Computational Neuroscience: Neuronal Dynamics of Cognition

ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

Decision models:
Competitive dynamics

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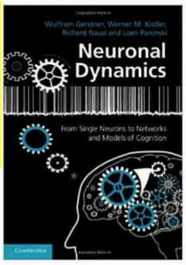
Reading for week 9:
NEURONAL DYNAMICS
Ch. 16 (except 16.4.2)



Cambridge Univ. Press

1. **Introduction**
   - decision making
2. **Perceptual decision making**
   - V5/MT
   - Decision dynamics: Area LIP
3. **Theory of decision dynamics**
   - competition via shared inhibition
   - effective 2-dim model
4. **Solutions**
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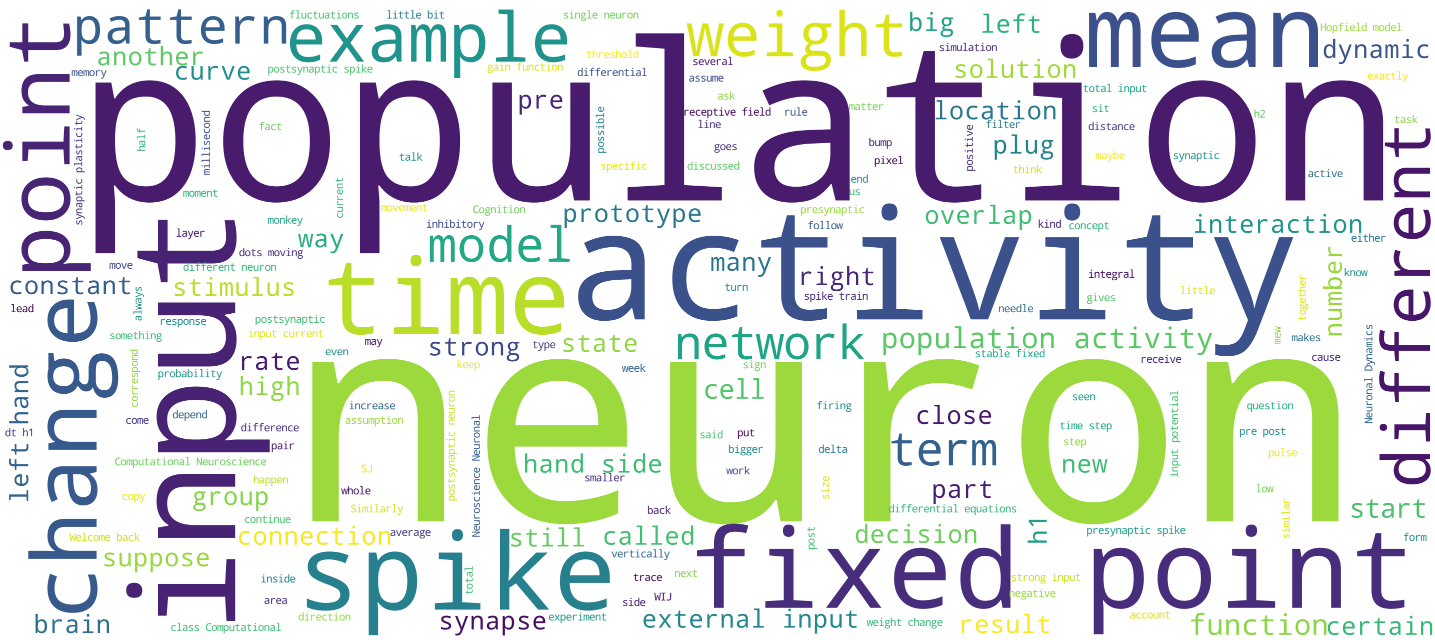
5. **Simulations and Experiments**
   - simulations and theory
   - simulations and experiments
6. **Decisions, actions, volition**
   - the problem of free will



