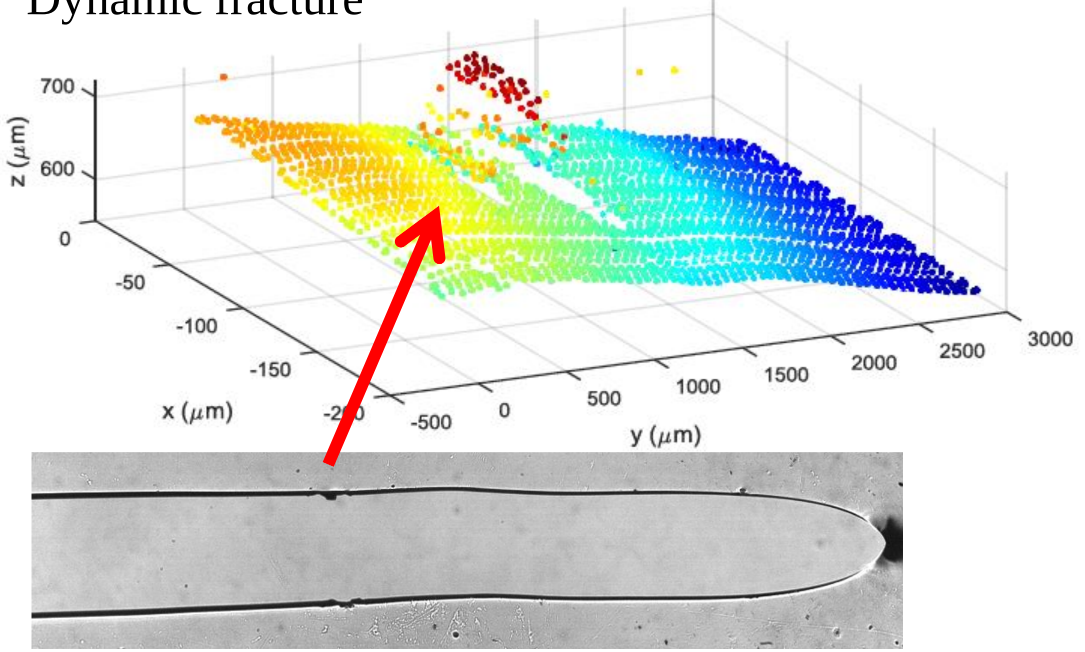
Migration by frustration (table-top relativity)



Droplet splashing & contact lines

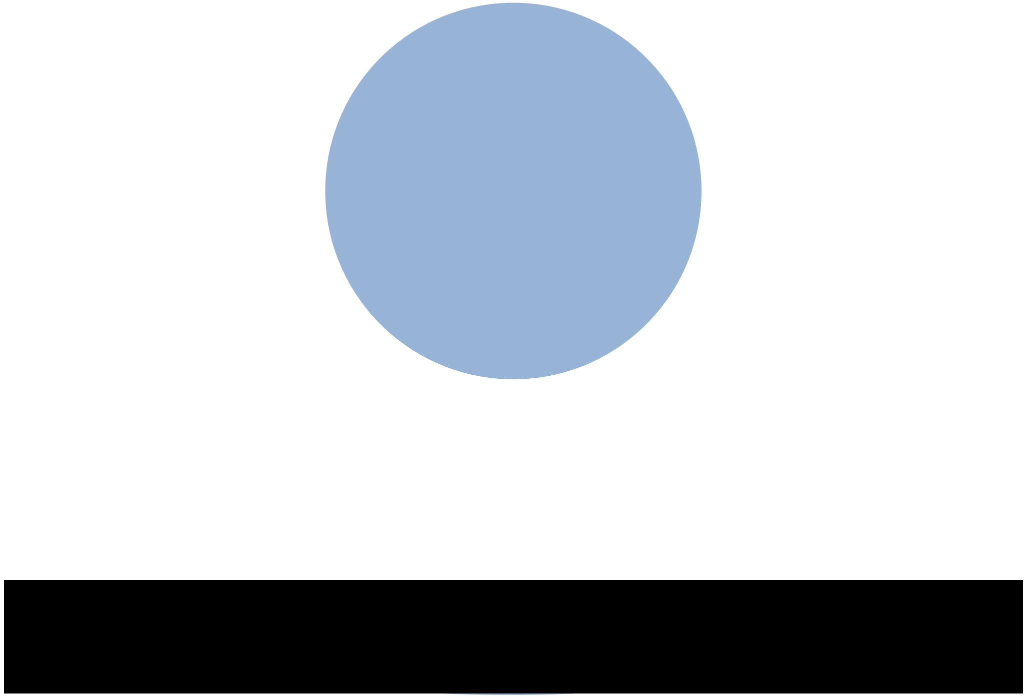


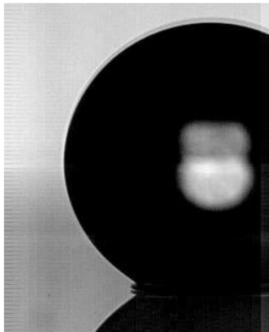
Dynamic fracture



Dynamic Instabilities at Interfaces:

propagating contact lines beneath impacting drops



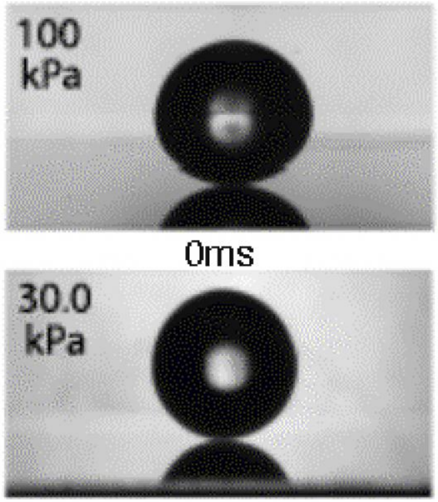


Work with Shmuel Rubinstein

No ambient pressure - No splash

Atmospheric pressure

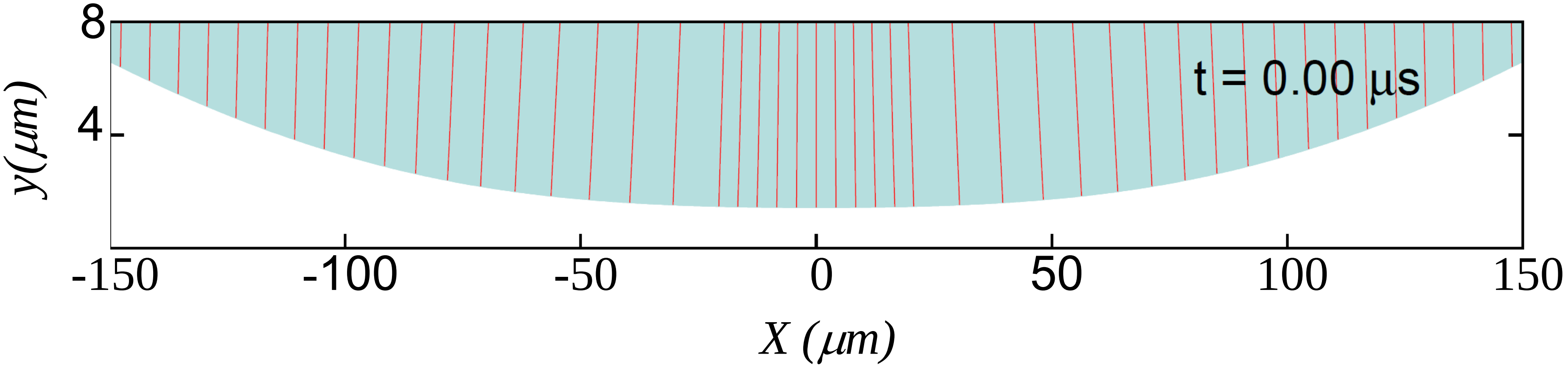
Reduced pressure



When pressure is reduced ‘*slightly*’ lower then atmospheric pressure corona splashing is *suppressed*

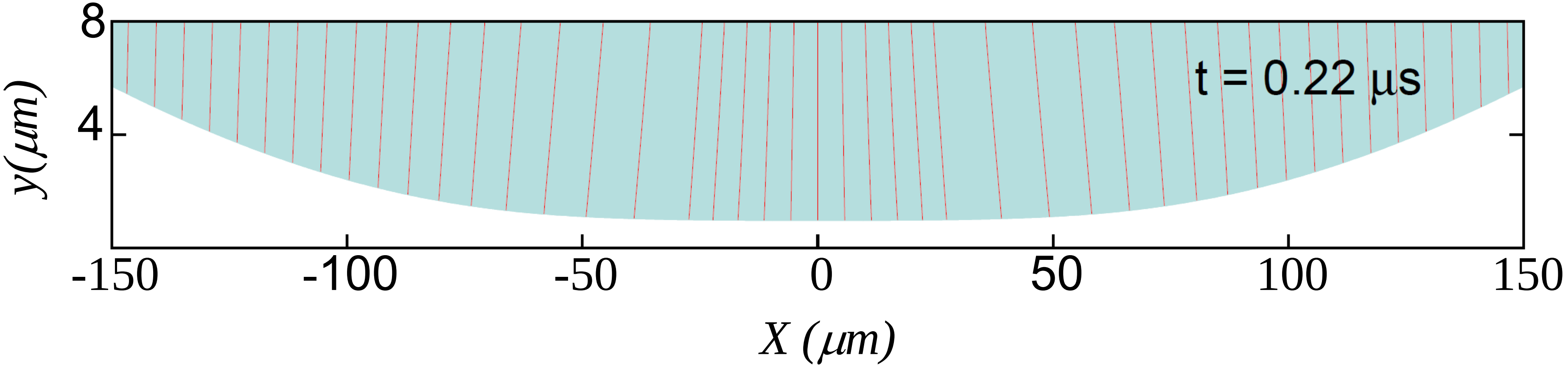
Xu et al. prl. 2005

Air pressure matters - ***Why***?



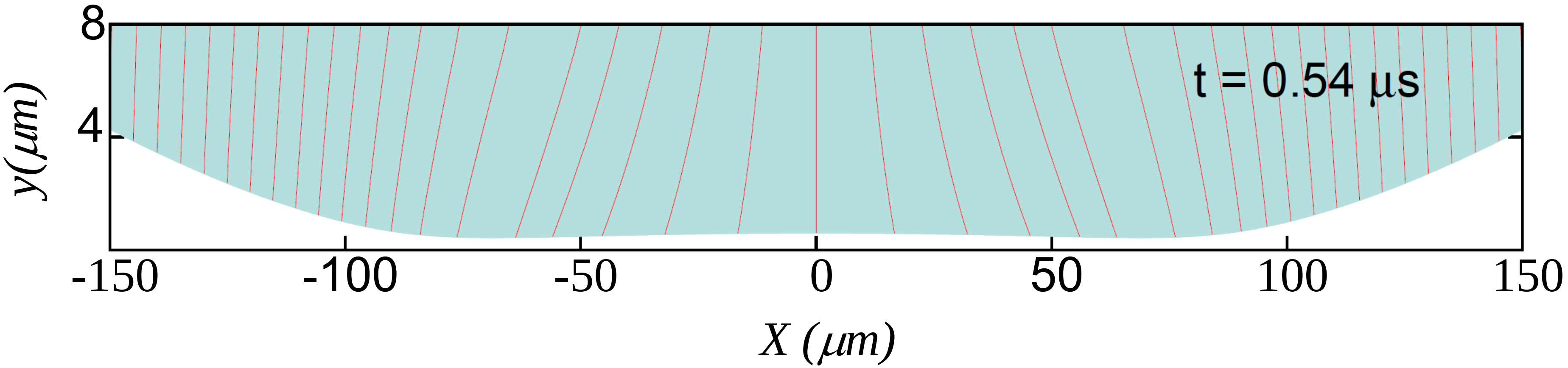
Mandre et al

Air pressure matters - ***Why***?



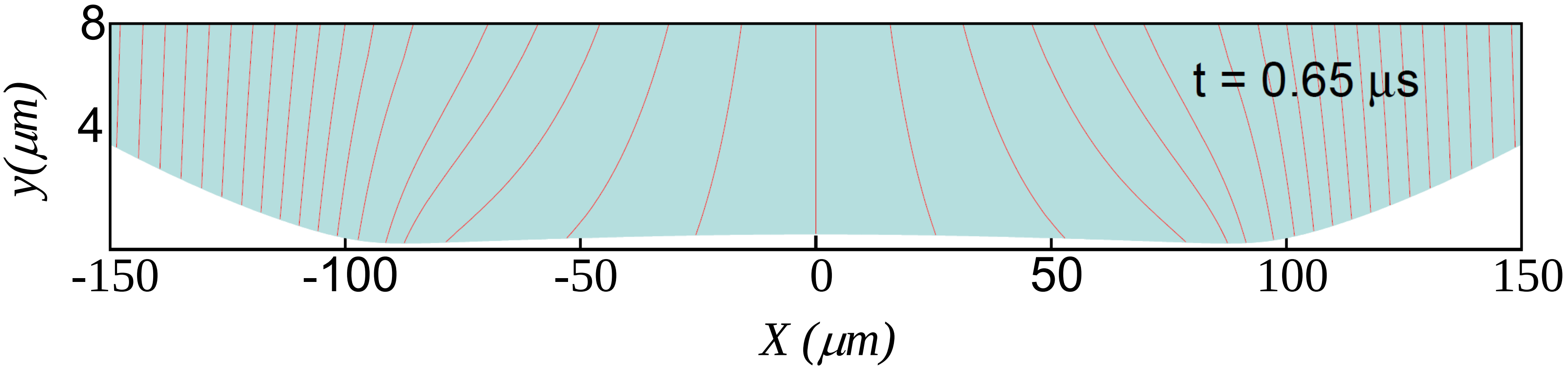
Mandre et al

Air pressure matters - ***Why***?



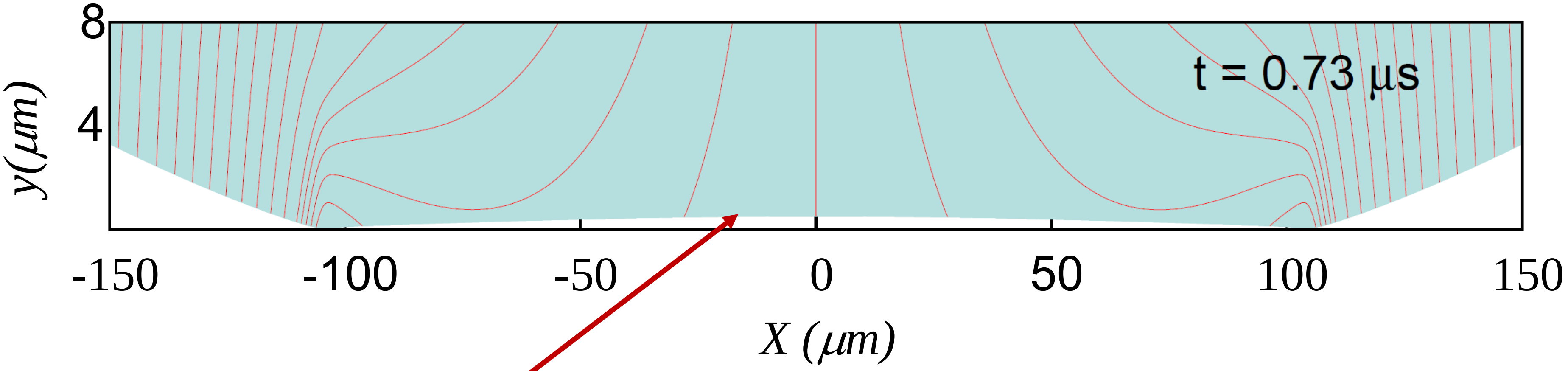
Mandre et al

Air pressure matters - ***Why***?



