ACTIVITY REPORT 2017/2018

Doctoral Program in Mathematics

The report covers the period from 1 September 2017 – 31 August 2018

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Members

Director	Victor Panaretos
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Prof.	Joachim Krieger	Prof.	Maryna Viazovska

EDMA Courses

Fall 2017

Scheme Theory - MATH-690 Z. Patakfalvi, M. Zdanowicz
Working Group in Topology I - MATH-726 K. Hess Bellwald

Block Courses Fall 2017

Advanced Topics in Computational Science for Multiphysics Problems - MATH-650 A. Quarteroni - 20.9.-26.10.2017
Working Group in High-Dimensional Statistics - MATH-692 S. Morgenthaler - 5.11.-18.12.2017

Spring 2018

Data Analysis for Science and Engineering - MATH-710 A. Davison, D. Goldstein, S. Morgenthaler, V. Panaretos
Extremal Combinatorics - MATH-689 I. Tomon
Optimal Control of Partial Differential Equations - MATH-694 H.-M. Nguyen
Sheaf Cohomology - MATH-693 M. Pieropan, M. Zdanowicz, Z. Patakfalvi
Topics in Geometric Analysis I - 731(1) M. Troyanov
Triangulated Categories: What, Where, Why ? - MATH-691 B. Sanders
Working Group in Topology II - MATH-726(2) K. Hess Bellwald

Candidacy Exams

Arwa Mrad	Numerical Simulation of Sediment Transport within Free Surface Flows
Date:	September 4, 2017
Thesis director:	Marco Picasso
Co-director:	Alexandre Caboussat
President of the Jury:	Fabio Nobile
Jury member:	Simone Deparis

Tomaś Rubin	Semi- and Nonparametric Statistical Inference for Stochastic Partial Differential Equations
Date:	September 12, 2017
Thesis director:	Victor Panaretos
President of the Jury:	Fabio Nobile
Jury member:	Stephan Morgenthaler

Gaspard Ohlmann	On well-posedness of quasilinear PDE's
Date:	November 1, 2017
Thesis director:	Joachim Krieger
President of the Jury:	Fabio Nobile
Jury member:	Hoài-Minh Nguyen

Fabian Mönkeberg	Essentially Non-Oscillatory Reconstruction Methods based on Radial Basis Functions
Date:	November 2, 2017
Thesis director:	Jan S. Hesthaven
President of the Jury:	Annalisa Buffa
Jury member:	Marco Picasso

Riccardo Puppi	Isogeometric methods on trimmed geometries
Date:	December 5, 2017
Thesis director:	Annalisa Buffa
President of the Jury:	Fabio Nobile
Jury member:	Marco Picasso

Dimitrios Gourzoulidis	Variational and adaptive methods for fully nonlinear equations
Date:	January 11, 2018
Thesis director:	Marco Picasso
Co-director:	Alexandre Caboussat
President of the Jury:	Annalisa Buffa
Jury member:	Simone Deparis

Boris Bonev	Efficient preconditioners for wave problems
Date:	February 26, 2018
Thesis director:	Jan S. Hesthaven
President of the Jury:	Annalisa Buffa
Jury member:	Marco Picasso

Luca Pegolotti	Reduced methods for non-conforming coupling of PDEs and application to blood flow in the coronary arterial tree
Date:	February 27, 2018
Thesis director:	Simone Deparis
President of the Jury:	Joachim Krieger
Jury member:	Annalisa Buffa

Lyne Moser	Model structures on enriched diagram categories and cellularizations of classifying spaces
Date:	March 9, 2018
Thesis director:	Kathryn Hess Bellwald
Co-director:	Jéôme Scherer
President of the Jury:	Marc Troyanov
Jury member:	Zsolt Patakfalvi

Giacomo Garegnani	Probabilistic methods for uncertainty quantification of the error in numerical solvers of differential equations
Date:	March 20, 2018
Thesis director:	Assyr Abdulle
President of the Jury:	Fabio Nobile
Jury member:	Marco Picasso

Alastair Hugh Flynn	Tight Configurations of Tubes
Date:	March 27, 2018
Thesis director:	John Maddocks
President of the Jury:	Annalisa Buffa
Jury member:	Pedro Reis
Jury member:	Eric Rawdon

Ursina Schweizer	Mori fiber spaces in low characteristics
Date:	April 11, 2018
Thesis director:	Zsolt Patakfalvi
President of the Jury:	Marc Troyanov
Jury member:	Philippe Michel

Thomas Antonin Zwahlen	Sequence dependent DNA mechanics in genomes
Date:	April 12, 2018
Thesis director:	John Maddocks
President of the Jury:	Victor Panaretos
Jury member:	Matteo Dal Peraro

Francesco Spadaro	Statistical mechanics and conformal field theory: connecting supersymmetry and CLE(16/5) through the critical Ising model
Date:	Aprll 17, 2018
Thesis director:	Clément Hongler
President of the Jury:	Joachim Krieger
Jury member:	João Miguel Penedones

Vasco Schiavo	Groups, cones and fixed points
Date:	May 30, 2018
Thesis director:	Nicolas Monod
President of the Jury:	Thomas Mountford
Jury member:	Boris Buffoni

