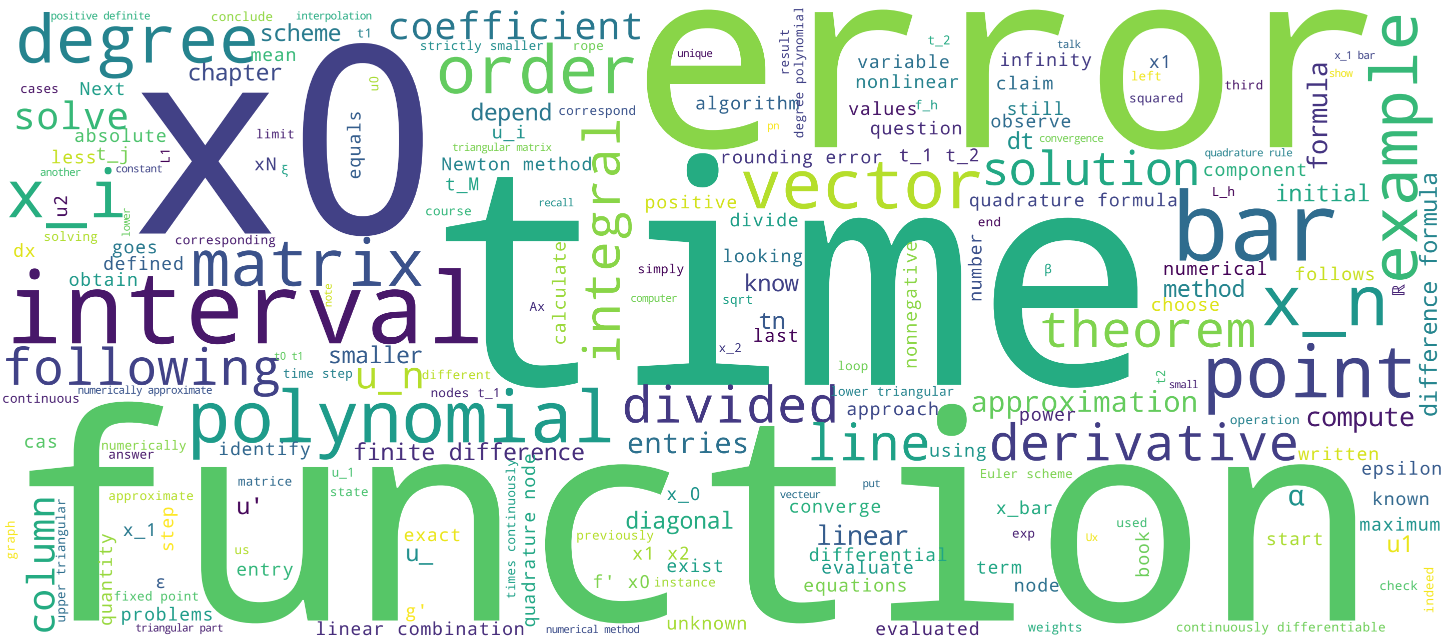
Chapitre 9 : Problèmes numériquement mal posés

Introduction à l'analyse numérique

Prof. Marco Picasso



Video

EPFL

Chap 9- Problèmes numériquement mal posés

Ex 9.1: $\begin{cases} \dot{u}(t) = 3u(t) - 3t \\ u(0) = \alpha \end{cases}$ $\quad u(t) = (\alpha - 1/3)\, e^{3t} + t + \frac{1}{3}$

$$\left.\begin{aligned} \alpha &= \tfrac{1}{3} & u(10) &= 10 + \tfrac{1}{3} \\ \alpha &= \tfrac{1}{3} + \varepsilon & u(10) &= \varepsilon e^{30} + 10 + \tfrac{1}{3} \end{aligned}\right) \qquad \varepsilon \approx 10^{-6} \quad \varepsilon e^{30} \sim 10^{7}$$

Une erreur de 10^{-6} sur la cond. initiale induit une erreur de 10^{7} après 10 s!

Before presenting some of the numerical methods, we shall exclude some cases. Consider example 9.1 of the book. We are looking for u such that u'(t)=3u(t)-3t with u(0)=α, a given real number. The solution to this problem is exp(3t) times α-1/3 + t+1/3. et puis α moins 1/3 qui est la bonne constante. So, take for example the case where α=1/3, in this case, let's evaluate the solution at t=10, u(10)=10+1/3. Consider a small error (for instance a rounding error) on the initial value α=1/3+ε. In that case, at the time 10, it is α-1/3, that is, ε^30+10+1/3. plus 10 plus 1/3. If ε=10^-6 for example, then ε^30 is of order 10^7, Donc voyez que dans ce cas-là, and the difference between the 2 solutions is of order 10^7. So we conclude that an error of 10^-6 on the initial value here, induces an error of 10^7 after 10 second. Donc, vous voyez bien que dans ce cas-là, It would be then completely useless to solve the problem numerically, In that case, we shall say that the problem is numerically ill-posed. In what follows, we will not consider such problems.

Notes

Summary

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