Mandre et al

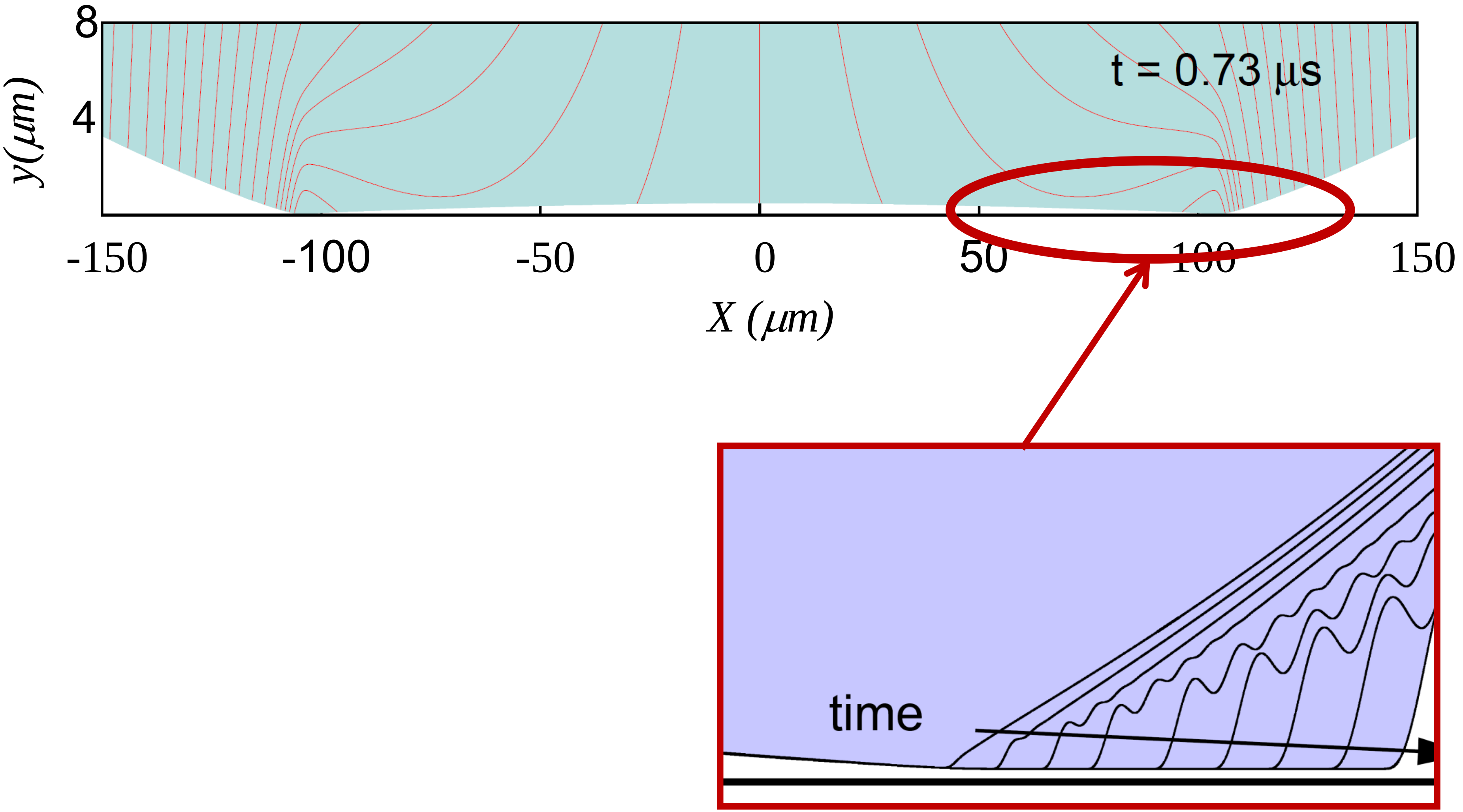
Air pressure matters - ***Why***?



Pocket of air

Mandre et al

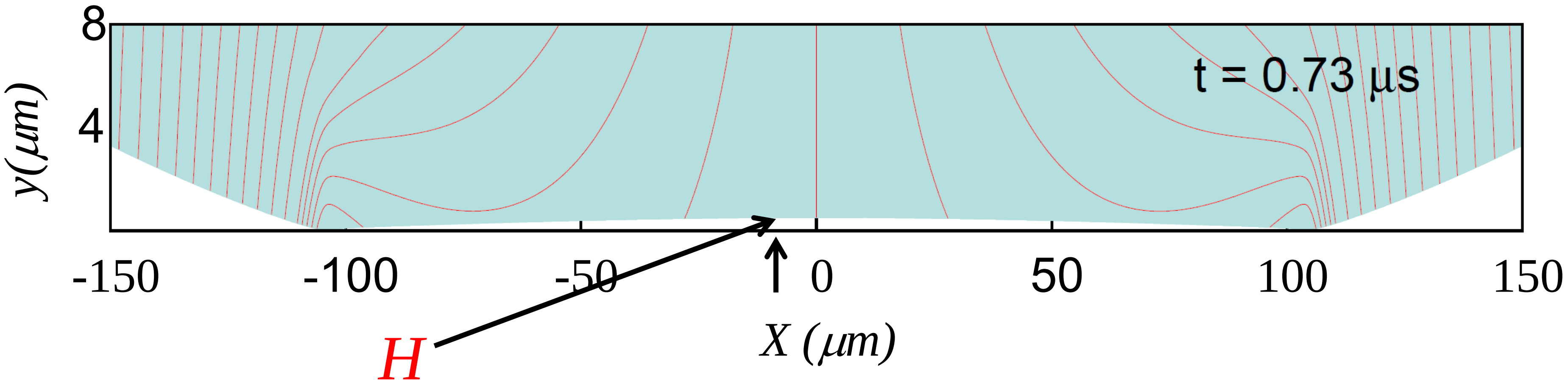
Air pressure matters - ***Why***?

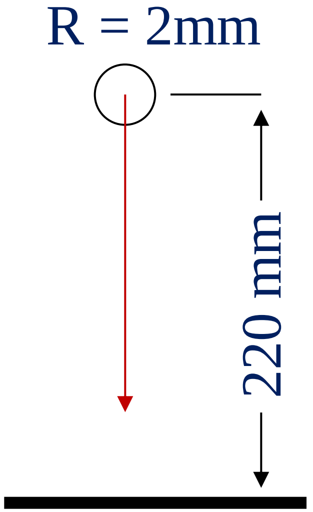


At the ***leading edge*** of the spreading droplet, the liquid skates over a ***nm-scale air film***

Mandre et al

Air pressure matters - ***Why***?





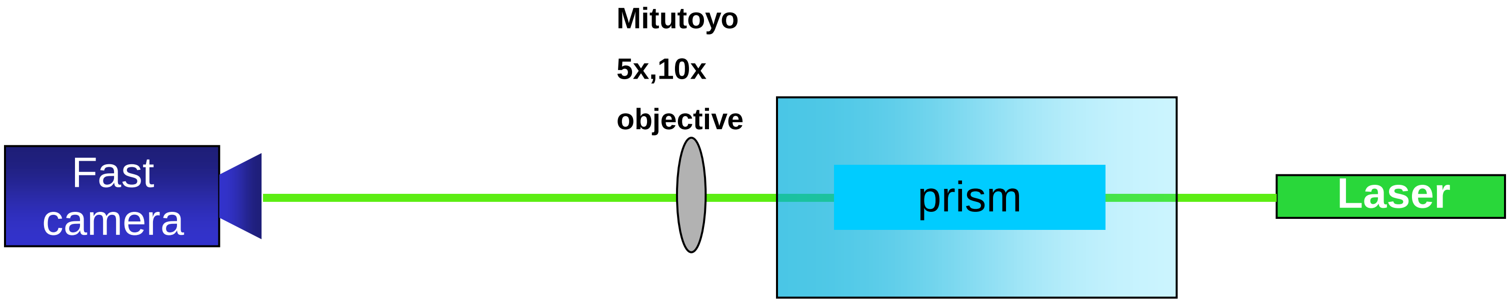
$$V = \sqrt{2\cdot g\cdot h} \approx 1.5 m/s$$

$$\Rightarrow \quad H \sim 1\mu m \quad \Rightarrow \quad \tau < 1\mu s$$

Mandre et. al.

… but how can we ***measure*** impact dynamics??

Rapid Evanescent microscopy



Mitutoyo

5x,10x

objective

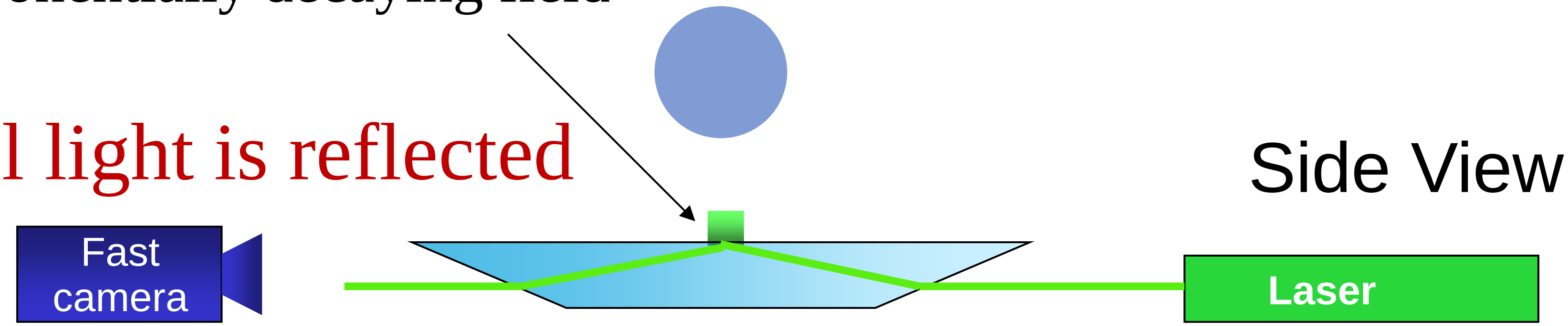
Fast camera

prism

Laser

The angle of incidence is ***greater*** than the critical angle for total internal reflection of a glass/air interface but ***smaller*** than that of a glass/Liquid interface.

Exponentially decaying field



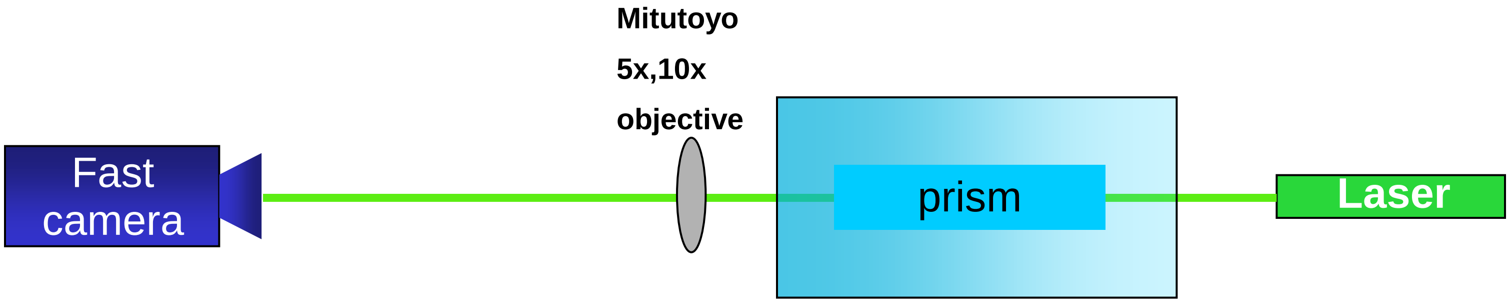
All light is reflected

Side View

Fast camera

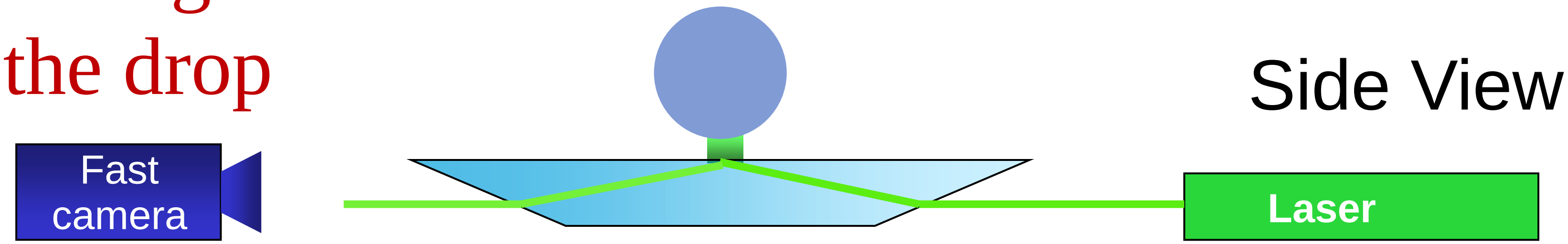
Laser

Rapid Evanescent microscopy



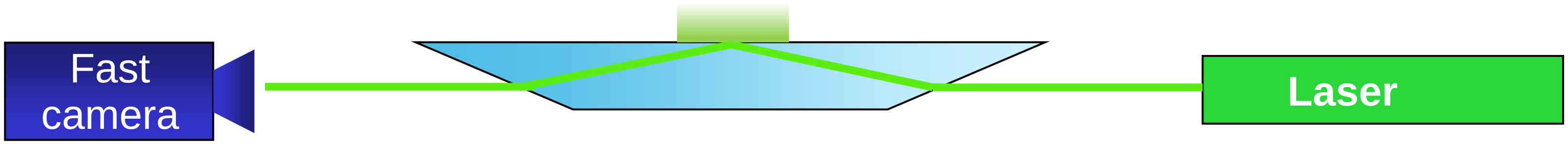
The angle of incidence is ***greater*** than the critical angle for total internal reflection of a glass/air interface but ***smaller*** than that of a glass/Liquid interface.

