Computational Neuroscience: Neuronal Dynamics of Cognition

ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

Neuronal Populations

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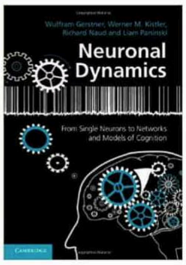
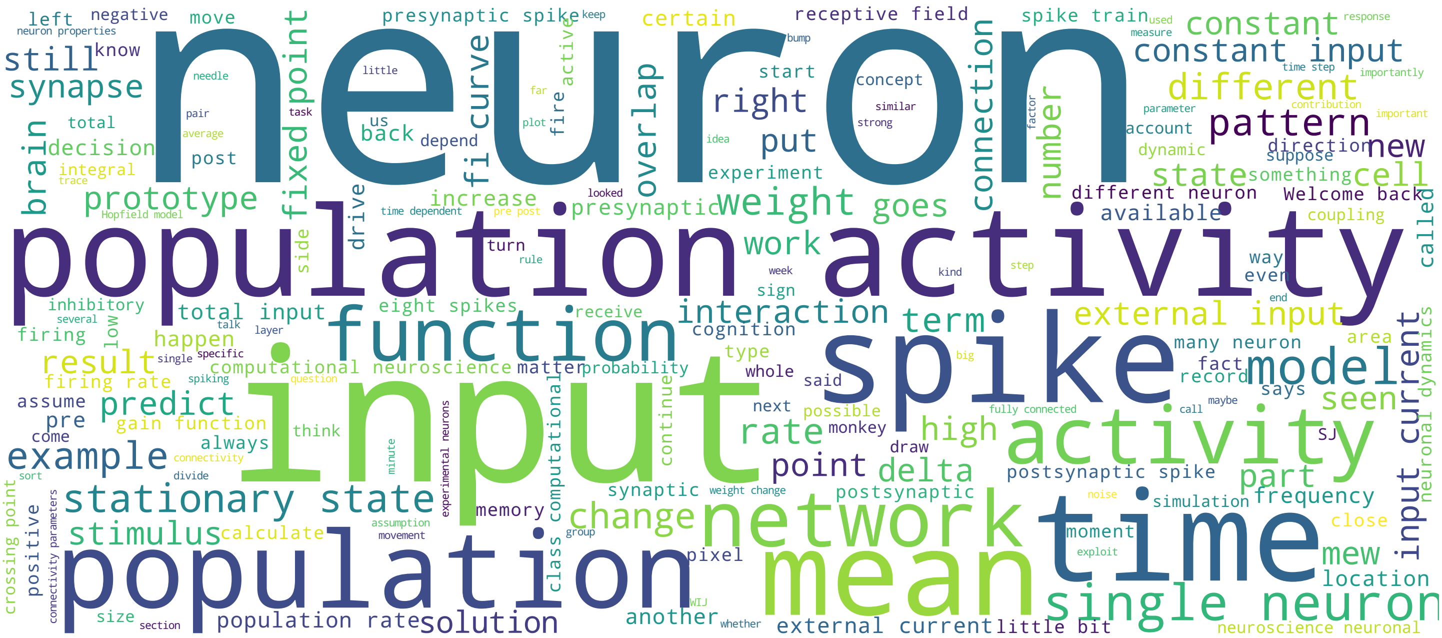
Reading:
NEURONAL DYNAMICS
- Ch. 12.1 – 12.4.3
(except Section 12.3.7)

Cambridge Univ. Press



1. **Population activity**
   - definition and aims
2. **Cortical Populations**
   - columns and receptive fields
3. **Connectivity**
   - cortical connectivity
   - model connectivity schemes
4. **Mean-field argument**
   - input to one neuron
5. **Stationary mean-field**
   - asynchronous state: predict activity
6. **Random Networks**
   - Balanced state



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Video

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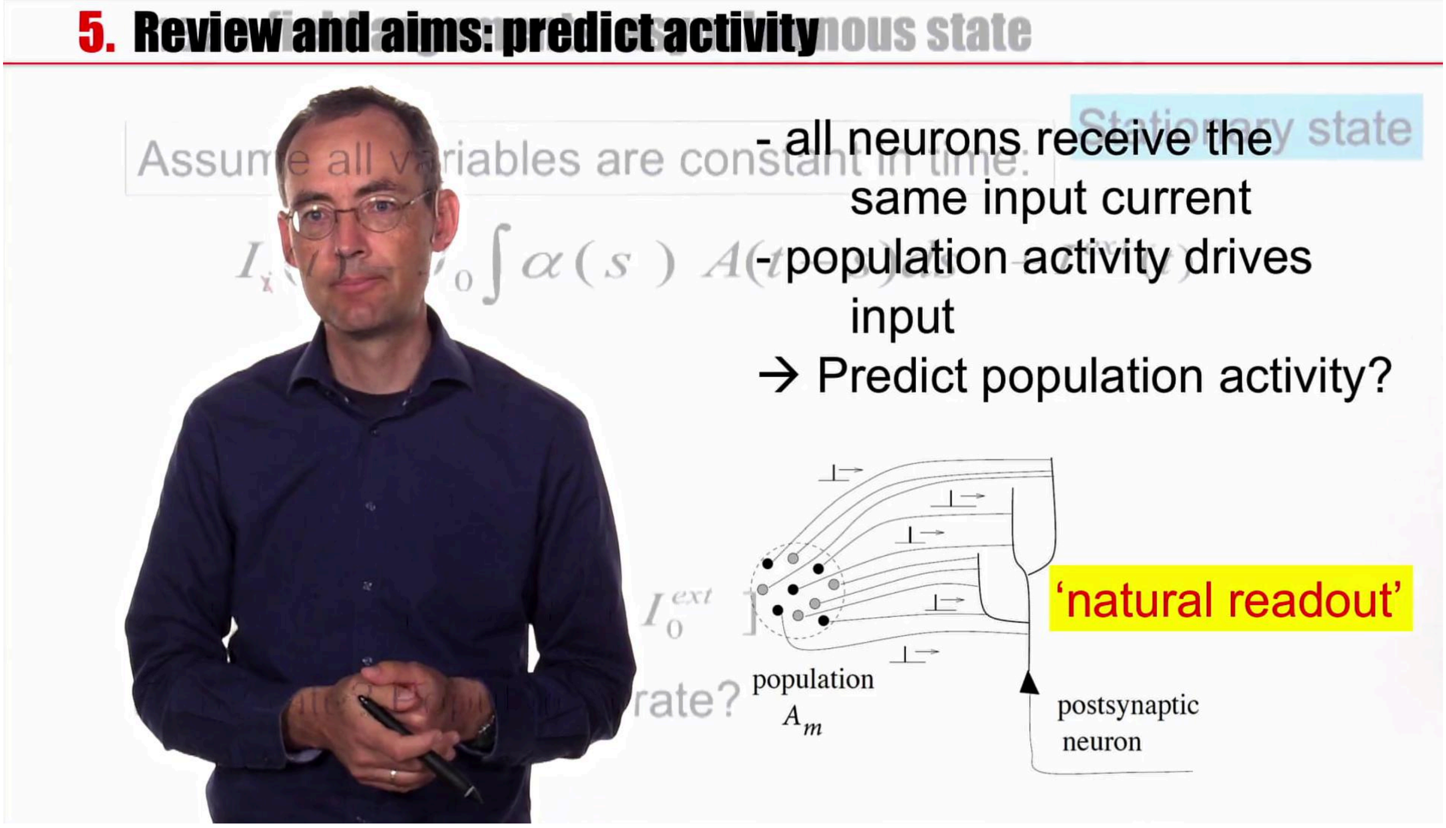