Search MOOC

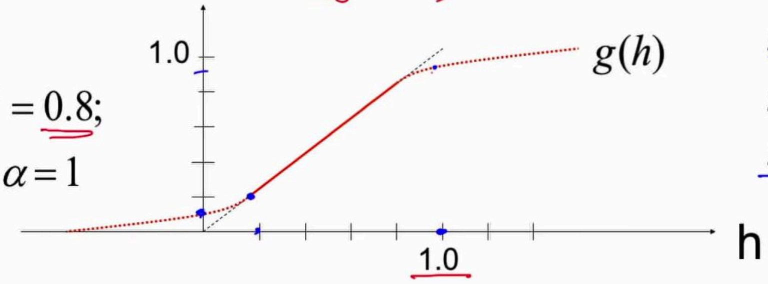
Video

4. Theory of decision dynamics

$$\tau \frac{d}{dt} h_1(t) = \underbrace{-h_1(t)}_{1} + \underbrace{h_1^{ext}(t)}_{0.8} + \underbrace{(w_{ee} - \alpha)}_{0.5} \underbrace{\overset{F}{g}(h_1(t))}_{0.9} - \underbrace{\alpha}_{1}\, g(h_2(t))$$

$h_1^{ext} = h_2^{ext} = 0.8;$

$w_{ee} = 1.5;\ \alpha = 1$



$g(h) = h$ for $0.2 < h < 0.8$

$g(0) = 0.1$

$g(1) = 0.9$

$\frac{d}{dt} h_1 = 0$

h_1	$g(h_2)$	h_2
1.0		
0.8		
0.2		
0.0		

$\frac{d}{dt} h_2 = 0$

h_2	$g(h_1)$	h_1
1.0		
0.8		
0.2		
0.0		

Welcome back to the class Computational Neuroscience: Neuronal Dynamics of Cognition. We are in the middle of our discussion of decision models and I would like now to discuss the solutions of the effective two-dimensional model with you. So we had equations of the type as indicated here, differential equations here for the first population. One of the terms is this transfer function which we can call big F or small g for gain function, and a typical shape for this gain function would look like this. Now, let's suppose we have a simple scenario where for a value of 1.0, I would have a readout of 0.9; or for 0, I would have a value of 0.1 -- or 0.2, I would have a value of 0.2. So I have a simplified curve, and now one of the tricks of the analysis of two-dimensional system is to identify the points where the derivative is 0. Okay? And now we just plug in one of these points. For example, we plug in the point h1 equal 1.0, that's this point here, we would have a result of 0.9. So, if I plug in 1, I have here 0.9, now I've chosen wee equal 1.5, equal 1, so this has a value of 0.5, then I have here equal 1, I plug in h1 equal 1, so this is 1, I have external input 0.8.

Notes

Summary

0m 00s

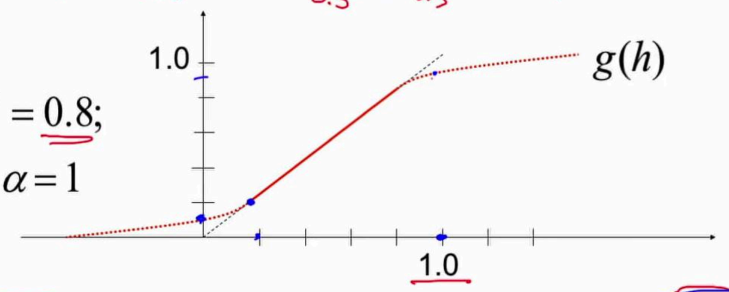
4. Theory of decision dynamics

$$\tau \frac{d}{dt} h_1(t) = -h_1(t) + h_1^{ext}(t) + \underbrace{(w_{ee} - \alpha)}_{0.5} \overset{F}{g(h_1(t))} - \alpha\, g(h_2(t))$$

0 1 0.8 0.5 0.9 1

$h_1^{ext} = h_2^{ext} = 0.8$;

