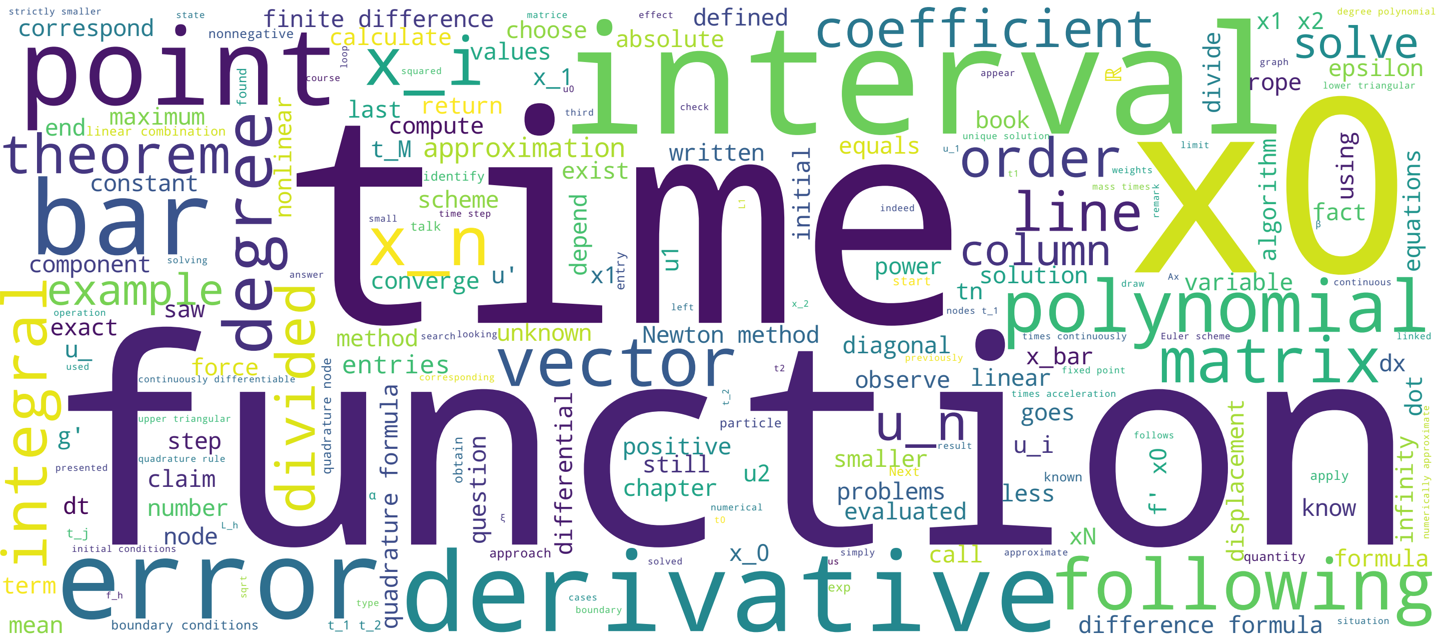
Chapitre 10 : Problèmes aux limites unidimensionnels (suite)