Emile T.P. Soutter	A mixture model for gas bubbles in an incompressible fluid in the scope of aluminium electrolysis
Date:	July 5, 2018
Thesis director:	Marco Picasso
Co-director:	Jacques Rappaz
President of the Jury:	Thomas Mountford
Jury member:	Simone Deparis

Juan Pablo Madrigal Cianci	Multi-Level and Multi-index Markov Chain Monte Carlo Sampling Techniques for Bayesian Inverse Problems with Applications to Seismic Source Inversion
Date:	July 19, 2018
Thesis director:	Fabio Nobile
President of the Jury:	Victor Panaretos
Jury member:	Marco Picasso

Aras Ergus	Homotopical Hopf–Galois extensions
Date:	August 14, 2018
Thesis director:	Kathryn Hess Bellwald
President of the Jury:	Marc Troyanov
Jury member:	Zsolt Patakfalvi

Haoqing Wu	Decomposing the homotopy theory of coactions
Date:	August 14, 2018
Thesis director:	Kathryn Hess Bellwald
President of the Jury:	Marc Troyanov
Jury member:	Zsolt Patakfalvi

Prizes and Awards

Mathematics Doctoral Thesis Award December 2017

given to

PHAM VAN THANG
PhD student of János Pach

for important contributions to algebraic combinatorics and geometry, particularly for proving important new incidence theorems for algebraic varieties and sum-product estimates in finite fields and quasi-fields, and thereby improving and extending classical results of Bourgain, Katz, Tao, and others.

Doctoral Program Thesis Distinction 2017

ONDREJ BUDAC
Phd student of Assyr Abdulle
Multiscale methods for Stokes flow in heterogeneous media

PHAM VAN THANG
Phd student of János Pach
Erdös Distinct Distances Problem and Extensions over Finite Spaces

Theses

ANDREA BARTEZZAGHI

Oral exam:	July 28, 2017
Public defense:	September 20, 2017
Thesis director:	Alfio Quarteroni
Thesis co-director:	Luca Dedé
President of the Jury:	Victor Panaretos
Thesis Nr:	7950

Isogeometric Analysis for High Order Geometric Partial Differential Equations with Applications

In this thesis, we consider the numerical approximation of high order geometric Partial Differential Equations (PDEs). We first consider high order PDEs defined on surfaces in the 3D space that are represented by single-patch tensor product NURBS. Then, we spatially discretize the PDEs by means of NURBS-based Isogeometric Analysis (IGA) in the framework of the Galerkin method. With this aim, we consider the construction of periodic NURBS function spaces with high degree of global continuity, even on closed surfaces. As benchmark problems for the proposed discretization, we propose Laplace-Beltrami problems of the fourth and sixth orders, as well as the corresponding eigenvalue problems, and we analyze the impact of the continuity of the basis functions on the accuracy as well as on computational costs. The numerical solution of two high order phase field problems on both open and closed surfaces is also considered: the fourth order Cahn-Hilliard equation and the sixth order crystal equation, both discretized in time with the generalized-alpha method. We then consider the numerical approximation of geometric PDEs, derived, in particular, from the minimization of shape energy functionals by L^2-gradient flows. We analyze the mean curvature and the Willmore gradient flows, leading to second and fourth order PDEs, respectively. These nonlinear geometric PDEs are discretized in time with Backward Differentiation Formulas (BDF), with a semi-implicit formulation based on an extrapolation of the geometry, leading to a linear problem to be solved at each time step. Results about the numerical approximation of the two geometric flows on several geometries are analyzed. Then, we study how the proposed mathematical framework can be employed to numerically approximate the equilibrium shapes of lipid bilayer biomembranes, or vesicles, governed by the Canham-Helfrich curvature model. We propose two numerical schemes for enforcing the conservation of the area and volume of the vesicles, and report results on benchmark problems. Then, the approximation of the equilibrium shapes of biomembranes with different values of reduced volume is presented. Finally, we consider the dynamics of a vesicle, e.g. a red blood cell, immersed in a fluid, e.g. the plasma. In particular, we couple the curvature-driven model for the lipid membrane with the incompressible Navier-Stokes equations governing the fluid. We consider a segregated approach, with a formulation based on the Resistive Immersed Surface method applied to NURBS geometries. After analyzing benchmark fluid simulations with immersed NURBS objects, we report numerical results for the investigation of the dynamics of a vesicle under different flow conditions.

DIMITRI ZAGANIDIS

Oral exam:	May 23, 2017
Public defense:	September 22, 2017

Thesis director:	Kathryn Hess Bellwald
President of the Jury:	Marc Troyanov
Thesis Nr:	7748

Towards an $(\infty,2)$-category of homotopy coherent monads in an infinity-cosmos

This thesis is part of a program initiated by Riehl and Verity to study the category theory of $(\infty,1)$-categories in a model-independent way. They showed that most models of $(\infty,1)$-categories form an infinity-cosmos $\mathcal{K}$, which is essentially a category enriched in quasi-categories with some additional structure reminiscent of a category of fibrant objects. Riehl and Verity showed that it is possible to formulate the category theory of $(\infty,1)$-categories directly with infinity-cosmos axioms. This should also help organize the category theory of $(\infty,1)$-categories with structure.