Welcome back to the class, computational neuroscience neuronal dynamics of cognition. We are in the middle of our discussion of population activity, and really just to a little bit more of math to continue. Our aim is to predict the population activity, just from neuron parameters and the connectivity parameters question is can we do this. Well, for time dependent input it's sort of difficult, but for stationary input, yes it's possible. To do this prediction, we will exploit that all neurons receive the same input current. And it's the population activity which drives this input. And from this input, we want to predict the population activity. So they get an enclosed sloop, and how to do this. This is the topic of this section.

Notes

Summary

0m 05s

5. mean-field arguments: asynchronous state

Stationary state

Assume all variables are constant in time:

$$I_x(t) = J_0 \int \alpha(s)\, A(t-s)\,ds + I^{ext}(\cancel{t})$$

$$= J_0 \cdot \underbrace{\int \alpha(s)\,ds}\; A_0 + I_0^{ext}$$

$$= J_0 \cdot q \cdot A_0 + I_0^{ext}$$

$$\| \quad (1) \quad I_0 = [J_0 q A_0 + I_0^{ext}] \quad \|$$

Firing rate? Population rate?

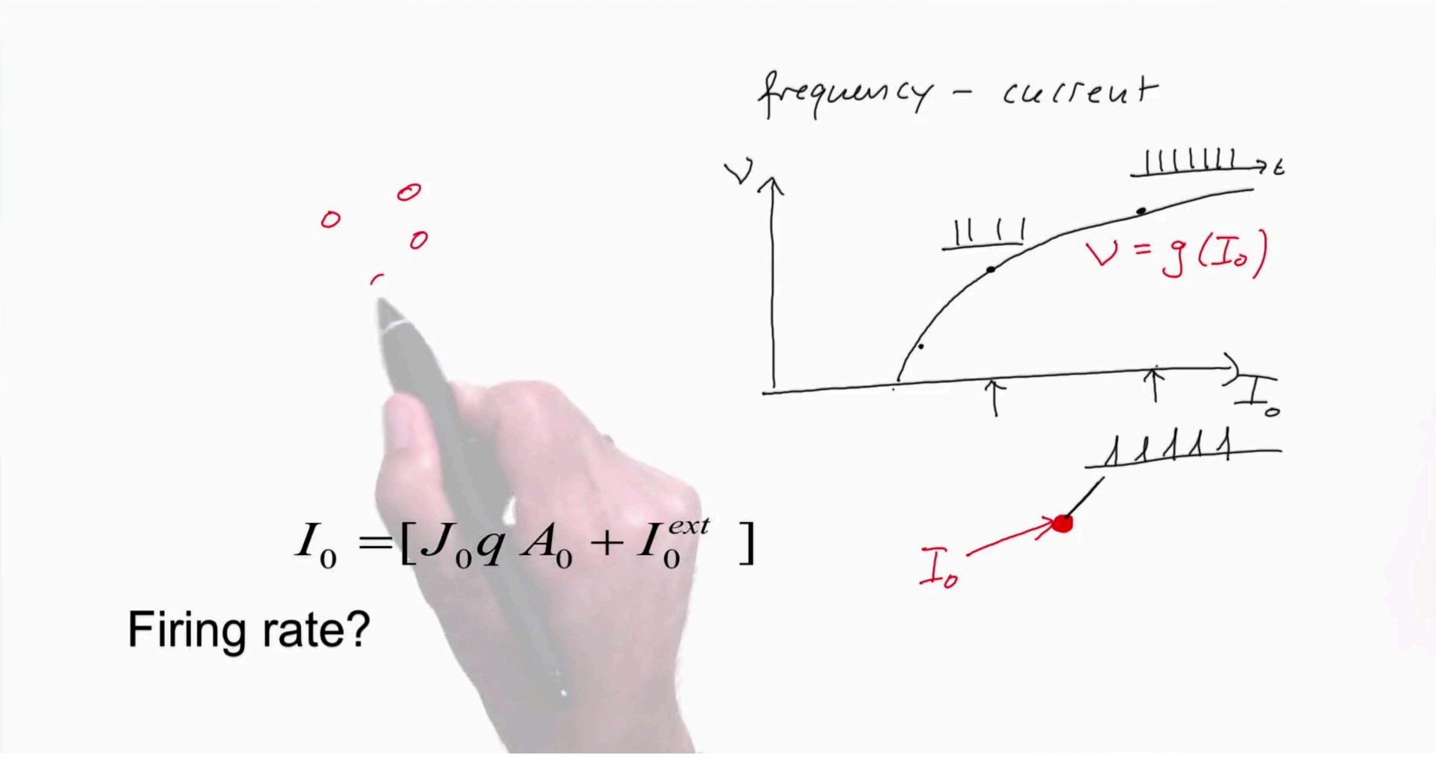
So, our assumption now is that we work with a stationary state. This implies that this activity is not time dependent but it's constant, constant added value, a zero. As a result, I can collect this integration of S DS easier no longer depends on time. There's no S anymore. So, I can write this off of S ES. And I still have to factor in front, and I still have a potential external input, which I assumed to also be constant, no time dependence. Okay. Now just let us define a parameter Q equal to alpha S DS. And so we have to resolve that J 0 QA zero + I 0 external is the total driving force. And this is my first equation, which i will use in a minute. Now, this is my input, this input will drive my neurons. Now what happens if you drive a neuron for the constant input?

