Computational Neuroscience: Neuronal Dynamics of Cognition

ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

Decision models:

Competitive dynamics

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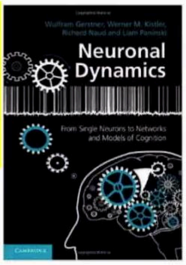
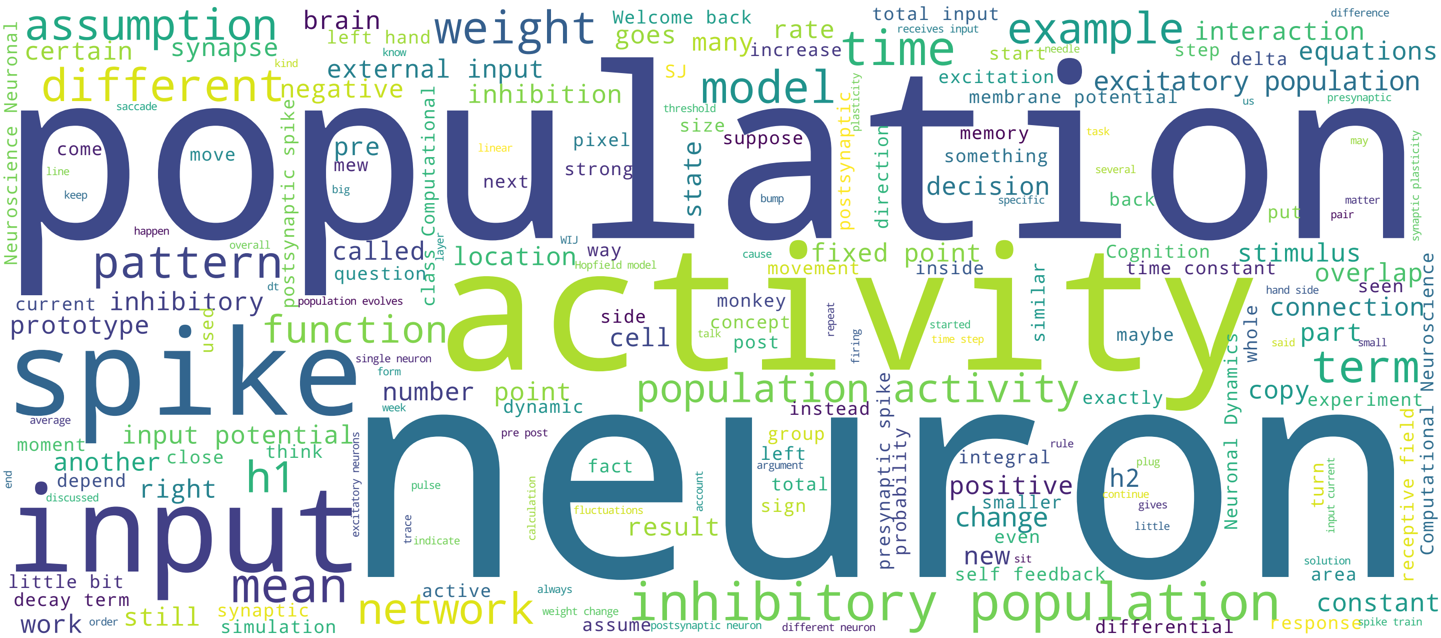
Reading for week 9:
NEURONAL DYNAMICS
Ch. 16 (except 16.4.2)



Cambridge Univ. Press

1. **Introduction**
   - decision making
2. **Perceptual decision making**
   - V5/MT
   - Decision dynamics: Area LIP
3. **Theory of decision dynamics**
   - competition via shared inhibition
   - effective 2-dim model
4. **Solutions**
   - symmetric case
   - biased case
5. **Simulations and Experiments**
   - simulations and theory
   - simulations and experiments
6. **Decisions, actions, volition**
   - the problem of free will