Given a category $\mathcal{K}$ enriched in quasi-categories, we build via a nerve construction a stratified simplicial set $\mathcal{N}_{\mathrm{Mnd}}(\mathcal{K})$ whose objects are homotopy coherent monads in $\mathcal{K}$. If two ∞-cosmoi are weakly equivalent, their respective stratified simplicial sets of homotopy coherent monads are also equivalent. We also provide an $(\infty,2)$-category $\mathbf{Adj}_r(\mathcal{K})$ whose objects are homotopy coherent adjunctions in $\mathcal{K}$, that we use to classify the 1-simplices of $\mathcal{N}_{\mathrm{Mnd}}(\mathcal{K})$ up to homotopy.

CLAUDIU A. VALCULESCU

Oral exam:	June 6, 2017
Public defense:	September 29, 2017
Thesis director:	János Pach
President of the Jury:	Kathryn Hess Bellwald
Thesis Nr:	7855

Algebraic and topological methods in combinatorics

The present thesis deals with problems arising from discrete mathematics, whose proofs make use of tools from algebraic geometry and topology. The thesis is based on four papers that I have co-authored, three of which have been published in journals, and one has been submitted for publication (and also appeared as a preprint on the arxiv, and as an extendend abstract in a conference). Specifically, we deal with the following four problems:

1. We prove that if $M \in \mathbb{C}^{2\times 2}$ is an invertible matrix, and $B_M : \mathbb{C}^2 \times \mathbb{C}^2 \to \mathbb{C}$ is a bilinear form $B_M(p, q) = p^T Mq$, then any finite set S contained in an irreducible algebraic curve C of degree d in $\mathbb{C}^2$ determines $\Omega_d(|S|^{4/3})$ distinct values of B_M, unless C is a line, or is linearly equivalent to a curve defined by an equation of the form $x^k = y^l$, with $k, l \in \mathbb{Z}\backslash\{0\}$, and $\gcd(k, l) = 1$.

2. We show that if we are given m points and n lines in the plane, then the number of distinct distances between the points and the lines is $\Omega(m^{1/5}n^{3/5})$, as long as $m^{1/2} \leq n \leq m^2$. Also, we show that if we are given m points in the plane, not all collinear, then the number of distances between these points and the lines that they determine is $\Omega(m^{4/3})$. We also study three-dimensional versions of the distinct point-line distances problem.

3. We prove the lower bound $\Omega(|S|^4)$ on the number of ordinary conics determined by a finite point set S in $\mathbb{R}^2$, assuming that S is not contained in a conic, and at most $c|S|$ points of S lie on the same line (for some $0 < c < 1$). We say that a conic is *ordinary* for $S \subset \mathbb{R}^2$ if it is determined by five points of S, and contains no other points of S. We also provide constructions, showing that our bound is the best possible.

4. Given nk points in general position in the plane, colored with two colors, with at least n points of each color, we prove that one can find n pairwise disjoint convex sets, each set containing precisely k of the points, not all of the same color. Also, we show that if P is a set of $n(d + 1)$ points in general position in $\mathbb{R}^d$, colored by d colors, with at least n points of each color, then one can always find n pairwise disjoint d-dimensional simplices with vertices from P, each simplex containing points of every color.

TIMOTHÉE N. POUCHON

Oral exam:	June 13, 2017
Public defense:	October 27, 2017
Thesis director:	Assyr Abdulle
President of the Jury:	Fabio Nobile
Thesis Nr:	7881

Effective models and numerical homogenization methods for long time wave propagation in heterogeneous media

Modeling wave propagation in highly heterogeneous media is of prime importance in engineering applications of diverse nature such as seismic inversion, medical imaging or the design of composite materials. The numerical approximation of such multiscale physical models is a mathematical challenge. Indeed, to reach an acceptable accuracy, standard numerical methods require the discretization of the whole medium at the microscopic scale, which leads to a prohibitive computational cost. Homogenization theory ensures the existence of a homogenized wave equation, obtained from the original problem by a limiting process. As this equation does not depend on the microscopic scale, it is a good target for numerical methods. Unfortunately, for general media, the homogenized equation may not be unique and no formulas are available for its effective data. Nevertheless, such formulas are known for media described by a locally periodic tensor. In that case, or more generally for problems with scale separation, methods such as the finite element heterogeneous multiscale method (FE-HMM) are proved to efficiently approximate the homogenized solution. For wave propagation in heterogeneous media, however, it is known that at large timescales the homogenized solution fails to describe the dispersive behavior of the original wave. Hence, a new equation that captures this dispersion is needed. In this thesis, we study such effective equations for long time wave propagation in heterogeneous media.