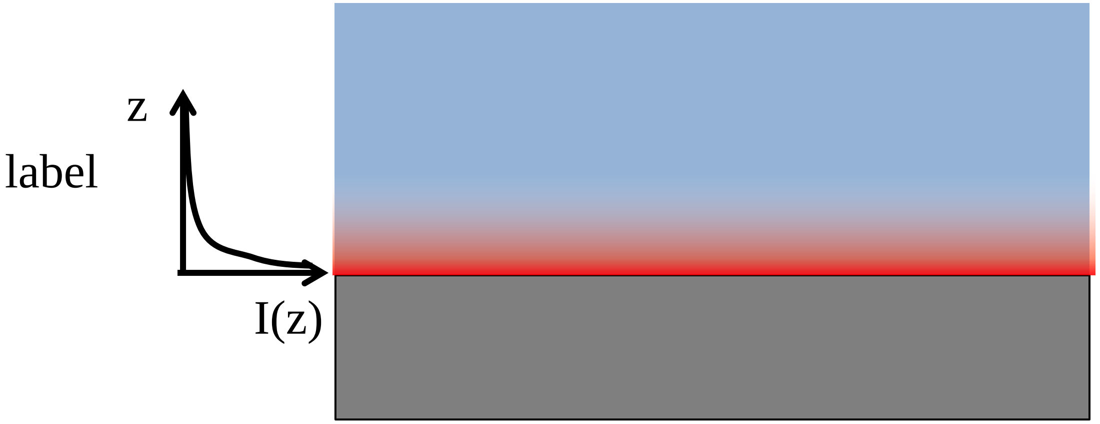
Some light tunnels to the drop



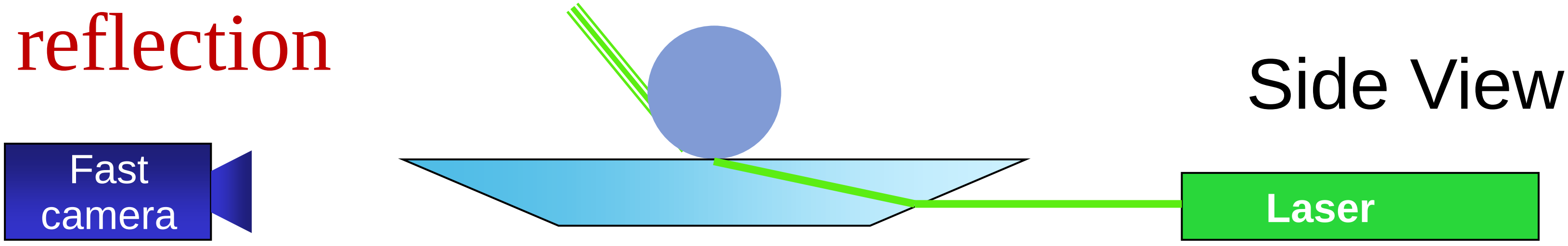
Rapid Evanescent microscopy

Transmission of light drops off ***exponentially*** with a decay-length of order ***100 nm***





No reflection



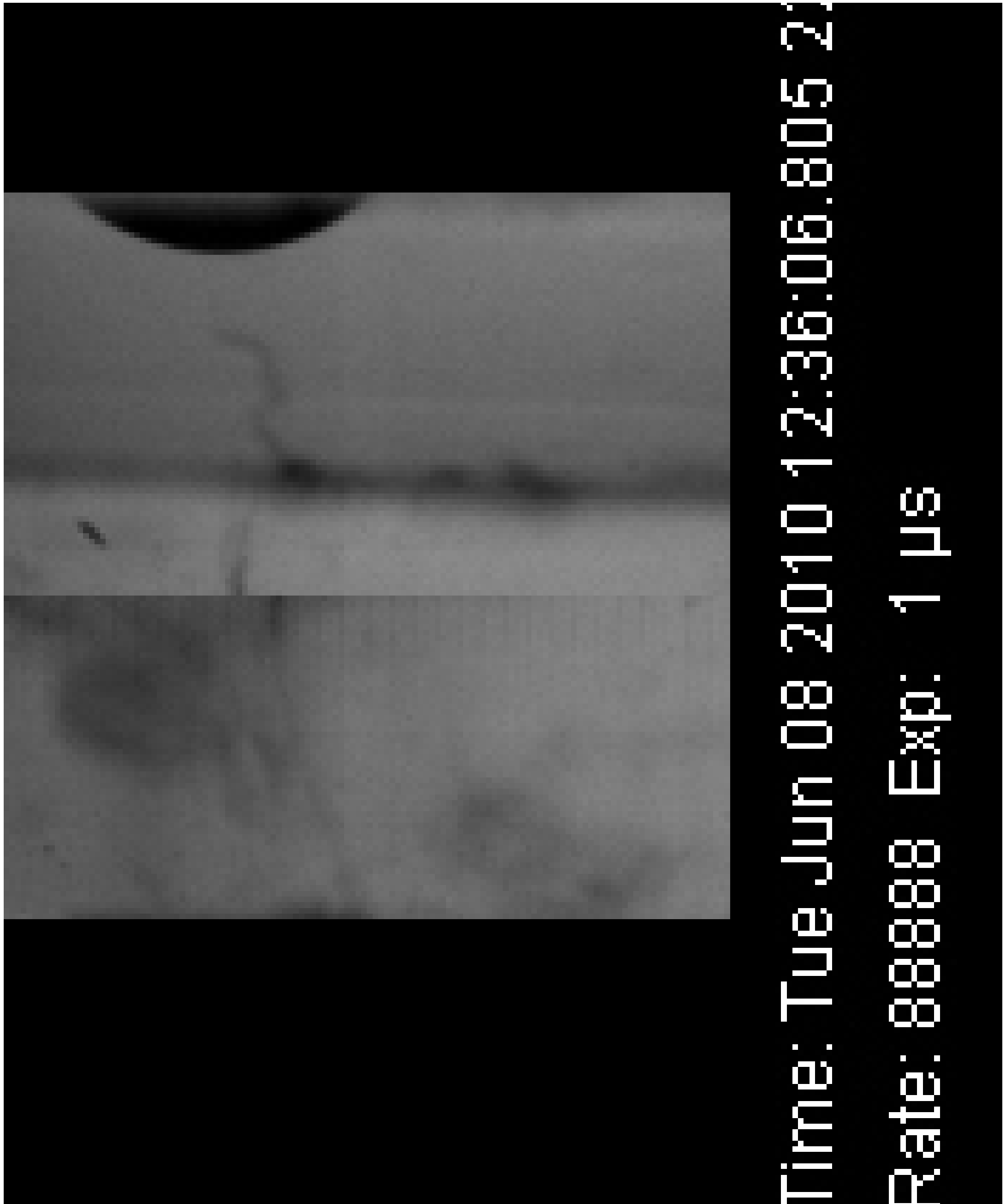
Side View

Rapid Evanescent microscopy

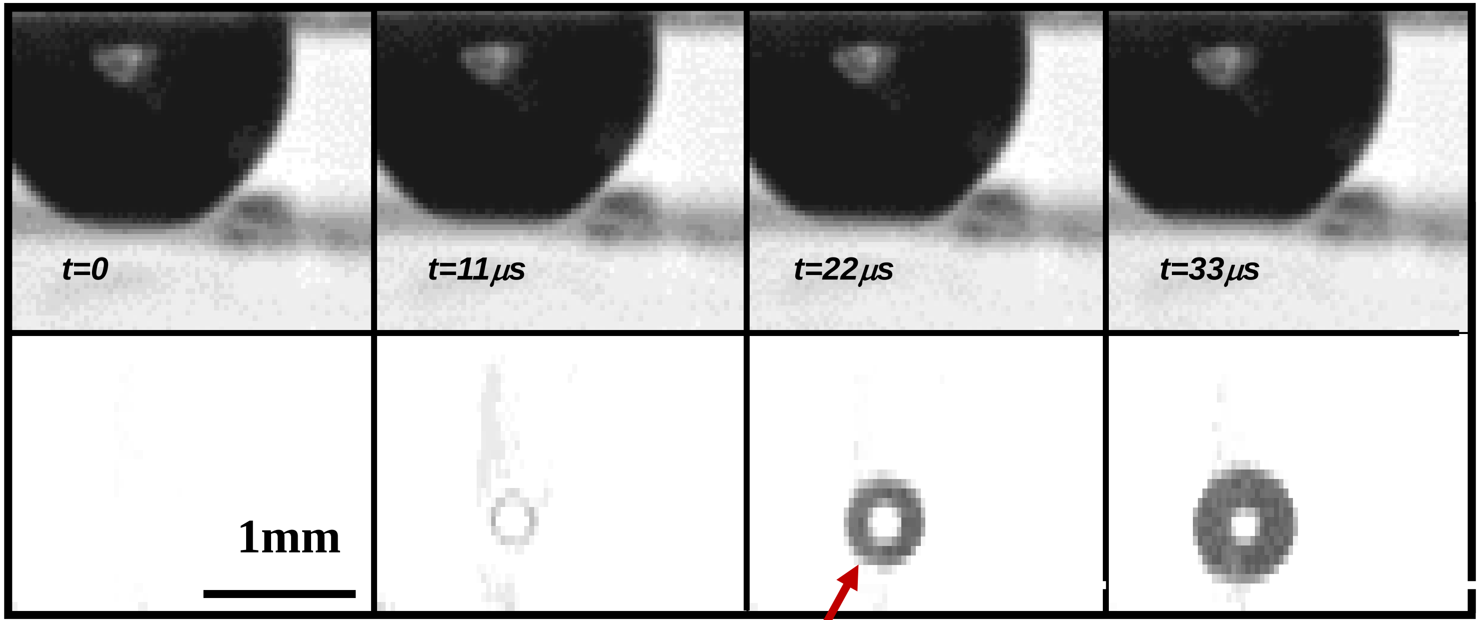
Side view

Bottom view

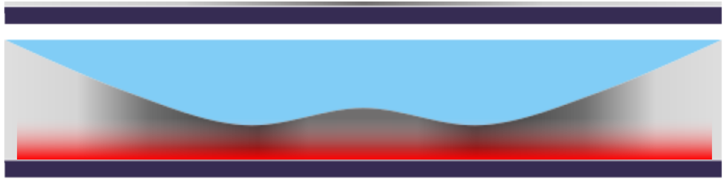
(evenescent)



Rapid Evanescent microscopy



Contact initially occurs in the shape of a ring



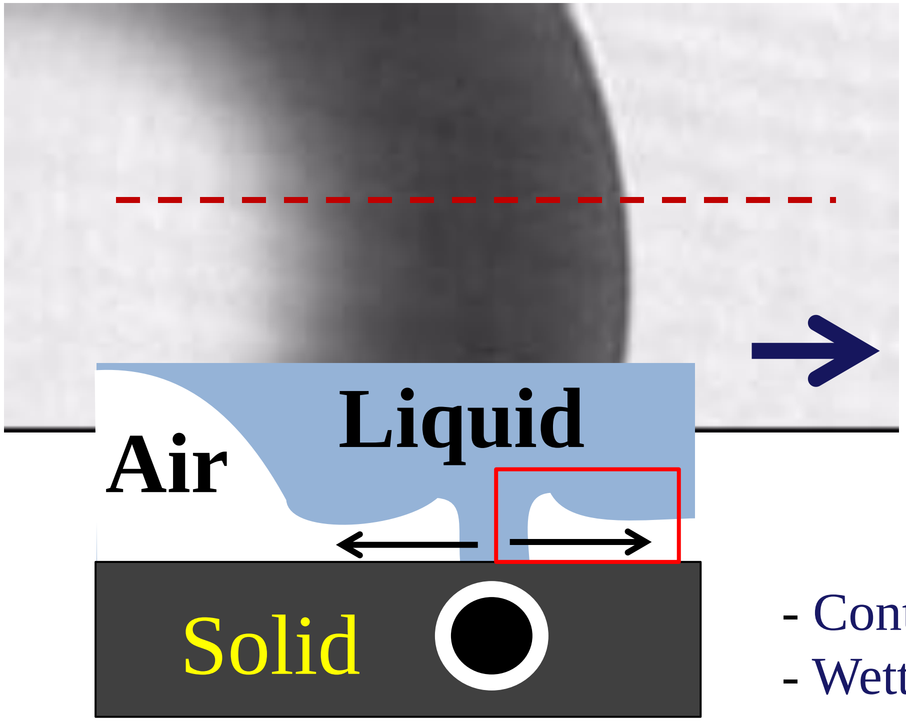
Mani et al. 2008, 2010

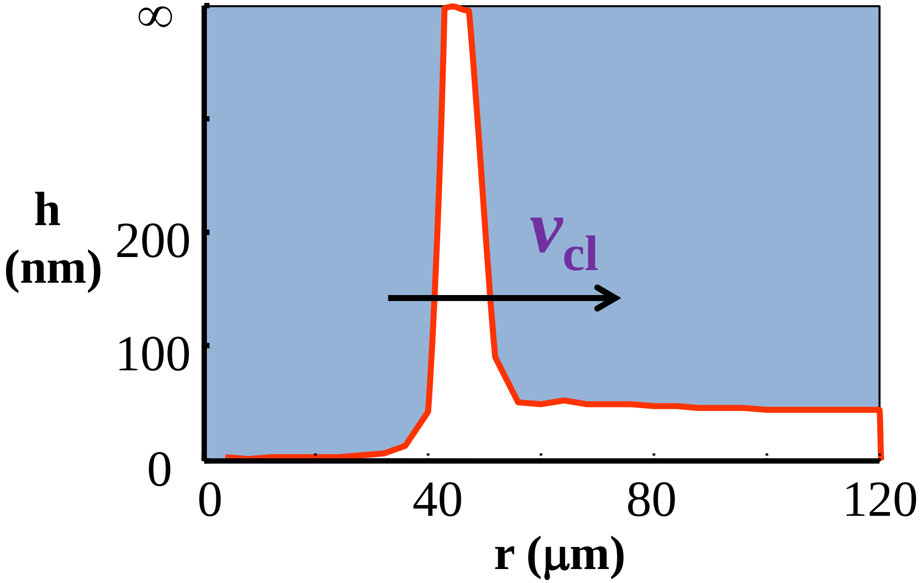
Contact line beneath the drop:

1 mm-radius 20 cSt water-glycerol droplet, V_{impact} =0.6 m/sec, 180 kHz

100 μm

What we see: ***focus on a single contact***

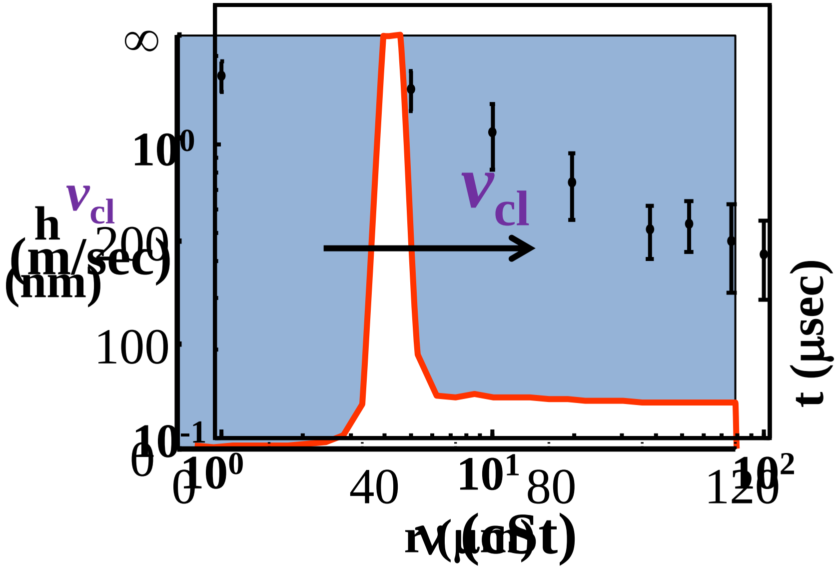


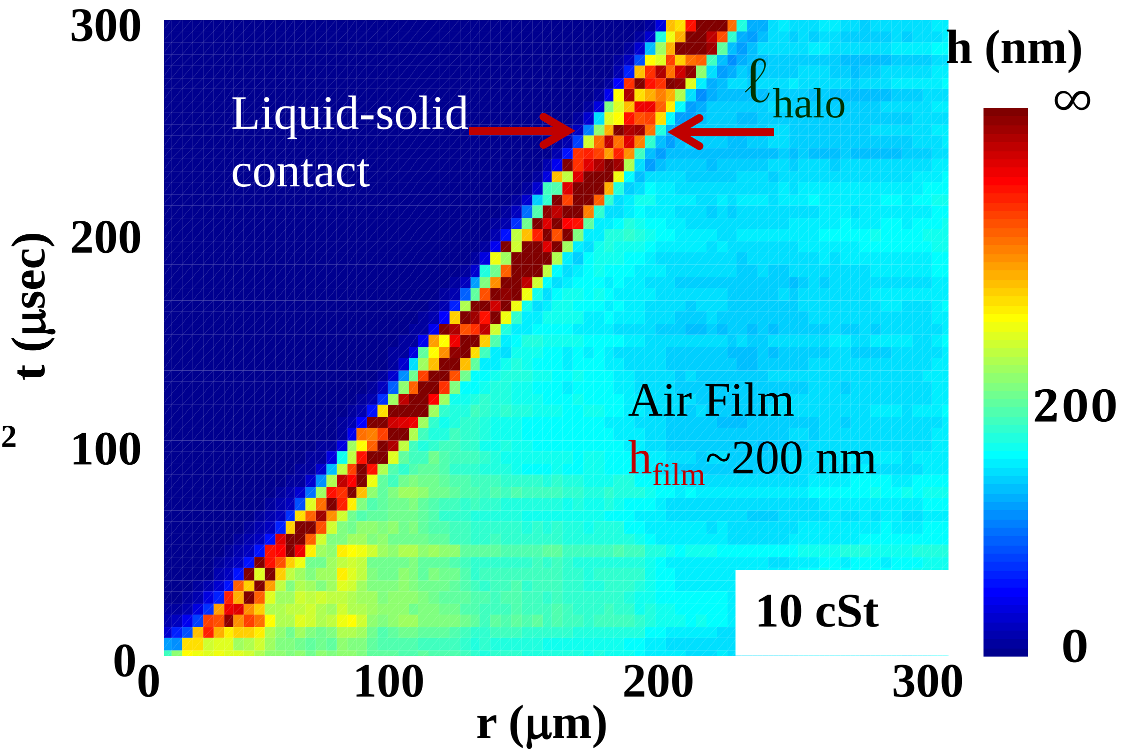


- Contact appears ***black***, encircled by a light ***`halo'***
- Wetting front spreads from nucleation point at v_{cl}

Contact line beneath the drop:

What are the dynamics of the advancing contact line?



