Chapitre 8 : Méthode de Newton

Introduction à l'analyse numérique

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Video

EPFL

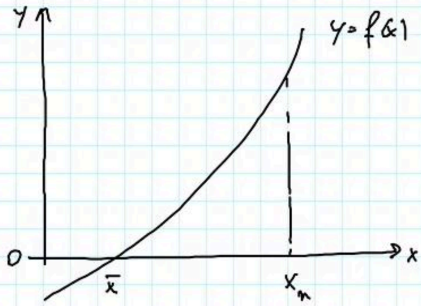
Chap 8 - Méthode de Newton

$\bar{x}$ tq $\bar{x}=g(\bar{x})$ $\quad x_0 \quad x_{n+1}=g(x_n)$ si $|g'(\bar{x})|<1$ et si x_0 suffisamment proche de $\bar{x}$ alors la suite converge

Question : comment s'affranchir de la cond. $|g'(\bar{x})|<1$? Méthode de Newton

$\bar{x}$ tq $f(\bar{x})=0$

x_n connu, x_{n+1} ?



Up to now, we have considered a method to find the fixed point of a function g. Let x bar such that x bar equals g(x bar), Starting from x_0, we have computed the sequence x_n from the scheme x_n+1=g(x_n). We have seen that if g'(x) in absolute value is strictly smaller than 1 and if x_0 is in a neighborhood of x bar, that means sufficiently close to x bar, -nous avons vu la définition mathématique précise de ce 'suffisamment proche'- then the sequence converges. However, I don't know g'(x_bar) since I don't know x_bar and what "sufficiently close" means. The Newton method will not allow us to improve the "sufficiently close" condition but will allow us to remove the condition g'(x bar) smaller than 1. So the question is how to remove the condition g'(x bar) smaller than 1? The answer is: use Newton's method. Newton's method is the following: coming back to my initial problem, which was to find x bar such that f(x) equals 0. I come back to the initial problem which was to find the zero of a function f, here is the graph of the function f, y equal to f(x), I seek x bar such that f(x bar) is equal to 0. Assume x_n is available: I have calculated the approximation x_n of x bar and now I want to compute x_n+1.

Notes

Summary

0m 03s

Chap 8 - Méthode de Newton

$\bar{x}$ tq $\bar{x} = g(\bar{x})$ $\quad x_0 \quad x_{n+1} = g(x_n)$ si $|g'(\bar{x})| < 1$ et si x_0 suffisamment proche de $\bar{x}$ alors la suite converge

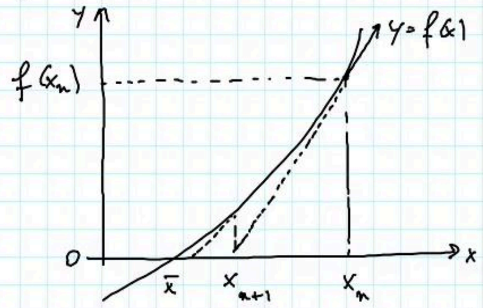
Question : comment s'affranchir de la cond. $|g'(\bar{x})| < 1$? Méthode de Newton

$\bar{x}$ tq $f(\bar{x}) = 0$

x_n connu, x_{n+1} ?

$$f'(x_n) = \frac{f(x_n) - 0}{x_n - x_{n+1}}$$

$$\boxed{x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}}$$



la méth de Newton est une méth de pt fixe

$$x_{n+1} = g(x_n) \quad \text{où} \quad g(x) = x - \frac{f(x)}{f'(x)} \qquad g'(x) = 1 - \frac{(f'(x))^2 - f(x)f''(x)}{(f'(x))^2}$$

The procedure is the following: I consider the derivative at point x_n and search for the intersection of this line with the x axis, this will be x_n+1. et ensuite je vais pouvoir calculer *x(n+2)* et ainsi de suite. What is the expression of x_n+1? Here is x_n, this is f(x_n) and the derivative f ' (x_n) is the increment in y divided by the increment in x. So the increment in y is f(x_n) minus 0, this distance here, and the increment in x, beware of the sign, is x_n minus x_n+1, thus x_n+1 is equal to (x_n minus f(x_n)) divided by f ' (x_n). à partir de*xn*. First remark: the Newton method is a fixed point method to find the zero of f. I can write this expression as x_n+1 equal to g(x_n), *x(n+1) = g(xn)* where the function g is defined for all x in R by g(x) equal to (x - f(x)) / f ' (x). If the function f is C2 and does not vanish then the denominator is not zero, I can compute g'(x). To check if this fixed point method converges I have to compute g' at x bar and check that is smaller than 1 in absolute value. I now compute g'(x) for any x, and I obtain (f ' (x))^2 in the denominator, for the numerator I get f ' (x) times the denominator that is (f ' (x))^2 minus the numerator f(x) times the derivative of the denominator f '' (x).

Notes

Summary

2m 04s

Chap 8 - Méthode de Newton

$\bar{x}$ tq $\bar{x} = g(\bar{x})$ x_0 $x_{n+1} = g(x_n)$ si $|g'(\bar{x})| < 1$ et si x_0 suffisamment proche de $\bar{x}$ alors la suite converge

Question : comment s'affranchir de la cond. $|g'(\bar{x})| < 1$? Méthode de Newton

$\bar{x}$ tq $f(\bar{x}) = 0$

x_n connu, x_{n+1} ?

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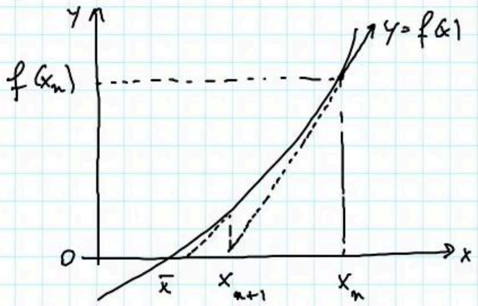

la méth de Newton est une méth de pt fixe

$x_{n+1} = g(x_n)$ où $g(x) = x - \frac{f(x)}{f'(x)}$ $\quad g'(x) = 1 - \frac{(f'(x))^2 - f(x) f''(x)}{(f'(x))^2}$

$g'(\bar{x}) = 1 - \frac{(f'(\bar{x}))^2}{(f'(\bar{x}))^2} = 0 < 1$ D'après le thm 8.3 : si x_0 suff. proche $\bar{x}$ alors la suite déf. par $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$ converge !

Now, I note that f(x bar) equals zero. Therefore when computing g' at x_bar this term vanishes since f(x bar) is null, it remains 1 - (f ' (x bar))^2 / (f ' (x bar))^2 that is 1-1 = 0. 0 is smaller than 1, so from theorem 8.3 I know that, if the starting point x_0 is sufficiently close to the x bar, the zero of f, then the sequence defined by x_n+1 equal to x_n - f(x_n) / f ' (x_n), this sequence converges to x bar. More precisely, I will state theorem 8.4 from the book.

Notes

Summary

4m 54s