Notes

Summary

0m 52s

5. mean-field argument: f-I curve of single neuron



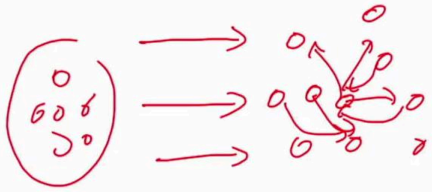
Well, let's just draw here, the totally input I 0 are constant input, and let's draw here, the firing right. Let's call it mew. So, this is the frequency of firing. And this is the current voice frequency versus current or fi curve of a neuron. So if I give a high current strong constant input current, the frequency of firing is high, which means as a function of time, then you and will emit many spikes. If I give a small input current, the neuron and will emit spikes at a somewhat reduced rate. And if I give an even smaller current the spikes are even more spaced. So the frequency will look something like this. The frequency is high for high input, its low for low input. And this is my function frequency as a function of input. Mew is a function of the input. So, this is a single neuron property. I have one single neuron. I inject the current I 0, and then I record in that neuron a spike train, and what I will see is a sequence of spikes, for the electrical pulses, and this is the frequency, as a function of this input current. Now we can do this for experimental neurons and also do this in model neurons. So this frequency current curve is available. But now let's go back to our network.

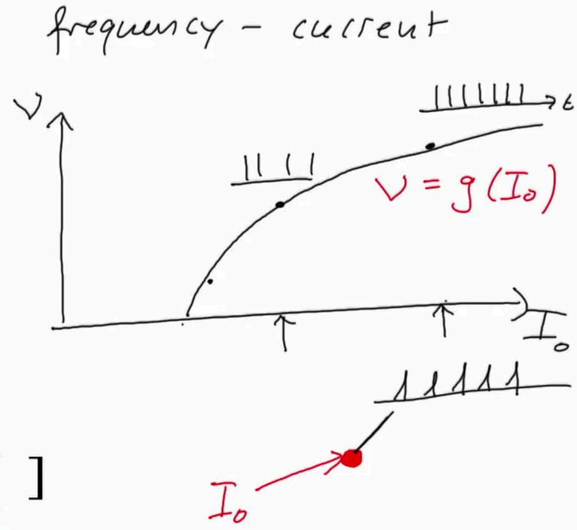
Notes

Summary

2m 16s

5. mean-field argument: f-I curve of single neuron





$$I_0 = [\underbrace{J_0 q\, A_0}_{\text{netw}} + \underbrace{I_0^{ext}}_{\text{ext}}]$$

Firing rate?

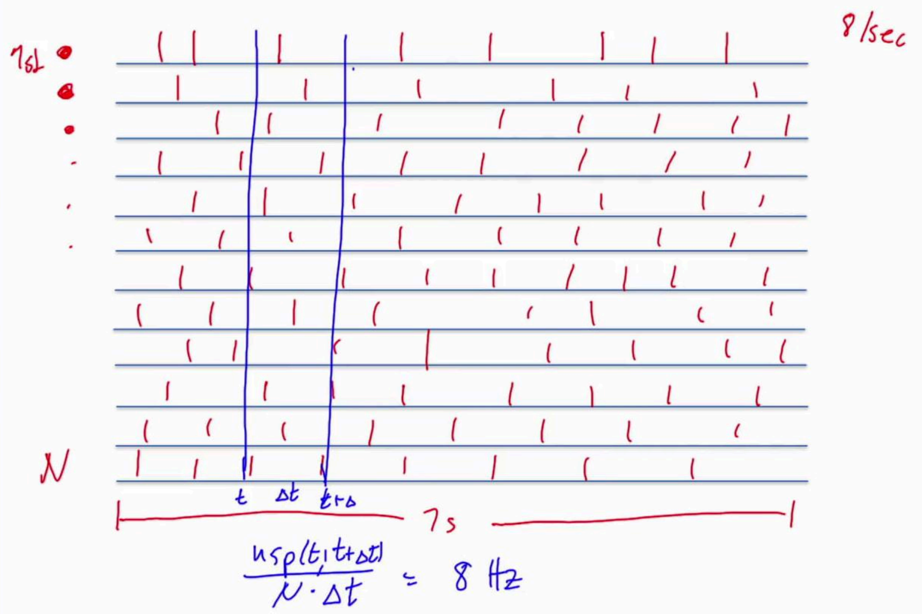
In our network, we have many neuron. And now we have seen that each neuron is driven by a total input, which is a contribution of this network input and the external input. This is the input coming through the synapses within the network. And then this is an additional external input maybe coming from other populations that we don't describe explicitly. So, each neuron receives the same input. So each neuron will fire at this rate.