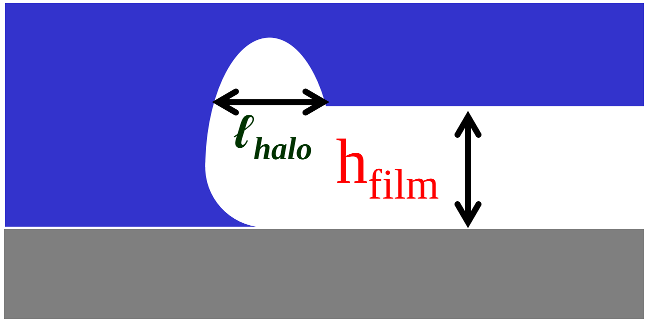


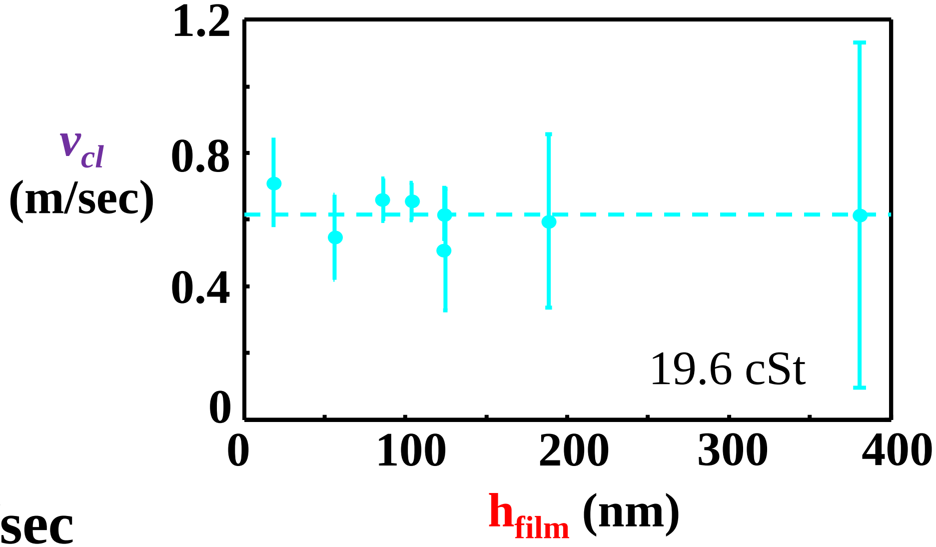
Each horizontal line is a ***single snap-shot***, taken every 5 μsec

- Front advances at ***steady velocity***; here, ***slope***
- Far-field air film thickness ***remains ~ constant***
- ℓ_{halo} leads advancing contact line; ℓ_{halo} ~ ***constant***

Contact line beneath the drop:

Can we understand this?





Capillary pressure ***pulls the fluid:***

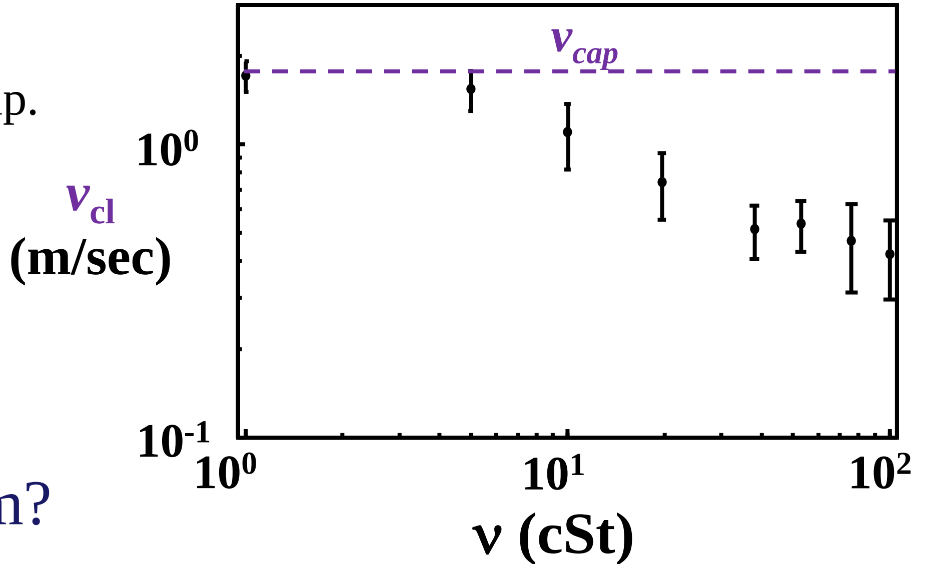
$$v_{cl} = \sqrt{\frac{\gamma}{\rho h_{film}}} \sim 32 \text{ m/sec!!} >> 2 \text{ m/sec}$$

… and depends strongly on film thickness.
Something is wrong here…

Recall: new length-scale, ℓ_{halo} : moderates cap. pressure!!

- This feature is a ***capillary wave:***

$$v_{cap} = \sqrt{\frac{\gamma}{\rho \ell_{halo}}} = 1.8 \text{ m/sec!!}$$



Why does the capillary wave form?

Contact line beneath the drop:

Why does the capillary disturbance form?

Recall - ***h_{film} steady*** at long times:

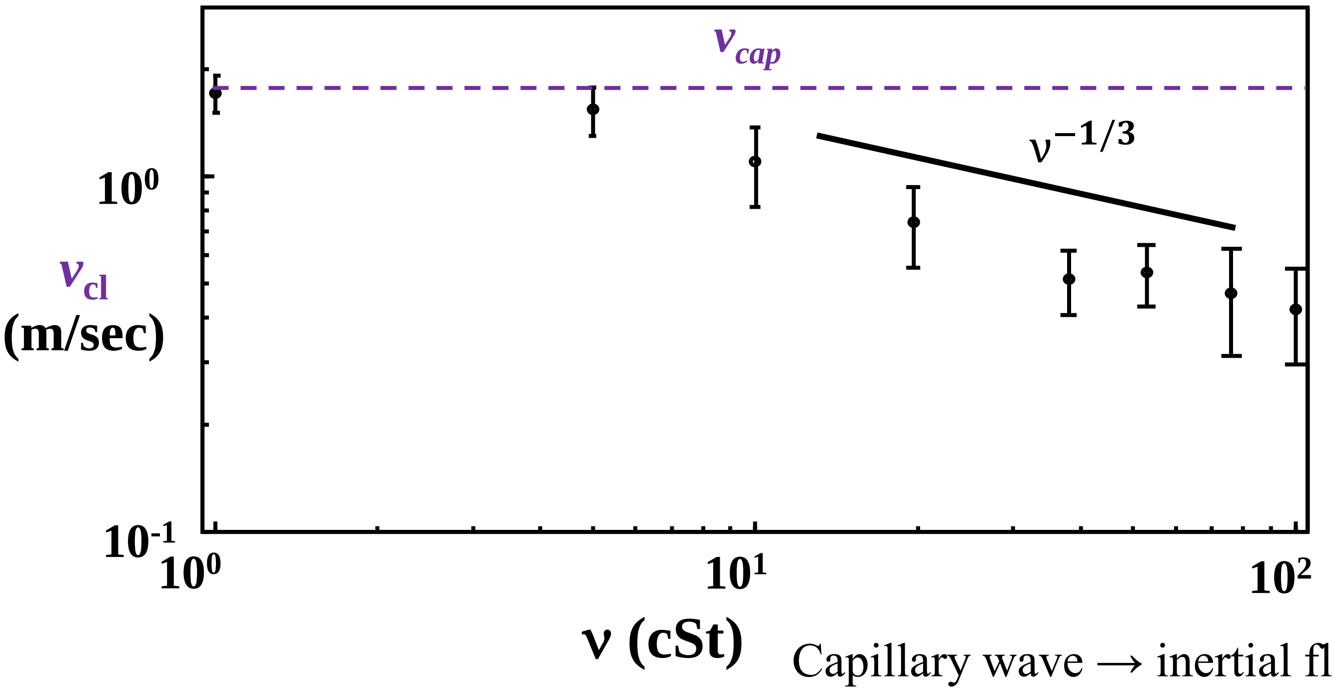
-Air flow is ***viscous*** flow between two plates, driven by ***capillary pressure,*** in halo $\frac{\gamma}{\ell_{halo}}$:

$$\bar{u}_{air} = 2.5\ mm/sec << 1m/sec\ (!)$$

- The air ***doesn't move,*** and instead ***accumulates in the expanding halo,*** inflating a toroidal bubble around the growing contact patch.
- The ***viscosity of the air*** modifies the ***fluid flow*** near the singularity at the contact line, ***significantly reducing*** v_{cl}

Contact line beneath the drop:

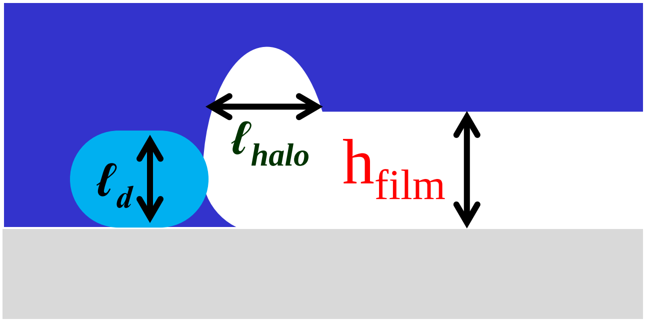
Where does liquid viscosity enter?



General trend:

$\uparrow\ \nu, \downarrow v_{\mathrm{cl}}$

Capillary wave → inertial flow;
Nevertheless, flow at contact line is ***viscous*** & localized to a ***boundary layer*** → $\ell_d \sim \sqrt{\nu\boldsymbol{\tau}}$, where $\boldsymbol{\tau} = \dfrac{\ell_{halo}}{v_{\mathrm{cl}}}$.



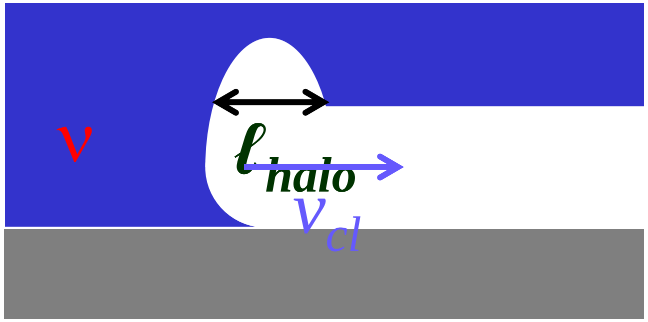
Balancing viscous and capillary stresses,

$$\frac{\gamma}{\rho\ell_{halo}} \sim \nu\frac{v_{\mathrm{cl}}}{\ell_d} \rightarrow v_{\mathrm{cl}} \sim \nu^{-1/3}$$

Observed scaling consistent with ***phenomenological theory***

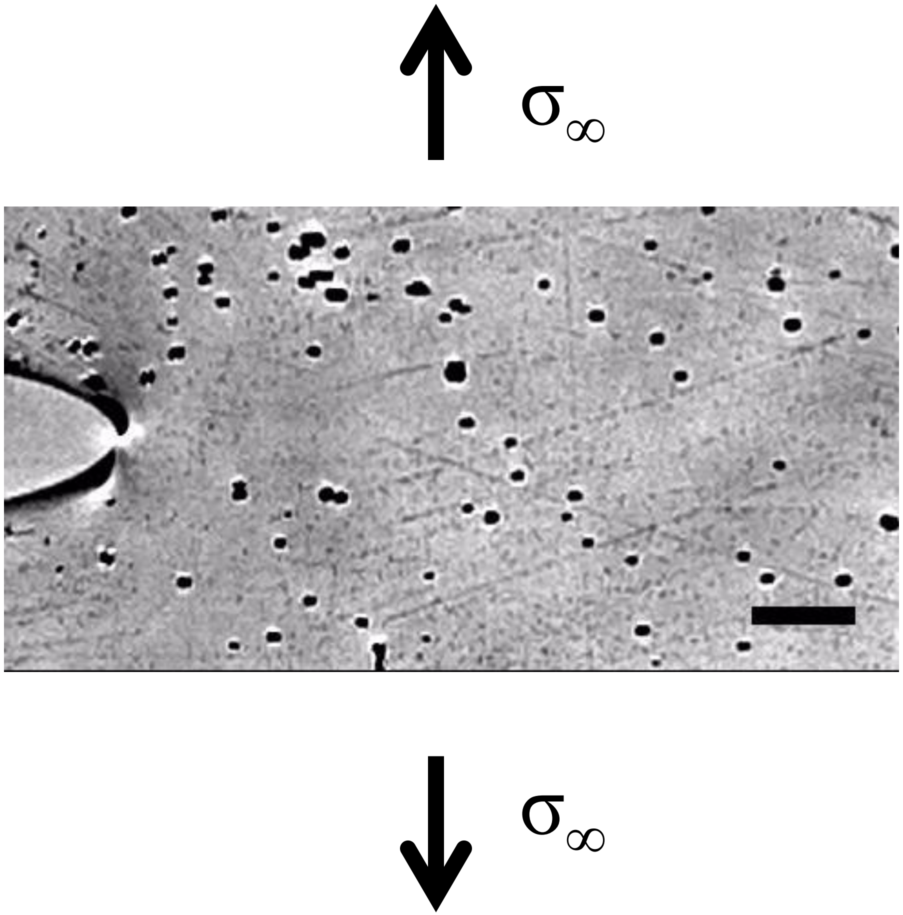
Contact line beneath the drop

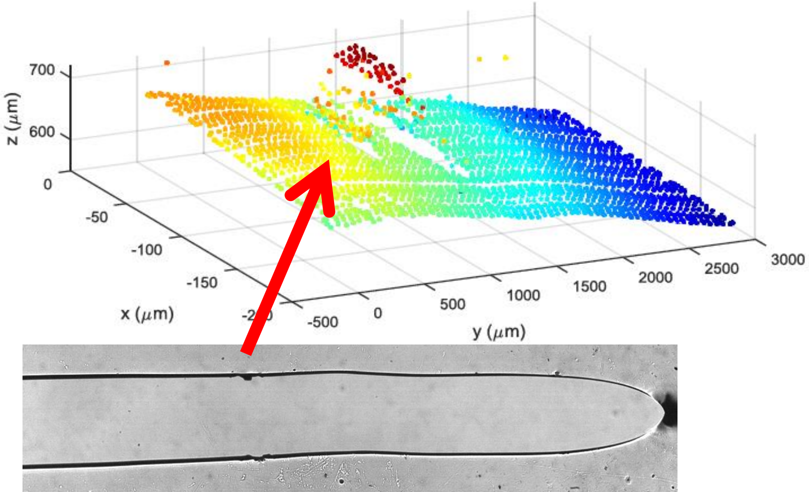
- First measurement of contact line dynamics
- New length-scale ℓ_{halo} explains observed c.l. velocities
- Measured and understood ν-dependence