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Video

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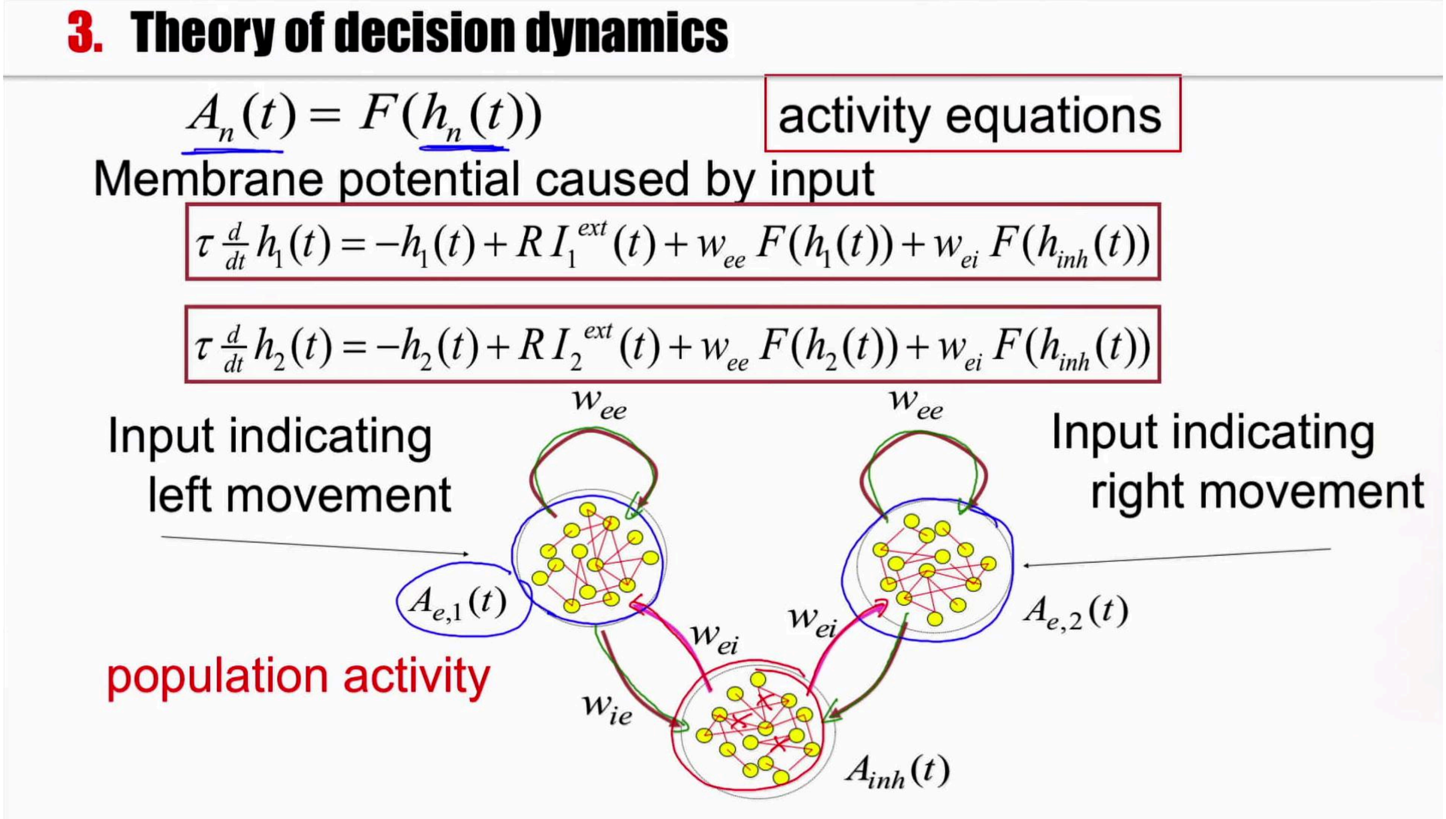
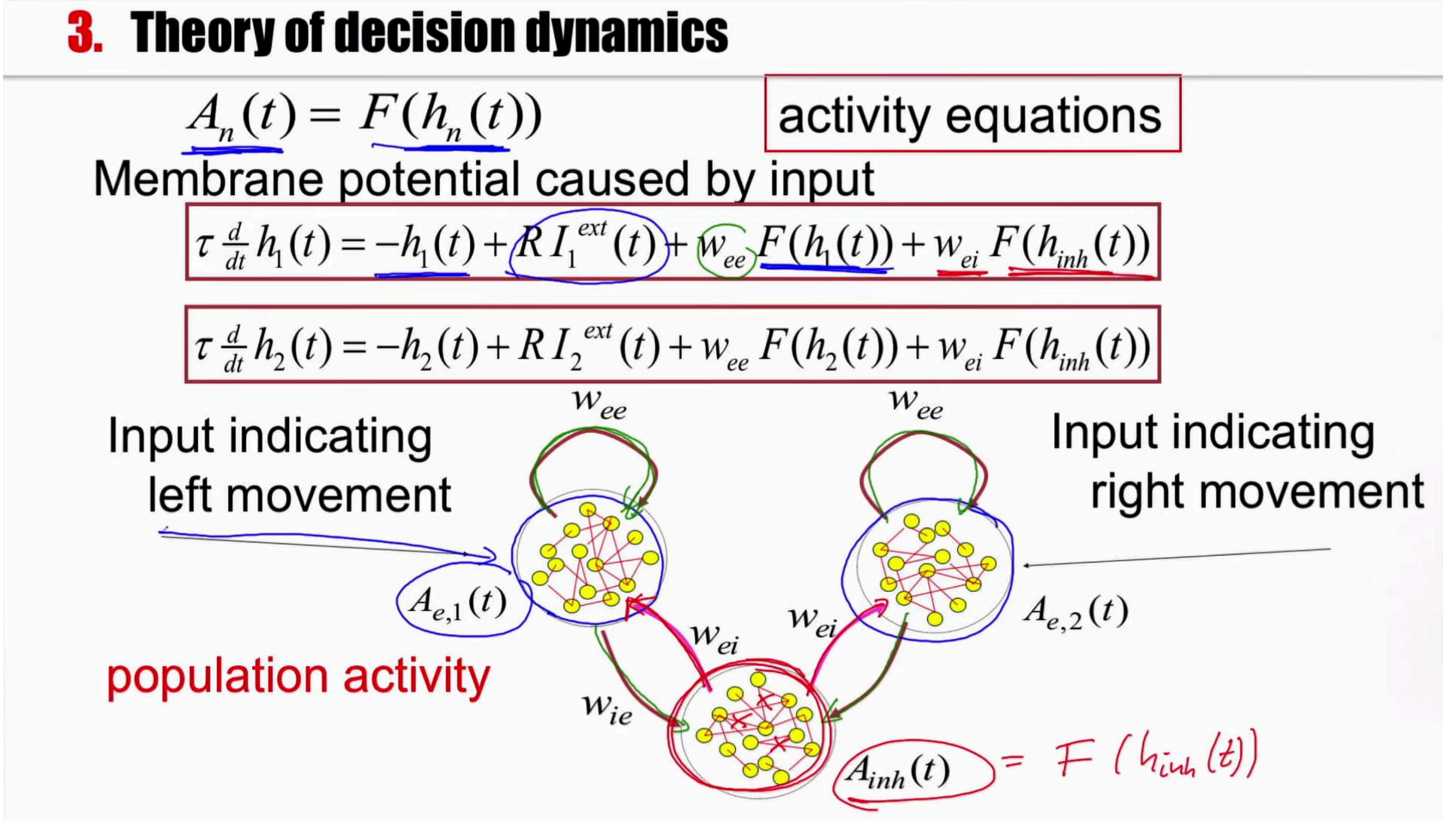