Notes

Summary

4m 04s

5. mean-field argument: population rate = single neuron rate



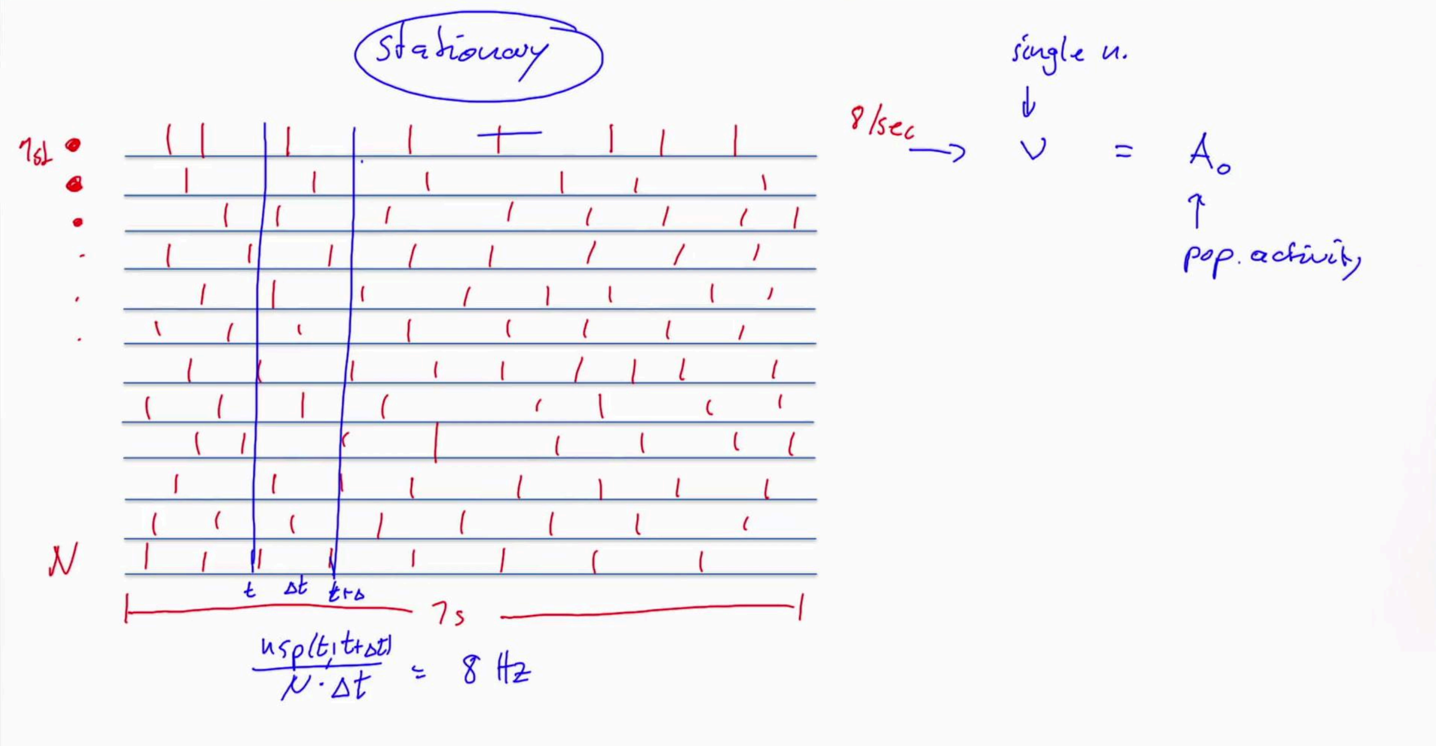
So let's look at one of the neurons. These are the spike trains of one neuron. Maybe like this. The absence of noise the spiking would be very regular if there's some noise the spiking can be irregular. This is my first neuron. Now for the second neuron that you'd also emit spikes. And if a third neuron that he made spikes, and so on that many neurons, each neurons, emits its spikes. And let me just draw a couple of these to illustrate the N neurons. This is my N neuron, so, as I said, in this population, each neuron receives the same constant input and for this constant input it emits a certain number of spikes. Say, we record for one second. This is one second, and the neurons fire with eight hertz. So typically, each neuron will emit exactly eight spikes. So if I do this average one on one in this direction I will get eight spikes, if I do it for another new and I also get eight spikes. To define rate as a temple average is eight spikes per second. Now my claim is that if you just look at the shorter interval. Now you count the spikes vertically in this interval between t and t plus delta t. If you divide by the number of neurons, and if you divide by this delta t, then it should also be eight hertz.