Dynamic Instabilities at Interfaces:

Dynamic cracks in *inhomogeneous* materials





Work with **Lital Levy and Jay Fineberg**

Slowing things down:

Cracks in polyacrylamide gels

Fracture of polyacrylamide gels enables us to view dynamic fracture in ***slow motion*** by reducing sound velocities by **2-3 orders of magnitude**

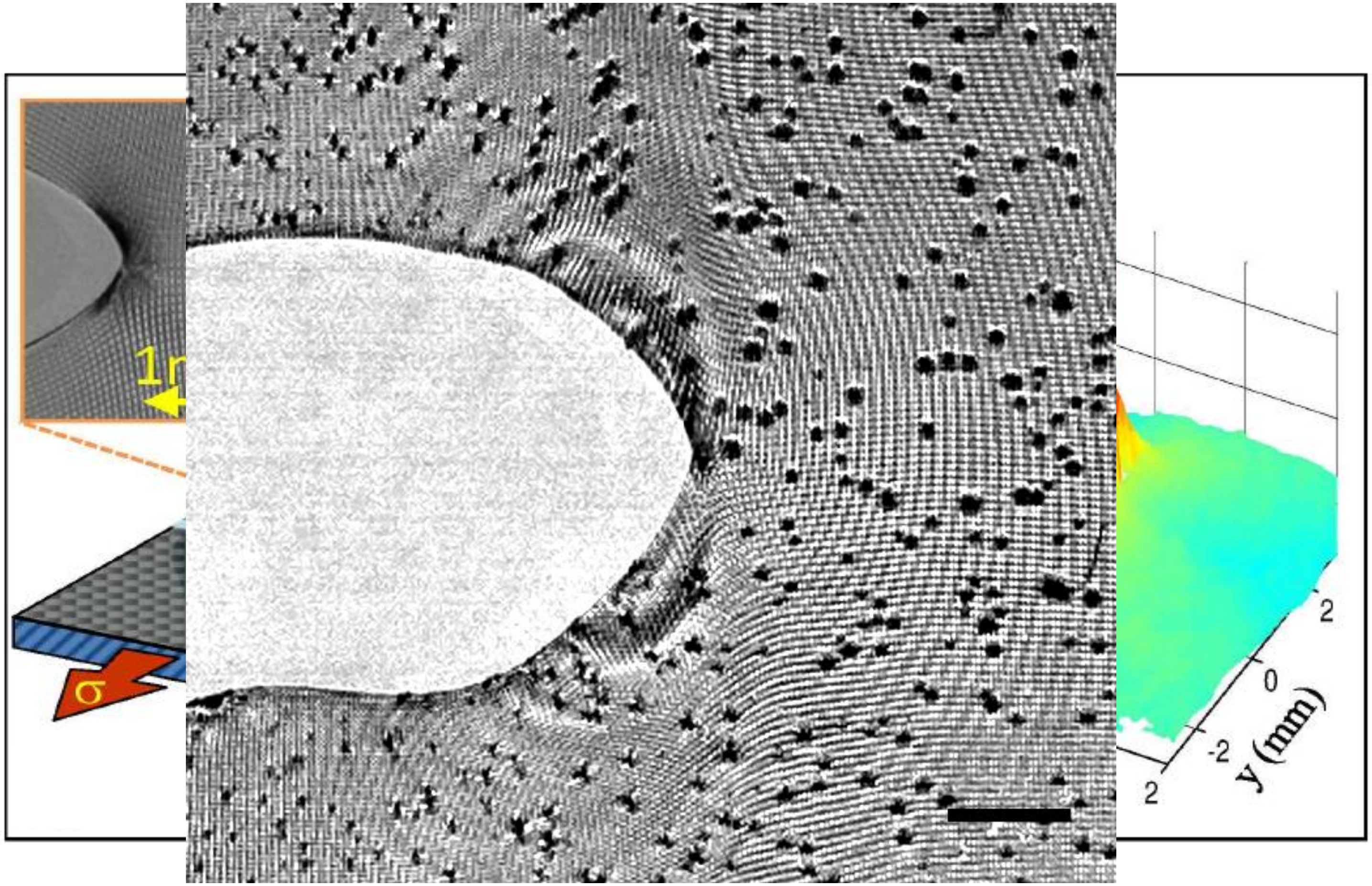
Material	**Young's Modulus (kPa)**	**Poisson ratio**	**C_R (m/s)**
Gel X% acrylamide Y% bis-acrylamide	100-1000	0.5	**5-14**
PMMA	3,900,000	0.35	**930**
Soda-Lime glass	70,000,000	0.22	**3340**

Change in the gel's composition → Change in elastic constants

Young's modulus $E=100\text{-}560$ kPa
Fracture energy $\Gamma=13\text{-}60$ J/m^2

* ***Same phenomenology as other brittle solids***

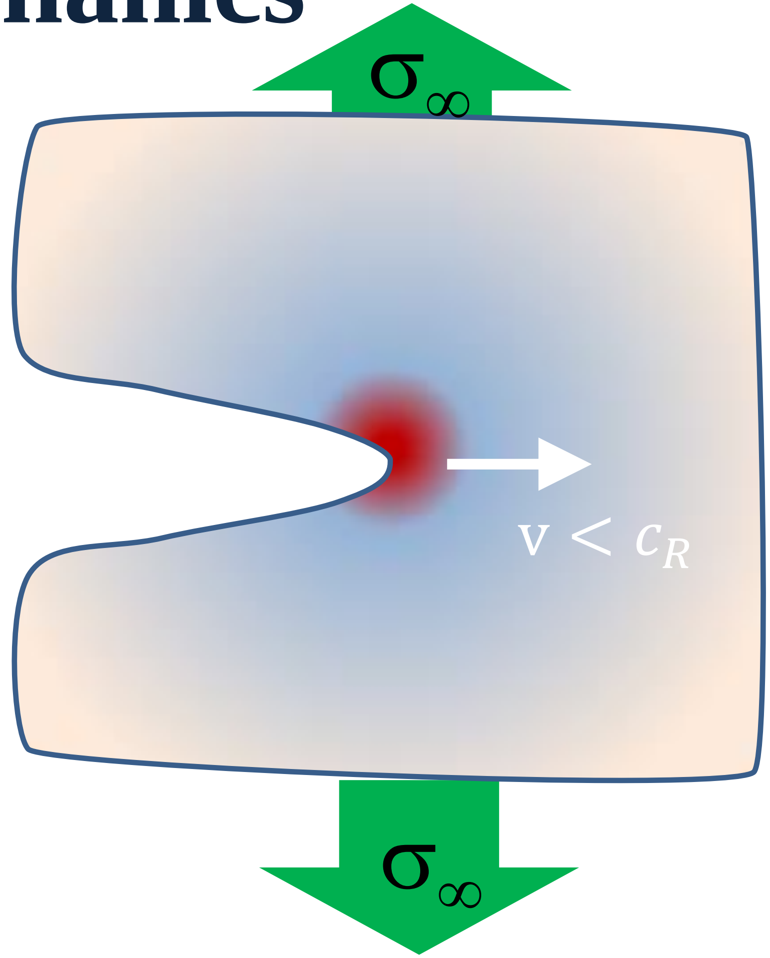
Fracture in *inhomogeneous* materials



Grid-cast gels enable ***dire measurement*** of strains in the ***material reference frame*** Goldman-Boué et. al. PRL 2015

Dynamic Fracture Mechanics

60-second summary



Stresses ***diverge*** at the crack tip:

$$\sigma \sim \frac{K}{\sqrt{2\pi r}}$$

$$K = k(\mathrm{v})K_0(loading, history)$$

Energy flux into the crack tip = dissipation

$$G = \Gamma$$

$G{\sim}g(\mathrm{v})\,K_0^2$ ➡ Crack equation of motion

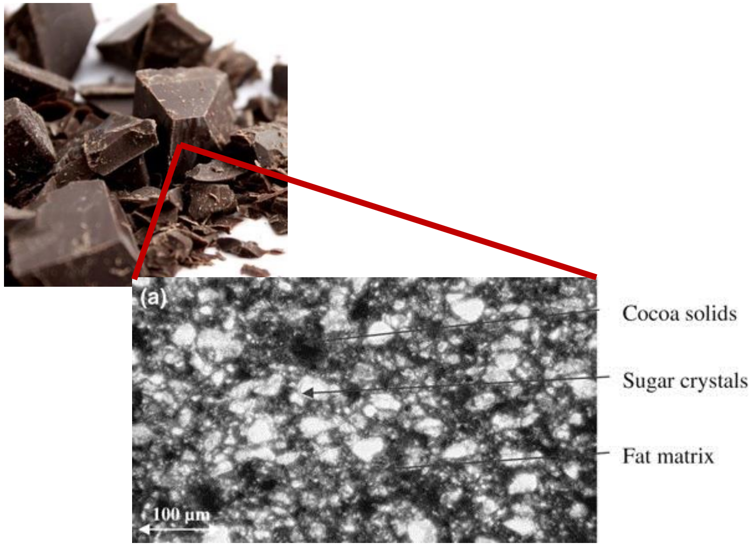
$\mathrm{v} < c_R$ - Rayleigh wave speed

Fracture in *inhomogeneous* materials

Typical materials ***are inhomogeneous:***

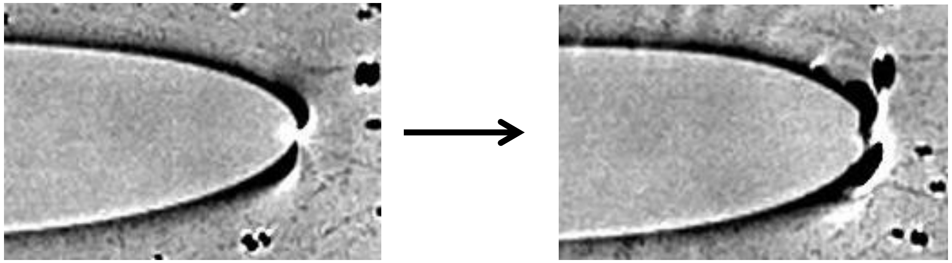
Concrete: Cement + Aggregate

Chocolate: Cocoa + sugar + fat



From Afoakwa et. al., 2007

Inhomogeneities:



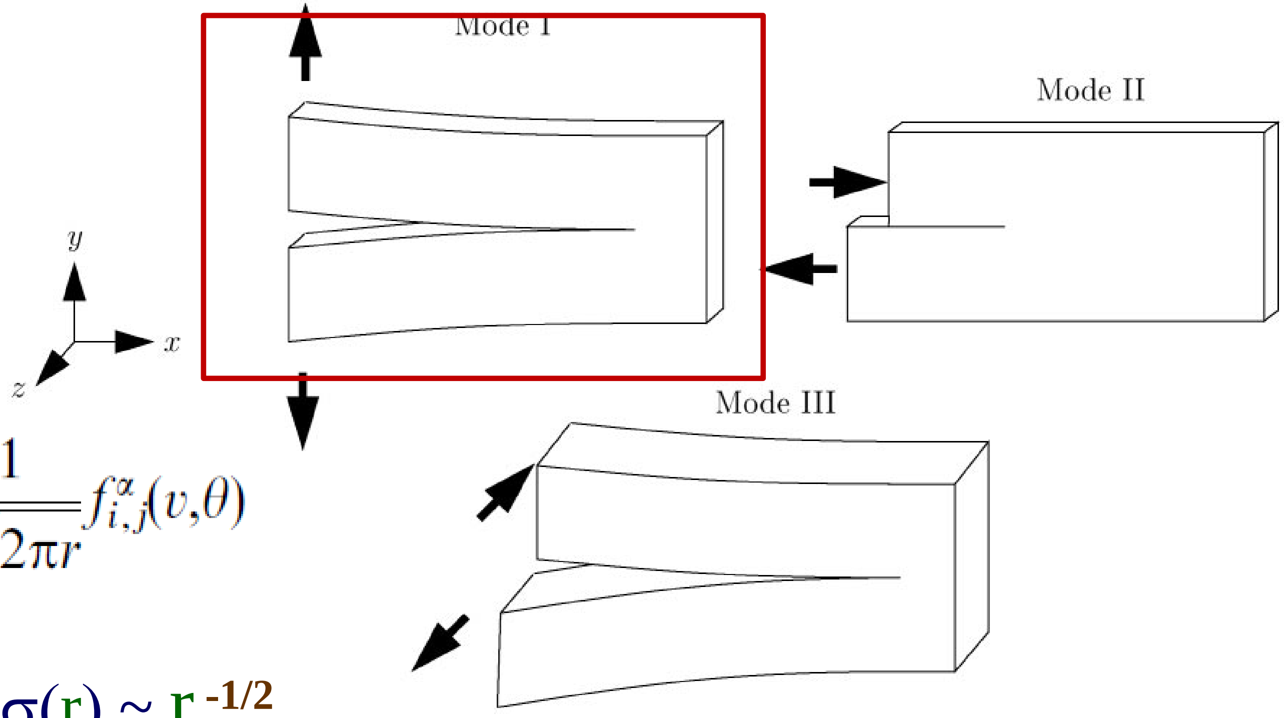
- ***Blunt*** the crack tip

  Alter ***geometry,*** increasing material toughness
- Fracture of inhomogeneous materials ***under-explored***

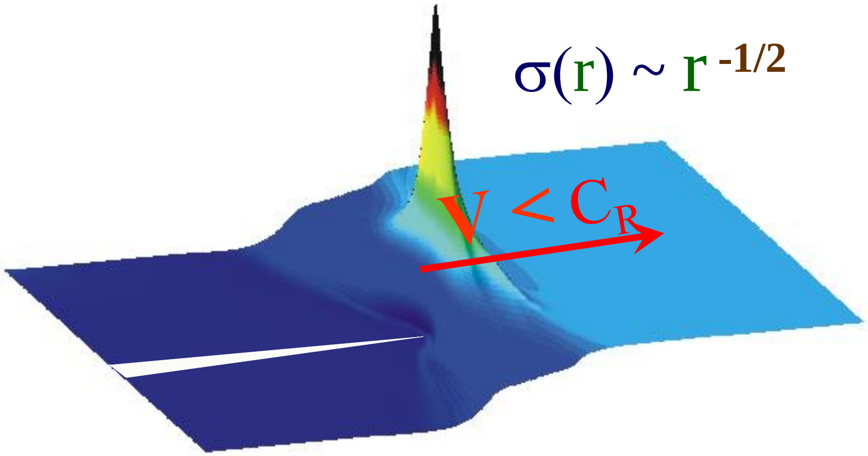
First, a brief overview:

dynamic fracture mechanics

There are three conventional fracture modes , which are characterized by the symmetry of the loading on the crack plane.