The first result that we present holds in periodic media. Using the technique of asymptotic expansion, we obtain the characterization of a whole family of equations that describes the long time dispersive effects of the oscillating wave. The validity of our derivation is ensured by rigorous a priori error estimates. We also derive a numerical procedure for the computation of the tensors involved in the first order effective equations. This leads to a numerical homogenization method for long time wave propagation in periodic media. The second result that we present generalizes the procedure for deriving effective equations to arbitrary timescales. This generalization is also useful, for example, for the homogenization of the wave equation with high frequency initial data. We also provide a numerical procedure allowing to compute effective tensors of arbitrary order. The third result is the generalization of the family of first order effective equations from periodic to locally periodic media. A rigorous a priori error analysis is also derived in this situation. This constitutes the first analysis of effective models for the long time approximation of the wave equation in locally periodic media. In a second part of the thesis, we derive numerical homogenization methods for the long time approximation of the wave equation in locally periodic media. In one dimension, we analyze a modification of the FE-HMM called the FE-HMM-L. In higher dimensions, we design a spectral homogenization method. For both methods, we prove error estimates valid for large timescales and in arbitrarily large spatial domains. In particular, we show that these numerical homogenization methods converge to effective solutions that approximate the highly oscillatory wave equation over long time.

DIMITRI S. WYSS

Oral exam:	September 11, 2017
Public defense:	November 3, 2017
Thesis director:	Tamás Hausel
President of the Jury:	Victor Panaretos
Thesis Nr:	8025

Motivic and p-adic Localization Phenomena

In this thesis we compute motivic classes of hypertoric varieties, Nakajima quiver varieties and open de Rham spaces in a certain localization of the Grothendieck ring of varieties. Furthermore we study the p-adic pushforward of the Haar measure under a hypertoric moment map μ. This leads to an explicit formula for the Igusa zeta function $\mathcal{I}_\mu(\int)$ of μ, and in particular to a small set of candidate poles for $\mathcal{I}_\mu(\int)$. We also study various properties of the residue at the largest pole of $\mathcal{I}_\mu(\int)$. Finally, if μ is constructed out of a quiver Γ we give a conjectural description of this residue in terms of indecomposable representations of Γ over finite depth rings.

The connections between these different results is the method of proof. At the heart of each theorem lies a motivic or p-adic volume computation, which is only possible due to some surprising cancellations. These cancellations are reminiscent of a result in classical symplectic geometry by Duistermaat and Heckman on the localization of the Liouville measure, hence the title of the thesis.

THOMAS M.J-B. HUMEAU

Oral exam: November 13, 2017
Public defense: December 8, 2017

Thesis director: Robert Dalang
President of the Jury: Stephan Morgenthaler
Thesis Nr: 8223

Stochastic partial differential equations driven by Lévy white noises: Generalized random processes, random field solutions and regularity

In this thesis, we study various aspects of stochastic partial differential equations driven by Lévy white noise. This driving noise, which is a generalization of Gaussian white noise, can be viewed either as a generalized random process or as an independently scattered random measure. After unifying these approaches and establishing appropriate stochastic integral representations, we show that a necessary and sufficient condition for a Lévy white noise to have values in $\mathcal{S}'(\mathbb{R}^d)$, the space of tempered Schwartz distributions, is that the underlying Lévy measure have a positive absolute moment.

In the case of a linear stochastic partial differential equation with a general differential operator and driven by a symmetric pure jump Lévy white noise, we show that when the mild solution is locally Lebesgue integrable, then it is equal to the generalized solution, and that a random field representation exists for the generalized solution if and only if the fundamental solution of the operator has certain integrability properties. In that case, we show that the random field representation is equal to the mild solution. For this purpose, a new stochastic Fubini theorem is proved. These results are applied to the linear stochastic heat and wave equations driven by a symmetric α-stable noise.

We then study the non-linear stochastic heat equation driven by a general type of Lévy white noise, possibly with heavy tails and non-summable small jumps. Our framework includes in particular the α-stable noise. In the case of the equation on the whole space $\mathbb{R}^d$, we show that the law of the solution that we construct does not depend on the space variable. Then we show in various domains $D \subset \mathbb{R}^d$ that the solution u to the stochastic heat equation is such that $t \mapsto u(t, \cdot)$ has a *càdlàg* version in a fractional Sobolev space of order $r < -\frac{d}{2}$. Finally, we show that $x \mapsto u(t, x)$ (respectively $t \mapsto u(t, x)$) at a fixed time (respectively fixed space-point) has a continuous version under some optimal moment conditions. In the α-stable case, we show that for the choices of α for which this moment condition is not satisfied, the sample paths of $x \mapsto u(t, x)$ (respectively $t \mapsto u(t, x)$) are unbounded on any non-empty open subset.