Notes

Summary

4m 44s

5. mean-field argument: population rate = single neuron rate

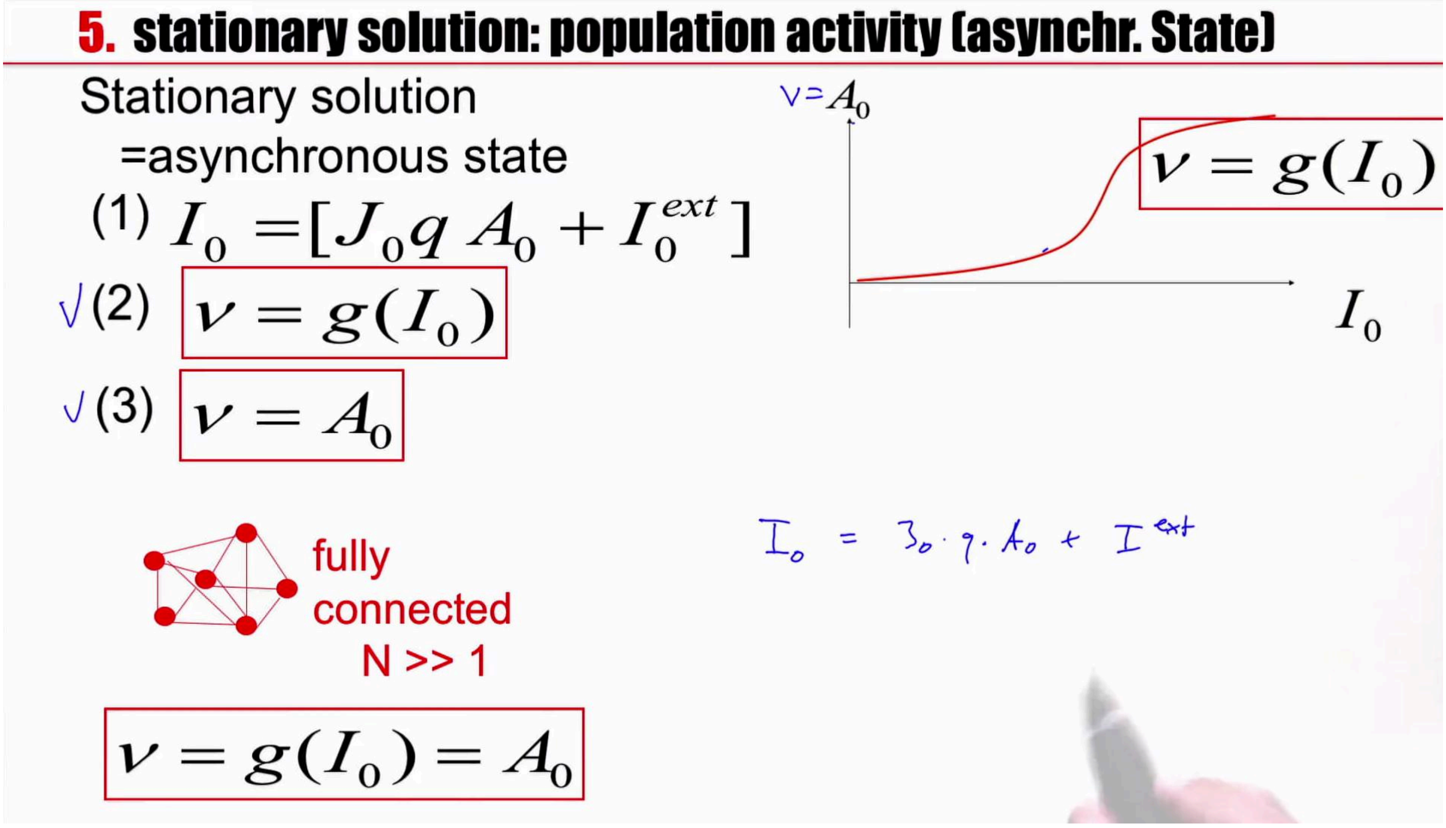


Well the reason is, if all neurons are the same. And if you're in a stationary state, then average for single neuron or spatial averaging is the same. So whether I have one single neuron and I look for one time and average over time, or I have one single neuron look for a shorter time and i do many, many repetitions of the same experiment, or besides n neurons and then look at a short time, I always get the same result. Which means that this fine rate for single neuron, which is the mew the single unifying rate is the same as the spatially average trying rate. A 0, which is the population rate, single year and temporal average many neurons spatial average population activity. And this holds in this stationary state. Now let's put things together.

Notes

Summary

6m 21s

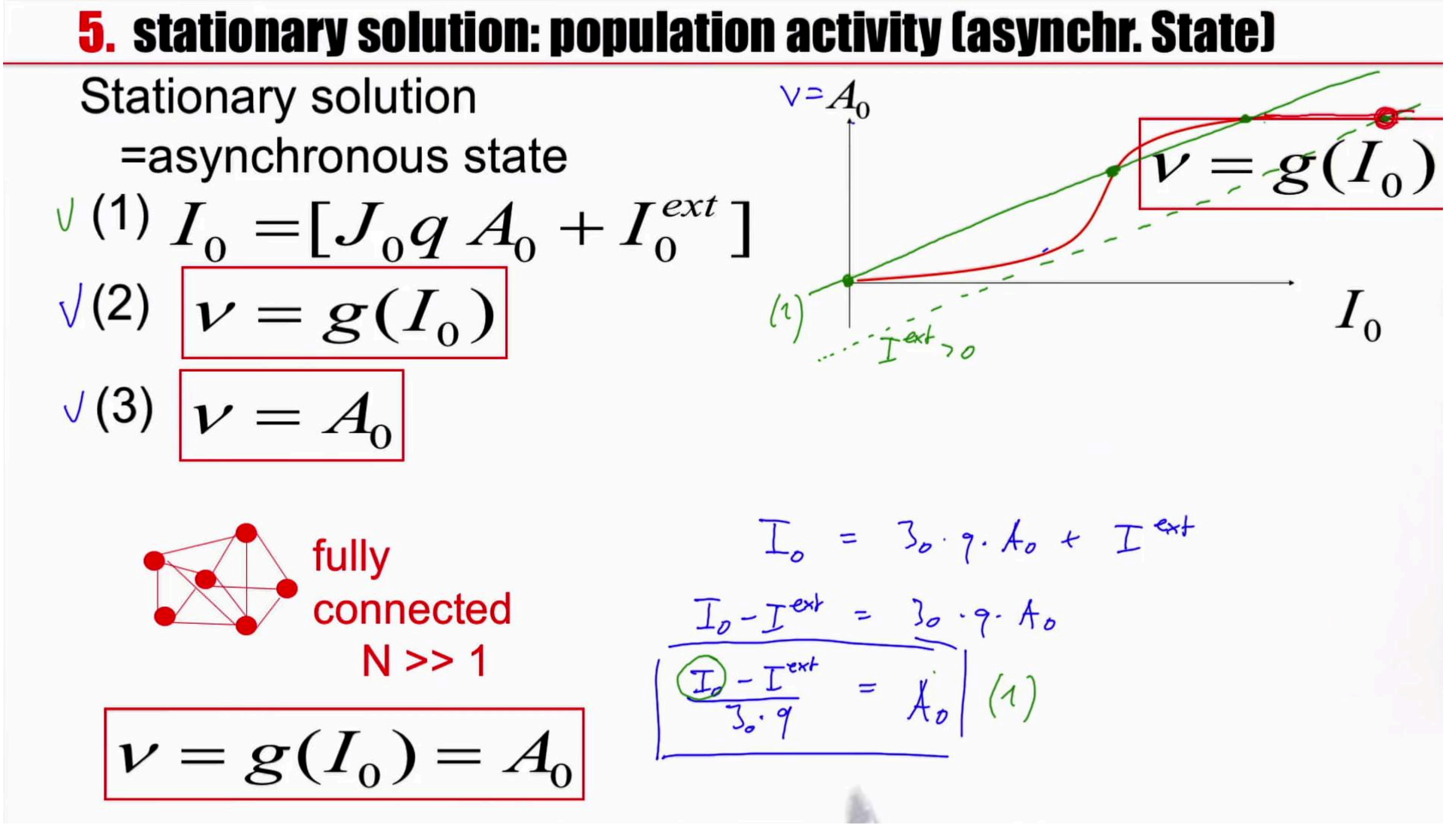


So we have a first result that says, in a stationary state, the input can be calculated from the stationary population activity, connectivity parameters, and some constant external input. Then we have a second result that says, The fi curve or gain function, tells me how much helps me define rate for a single neuron for given constant input, then a third result that says the single neuron right is equal to put to the population rate. Again this third result is just based on the idea that temporal averaging should give the same as spatial averaging. If you're on a stationary state. And if the network is homogeneous, which means all neurons are the same. So let's put all three equations together. The single neuron firing rate might be for example this curve here, could be a slightly different curve, and depends on the neuron type and the neuron model. This is my equation two. I have taken this into account. Then I've my equation three, which tells me that the vertical axis, which normally is the single neuron firing rate, can also be interpreted as the population rate. And what's still missing is equation one. So let's work on integration one. I have, I 0, just copied J0 or Q A0 plus I. external.

