Chapitre 9 : Schémas d'ordre supérieur

Introduction à l'analyse numérique

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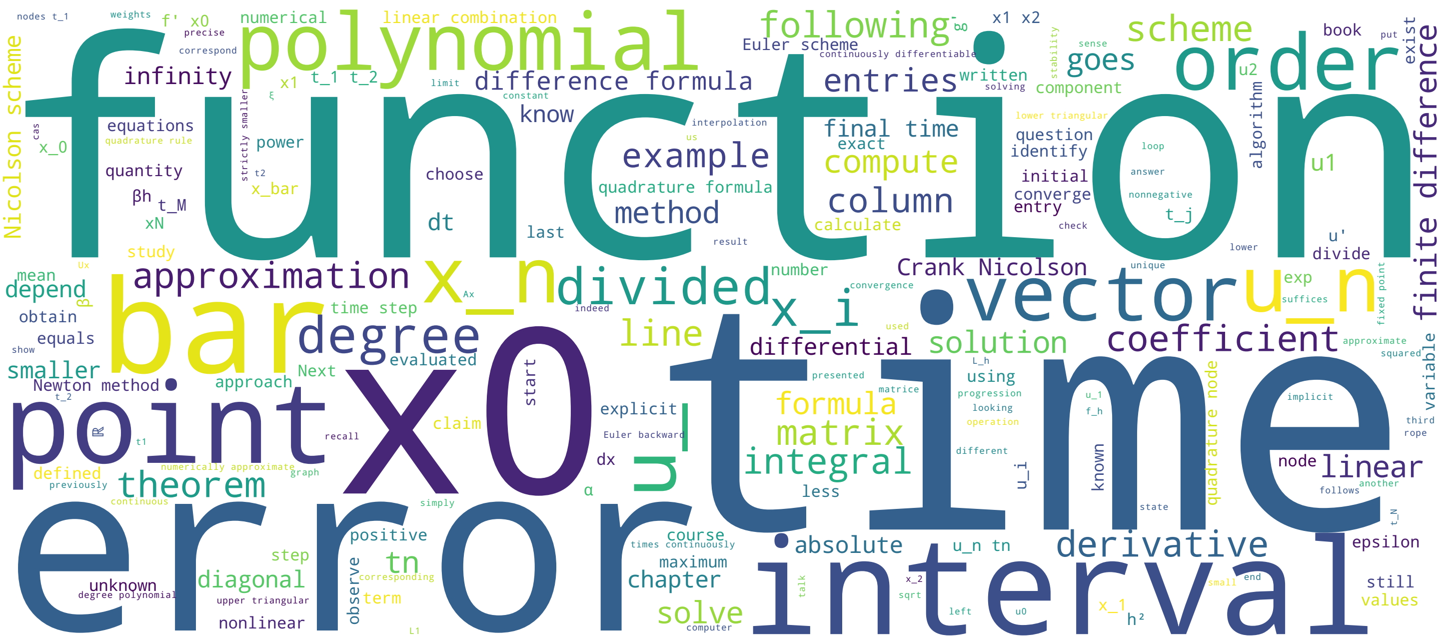

Video

Chap 9 - Schémas d'ordre supérieur

Sch Euler progr : $\frac{u^{n+1}-u^n}{h} = f(u^n, t_n)$

— rétr $\frac{u^{n+1}-u^n}{h} = f(u^{n+1}, t_{n+1})$

$$\frac{u^{n+1}-u^n}{h} = \frac{1}{2}\left[f(u^n,t_n) + f(u^{n+1},t_{n+1})\right] \quad \text{Crank-Nicolson}$$

ordre 2 en h : $|u(t_N) - u^N| = O(h^2)$ Si h div. par 2 et N mult. par 2, l'erreur est divisée par 4.

Stabilité : $\frac{u^{n+1}-u^n}{h} = \frac{1}{2}\left(-\beta u^n - \beta u^{n+1}\right) \qquad u^{n+1} = u^n \dfrac{1-\frac{\beta h}{2}}{1+\frac{\beta h}{2}}$

There are many schemes more precise than Euler's schemes. Schemes of order 2,3,4 and so on, there are books dedicated to the study of such schemes, I will only give one here, the Crank-NIcolson's scheme. In Euler's schemes we have the term (u_(n+1)-u_n)/h -so the approximation of the derivative equals f(u_n,tn). or qui s'écrit *(u^(n+1) - u^n)/h* f(u_(n+1),t(n+1)), and so if we average both schemes we get (u_(n+1)-u_n)/h =1/2*[f(u_n,tn) +f(u_(n+1),t(n+1))]. This is nothing but Crank-NIcolson's scheme. This scheme is of order 2 in h, in the sense that si on refait l'expérience précédente, if we approach u at some final time T=t_N, we note u_N this approximation, then the approximation at the final time is in O(h²), the error at final time is in O(h²) which means that if h is divided by 2 and the number of time step is multiplicated by 2, the error at the same final time, the error u(t_N)-u_N is divided this time by 4. Let's study the stability of this scheme, still on the differential equation u'=-βu where β is nonnegative. Crank-Nicolson's scheme is written in this case : (u_(n+1) - u_n)/h =1/2(-βu_n -βu_(n+1)). So I can write the scheme as following: I have u_(n+1)=u_n times 1-βh/2 divided by 1+ βh/2.

Notes

Summary

0m 03s

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$$\frac{u^{n+1}-u^n}{h} = \frac{1}{2}\left(f(u^n,t_n) + f(u^{n+1},t_{n+1})\right) \quad \text{Crank-Nicolson}$$

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Stabilité : $\frac{u^{n+1}-u^n}{h} = \frac{1}{2}\left(-\beta u^n - \beta u^{n+1}\right)$ $\quad u^{n+1} = u^n \frac{1-\frac{\beta h}{2}}{1+\frac{\beta h}{2}}$ $\quad u^{n+1} = u^0 \left(\frac{1-\frac{\beta h}{2}}{1+\frac{\beta h}{2}}\right)^{n+1}$

$\underset{n\to\infty}{u^{n+1} \to 0} \iff \left|\frac{1-\frac{\beta h}{2}}{1+\frac{\beta h}{2}}\right| < 1 \Rightarrow$ toujours le cas $\forall h > 0$

le sch. de CN est inconditionnellement stable (idem ER)

Autres schémas : BDF 2, Runge !

So by induction, u_(n+1) is equal to the initial value, u_0 times the progression, here (1-βh/2)/(1 + βh/2)^(n+1) élevé à la puissance *(n+1)* and u_(n+1) tends to 0 when n goes to the infinity, if and only if the common ratio is strictly less than 1 in absolute value. (stricly less than 1). So it suffices to study the function (1-x)/(1+x) for positive x, when x=0 this function equals 1, when x goes to +infinity this function, (1-x)/(1+x) is -1, and so for all h ceci est toujours le cas, sous-entendu pour tout *h* positif, this quantity is strictly lesser than 1. Je vous rappelle ici que dans l'étude de la stabilité, *β* est strictement positif. So Crank Nicolson's scheme is, as Euler's backward method, unconditionally stable. comme le schéma d'Euler, rétrograde, So stable without any condition on h. There are obviously other schemes, for example the BDF2 scheme, Runge Kutta schemes, that can be implicit or explicit, of order 1,2,3 and so on. What I have presented in this course is just a tiny part of the available litterature.

Notes

Summary

2m 41s