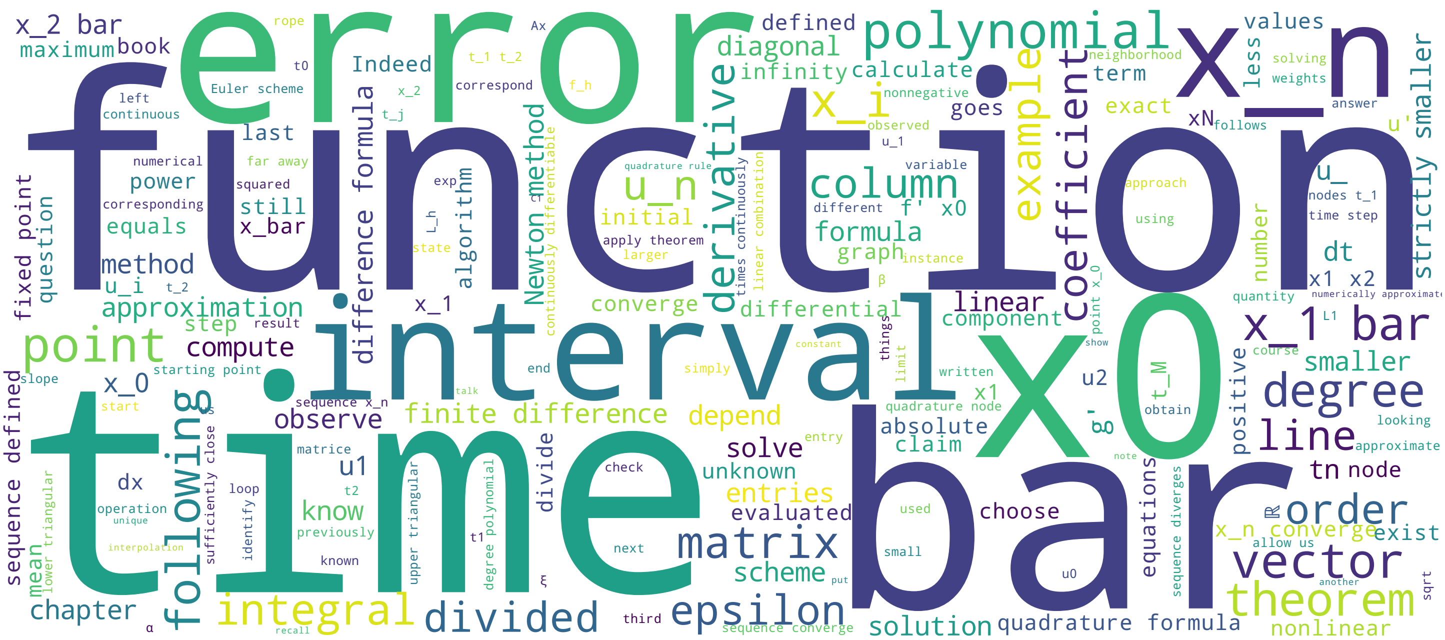
Chapitre 8 : Méthode de point fixe (exemple)

Introduction à l'analyse numérique

Prof. Marco Picasso



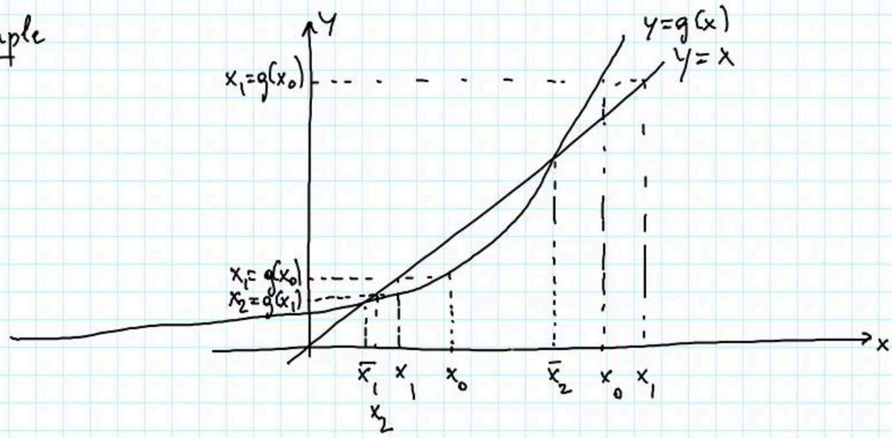
Video

EPFL

Chap 8 - Méthode de point fixe

$g:\mathbb{R}\to\mathbb{R}$ cont $\quad \bar{x} = g(\bar{x}) \quad x_0$ donné $\quad n=0,1,2,\dots \quad x_{n+1} = g(x_n)$ Question $(x_n)_n$ converge ?

Exemple



$\bar{x}_i = g(\bar{x}_i) \quad i=1,2$

Si $x_0 < \bar{x}_2$ on obs.

$\lim_{n\to\infty} x_n = \bar{x}_1$

Si $x_0 > \bar{x}_2$ on obs.