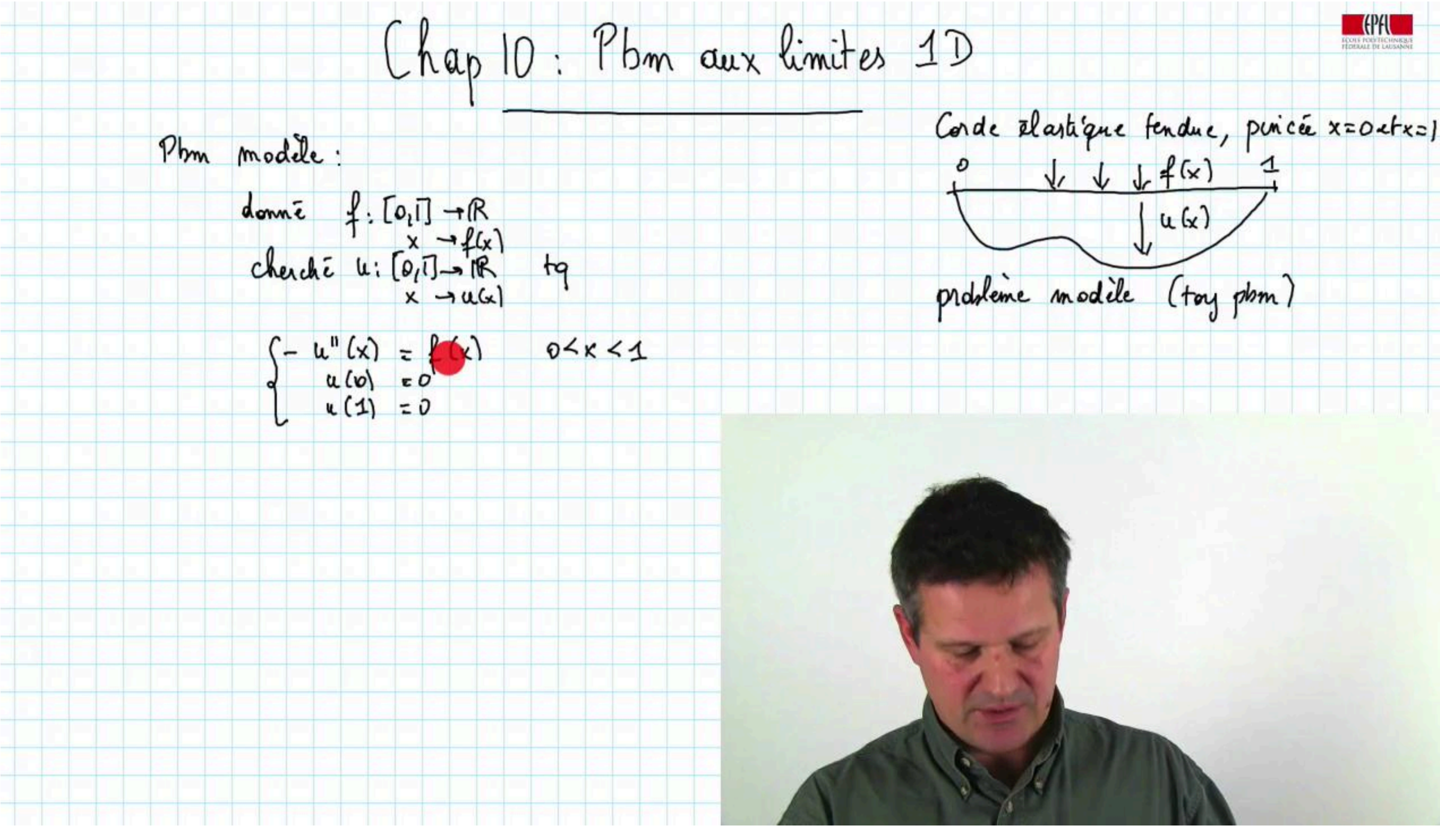
Introduction à l'analyse numérique

Prof. Marco Picasso



Video

EPFL



The problem I want to solve is the following. Toy problem: for a given function f, defined on the interval [0,1], with values in R, which for a value x returns f(x), search for a function u, the unknown of the problem, defined on the interval [0,1] in R, which for x returns u(x), this function u must satisfy the following differential equation: minus the second derivative of u, here u second, is the second derivative with respect to the only variable present which is x, so minus u '' (x) = f(x), x is in the (0,1), with conditions: u in x = 0 equals 0, and u in x = 1 also equal to 0. The problem corresponds to the following physical situation: we consider an elastic cord, stretched and clamped in the ends. The ends are x = 0 and x = 1. I can draw the following figure, here is the interval [0,1], by pushing on this elastic rope with a force f(x) in point x, under the effect of this force, the rope will deform, I call u(x) the displacement, as I am holding the rope in the ends, which are x = 0 and x = 1. the displacement is zero in x = 0 and x = 1. This problem is a toy problem as is it a simplification of reality, a model problem. We must remark that this problem can be solved by hand, by integrating twice.