So welcome back to the class Computational Neuroscience: Neuronal Dynamics of Cognition. I talked a little bit about experimental data related to decisions, but now let's turn to the theory of decision dynamics, and we'll work with populations of neurons that share inhibition. So here's the basic model. I have one population of excitatory neurons that would indicate left movement or a saccade performed to the left, and there's another population that would indicate a saccade to the right or a movement to the right. Now each of these neuronal populations has interactions with itself, so neurons communicate with each other, inside the population, but each of these excitatory populations also sends activity, it sends spikes to a further population, and this population is the inhibitory population. Now for this inhibitory population, I will have no lateral interactions, so there's no self-feedback, but the inhibitory population will send spikes, will send activity symmetrically back to the other two populations. So now let's look at the activity of this population here. The activity depends on the membrane potential, more precisely on the input potential, the part of the membrane potential that's caused by the input.

Notes

Summary

0m 00s



And we used the differential equation for the population activity, so we can say that the derivative of the input potential, the temporal derivative, is given by decay term and the external input, the external input into this population that we are looking at, my first population here, then it depends on the self-feedback, that's this term here, the self-feedback, and what's sent over here is the activity, but the activity is given by the input potential. So I can -- F of h1 is the activity of that population, but then this population also receives input from the inhibitory population via these links here and this would be the activity of the inhibitory population. Now I can repeat the same argument for the second population. The second population has self-feedback, it receives input from the inhibitory population, external stimuli and its differential equation will have a decay term.

