

Computational Neuroscience: Neuronal Dynamics of Cognition



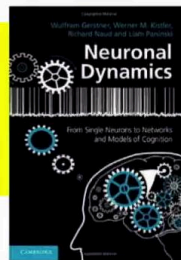
A: ASSOCIATIVE MEMORY

Wulfram Gerstner

EPFL, Lausanne, Switzerland

Reading for this week:
NEURONAL DYNAMICS
- Ch. 17.1 - 17.2.4

Cambridge Univ. Press



1 Introduction

- networks of neuron
- systems for computing
- associative memory

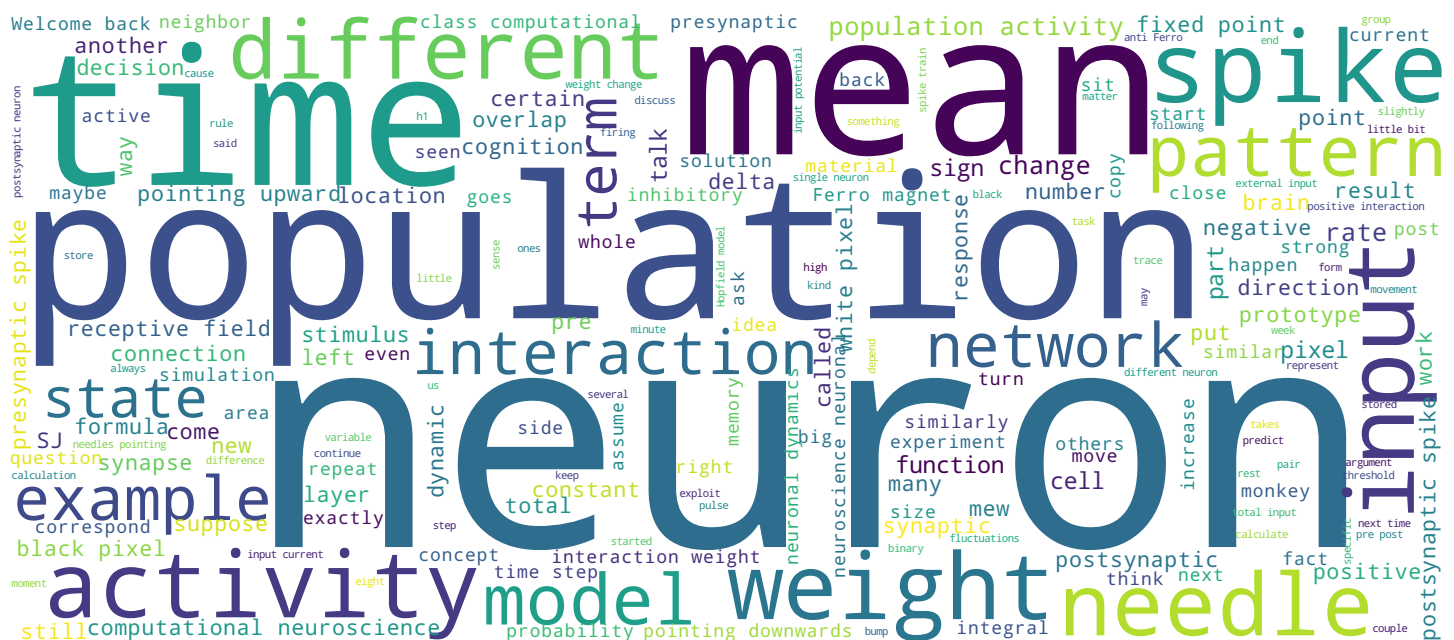
2 Classification by similarity

3 Detour: Magnetic Materials

4 Hopfield Model

5 Learning of Associations

6 Storage Capacity



Search MOOC

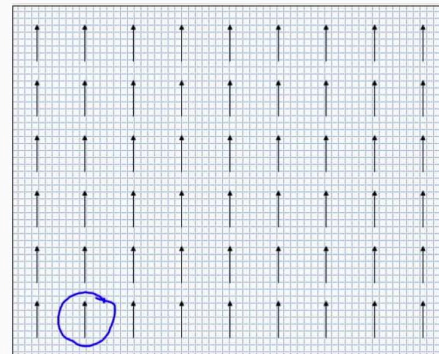
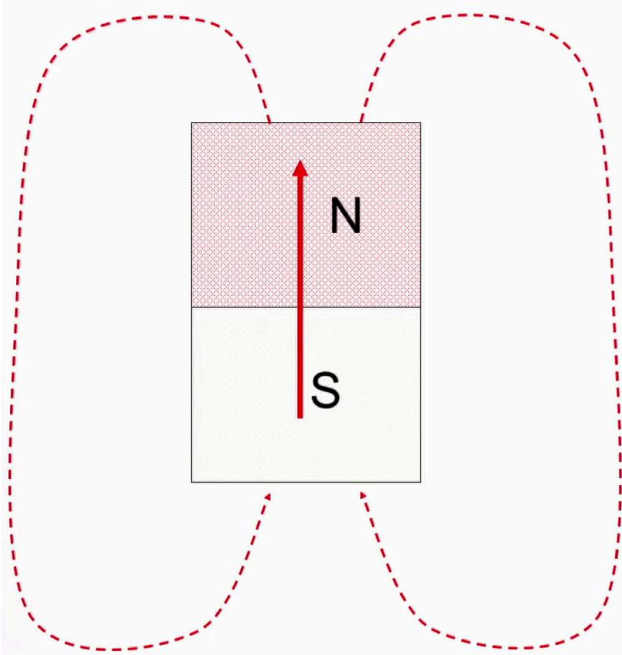


Video



EPFL

3. Detour: magnetism



Welcome back to the class, computational neuroscience neuronal dynamics of cognition. Before we continue with associative memories in a network of neurons, I would like to make a short detour and discuss magnetic materials. You may remember that a big magnet, in fact, consists of many small magnets in the sense that there are little needles inside the material. And in a very strong magnet, all these needles will point in the same direction.

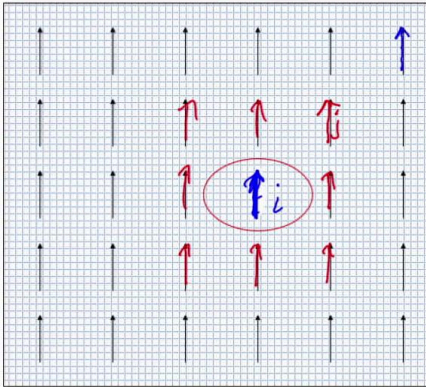
Notes

Summary



0m 16s

3. Detour: magnetism



Elementary magnet

$$\uparrow S_i = +1$$

$$\downarrow S_i = -1$$

dynamics

$$\underline{S_i(t+1)} = \text{sgn} \left[\sum_j \underline{S_j(t)} \right]$$

Sum over all
interactions with i

$$\begin{aligned} i \quad S_i(t+1) &= \text{sgn} \left[\sum_j S_j(t) \right] \\ &= \text{sgn} [8 \cdot (+1)] \\ &= +1 = \uparrow \end{aligned}$$

Now, one or few of the needles can point in the wrong direction, but then it's just the interaction between these different needles, that reconstitutes a pure magnet. And this is what I would like to discuss with you now. So I will describe each little needle inside the magnet by a variable, which only takes two states, it's either positive, then