$w_{ee} = 1.5$; $\alpha = 1$



$g(h) = h \;\; for \;\; 0.2 < h < 0.8$

$g(0) = 0.1$

$g(1) = 0.9$

$\frac{d}{dt} h_1 = 0$

h_1	$g(h_2)$	h_2
1.0	0.25	0.25
0.8	0.4	0.4
0.2		
0.0		

$\frac{d}{dt} h_2 = 0$

h_2	$g(h_1)$	h_1
1.0		
0.8		
0.2		
0.0		

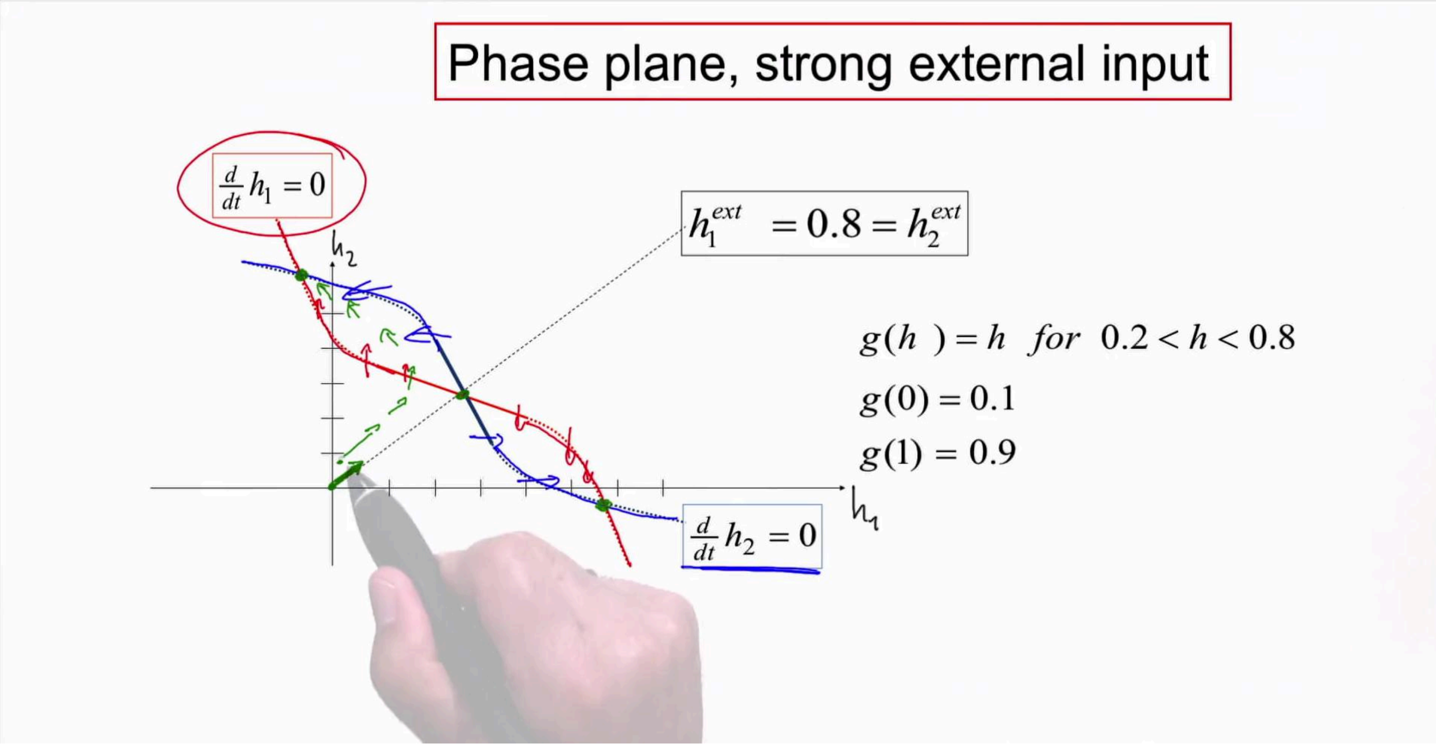
So what would be the g of h2 in this case? So the left-hand side is, I look for left-hand side 0, so we have the left-hand side 0, we have minus 1 plus 0.8 makes minus 0.2 plus 0.45 gives the value of 0.25, I readout that 0.25 have 0.25. Similarly, if I plug in 0.8, I have minus 0.8, minus 0.8, I have 0.5 times 0.8, I would have the result of 0.4, 0.4. And so you continue this way, and then you do the same thing for the other differential equation and fill in that table as well.