Notes

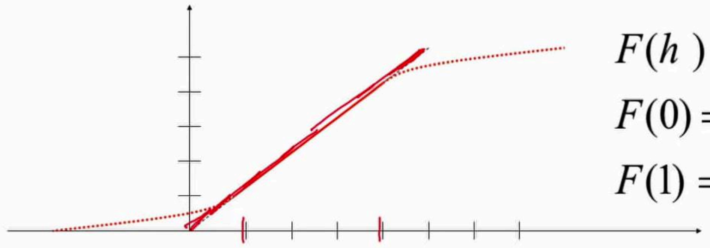
Summary

1m 37s

Population activity

$$A_n(t) = F(h_n(t))$$

activity equations



$F(h) = h \quad for \quad 0.2 < h < 0.8$

$F(0) = 0.1$

$F(1) = 0.9$

Inhibitory Population

$$A_{inh}(t) = F(h_{inh}(t)) = h_{inh}(t)$$

Assumption 1: linear

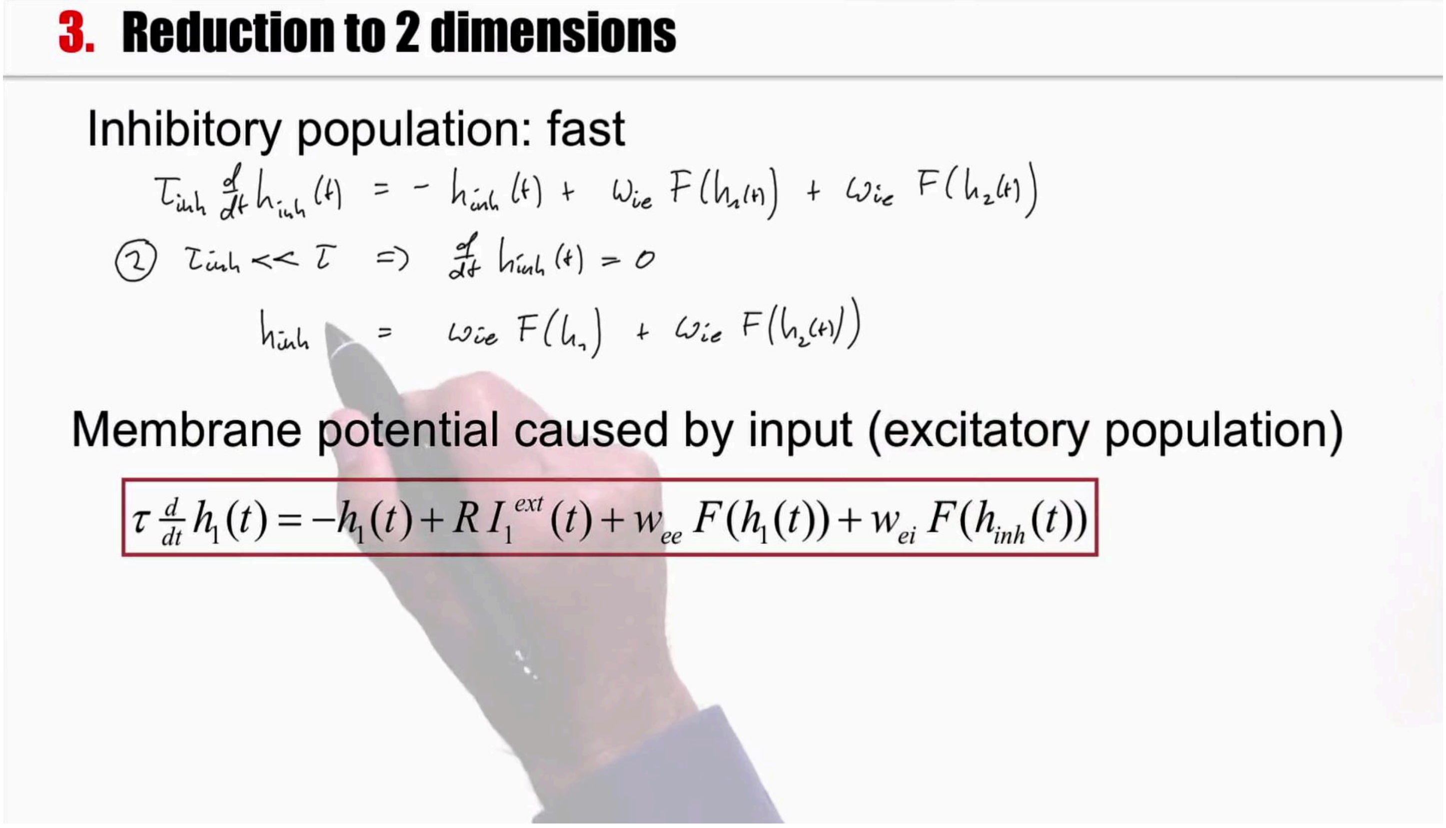
Assumption 2: fast

Now in order to proceed with the analysis of these equations, I make two assumptions. The first one is that this activity function for the inhibitory population is just linear. So I just take a linear line here instead of this full bended curve. Now this is an assumption, this assumption would be valid in some limited regime. Okay? Now the other assumption is that I say the inhibitory population is, in fact, fast.

Notes

Summary

2m 53s



So, what does this mean? For the inhibitory population, we have an equation very similar to this one here. So let me copy this here. It has a time constant for the inhibitory population. It has d/dt h for the inhibitory population, I have the decay term for the inhibitory population, I have no external stimulus for the inhibitory population, it's our first assumption, and then I would have the input to the inhibitory population coming from the excitatory population which would be F of h1 of t plus the input from the other excitatory population F of h2 of t. Now my assumption is that, with assumption two, that the inhibitory population is fast, and I express this by saying the time constant for inhibition is much, must smaller than the time constant for the excitatory population. And this means that while the excitatory population evolves, the inhibitory population can always rapidly approach the state that's quasi-stationary. So we can set at each moment in time, on this slow time scale which the excitatory population evolves, the derivative is 0. Now I bring that, so, I have zero on the left-hand size, I bring this to the other side, and I can say h inhibitory is equal to wie F of h1 plus wie F of h2 of t.

