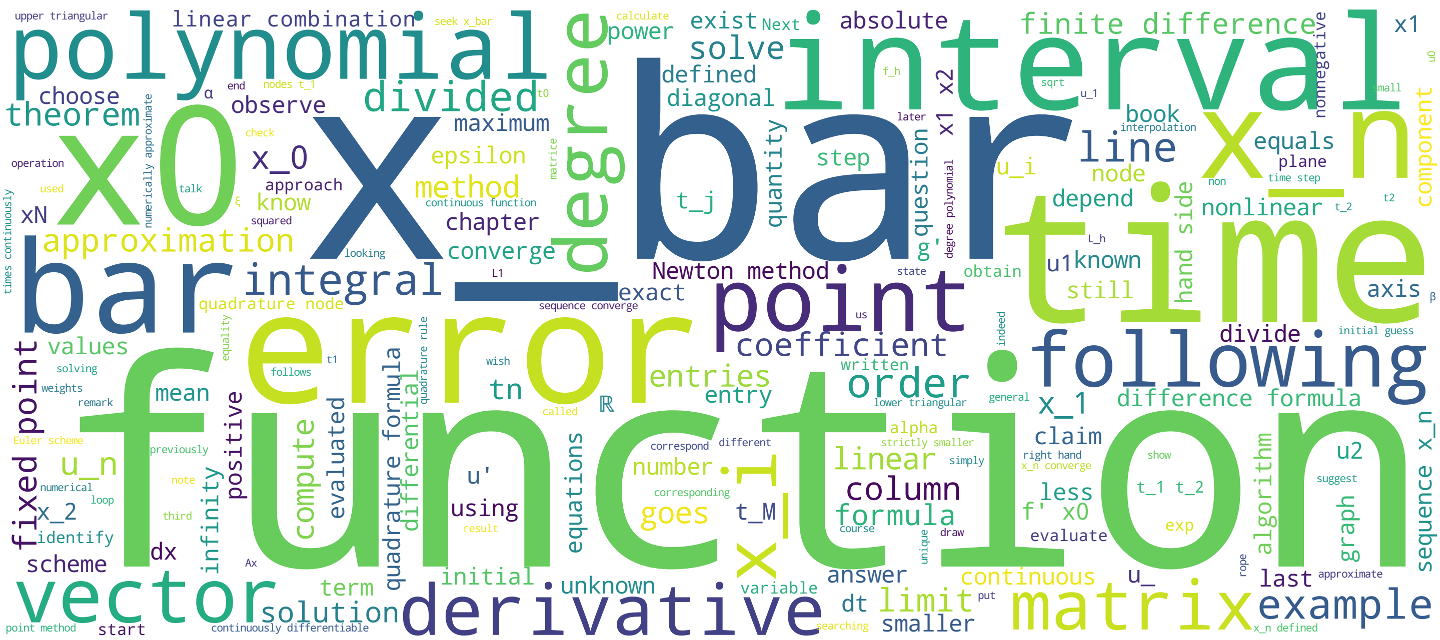
Chapitre 8 : Equations non linéaires – Position du problème

Introduction à l'analyse numérique

Prof. Marco Picasso



Video

Chap 8 - Equations non linéaires - Position du problème

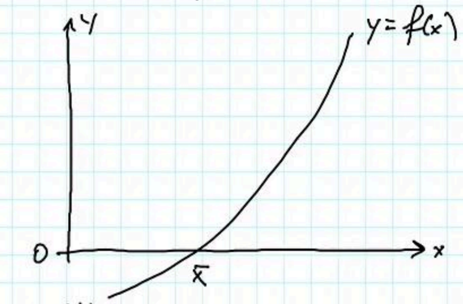
Donné $f: \mathbb{R} \to \mathbb{R}$ continue, cherche

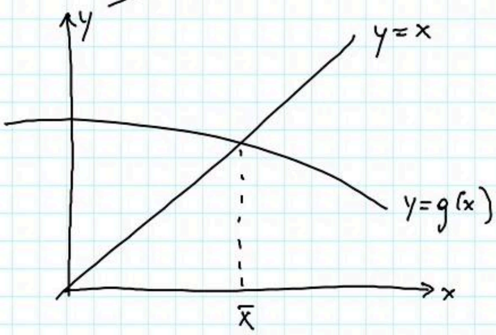
$$(\bar{x} \text{ tq } f(\bar{x}) = 0) \Leftrightarrow (\bar{x} \text{ tq } \bar{x} = g(\bar{x}))$$