Notes

Summary

1m 57s

4. Theory of decision dynamics

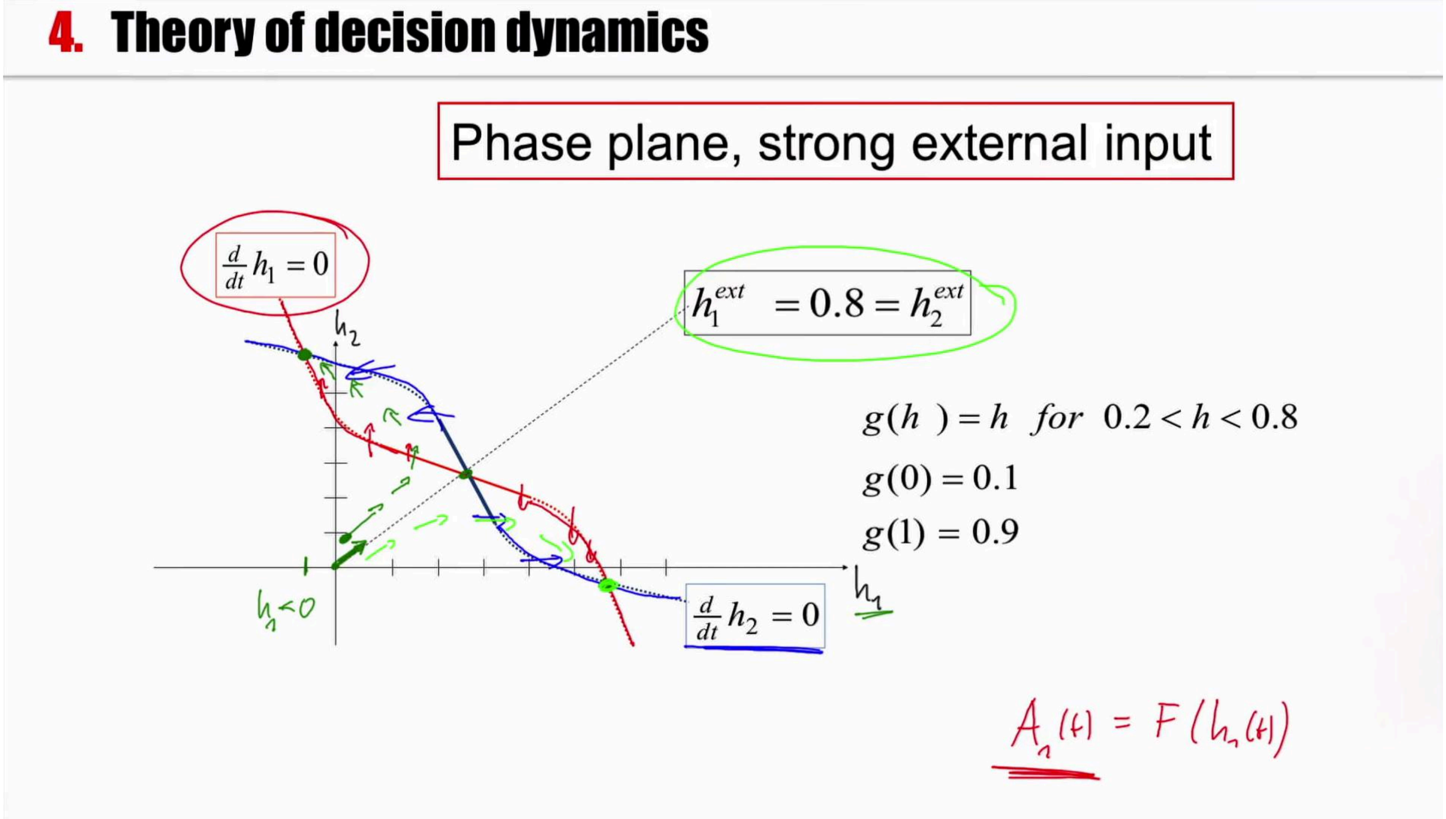


And this way, you can construct the position of all the points that have d/dt h1 equal 0. So this is the axis h1, this is the axis h2, and all the points that have d/dt h1 equal 0 lie on this line here. Now all the points that lie on the other function are the points that have d/dt h2 equal 0. I've two differential equations, and there are now three points where both differential equations are 0, and these are the fixed points of the system, that means there is no change. If the system is here, there is no change. Now suppose we start here. Then there is a change. And I can evaluate the differential equation and then find that the change is symmetric and moving upwards. Now, if I'm on one of the curves, for example the curve d/dt h1, then there is no change in the h1 direction, which means the arrows are vertically, they are pointing upwards, because I have the upwards component down here. And they are pointing downwards on the other side, and I have