Notes

Summary

7m 29s

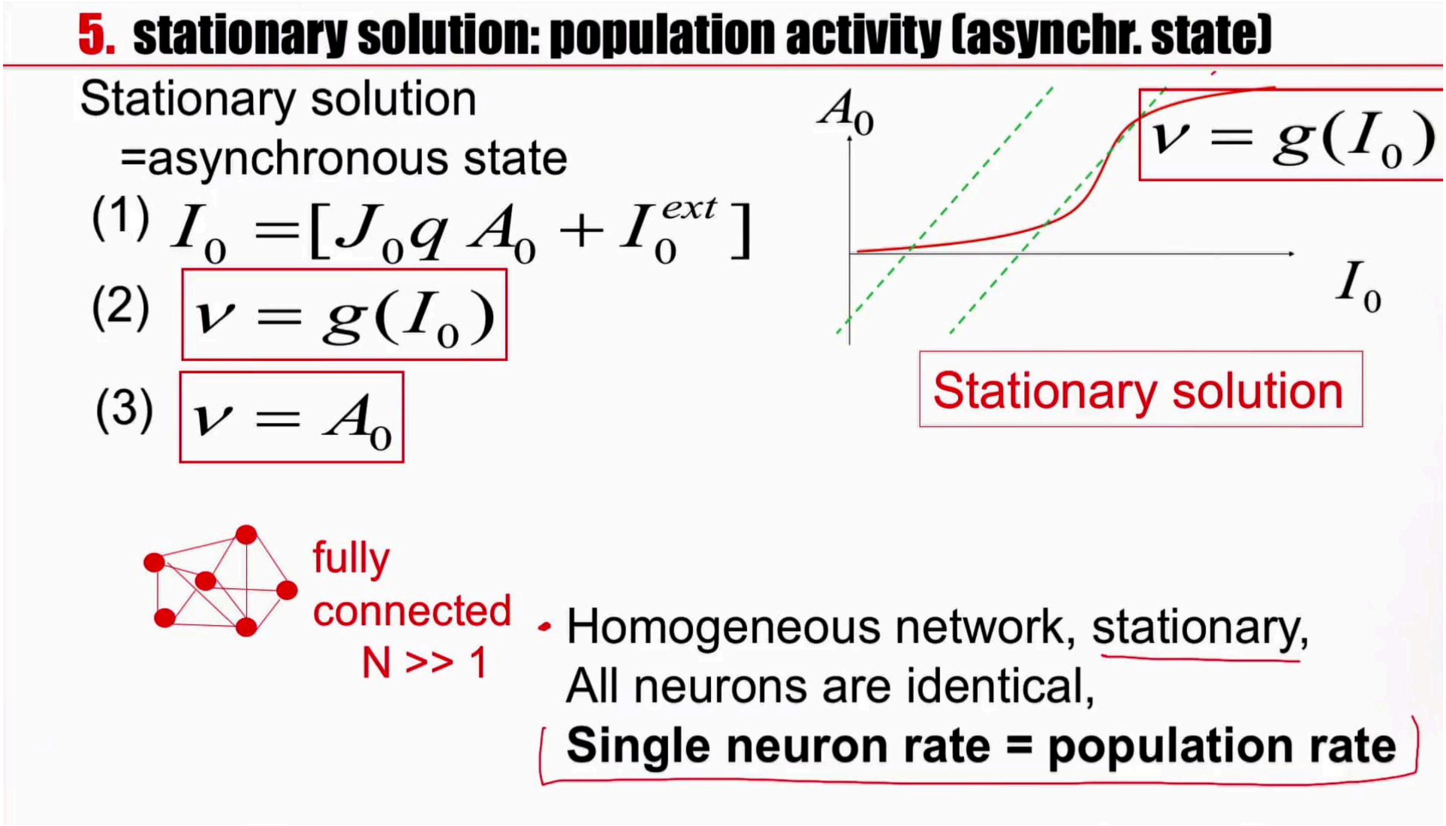


I would like to plot it in the same plot. So I would like to pull out the A0 to have right I 0 minus I external like move this to the other side is J zero QA zero or a zero is zero. I external divided by J0Q. Now here I have A zero as a function of I zero, it's linear and I 0. And if, for the moment I have no external current, then it goes through the origin. So what do you find is, there's a line and A 0.is a linear function of I 0 it's the green function. This is the result of equation one. Now at the same time, because equation two and three, A 0 is also the red curve as a function of the total input. If all equations have to hold at the same time. Then, the solution is at the crossing points. These are potential solutions, or the network is in a stationary state of a synchronous firing. There is a lower activity state. There's a higher activity state. There's another one which turns out to be unstable. Now, what happens if we change the external current if you increase the external current, you get a negative offset. So this next external current, bit bigger, we would only get the high fixed point up here. If I have a negative external current, then the green curve would shift upward and only find crossing point down here.