this needle has value plus one, or it's not negative, then it has value minus one. And now there's a dynamics. So these dynamics say that that state at the next time step is given by the interactions with its neighbors. So the question now is? What's going this needle? Now, this needle at precision I have a value S_i at time t plus one, which is the sign-off, and I just copy this formula, some over j . And then I have two different S_j s. Now, what are this S_j s I have here, for example, and the needle S_j ? And this has a value of plus one, that's my S_j . But the other needles in the neighborhood also have this value of plus one. So it's just 8 times plus one. So I will have this sign of eight times my value, plus one. And this means as a result of plus one, and this means that the needle is pointing upwards, so the new state is indeed aligned with these others.

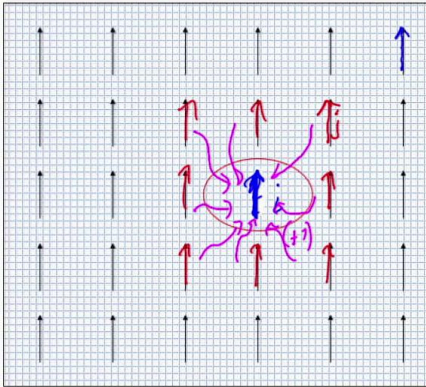
Notes

Summary



0m 31s

3. Detour: magnetism



Elementary magnet

$$\uparrow S_i = +1$$

$$\downarrow S_i = -1$$

$$i \quad S_i(t+1) = \text{sgn} \left[\sum_j w_{ij} S_j(t) \right]$$

$$= \text{sgn} [8 \cdot (+1)]$$

$$= +1 = \uparrow$$

dynamics

$$S_i(t+1) = \text{sgn} \left[\sum_j w_{ij} S_j(t) \right]$$

Sum over all interactions with i

Now I can discuss this in a slightly different fashion, I can say, well, what's what does this dynamics really mean? It means that the neighbors talk to this needle here. And this interaction, Vader's plus one, this interaction wait tells the needle here, while you should be aligned with your neighbor, this neighbor is pointing others; therefore, you should also point upward. So I can introduce here, interaction weights W_{IJ} , from your neuron J to neuron I . And these W , I , JS , would all have a value plus one. And now with this in mind, and we can turn to a slightly more complicated configuration.