horizontal arrows on the blue graph. And now we can follow the dynamics. Suppose we start at the point, somewhere here. Well, we are close to this arrow, so we would probably move upwards, and then we cross here vertically, and then we go over here.

Notes

Summary

2m 41s



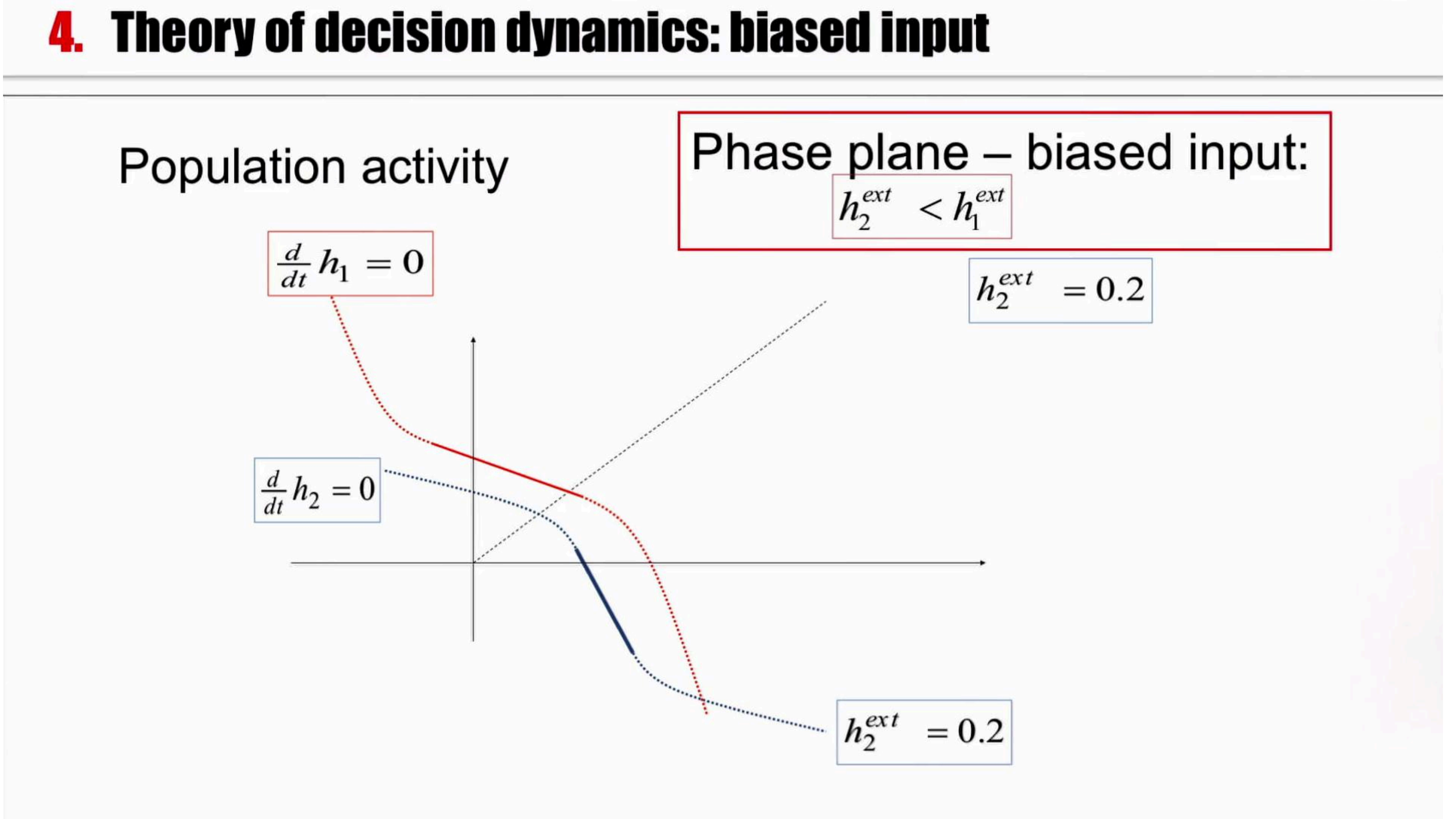
If I have an initial position here, the dynamics will lead me to a stable state, to the fixed point over here. Now what is this fixed point? It has very low h1, so h1 is smaller than 0, and h2 is big. But the activity of the first group at time t is just a function of h1 of t. So if h1 is 0, then the activity will be very close to 0. If h2 is big, then the activity of this second group will be very high. So this is a point where the activity of the second group is big and the activity of the first group is low. Now, if you follow a point starting here, it would look like this, and we end up at the other point where the activity of the first population is very high. So this is a scenario where I had strong external input to both populations.