Notes

Summary

9m 05s

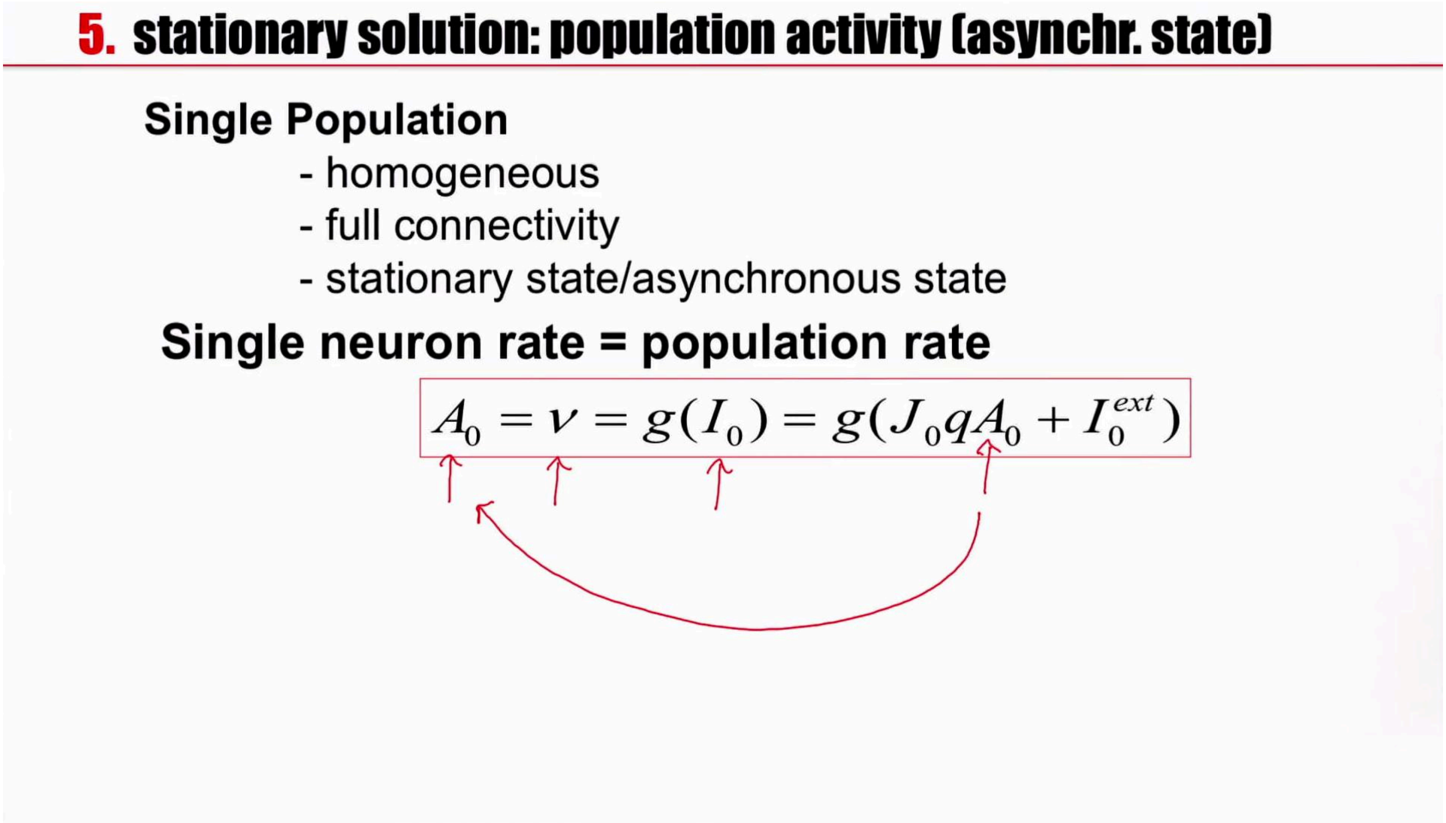


Now what happens if the coupling is weak two where's the coupling here. The coupling is the J 0 is the coupling constant. If I change J 0, make the coupling smaller, then the slope of this line gets steeper and I lose the other crossing points. So, for weak coupling in the network. Only a population activity at a very low level is possible, unless I have a strong external input, in which case, I can still find my solutions that have a much larger value. The message here is that in the stationary state we can graphically construct the solutions of the population activity, which means we can predict the population activity for the connectivity. The j zero, and you can predict the stationary population activity, if we know the connectivity J 0 are, and if we know the single neuron properties that are completely summarized by this Fi curve, G of is zero. Let me emphasize this. We have had a homogeneous network, which means all near it's all the same. Yeah, looking at the stationary solution. And these conditions allow us to exploit the fact that the single human rate is equal to the population rate, then I can go for graphical solution, or, alternatively, I can find the solution numerically.

Notes

Summary

10m 50s

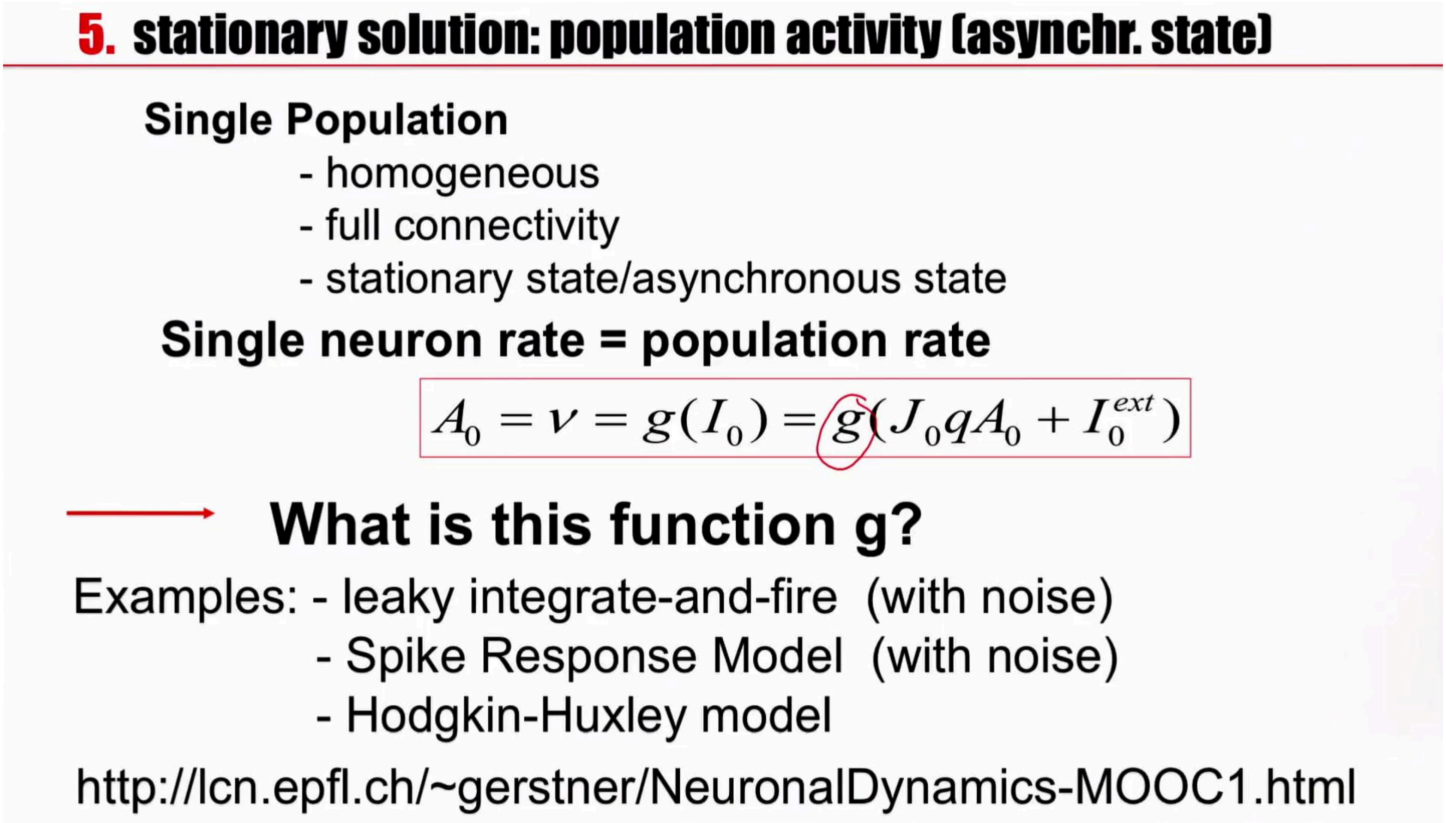


The activity is equal to the single unifying rate, which is a function of I 0 having this expression, but this expression contains the population activity. So I have a loop from the population activity, which gives rise to the population activity, what I've shown you is the graphical solution. But if you're on a computer, you can also numerically self for the stationary states of asynchronous synchronous firing, in this homogeneous population that's fully connected.