Notes

Summary

3m 29s

3. Reduction to 2 dimensions

Inhibitory population: fast

$$\tau_{inh} \frac{d}{dt} h_{inh}(t) = -h_{inh}(t) + w_{ie} F(h_1(t)) + w_{ie} F(h_2(t))$$

(2) $\tau_{inh} << \tau \Rightarrow \frac{d}{dt} h_{inh}(t) = 0$

$$h_{inh}(t) = w_{ie} F(h_1(t)) + w_{ie} F(h_2(t))$$

(1) $h_{inh}(t) = F(h_{inh}(t))$

Membrane potential caused by input (excitatory population)

$$\boxed{\tau \frac{d}{dt} h_1(t) = -h_1(t) + R I_1^{ext}(t) + w_{ee} F(h_1(t)) + w_{ei} F(h_{inh}(t))}$$

(1) (2)

$$= -h_1(t) + R I_1^{ext}(t) + w_{ee} F(h_1(t)) + \underbrace{\underbrace{w_{ei}}_{<0} \underbrace{w_{ie}}_{>0}}_{<0} F(h_1(t)) + w_{ei} w_{ie} F(h_2(t))$$

$- \alpha$

And this is true for all points t where t is now the slow time scale on which the membrane potential of the excitatory population evolves. So, this is my second assumption. Now the first assumption was that h inhibitory of t is the same as F of h inhibitory of t because this function here is linear. Okay? And so, this is now what I exploit here, this is the same as h inhibitory, but h inhibitory is this. So because of assumptions one and two, I can write this as wei, from inhibition to excitation, and then I plug in this term, so I have wie F of h1 of t plus, I copy this, wei wie, I have this, I take this F of h2 of t. So, this is one, and this is two. And the rest I just copy. Minus h1 of t plus RI, external for the first population, plus wee F of h1 of t. Now let me continue with a few remarks. So, this is the connection from excitation to inhibition. So this is a positive term. This is from inhibition back to excitation, so this is the inhibitory coupling constant which is negative. So overall, I have here something which is negative, because it's a multiplication, and therefore I write this as new parameter, minus, with a positive. Okay?