Notes

Summary

0m 03s

Chap 10 : Pbm aux limites 1D

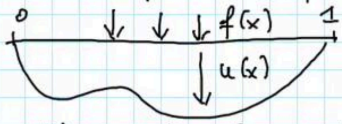
Pbm modèle :

donné $f : [0,1] \to \mathbb{R}$, $x \to f(x)$

cherché $u : [0,1] \to \mathbb{R}$, $x \to u(x)$ tq

$$\begin{cases} -u''(x) = f(x) & 0<x<1 \\ u(0) = 0 \\ u(1) = 0 \end{cases}$$

Corde élastique tendue, pincée x=0 et x=1



problème modèle (toy pbm)

$$-u''(x) + c(x)\,u(x) = f(x) \qquad c \geq 0$$

$$-\frac{d}{dx}\left(c(x)\,\frac{du}{dx}(x)\right) = f(x) \qquad c(x) > 0$$

Attention ne pas confondre avec pbm à valeur initiale chap 9 :

I have two constants which appear during this integration. These two constants are found by using the boundary conditions. Nevertheless, this problem is interesting enough from the numerical point of view, and a numerical method will be presented. More interesting problems are some of the following: for example, I consider - u "(x) plus c(x) times, which is a function of one variable x, times u(x) which is equal to f(x) given. This problem is linked to the deformation of beams. If c(x) is positive for all x, then the problem is well posed and admits a unique solution. Another boundary value problem with second derivatives is the following: d/dx of c(x) times du/dx equal to f(x). This time, I impose that c(x) is strictly positive in all points, and this problem could correspond to a situation where the rope has different properties depending on x, for x in [0,1]. We call these problems boundary value problems since there are conditions on the boundaries. This problem must not be mistaken with initial value problems which we saw in chapter 9. Beware ! Do not get confused with initial value problems. The initial value problem we saw in chapter 9 was the following problem: it was mass times acceleration equal to force.

Notes

Summary

2m 14s

Chap 10 : Pbm aux limites 1D

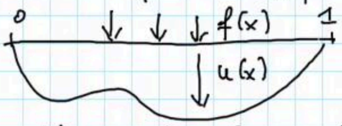
Pbm modèle :

donné $f : [0,1] \to \mathbb{R}$, $x \to f(x)$

cherché $u : [0,1] \to \mathbb{R}$, $x \to u(x)$ tq

$$\begin{cases} -u''(x) = f(x) & 0 < x < 1 \\ u(0) = 0 \\ u(1) = 0 \end{cases}$$

Corde élastique tendue, pincée $x=0$ et $x=1$



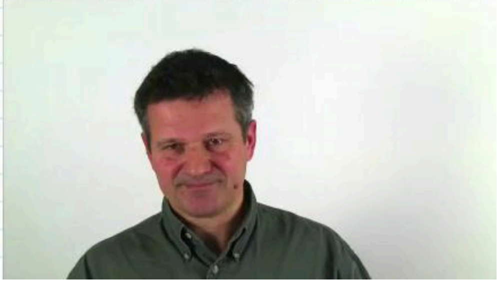
problème modèle (toy pbm)

$$-u''(x) + c(x)\,u(x) = f(x) \qquad c \geq 0$$

$$-\frac{d}{dx}\left(c(x)\frac{du}{dx}(x)\right) = f(x) \qquad c(x) > 0$$

Attention ne pas confondre avec pbm à valeur initiale chap 9 :

$$\begin{cases} \ddot{u}(t) = f(u(t), \dot{u}(t), t) & t > 0 \\ u(0) \\ \dot{u}(0) \end{cases}$$



A problem of type u dot-dot equal to the forces which I apply to a particle, these forces can depend on u(t), maybe u dot. (where u dot is the derivative with respect to time) for all t positive. So mass times acceleration is equal to the external forces applied on the particle with two initial conditions: u(0) is given and u dot in 0 is also given. You see here that there is also a second derivative, which is written "dot" in the physics literature. Here the derivative is written " ' ". Therefore, in both cases there are second derivatives. But in this chapter we talk about boundary value problems where we have specified two boundary conditions which correspond to the fact that the rope is held at both ends. Here, there are two initial conditions which correspond to the fact that, when a free the object, I must specify the position and velocity.

Notes

Summary

4m 15s