Notes

Summary

4m 18s

4. Theory of decision dynamics: biased input



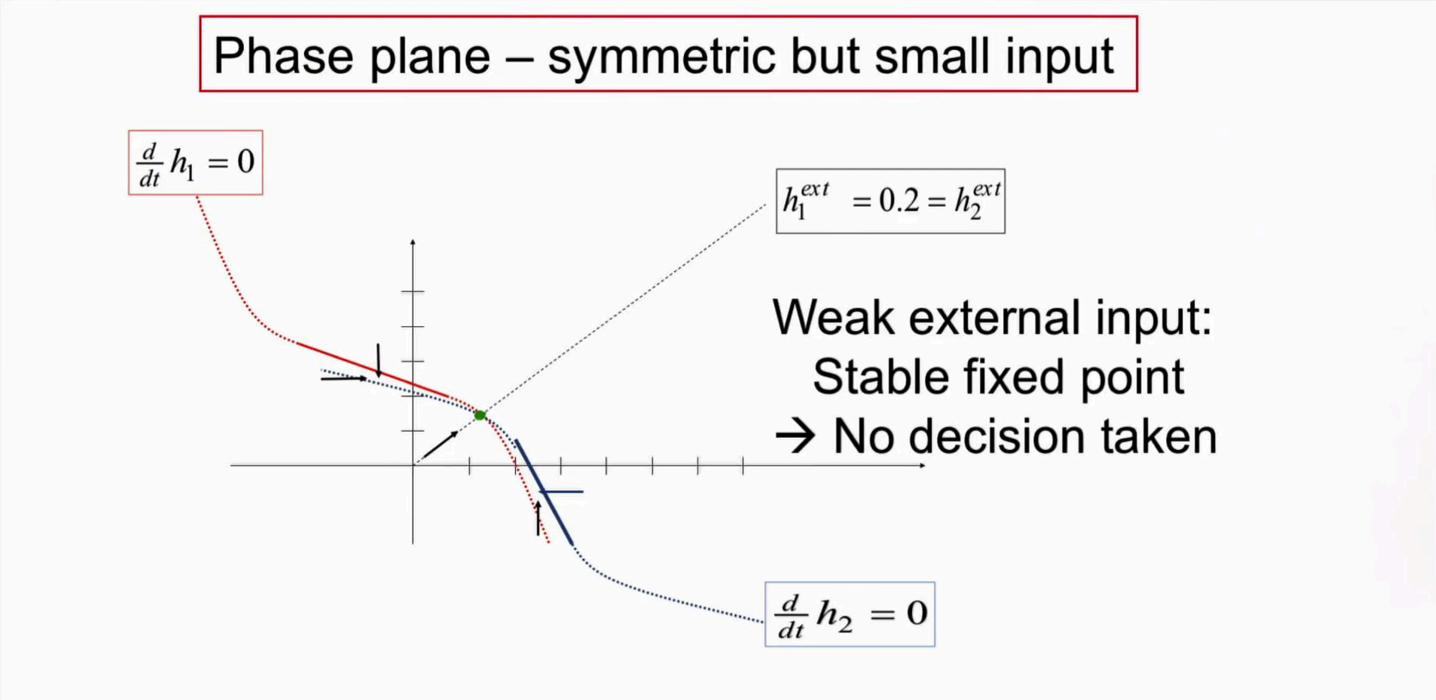
Now I can keep the strong external input to the first population while reducing the input to the second population, and what happens is that this second curve is now shifted downward. If you now look again at the fixed points, we see that there's only one single fixed point left, wherever I start, so these are the flow arrows, wherever I start, I necessarily end up at this fixed point. So in this case, if you interpret the final state, this here means that the activity of the first population is big, h1 is big, therefore the activity of the first population is also big, while the other population is completely suppressed. So here I had a strong input, h1 is much bigger than h2, h1 was 0.8, h2 was 0.2, the external input, and so only one solution is possible.