Notes

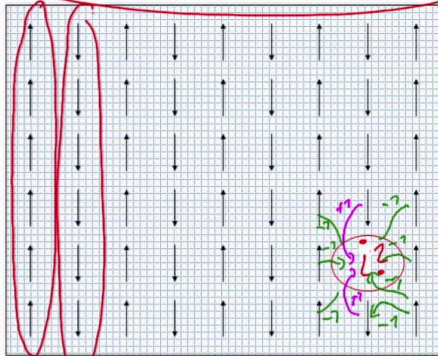
Summary



2m 18s

3. Detour: magnetism

Anti-ferromagnet



$$S_i(t+1) =$$

Elementary magnet

$$\uparrow S_i = +1$$

$$\downarrow S_i = -1$$

$$\uparrow \uparrow w_{ij} = +1$$

$$\uparrow \downarrow w_{ij} = -1$$

dynamics

$$S_i(t+1) = \text{sgn}\left[\sum_j w_{ij} S_j(t)\right]$$

Sum over all
interactions with i

In nature, you not only find Ferro-magnets, but you also find anti-Ferro magnets. These are usually composite materials that consist of two different types of molecules that sit in the material in layers. So this is molecule type A, this is molecule type B, and then comes another A another B now the needles in one layer point upwards, the needle in the next layer point downward. And this is a natural configuration for this anti-Ferro magnet. And now I would like to repeat the calculation from the previous slide in the ask this needle I sitting here, what is this going to do. And now I exploit the idea that neighbors talk to each other. So as before I can I have positive interaction ways, I can say this neighbor talks to this one, with weight plus one, this neighbor talks to this one with weight plus one. However, there are also other ways that means a needle pointing downwards talks to a needle pointing upward with the weight of minus one. And that means this needle also talks to this one, with a weight of minus one, this one with a weight minus one, this one is a weight of minus one. And also similarly from the other side and I can calculate the state of my needle at a position I at time t plus one.

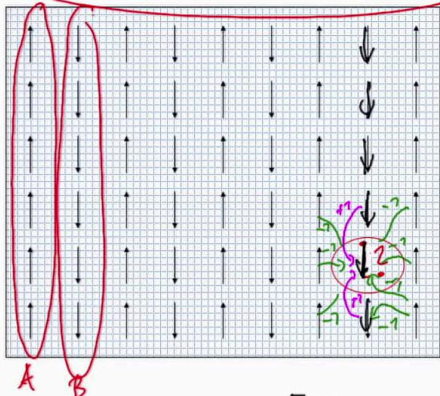
Notes

Summary



3. Detour: magnetism

Anti-ferromagnet



Elementary magnet

$$\uparrow S_i = +1$$

$$\downarrow S_i = -1$$

$$\uparrow \uparrow w_{ij} = +1$$

$$\uparrow \downarrow w_{ij} = -1$$

dynamics

$$S_i(t+1) = \text{sgn}\left[\sum_j w_{ij} S_j(t)\right]$$

Sum over all
interactions with i

$$\begin{aligned} S_i(t+1) &= \text{sgn}\left[\sum_j w_{ij} S_j(t)\right] \\ &= \text{sgn}\left[6 \cdot (-1) \cdot (+1) + 2 \cdot (+1) \cdot (-1)\right] \\ &= \text{sgn}[-8] = -1 = \downarrow \end{aligned}$$

And it just takes this formula, it's the sign-off and I have, this big sum of all neighbors $[w_{ij} S_j(t)]$. This is just a copy of the formula here. And now let's look at these; I have a couple of neighbors that have a value plus one. Out of these ones and they come in with greenening the actions and the greenening actions are minus one. How many of these do I have 123456? And then I have two others, they have a value of minus one. And they come in with a weight that's positive. So in total, I have my sign off, minus six, minus two, minus eight, for the sign of minus eight is minus one. And this means that this needle is pointing downwards, and therefore it's aligned with all the other needles in the same layer. So here, its nature, which has chosen these interactions, we have negative interactions, and we have positive interactions. And because of these interactions, natural state of these materials is that we have layers with needles pointing upwards, followed by needles pointing downwards pointing upwards pointing downwards. So we get a rather interesting, organized structure of the needle positions just by the interactions. And this is the idea we're going to exploit in the following.