$$\sigma_{i,j} = \sum_{\alpha=1}^{3} K_\alpha \frac{1}{\sqrt{2\pi r}} f_{i,j}^{\alpha}(v,\theta)$$



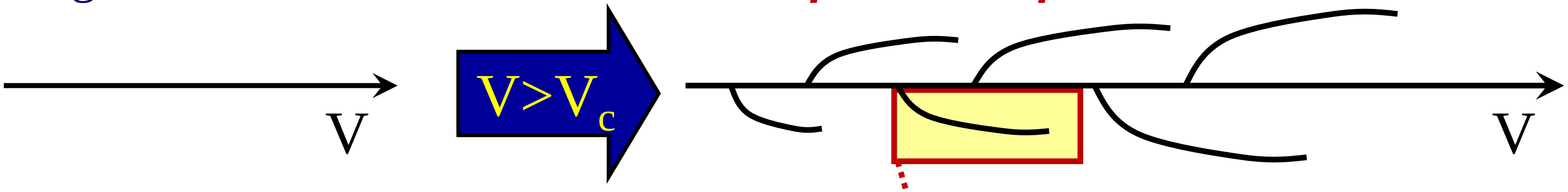
$\sigma(r) \sim r^{-1/2}$

Our ***applied loading*** is ***tensile*** loading → Mode I

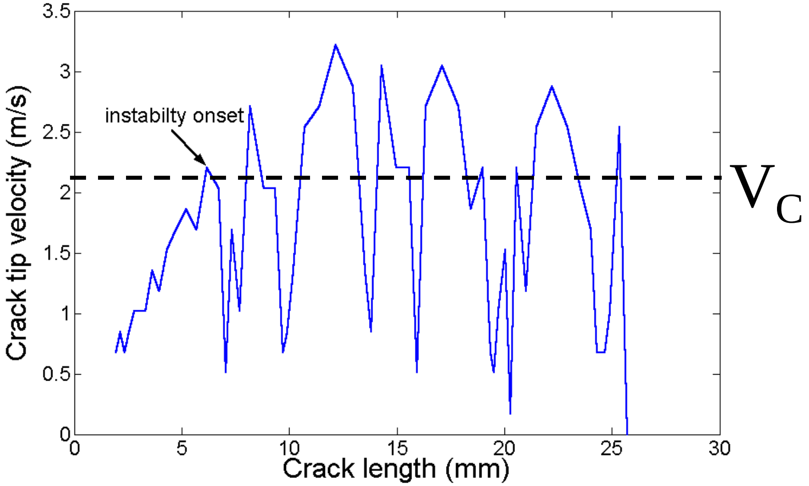
Instabilities in `clean' materials

At a critical velocity a single crack **may** become *unstable* to frustrated micro-branches

- In gels, Micro-branches have the ***same functional form*** as in other brittle materials

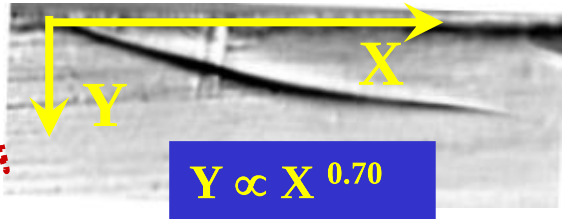


Gels



A. Livne, et. al., PRL (2005)
B. A. Livne, et. al. (2007)

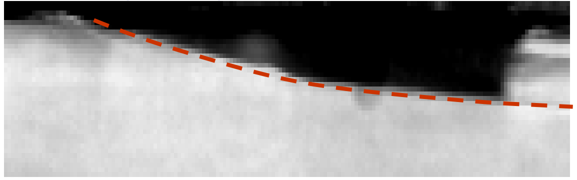
PMMA



$Y \propto X^{0.70}$

Glass

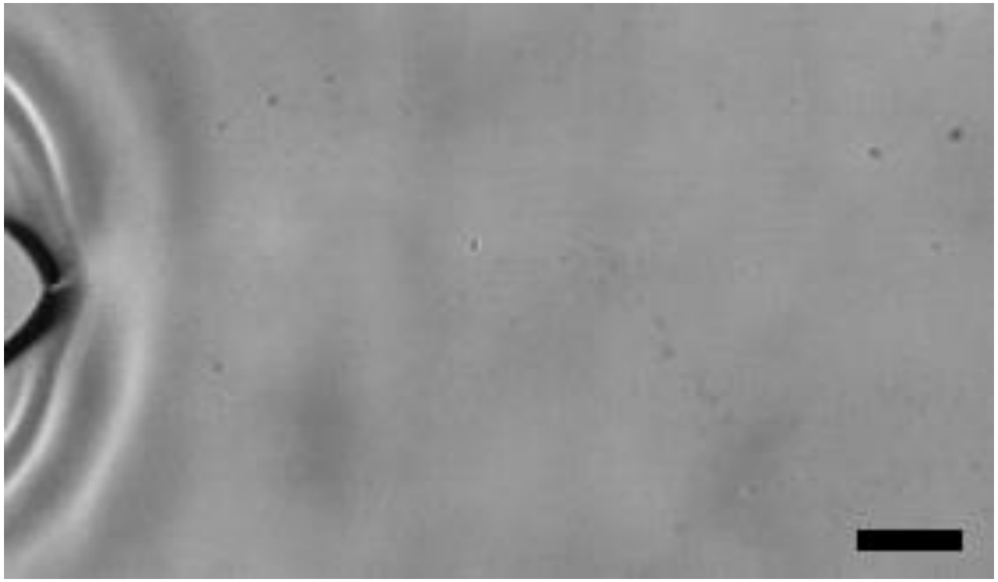
Polyacrylamide Gels



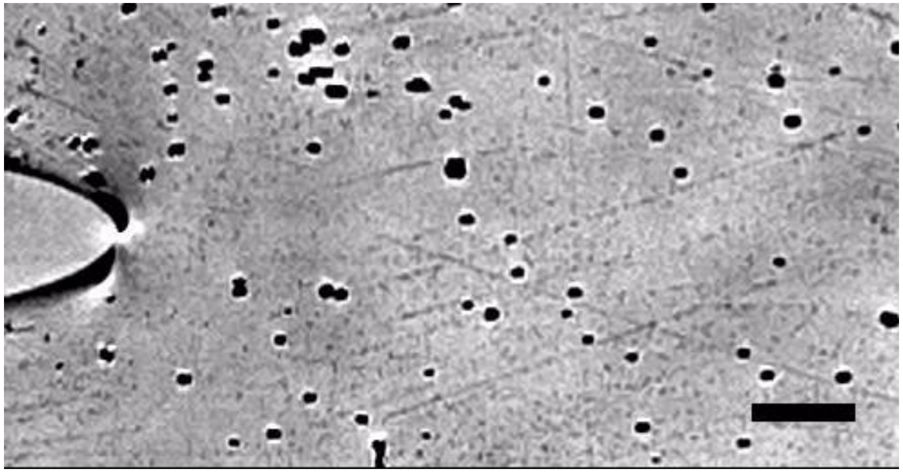
Fracture in *inhomogeneous* materials

Familiar dynamics from new ***inclusions***

2.7 % bis-acrylamide – `clean' material



2.7 % bis-acrylamide – mono-disperse inclusions

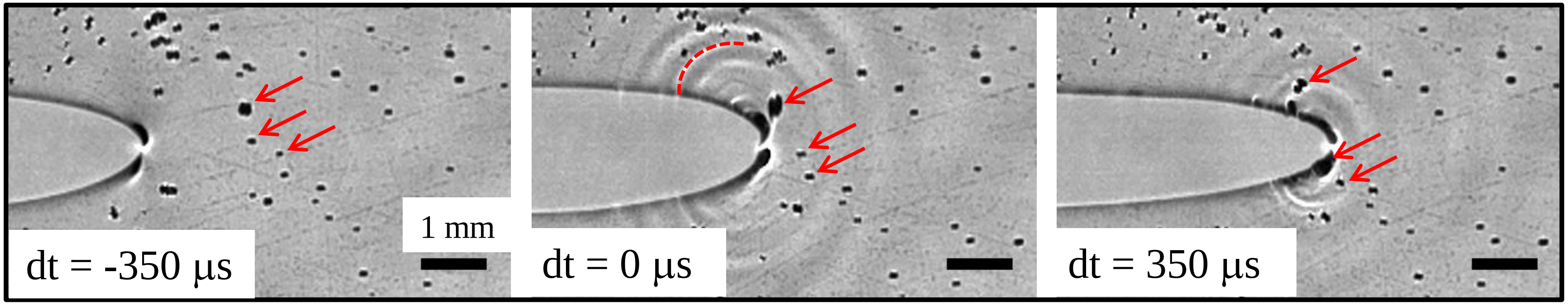


For the ***same*** gel composition, ***microbranching*** dynamics appear similar to ***instability*** triggered by inclusions

Inclusions ***blunt the crack tip***, and ***slow the crack***

Fracture in *inhomogeneous* materials

How do particles affect a dynamic crack?



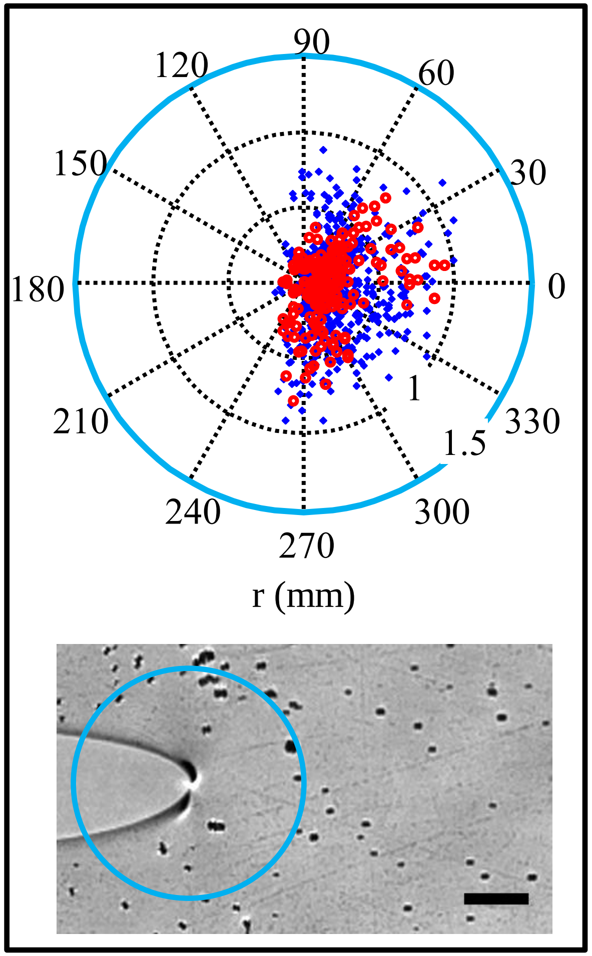
The crack ***interacts*** with the particles, and ***emits elastic waves***…

… we ***analyze*** these events using particle tracking software, and ***collect statistics*** on particle locations when they occur

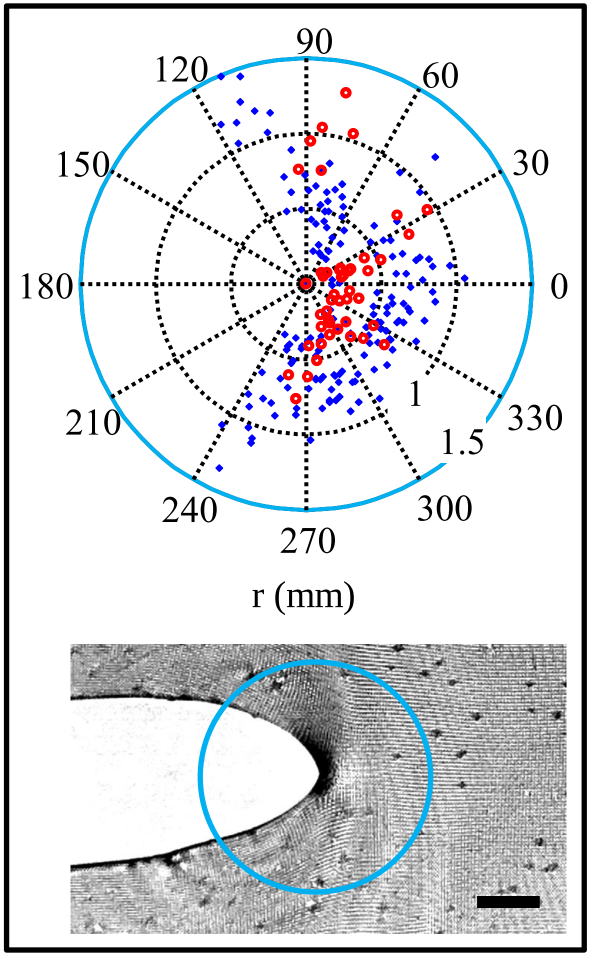
Fracture in ***inhomogeneous*** materials

How do particles affect a dynamic crack?

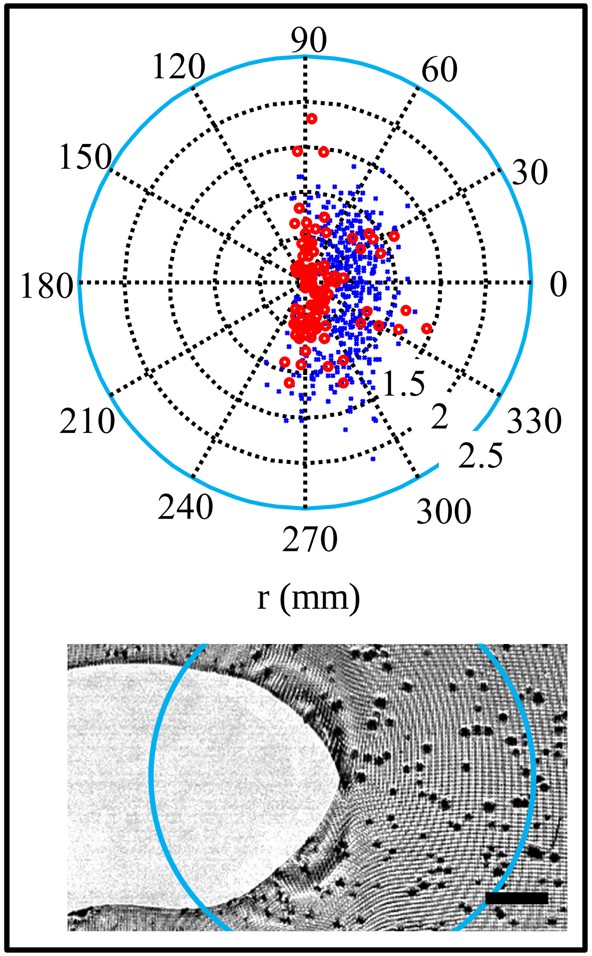
$v/c_s = 0.42$



$v/c_s = 0.75$



$v/c_s = 0.88$



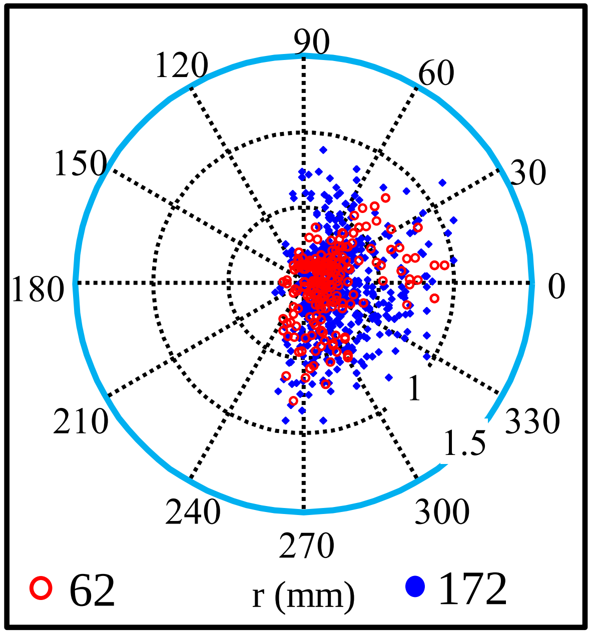
Key:
- ● ***Do not*** affect the crack tip
- ○ Affect the crack tip

Fracture in *inhomogeneous* materials

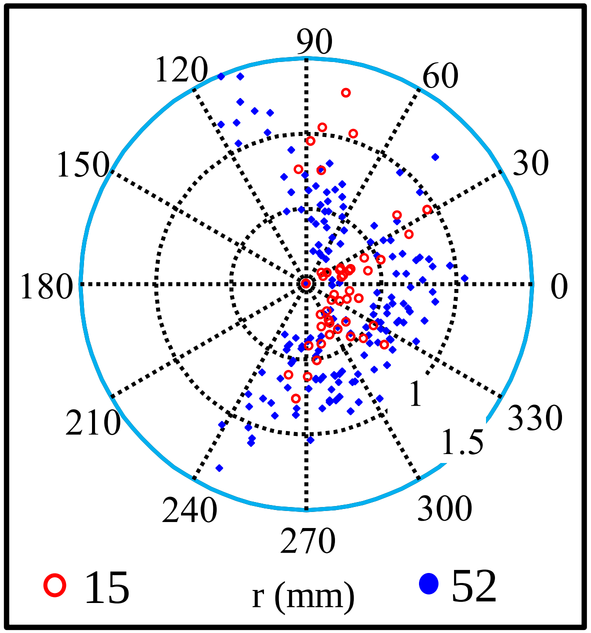
How do particles affect a dynamic crack?