Notes

Summary

5m 19s

4. Theory of decision dynamics: unbiased weak



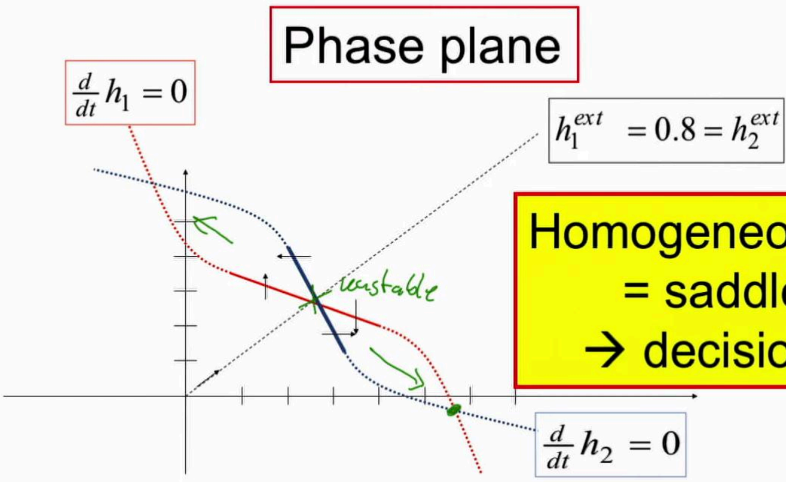
Now if I also decrease the first input, then I find another fixed point. This is now a fixed point sitting here, it's again a symmetric fixed point where both populations had very weak activity. So, let me summarise. In the experiment, I would start with no input or weak external input. This means small input that's completely symmetric, no bias either one way or another. And I only have a stable fixed point.

Notes

Summary

6m 30s

4. Theory of decision dynamics: unbiased strong



Homogeneous solution
= saddle point
→ decision must be taken

Now if I give a strong input to both, it's like having many dots moving to the left, and many dots moving to the right, then two other fixed points emerge, these fixed points here and these are stable fixed points and the dynamics will move to one or the other of these fixed points. If only one input is strong, so I have only dots moving to the left, then only one solution is possible and that's the one corresponding to where the population representing left movement is strongest. So, for biased input, the stable fixed point reflects the input, reflects the bias in the input. If I have strong input, that's unbiased, a decision must be taken, and that's represented by the fact that the symmetric solution is unstable, and therefore it's going either to the fixed point on the left, or to the fixed point on the right.

Notes

Summary

7m 00s

Quiz: Competitive dynamics

Which of the following statements are correct?

[ ] For symmetric weak input, our competitive model system has a single fixed point.
[ ] For symmetric strong input, our competitive model system has a single fixed point.
[ ] For biased strong input, our competitive model system has a single fixed point.
[ ] For symmetric strong input, our competitive model system must take a decision
[ ] For biased strong input, our competitive model system must take a decision
[ ] Taking a decision corresponds to a movement toward an asymmetric stable fixed point of the dynamics

Notes

Summary

8m 04s