Notes

Summary

12m 33s



Now an important role plays this function G. But what is this function G. Well, you can calculate it. For example for leaky you can fire a neuron, without or without noise. You can calculate for spike response model another form of integrated fire model, you can calculate for the Hodgkin Huxley model, you can simulate. Most importantly, for real neurons experimental neurons. You can also measure this gain function. So of all the many different neuron properties, once you're in a stationary state. The only thing that matters is this fi curve, the frequency current curve. What matters is this function g.

Notes

Summary

13m 10s

5. gain function is noise-dependent

Gain-function *g* =frequency-current relation = f-I curve

function *g* can be calculated analytically or measured in **single-neuron simulations/single-neuron experiments**

$$\boxed{\nu = g_\sigma(I_0)}$$

different noise levels

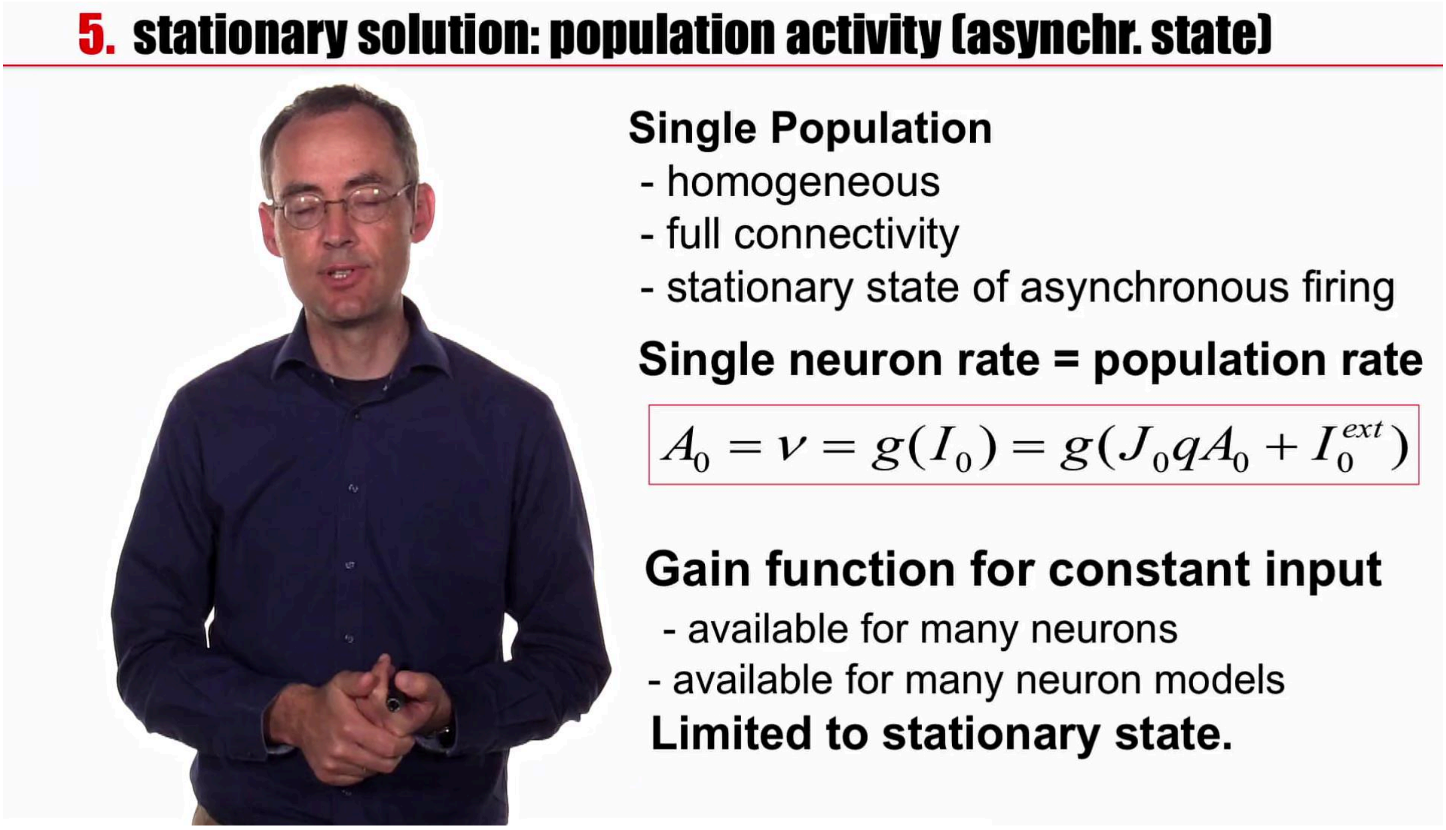
http://lcn.epfl.ch/~gerstner/NeuronalDynamics-MOOC1.html

So once you have the connectivity parameters of a neuronal population. You can predict the population activity based on single neuron properties and the relevant single human property is the fi curve, which itself may depend on the noise level that you have in your network. So let me summarize what we have seen so far. If you look at a single population. If assume that is homogeneous which means all neurons are the same, they have the same problem because they have the same type, we have assumed full connectivity.

Notes

Summary

13m 53s



And we looked at the stationary state for asynchronous firing. And then we exploited that the single mean rate is equal to population rate, and you could just write down the formula for the population activity, and we could solve it graphically. Now, the important thing that you need is the gain function for single neurons for constant input, and that's available, that's available for most neuron models, and importantly is also available for most experimental neurons. However, please note the whole approach that we have used here in this section is limited to stationary state constant activity. Now, in reality, the brain will never be in a stationary state, but it's nevertheless interesting. That is possible to predict these values of asynchronous firing in a stationary state, just for model parameters. The second limitation has been that we looked at a fully connected network. And this is the restriction that we will drop in the next section. So please stay on, and let's continue with section six.

Notes

Summary

14m 27s