$\lim_{n\to\infty} x_n = +\infty$

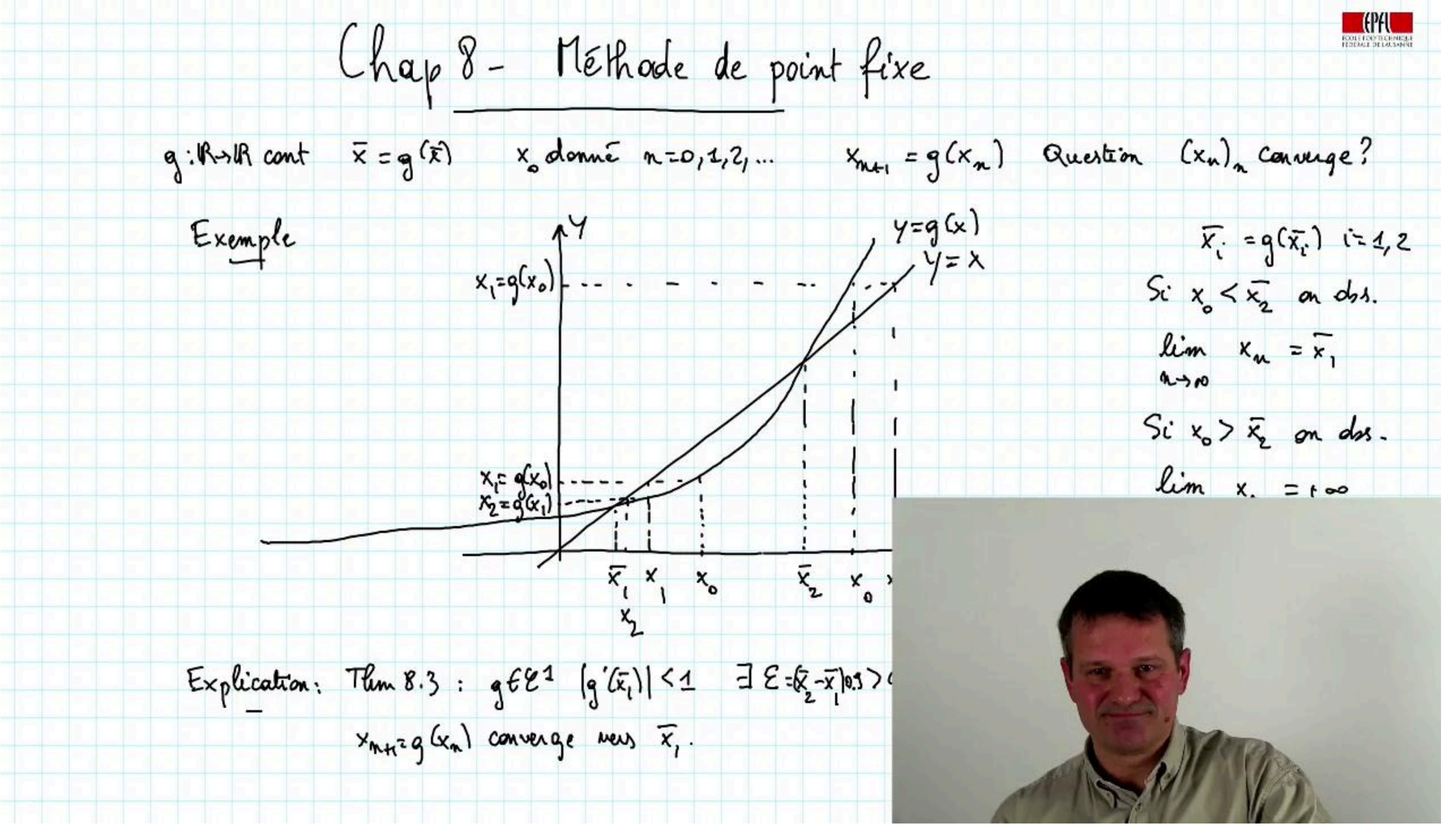
Explication: Thm 8.3 : $g \in \mathcal{C}^1 \quad |g'(\bar{x}_1)| < 1 \quad \exists\, \varepsilon = (\bar{x}_2 - \bar{x}_1)0.9 > 0 \quad \forall\ \bar{x} - \varepsilon \leq x_0 \leq \bar{x} + \varepsilon$ la suite déf. par $x_{n+1} = g(x_n)$ converge vers $\bar{x}_1$.

The example which was considered previously was the following. Here is the graph of the function g, the two fixed points x_1 bar and x_2 bar. We observed that if x_0 was chosen smaller than x_2 bar, then the sequence x_n+1 equal to g(x_n) converges to x_1 bar. If x_0 is larger than x_2 bar, the sequence defined by x_n+1 = g(x_n) diverges. Now we can apply theorem 8.3 to this function g. This function g is C1, I observe that g' at x_1 bar is strictly smaller than 1, since the slope of the first bisectrix is 1. la dérivée est strictement plus petite. Now I can apply theorem 8.3. According to theorem 8.3, there exists a positive epsilon, I will quantify it shortly, such that for all x_0 chosen in the interval x bar plus epsilon to x bar minus epsilon, well the sequence defined by x_n+1 equal to g(x_n) converges to x_1 bar. This is what I observed since I took x_0 smaller than x_2 bar, I can quantify epsilon, which must be smaller than the difference between x_2 bar and x_1 bar. I will take a fraction of this difference, say 0.9 times the difference for instance. Here is epsilon, and if I choose x_0 in this neighborhood, x bar minus epsilon to x bar plus epsilon, then the sequence converges to x_1 bar.

Notes

Summary

0m 03s



On the other hand, in the case when x_0 is too far away from x_1 bar, I cannot say anything. But I observe in this case that the sequence diverges. The theorem does not say that the sequence diverges when x_0 is far away from x_1 bar. It says only that if the starting point x_0 is sufficiently close to x_1 bar, then the sequence will converge. In the next part of this chapter, we will try to improve this method. ce résultat, parce que le point critique ici c'est, Indeed, there are two things I don't master in theorem 8.3. Is g'(x bar) strictly smaller than 1? What is epsilon? Eh bien, ces deux grandeurs ne me sont pas données de manière précise. We shall now see Newton's method which will allow us to remove the condition that g'(x bar) must be strictly smaller than 1.

Notes

Summary

2m 12s