Notes

Summary

5m 20s

3. Reduction to 2 dimensions

Inhibitory population: fast

$$\tau_{inh} \frac{d}{dt} h_{inh}(t) = -h_{inh}(t) + w_{ie} F(h_1(t)) + w_{ie} F(h_2(t))$$

(2) $\tau_{inh} << \tau \Rightarrow \frac{d}{dt} h_{inh}(t) = 0$

$$h_{inh}(t) = w_{ie} F(h_1(t)) + w_{ie} F(h_2(t))$$

(1) $h_{inh}(t) = F(h_{inh}(t))$

Membrane potential caused by input (excitatory population)

$$\boxed{\tau \frac{d}{dt} h_1(t) = -h_1(t) + R I_1^{ext}(t) + w_{ee} F(h_1(t)) + w_{ei} F(h_{inh}(t))}$$

(1) ↓ (2)

$$= -h_1(t) + R I_1^{ext}(t) + w_{ee} F(h_1(t)) + \underbrace{\underbrace{w_{ei}}_{<0} \underbrace{w_{ie}}_{>0}}_{<0} F(h_1(t)) + \underbrace{w_{ei} w_{ie}}_{} F(h_2(t))$$

$$\tau \cdot \frac{d}{dt} h(t) = -h_1(t) + \underbrace{R \cdot I_1^{ext}(t)}_{h^{ext}(t)} + (w_{ee} - \alpha) F(h_1(t)) - \alpha F(h_2(t))$$

Second remark, I see that I have here F of h1 and I have the same F of h1 here again, so I can sum these up, I can say I have plus wee minus, F of h1 of t, and then I have here F of h2, and this is the same constant as before, so it also gives to minus. I copy the rest, h1 of t, this is the external input into population one, I can also describe this as a external input potential, contribution to the total input potential. And on the left-hand side, I have just my d/dt h of t. So this is now my equation for this first population. And I can repeat exactly the same calculation for the second population.

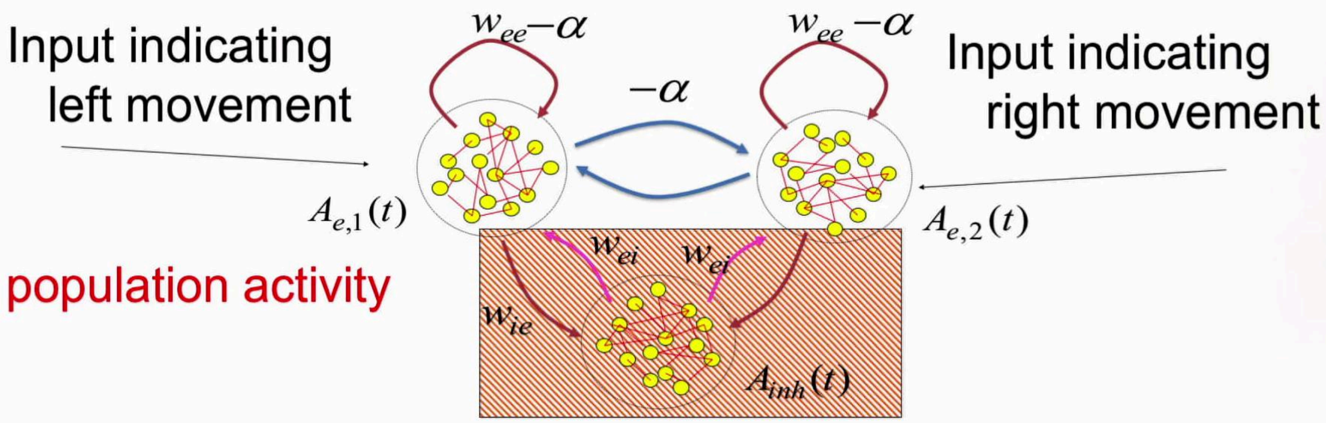
Notes

Summary

7m 39s

3. Effective 2-dim. model

- two populations 'compete':
  if one shows high activity, it inhibits the other
- effective inhibitory interaction



The result would look like this. It's the same equation, just with indices exchange, whereby I had h1 I write h2 and vice versa. Now what has happened? We started off with three populations, we had two excitatory populations and one inhibitory populations. Now by these mathematical tricks that I've used on the previous slide, I was able to remove the inhibitory population, it has disappeared. And instead, I have an effective inhibitory coupling between the populations. So it's as if population one has an inhibitory influence on population two. That's this term here. Population one has a negative influence on the input potential of population two. And the same holds in the other direction. Thus we have been able to reduce the number of equations from three equations to two equations. And these two populations compete. If one population shows high activity, it will inhibit the other population, and vice versa. Now note that this is an effective inhibitory interaction. In the reality, excitatory never send out inhibitory interaction. And that's why we start with separated excitation and inhibition. But the mathematical manipulation show that this excitatory population can effectively inhibit the other line. And that's why the two populations compete.

Notes

Summary

8m 54s

Notes

Summary

10m 36s