HOSSEIN S. NASSAJIANMOJARRAD

Oral exam:	October 30, 2017
Public defense:	December 15, 2017
Thesis director:	János Pach
President of the Jury:	Kathryn Hess Bellwald
Thesis Nr:	8115

On some algebraic and extremal problems in discrete geometry

In the present thesis, we delve into different extremal and algebraic problems arising from combinatorial geometry. Specifically, we consider the following problems. For any integer $n \geq 3$, we define $e(n)$ to be the minimum positive integer such that any set of $e(n)$ points in general position in the plane contains n points in convex position. In 1935, Erdős and Szekeres proved that $e(n) \leq \binom{2n-4}{n-2} + 1$ and later in 1961, they obtained the lower bound $2^{n-2} + 1 \leq e(n)$, which they conjectured to be optimal. We prove that $e(n) \leq \binom{2n-5}{n-2} - \binom{2n-8}{n-3} + 2$. In a recent breakthrough, Suk proved that $e(n) \leq 2^{n+O\left(n^{2/3}\log n\right)}$. We strengthen this result by extending it to pseudo-configurations and also improving the error term. Combining our results with a theorem of Dobbins et al., we significantly improve the best known upper bounds on the following two functions, introduced by Bisztriczky and Fejes Tóth and by Pach and Tóth, respectively. Let $c(n)$ (and $c'(n)$) denote the smallest positive integer N such that any family of N pairwise disjoint convex bodies in general position (resp., N convex bodies in general position, any pair of which share at most two boundary points) has an n members in convex position. We show that $c(n) \leq c'(n) \leq 2^{n+O\left(\sqrt{n\log n}\right)}$. Given a point set P in the plane, an ordinary circle for P is defined as a circle containing exactly three points of P. We prove that any set of n points in the plane, not all on a line or a circle, determines at least $\frac{1}{4}n^2 - O(n)$ ordinary circles. We determine the exact minimum number of ordinary circles for all sufficiently large n, and characterize all point sets that come close to this minimum. We also consider the orchard problem for circles, where we determine the maximum number of circles containing four points of a given set and describe the extremal configurations. A special case of the Schwartz-Zippel lemma states that given an algebraic curve $C \subset \mathbb{C}^2$ of degree d and two finite sets $A, B \subset \mathbb{C}$, we have $|C \cap (A \times B)| = O_d(|A| + |B|)$. We establish a two-dimensional version of this result, and prove upper bounds on the size of the intersection $|X \cap (P \times Q)|$ for a variety $X \subset \mathbb{C}^4$ and finite sets $P, Q \subset \mathbb{C}^2$. A key ingredient in our proofs is a two-dimensional version of a special case of Alon's combinatorial Nullstellensatz. As corollaries, we generalize the Szemerédi-Trotter point-line incidence theorem and several known bounds on repeated and distinct Euclidean distances. We use incidence geometry to prove some sum-product bounds over arbitrary fields. First, we give an explicit exponent and improve a recent result of Bukh and Tsimerman by proving that $\max\{|A + A|, |f(A, A)|\} \gg |A|^{6/5}$ for any small set $A \subset \mathbb{F}_p$ and quadratic non-degenerate polynomial $f(x, y) \in \mathbb{F}_p[x, y]$. This generalizes the result of Roche-Newton et al. giving the best known lower bound for the term $\max\{|A + A|, |A \cdot A|\}$. Secondly, we improve and generalize the sum-product results of Hegyvári and Hennecart on $\max\{|A + B|, |f(B, C)|\}$, for a specific type of function f. Finally, we prove that the number of distinct cubic distances generated by any small set $A \times A \subset \mathbb{F}_p^2$ is $\Omega(|A|^{8/7})$, which improves a result of Yazici,Murphy, Rudnev, and Shkredov.

CARLO CICCARELLA

Oral exam:	November 17, 2017
Public defense:	December 15, 2017
Thesis director:	Robert Dalang
President of the Jury:	Victor Panaretos
Thesis Nr:	8149

Optimal solution and asymptotic properties of a stochastic control problem arising in sailboat trajectory optimization

We study the optimal strategy for a sailboat to reach an upwind island under the hypothesis that the wind direction fluctuates according to a Brownian motion and the wind speed is constant. The work is motivated by a concrete problem which typically arises during sailing regattas, namely finding the best tacking strategy to reach the upwind buoy as quickly as possible. We assume that there is no loss of time when tacking.

We first guess an optimal strategy and then we establish its optimality by using the dynamic programming principle. The Hamilton Jacobi Bellmann equation obtained is a parabolic PDE with Neumann boundary conditions. Since it does not admit a closed form solution, the proof of optimality involves an intricate estimate of derivatives of the value function.

We explicitly provide the asymptotic shape of the value function. In order to do so, we prove a result on large time behavior for solutions to time dependent parabolic PDE using a coupling argument. In particular, a boat far from the origin approaches the origin at $\frac{1}{2} + \frac{\sqrt{2}}{\pi} = 95.02\%$ of the boat's speed.

FRANCISCO SANTOS PAREDES QUARTIN DE MACEDO

Oral exam and Public defense:	January 8, 2018
Thesis director:	Daniel Kressner
Thesis co-director:	Pacheco Pires António Manuel, IST Portugal
President of the Jury:	Victor Panaretos
Thesis Nr:	7718

Low-rank tensor methods for large Markov chains and forward feature selection methods

