Chapitre 8 : Méthode de point fixe

Introduction à l'analyse numérique
Prof. Marco Picasso

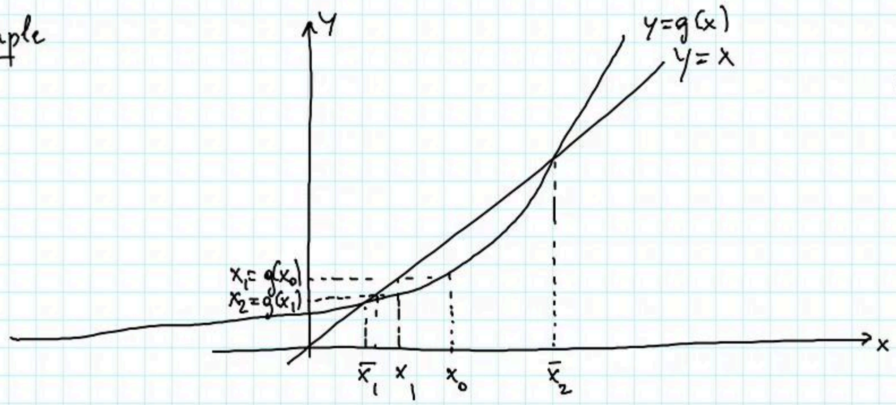
Video

EPFL

Chap 8 - Méthode de point fixe

$g : \mathbb{R} \to \mathbb{R}$ cont $\quad \bar{x} = g(\bar{x}) \quad x_0$ donné $n = 0, 1, 2, \dots \quad x_{n+1} = g(x_n)$ Question $(x_n)_n$ converge?

Exemple



$\bar{x}_i = g(\bar{x}_i) \quad i = 1, 2$

Si $x_0 < \bar{x}_2$

I remind you that g is a given, continuous, function, and that I search x_bar which is a fixed point of g, hence x_bar = g(x_bar). The method is the following: we give ourselves x_0 and then for n=0,1,2... we calculate x_n+1 from x_n simply by setting x_n+1 equal to g(x_n). The question is: does the sequence x_n converge? If it does converge, and since the function g is continuous, then it converges towards x_bar such that g(x_bar) is equal to 0. I suggest a little graphical illustration: I choose a function g which has the following aspect. So here is the x-y plane, the first bisectrix, and the function g, which has two fixed points. Here is the graph of the function g. There exist two x_bar such that x_bar = g(x_bar). The first is located here, I'll denote it x_1 bar, the second is found here, denoted x_2 bar. I therefore have x_i bar equal to g(x_i bar) for i=1,2. I start from a point x_0 which is smaller than x_2 bar. it is even between x_1 bar and x_2 bar, so if x_0 is strictly smaller than x_2 bar, what happens? I calculate x_1, here x_1 equal to g(x_0), I report x_1 on the x axis, I calculate afterwards x_2 equal to g(x_1), again I report the new value on the x axis.

Notes

Summary

0m 04s

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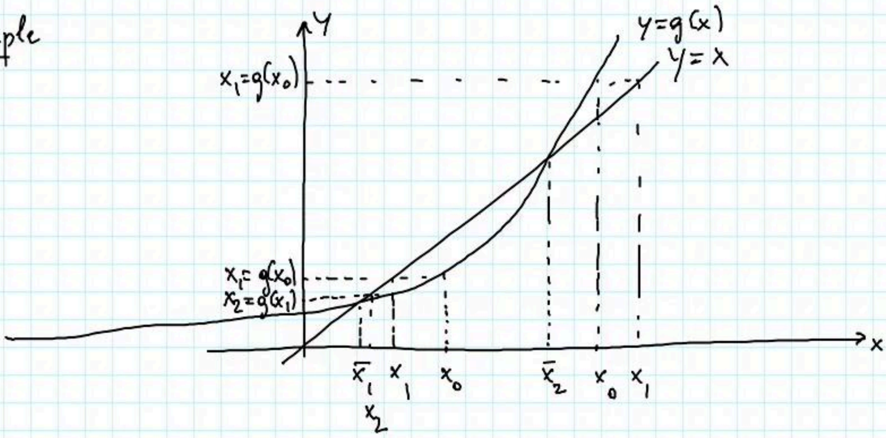
$g:\mathbb{R}\to\mathbb{R}$ cont $\quad \bar{x}=g(\bar{x}) \quad$ x_0 donné $\quad n=0,1,2,\dots \quad$ $x_{n+1}=g(x_n)$ $\quad$ Question $(x_n)_n$ converge ?

Exemple



$\bar{x}_i = g(\bar{x}_i) \quad i=1,2$

Si $x_0 < \bar{x}_2$ on obs.

$\lim_{n\to\infty} x_n = \bar{x}_1$

Si $x_0 > \bar{x}_2$ on obs.

$\lim_{n\to\infty} x_n = +\infty$

We observe that the sequence converges as n approaches infinity, towards x_1 bar. Therefore the limit as n approaches infinity of x_n equals x_1 bar. Starting from x_0, I computed x_1, x_2 and the x_n, when n increases, get closer and closer to x_1 bar. Now, what happens if I choose x_0 larger than x_2 bar ? If x_0 is beyond x_2 bar, what happens ? Well, for example, this x_0, I compute x_1 equal to g(x_0) and then carry it on the x axis, here it is, next I compute x_2 equal to g(x_1) and I observe that x_2 is beyond x_1 and so on. Well on this case, we observe that the limit as n goes to infinity of x_n equals plus infinity; the sequence diverges. How to explain this experiment? Well we need to study theorem 8.3 in the book.

Notes

Summary

2m 06s