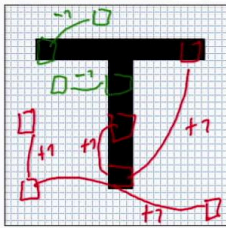
Notes

Summary



4m 40s

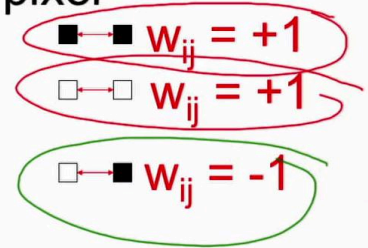
3. Magnetism and memory patterns



Elementary pixel

$$\blacksquare S_i = +1$$

$$\square S_i = -1$$



dynamics

$$S_i(t+1) = \text{sgn}\left[\sum_j w_{ij} S_j(t)\right]$$

Sum over all
interactions with i

Hopfield model:
Several patterns → next section

Now instead of needles pointing upwards and downwards, we'll talk about pixels. And the black pixel has a binary value plus one, a white pixel has a binary value minus one. Now, if I want to store a certain configuration of pixels, I will choose interactions such that I have positive interactions between pixels that should both be black, it means this pixel talks to that pixel with the interaction rate plus one, it also talks to this pixel Whitney action rate plus one. Similarly, the white pixels talk to each other with an interaction weight plus one. Now in contrast to the physical magnet, these interactions are supposed to represent axons, the neuronal cables that started one neuron into the other neuron, and therefore interactions can be long-range, a neuron here corresponds to a pixel representing the white pixel here can talk to another neuron that's quite far away. And the interaction is plus one if both neurons should be inactive. Now the final point is that neurons that represent black pixels, talk to neurons that represent white pixels with an interaction weight minus one. And similarly a white peaceful neuron talks to a black pixel neuron with the interaction by minus one.

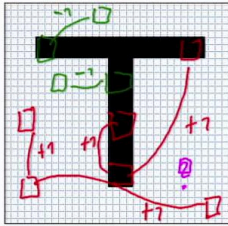
Notes

Summary



6m 22s

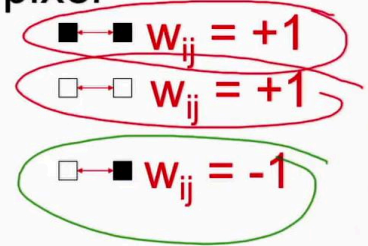
3. Magnetism and memory patterns



Elementary pixel

$$\blacksquare S_i = +1$$

$$\square S_i = -1$$



dynamics

$$S_i(t+1) = \text{sgn}\left[\sum_j w_{ij} S_j(t)\right]$$

Sum over all
interactions with i

Hopfield model:
Several patterns → next section

So now, you can repeat the argument that we did for the anti-Ferro magnet. And we can ask now, suppose I have stored this pattern. That's a neuron that's representing the pixel I, this pixel is I. And for which I don't know whether it's going to be in this black state or the white state, but they will have a value plus one or minus one. For such a neuron, can I predict the value it will take?

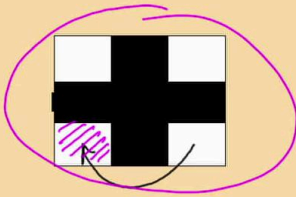
Notes

Summary



8m 01s

Exercise 1: Associative memory (1 pattern)



Elementary pixel

■ $S_i = +1$

□ $S_i = -1$

■ \longleftrightarrow ■ $w_{ij} = +1$

□ \longleftrightarrow □ $w_{ij} = +1$

9 neurons, connected all-to-all

- define appropriate weights:
what is the weight

$w_{79} = ?$

- what happens if neuron 7 is +1?
- what happens if 3 neurons wrong?

dynamics

$$S_i(t+1) = \text{sgn}\left[\sum_j w_{ij} S_j(t)\right]$$

Sum over all
interactions with i

And this is what I asked you to do. In the first exercise, we represent a simple pattern corresponding to nine pixels. Some of them are white; five of the nine are black. And I would like you to construct the interaction weights. What happens if the neuron at pixel number seven currently has the state plus one, which is the wrong state? Would that be corrected based on these interactions? Please take a couple of minutes to walk yourself through this exercise. This will help you to understand the rest of today's lecture.

Notes

Summary



8m 34s