In the first part of this thesis, we present and compare several approaches for the determination of the steady-state of large-scale Markov chains with an underlying low- rank tensor structure. Such structure is, in our context of interest, associated with the existence of interacting processes. The state space grows exponentially with the number of processes. This type of problems arises, for instance, in queueing theory, in chemical reaction networks, or in telecommunications. As the number of degrees of freedom of the problem grows exponentially with the number of processes, the so-called *curse of dimensionality* severely impairs the use of standard methods for the numerical analysis of such Markov chains. We drastically reduce the number of degrees of freedom by assuming a low-rank tensor structure of the solution. We develop different approaches, all considering a formulation of the problem where all involved structures are considered in their low-rank representations in tensor train format. The first approaches that we will consider are associated with iterative solvers, in particular focusing on solving a minimization problem that is equivalent to the original problem of finding the desired steady state. We later also consider tensorized multigrid techniques as main solvers, using different operators for restriction and interpolation. For instance, aggregation/disaggregation operators, which have been extensively used in this field, are applied. In the second part of this thesis, we focus on methods for feature selection. More concretely, since, among the various classes of methods, sequential feature selection methods based on mutual information have become very popular and are widely used in practice, we focus on this particular type of methods. This type of problems arises, for instance, in microarray analysis, in clinical prediction, or in text categorization. Comparative evaluations of these methods have been limited by being based on specific datasets and classifiers. We develop a theoretical framework that allows evaluating the methods based on their theoretical properties. Our framework is based on the properties of the target objective function that the methods try to approximate, and on a novel categorization of features, according to their contribution to the explanation of the class; we derive upper and lower bounds for the target objective function and relate these bounds with the feature types. Then, we characterize the types of approximations made by the methods, and analyse how these approximations cope with the good properties of the target objective function. We also develop a distributional setting designed to illustrate the various deficiencies of the methods, and provide several examples of wrong feature selections. In the context of this setting, we use the minimum Bayes risk as performance measure of the methods.

THOMAS LUGRIN

Oral exam: March 26, 2018
Public defense: April 26, 2018

Thesis director: Anthony C. Davison
Thesis co-director: Jonathan Tawn
President of the Jury: Thomas Mountford
Thesis Nr: 8349

Semiparametric Bayesian Risk Estimation for Complex Extremes

Extreme events are responsible for huge material damage and are costly in terms of their human and economic impacts. They strike all facets of modern society, such as physical infrastructure and insurance companies through environmental hazards, banking and finance through stock market crises, and the internet and communication systems through network and server overloads. It is thus of increasing importance to accurately assess the risk of extreme events in order to mitigate them. Extreme value theory is a statistical approach to extrapolation of probabilities beyond the range of the data, which provides a robust framework to learn from an often small number of recorded extreme events.

In this thesis, we consider a conditional approach to modelling extreme values that is more flexible than standard models for simultaneously extreme events. We explore the sub-asymptotic properties of this conditional approach and prove that in specific situations its finite-sample behaviour can differ significantly from its limit characterisation. For modelling extremes in time series with short-range dependence, the standard peaks-over-threshold method relies on a pre-processing step that retains only a subset of observations exceeding a high threshold and can result in badly-biased estimates. This method focuses on the marginal distribution of the extremes and does not estimate temporal extremal dependence. We propose a new methodology to model time series extremes using Bayesian semiparametrics and allowing estimation of functionals of clusters of extremes. We apply our methodology to model river flow data in England and improve flood risk assessment by explicitly describing extremal dependence in time, using information from all exceedances of a high threshold.

We develop two new bivariate models which are based on the conditional tail approach, and use all observations having at least one extreme component in our inference procedure, thus extracting more information from the data than existing approaches. We compare the efficiency of these models in a simulation study and discuss generalisations to higher-dimensional setups. Existing models for extremes of Markov chains generally rely on a strong assumption of asymptotic dependence at all lags and separately consider marginal and joint features. We introduce a more flexible model and show how Bayesian semiparametrics can provide a suitable framework allowing simultaneous inference for the margins and the extremal dependence structure, yielding efficient risk estimates and a reliable assessment of uncertainty.

NICCOLÒ DAL SANTO

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Thesis director:	Alfio Quarteroni
Thesis co-director:	Andrea Manzoni
President of the Jury:	Fabio Nobile
Thesis Nr:	8553

Multi space reduced basis preconditioners for parametrized partial differential equations

The multiquery solution of parametric partial differential equations (PDEs), that is, PDEs depending on a vector of parameters, is computationally challenging and appears in several engineering contexts, such as PDE-constrained optimization, uncertainty quantification or sensitivity analysis. When using the finite element (FE) method as approximation technique, an algebraic system must be solved for each instance of the parameter, leading to a critical bottleneck when we are in a multiquery context, a problem which is even more emphasized when dealing with nonlinear or time dependent PDEs. Several techniques have been proposed to deal with sequences of linear systems, such as truncated Krylov subspace recycling methods, deflated restarting techniques and approximate inverse preconditioners; however, these techniques do not satisfactorily exploit the parameter dependence. More recently, the reduced basis (RB) method, together with other reduced order modeling (ROM) techniques, emerged as an efficient tool to tackle parametrized PDEs. In this thesis, we investigate a novel preconditioning strategy for parametrized systems which arise from the FE discretization of parametrized PDEs. Our preconditioner combines multiplicatively a RB coarse component, which is built upon the RB method, and a nonsingular fine grid preconditioner. The proposed technique hinges upon the construction of a new Multi Space Reduced Basis (MSRB) method, where a RB solver is built at each step of the chosen iterative method and trained to accurately solve the error equation. The resulting preconditioner directly exploits the parameter dependence, since it is tailored to the class of problems at hand, and significantly speeds up the solution of the parametrized linear system. We analyze the proposed preconditioner from a theoretical standpoint, providing assumptions which lead to its well-posedness and efficiency. We apply our strategy to a broad range of problems described by parametrized PDEs: (i) elliptic problems such as advection-diffusion-reaction equations, (ii) evolution problems such as time-dependent advection-diffusion-reaction equations or linear elastodynamics equations (iii) saddle-point problems such as Stokes equations, and, finally, (iv) Navier-Stokes equations. Even though the structure of the preconditioner is similar for all these classes of problems, its fine and coarse components must be accurately chosen in order to provide the best possible results. Several comparisons are made with respect to the current state-of-the-art preconditioning and ROM techniques. Finally, we employ the proposed technique to speed up the solution of problems in the field of cardiovascular modeling.