Key: ○ Affect
● ***Do not*** affect

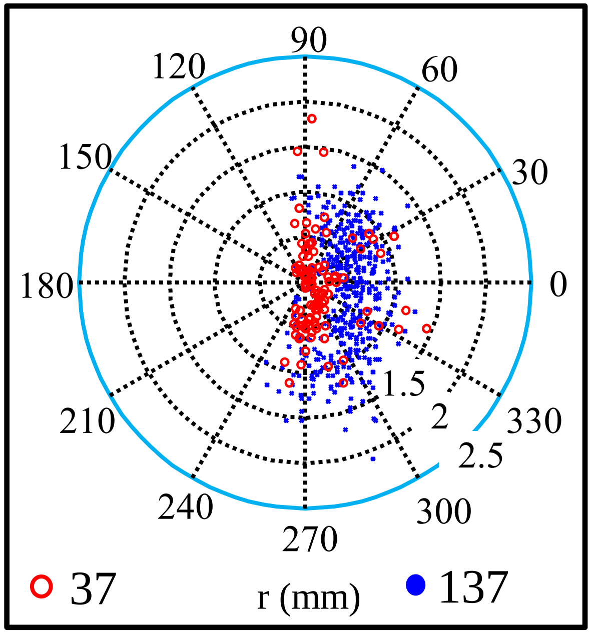
$v/c_s = 0.42$



$v/c_s = 0.75$



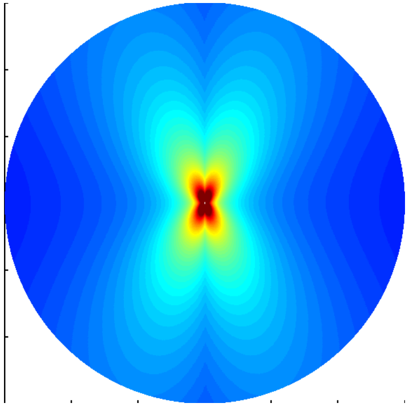
$v/c_s = 0.88$



- ***Most*** particles ***don't*** trigger an event
- The ones that ***do:***
  - **o** ***further*** as v ↑
  - **o** ***non-uniformly distributed*** in θ

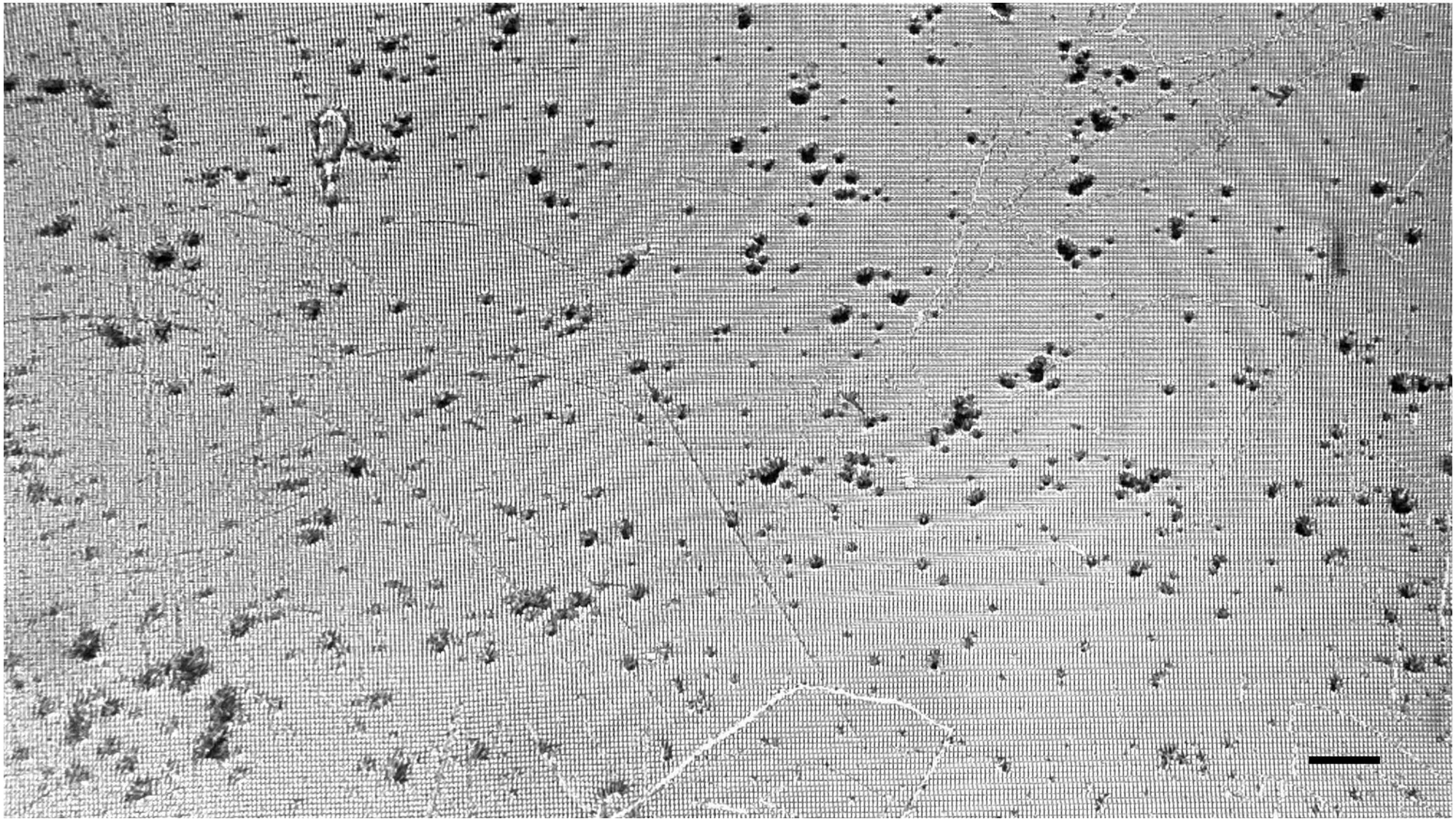
LEFM:

ε_{yy}, v/cs = 0.7



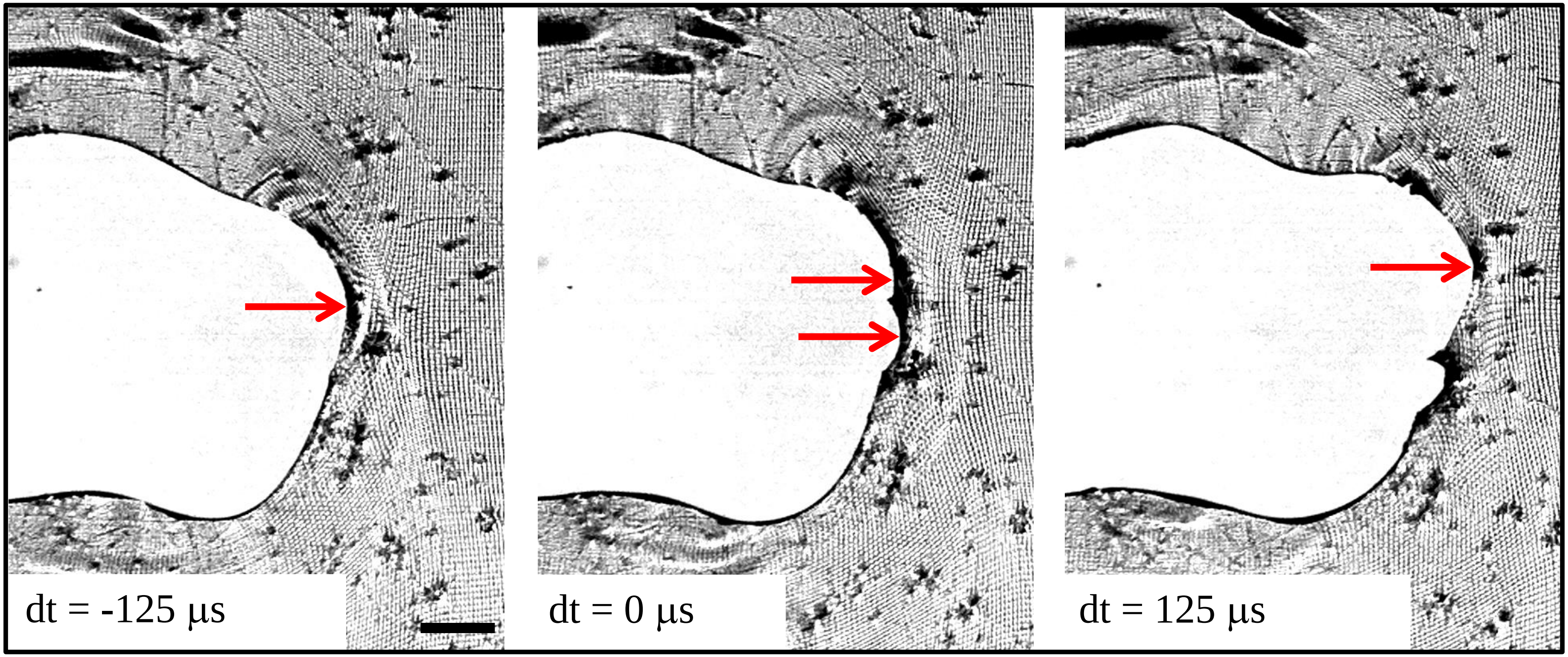
Fracture in *inhomogeneous* materials

Perturbing an oscillating crack



Fracture in *inhomogeneous* materials

Perturbing an oscillating crack

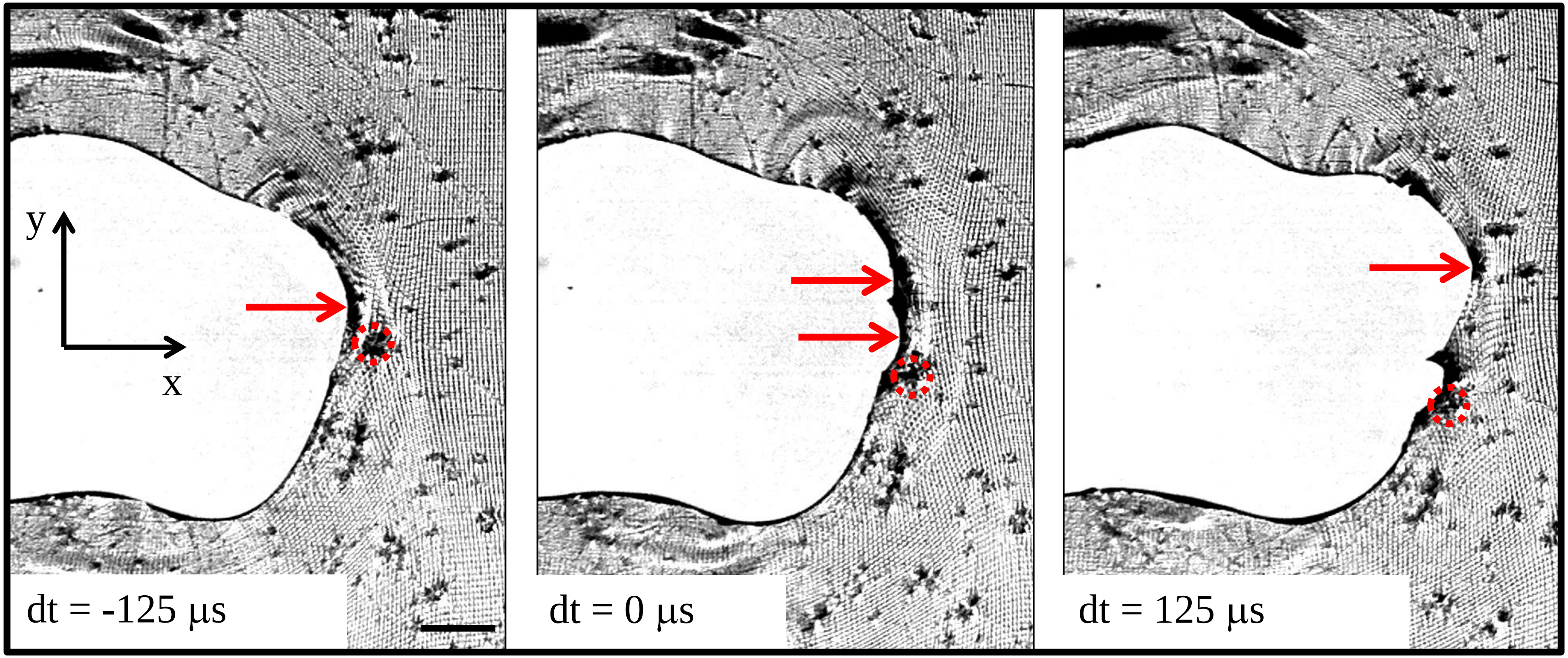


Macro – branch: crack splits in ***two***, indicated by arrows at dt=0

- Typically occurs when elastic energy is ~ 25% ***larger*** for clean samples

Fracture in *inhomogeneous* materials

Perturbing an oscillating crack

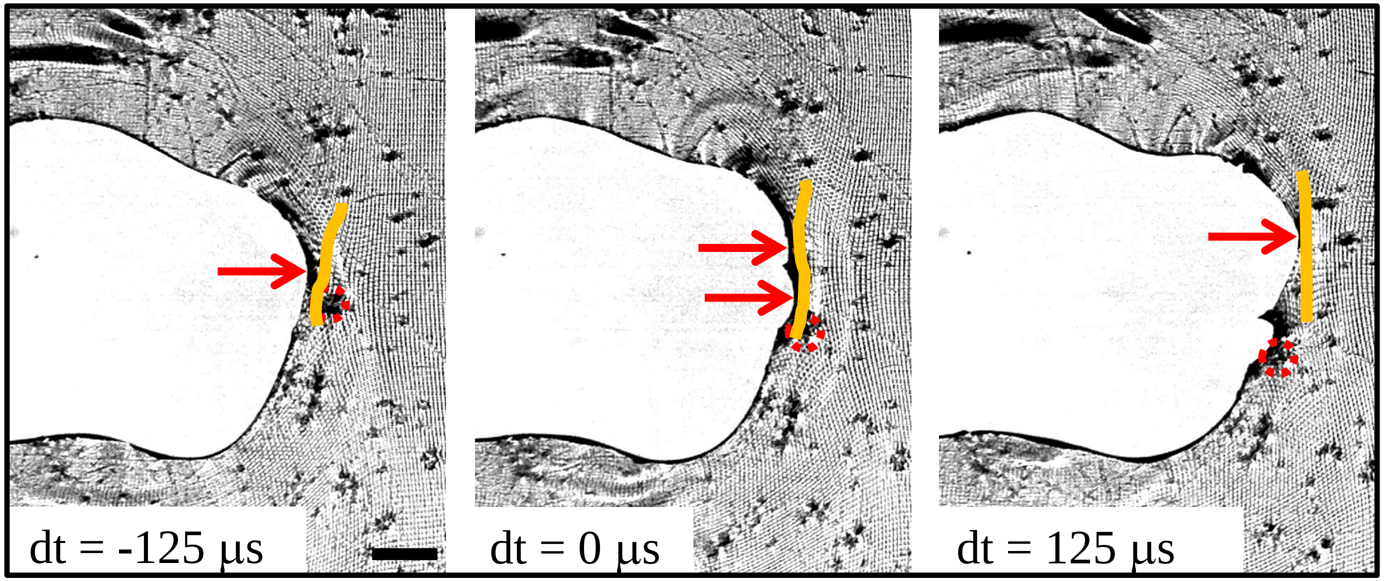


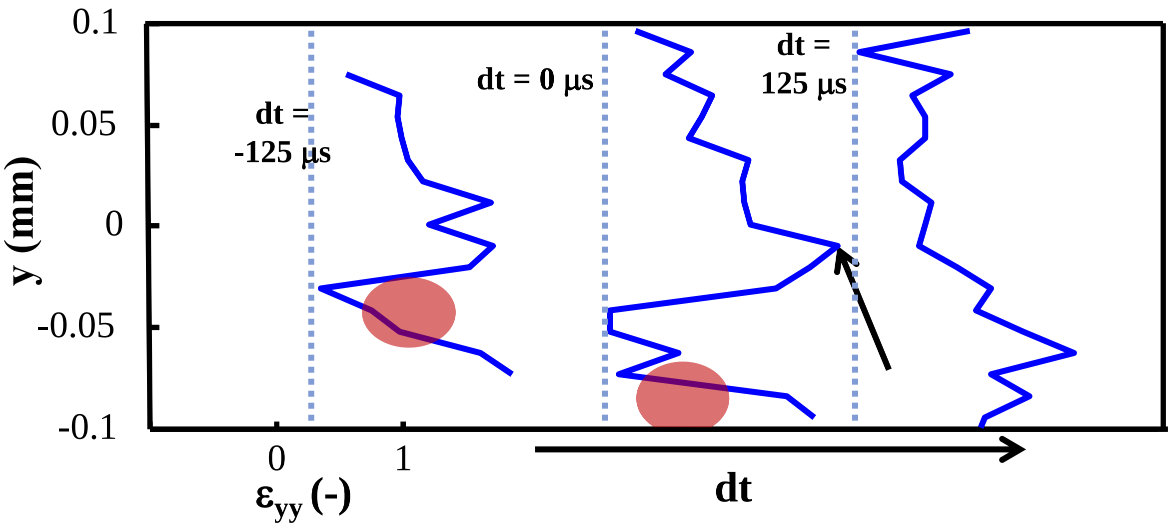
This particle is ***exactly*** aligned with the y-direction from the crack tip

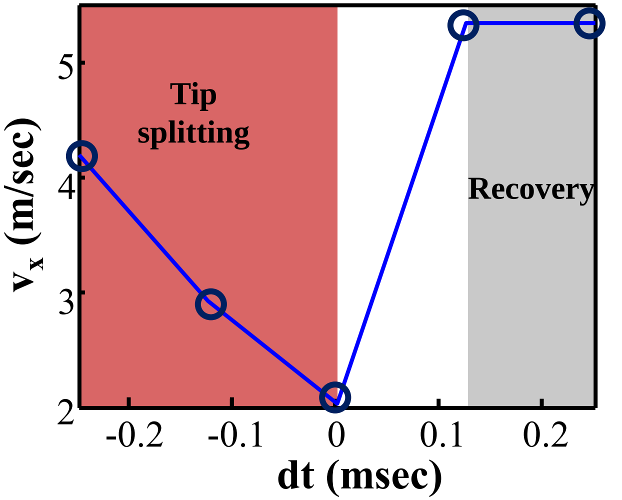
Distortions to material are ***visible in the grid***; let’s take a closer look:

Fracture in *inhomogeneous* materials

Perturbing an oscillating crack

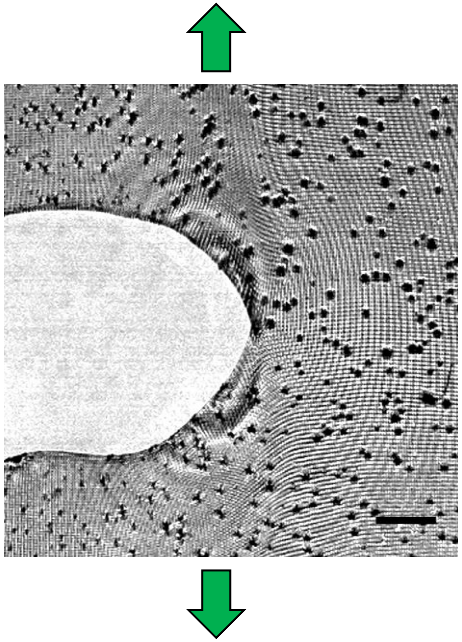






Fracture in *inhomogeneous* materials

Particle inclusions in gels



Inclusions can alter ***geometry*** at the crack tip – but ***not always***

Looking at the particles that trigger branching will enable us to better understand:

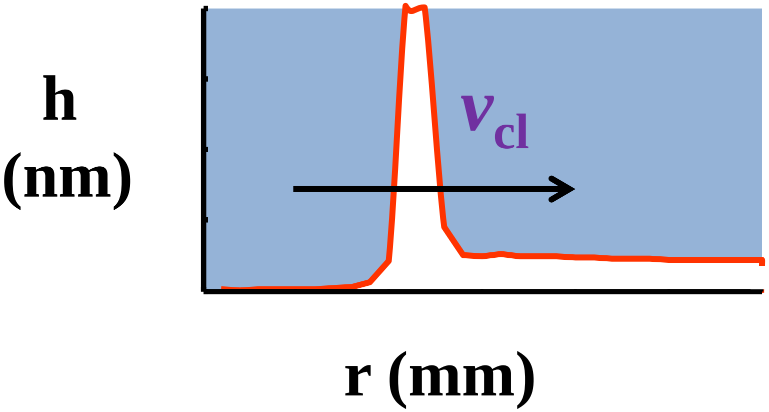
- -instabilities in ***homogeneous*** materials
- -toughening mechanisms ***due to inhomogeneities***
- ***-fracture dynamics* of *intrinsically*** inhomogeneous materials
- -Macro-branching

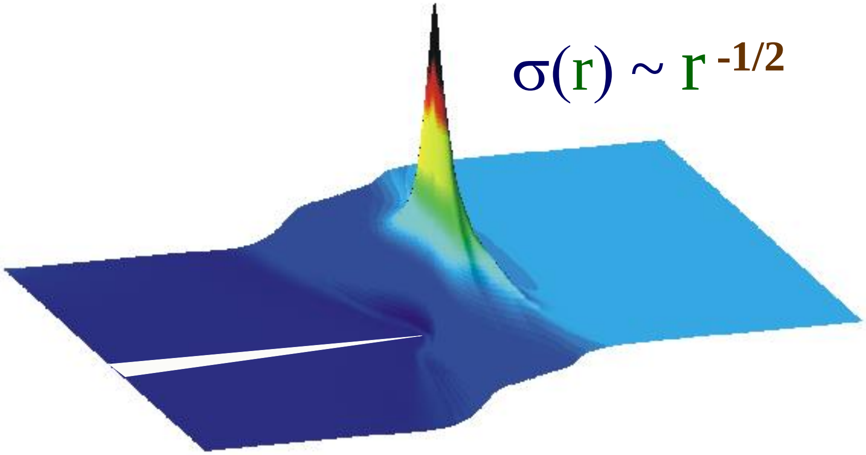
Going beyond preliminary results:

- Measure ***entire*** strain field around crack tip & particles, calculate J-integral / energy flux
- Change ***properties of particles***: density in sample, size, chemistry …

Dynamic Instabilities at Interfaces

propagating contact lines and *dynamic fracture mechanics*





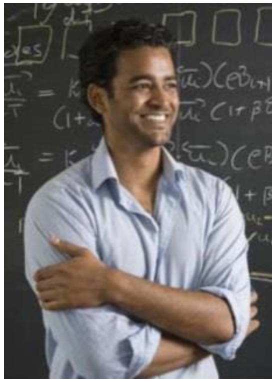
- Both contact lines and dynamic cracks are ***propagating geometric singularities***
- In both cases, the ***stress diverges*** at the tip of the advancing singularity; for contact lines as 1/r, for cracks as $1/r^{1/2}$

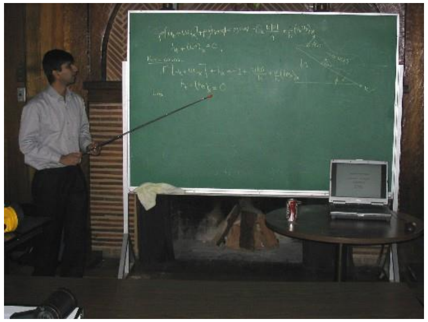
Thank you:

Droplet impacts:

- **Shmuel Rubinstein - Harvard**
- **L. Mahadevan - Harvard**
- **Aaron Mowitz – U. Chicago**
- **Shreyas Mandre – Brown University**
- **Michael Brenner - Harvard**
- **Dave Weitz - Harvard**
- **Madhav Mani – Northwestern U.**



Dynamic Fracture:

- **Jay Fineberg – HUJI**
- **Tami Goldman – HUJI**
- **Lital Levy – HUJI**
- **Tal Sheaffer - HUJI**




Contact line beneath the drop:

Why does the capillary disturbance form?

Capillary number ***of the air***:

$$Ca = \frac{\mu v}{\gamma} = \frac{2e - 5\ 1}{0.1} = 2e - 4$$

This suggests ***air viscosity is negligible (!)***

… but does <u>not</u> account for the thin gap.

Recall that h_{film} remains steady for long times

Estimate the velocity of the air: *viscous flow between two plates (plane Poiseuille flow):*

$$\frac{dp}{dx} = \mu \frac{d^2u}{dy^2} \rightarrow u_{air} = \frac{1}{2\mu}\frac{dp}{dx}\left(y^2 - h_{film}y\right)$$

Thus,

$$\bar{u}_{air} = -\frac{1}{12\mu}\frac{dp}{dx}h^2$$

For typical values in our experiment, we estimate:

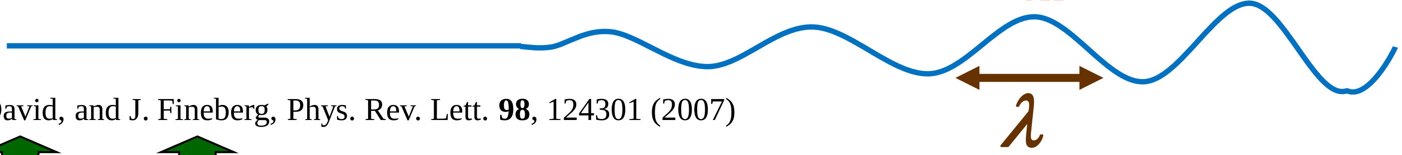
$$\bar{u}_{air} = 2.5\ mm/sec!! << 1m/sec$$

of ***wetting front.***

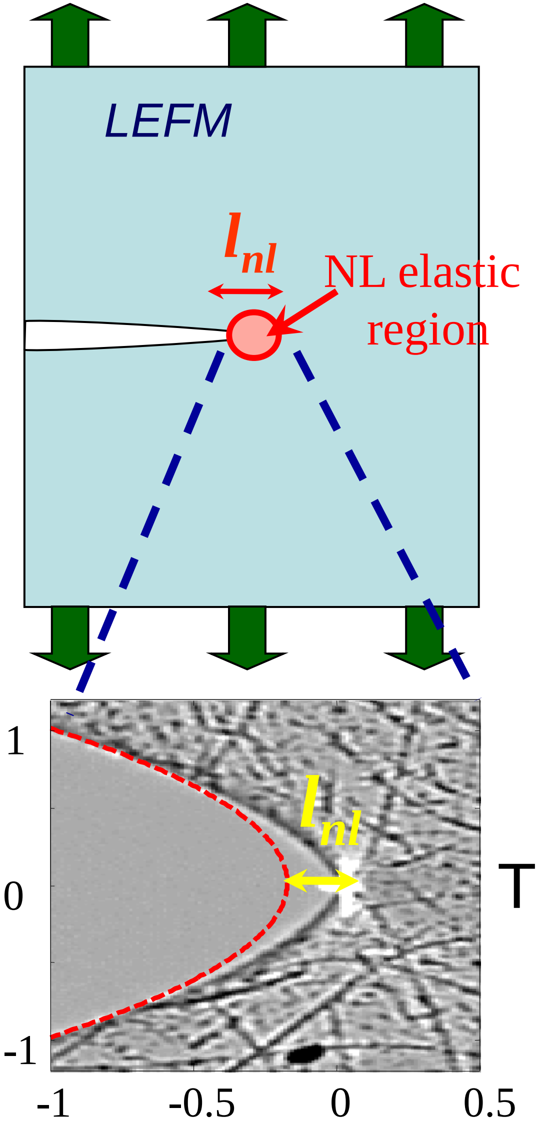
The air ***doesn't move,*** and instead ***accumulates in the expanding halo,*** inflating a toroidal bubble around the growing contact patch.

Oscillatory instabilities $v>0.9C_S$

Is the oscillatory instability related to ***l_{nl}* ??**



A. Livne, O. Ben-David, and J. Fineberg, Phys. Rev. Lett. **98**, 124301 (2007)



Theory (Bouchbinder 2009)

Assume ***retarded potentials*** in nonlinear region:

$$K_{tip}(t) = K_{LEFM}(t - l_{nl}/c_S)$$

Look for ***instability*** to a shear **(Mode II)** perturbation ***driving the crack out of the fracture plane***

$K_{II}(t)$ is a distance l_{nl} from the ➔ delay time τ_d

τ_d scales with l_{nl}

$$\Rightarrow \lambda \sim l_{nl}$$

The out-of-phase response drives **oscillations**!

E. Bouchbinder, PRL **103**, 164301 (2009)