MIKKO T. KORHONEN

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Thesis director: Donna Testerman
President of the Jury: Kathryn Hess Bellwald
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Reductive overgroups of distinguished unipotent elements in simple algebraic groups

Let G be a simple linear algebraic group over an algebraically closed field K of characteristic $p \geq 0$. In this thesis, we investigate closed connected reductive subgroups $X < G$ that contain a given distinguished unipotent element u of G. Our main result is the classification of all such X that are maximal among the closed connected subgroups of G.

When G is simple of exceptional type, the result is easily read from the tables computed by Lawther (J. Algebra, 2009). Our focus is then on the case where G is simple of classical type, say $G = \mathrm{SL}(V)$, $G = \mathrm{Sp}(V)$, or $G = \mathrm{SO}(V)$. We begin by considering the maximal closed connected subgroups X of G which belong to one of the families of the so-called *geometric subgroups*. Here the only difficult case is the one where X is the stabilizer of a tensor decomposition of V. For $p = 2$ and $X = \mathrm{Sp}(V_1) \otimes \mathrm{Sp}(V_2)$, we solve the problem with explicit calculations; for the other tensor product subgroups we apply a result of Barry (Comm. Algebra, 2015).

After the geometric subgroups, the maximal closed connected subgroups that remain are the $X < G$ such that X is simple and V is an irreducible and tensor indecomposable X-module. The bulk of this thesis is concerned with this case. We determine all triples (X, u, φ) where X is a simple algebraic group, $u \in X$ is a unipotent element, and $\varphi : X \to G$ is a rational irreducible representation such that $\varphi(u)$ is a distinguished unipotent element of G. When $p = 0$, this was done in previous work by Liebeck, Seitz and Testerman (Pac. J. Math, 2015).

In the final chapter of the thesis, we consider the more general problem of finding all connected reductive subgroups X of G that contain a distinguished unipotent element u of G. This leads us to consider connected reductive overgroups X of u which are contained in some proper parabolic subgroup of G. Testerman and Zalesski (Proc. Am. Math. Soc, 2013) have shown that when u is a regular unipotent element of G, no such X exists. We give several examples which show that their result does not generalize to distinguished unipotent elements. As an extension of the Testerman-Zalesski result, we show that except for two known examples which occur in the case where $(G, p) = (C_2, 2)$, a connected reductive overgroup of a distinguished unipotent element of order p cannot be contained in a proper parabolic subgroup of G.

FEI PU

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Thesis director: Robert Dalang
President of the Jury: Thomas Mountford
Thesis Nr: 8695

The stochastic heat equation: hitting probabilities and the probability density function of the supremum via Malliavin calculus

In this thesis, we study systems of linear and/or non-linear stochastic heat equations and fractional heat equations in spatial dimension 1 driven by space-time white noise. The main topic is the study of hitting probabilities for the solutions to these systems.

We first study the properties of the probability density functions of the solution to non-linear systems of stochastic fractional heat equations driven by multiplicative space-time white noise. Using the techniques of Malliavin calculus, we prove that the one-point probability density function of the solution is infinitely differentiable, uniformly bounded and positive everywhere. Moreover, a Gaussian-type upper bound on the two-point probability density function is obtained by a detailed analysis of the small eigenvalues of the Malliavin matrix. We establish an optimal lower bound on hitting probabilities for the (non-Gaussian) solution, which is as sharp as that for the Gaussian solution to a system of linear equations.

We develop a new method to study the upper bound on hitting probabilities, from the perspective of probability density functions. For the solution to the linear stochastic heat equation, we prove that the random vector, which consists of the solution and the supremum of a linear increment of the solution over a time segment, has an infinitely differentiable probability density function. We derive a formula for this density and establish a Gaussian-type upper bound. The smoothness property and Gaussian-type upper bound for the density of the supremum of the solution over a space-time rectangle touching the $t = 0$ axis are also studied. Furthermore, we extend these results to the solutions of systems of linear stochastic fractional heat equations. For a system of linear stochastic heat equations with Dirichlet boundary conditions, we present a sufficient condition for certain sets to be hit with probability one.

ANTONELLO GERBI

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Thesis director:	Alfio Quarteroni
Thesis co-director:	Luca Dedé
President of the Jury:	Joachim Krieger
Thesis Nr:	8731

Numerical approximation of cardiac electro-fluid-mechanical models: coupling strategies for large-scale simulation

The mathematical modeling of the heart involves several challenges, which are intrinsically related to the complexity of its function. A satisfactory cardiac model must be able to describe a wide range of different processes, such as the evolution of the transmembrane potential in the myocardium, the deformation caused by the muscles contraction, and the dynamics of the blood inside the heart chambers. In this work, we focus on the coupling of the electrophysiology, the active and the passive mechanics, and the fluid dynamics of the blood in the left ventricle (LV) of the human heart. The models describing the previously mentioned processes are called "single core models", and can be regarded as the building blocks of an "integrated model".

In this thesis, we first review the isolated single core mathematical models for the description of the LV function, and discuss their space and time discretizations with particular emphasis on the coupling conditions. We consider both implicit and semi-implicit schemes for the time discretization. The fully discretized single core problems thus obtained are then combined to define integrated electromechanics and electrofluidmechanics problems. We then focus on the numerical coupling strategy for the electromechanics solver in the framework of the active strain formulation. First, we propose a monolithic strategy where the discretized core models are solved simultaneously; then, several novel segregated strategies, where the discretized core models are solved sequentially, are proposed and systematically compared with each other. The segregated strategies are obtained by exploiting a Godunov splitting scheme, which introduces a first order error on the solution. We show that, while the monolithic approach is more accurate and more stable for relatively large timesteps, segregated approaches allow to solve the integrated problem much more efficiently in terms of computational resources. Moreover, with segregated approaches, it is possible to use different timesteps for the different core models in a staggered fashion, thus further improving the computational efficiency of the schemes.

The monolithic and the segregated strategies for the electromechanics are used to solve a benchmark problem with idealized geometry: the results are then compared in terms of accuracy and efficiency. We numerically confirm that the segregated strategies are accurate at least of order one. In light of the results obtained, we employ the proposed strategies to simulate the electromechanics of a subject-specific LV for a full heartbeat. We simulate both healthy and pathological scenarios: in the latter case, we account for an ischemic necrosis of the tissue and analyze several clinical indicators such as pressure-volume loops and the end systolic pressure-volume relationship. Finally, we use the proposed strategies to simulate the electrofluidmechanics of a realistic LV during the systolic phase of the heartbeat.

When defining the integrated cardiac models, we establish a preprocess pipeline aimed at preparing geometries and data for both idealized and subject-specific simulations. The pipeline is succesfully used for the setting up of large scale simulations in a high performance computing framework, where the (strong and weak) scalability of the proposed coupling strategies is assessed.

ANA SUSNJARA

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President of the Jury:	Thomas Mountford
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Fast hierarchical solvers for symmetric eigenvalue problems

In this thesis we address the computation of a spectral decomposition for symmetric banded matrices. In light of dealing with large-scale matrices, where classical dense linear algebra routines are not applicable, it is essential to design alternative techniques that take advantage of data properties. Our approach is based upon exploiting the underlying hierarchical low-rank structure in the intermediate results. Indeed, we study the computation in the *hierarchically off-diagonal low rank* (HODLR) format of two crucial tools: QR decomposition and spectral projectors, in order to devise a fast spectral divide-and-conquer method.

In the first part we propose a new method for computing a QR decomposition of a HODLR matrix, where the factor R is returned in the HODLR format, while Q is given in a compact WY representation. The new algorithm enjoys linear-polylogarithmic complexity and is norm-wise accurate. Moreover, it maintains the orthogonality of the Q factor.

The approximate computation of spectral projectors is addressed in the second part. This problem has raised some interest in the context of linear scaling electronic structure methods. There the presence of small spectral gaps brings difficulties to existing algorithms based on approximate sparsity. We propose a fast method based on a variant of the QDWH algorithm, and exploit that QDWH applied to a banded input generates a sequence of matrices that can be efficiently represented in the HODLR format. From the theoretical side, we provide an analysis of the structure preservation in the final outcome. More specifically, we derive a priori decay bounds on the singular values in the off-diagonal blocks of spectral projectors. Consequently, this shows that our method, based on data-sparsity, brings benefits in terms of memory requirements in comparison to approximate sparsity approaches, because of its logarithmic dependence on the spectral gap. Numerical experiments conducted on tridiagonal and banded matrices demonstrate that the proposed algorithm is robust with respect to the spectral gap and exhibits linear-polylogarithmic complexity. Furthermore, it renders very accurate approximations to the spectral projectors even for very large matrices.

The last part of this thesis is concerned with developing a fast spectral divide-and-conquer method in the HODLR format. The idea behind this technique is to recursively split the spectrum, using invariant subspaces associated with its subsets. This allows to obtain a complete spectral decomposition by solving the smaller-sized problems. Following Nakatsukasa and Higham, we combine our method for the fast computation of spectral projectors with a novel technique for finding a basis for the range of such a HODLR matrix. The latter strongly relies on properties of spectral projectors, and it is analyzed theoretically. Numerical results confirm that the method is applicable for large-scale matrices, and exhibits linear-polylogarithmic complexity.

HODLR format, low-rank structure, QR decomposition