$\bar{x}$ zéro de f $\qquad$ $\bar{x}$ point fixe de g

On pose $g(x) = x - f(x)$

ou $g(x) = x - \alpha f(x)$ $\quad \alpha \in \mathbb{R}$





Lets start by a nonlinear equation. The problem we wish to solve is the following: Let f be a continuous function defined from R to R, we are searching for x_bar such that f(x_bar) is equal to 0. Here is the graph of the function f in the x-y plane: y is equal to f(x). I search x_bar such that f(x_bar) is equal to 0. We say that x_bar is the zero of the function f, I will write the problem in another equivalent way: that is I seek x_bar, the same x_bar as this one, such that x_bar = g(x_bar). In this case we say that x_bar is a fixed point of the function g. I draw the graph of the function g in the x-y plane. You see here the first bisectrix, and here the graph of the function g, and I seek x_bar such that x_bar is equal to g(x_bar). How to get from the function f to the function g? Well, for example, we define g(x) equal to x minus f(x), so that when I evaluate this function in x_bar, I get g(x_bar) equal to x_bar minus f(x_bar) which is null. I could also define g(x) equal to x minus alpha times f(x) for any non-zero alpha. Well in this case, like previously, x_bar is a zero of f(x) if and only if x_bar is a fixed point of g.

Notes

Summary

0m 04s

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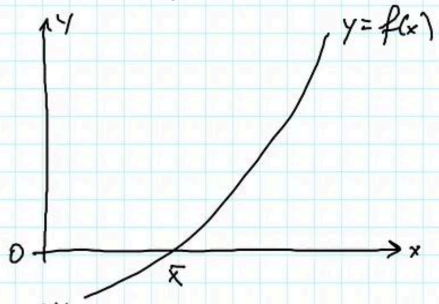


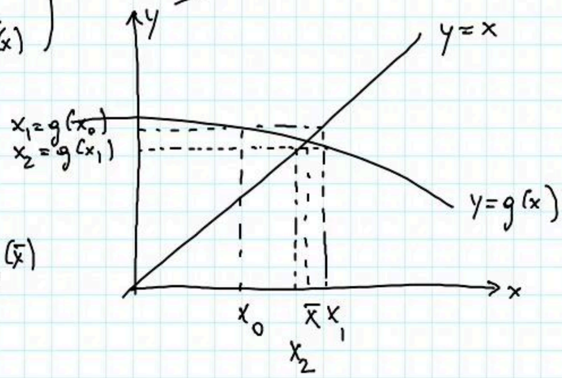
On pose $g(x) = x - f(x)$

ou $g(x) = x - \alpha f(x)$ $\quad \alpha \in \mathbb{R}$ $\left(\text{Newton } \alpha = \frac{1}{f'(x)} \right)$

Méthode de pt fixe: x_0 donné

$$x_1 = g(x_0)$$
$$x_2 = g(x_1)$$
$$\vdots$$
$$x_{n+1} = g(x_n)$$

Si la suite $g(x_n)_n$ converge et si g continue, alors $\bar{x} = g(\bar{x})$



In the case of Newton's method we can observe that, and this will be done later on, that alpha is nothing but 1 over the derivative of f. The method I suggest to find x_bar a fixed point of g is the following. It is called a fixed point method. Given x_0 in R, which is an approximation of x_bar, for example, here is x_0. We shall calculate x_1 equal to g(x_0). Here we have x_0, and this is x_1 equal to g(x_0). I carry x_1 on to the x axis, here is x_1. Next I calculate x_2 equal to g(x_1), and by continuing on this figure, x_2 equal to g(x_1) will be here. I carry on the x-axis the value of x_2. In general, for a given x_n, I can calculate x_n+1 defined as g(x_n). The first remark is that the sequence x_n converges, and if the function g is continuous then the limit when n goes to infinity of x_n satisfies, let x_bar be this limit, x_bar is the limit of x_n as n goes to infinity, then x_bar is equal to g(x_bar). This means I have effectively found x_bar which is a fixed point of g. Why? well it is sufficient to take the limit in the left hand side and the limit in the right hand side of this equality, since g is continuous I can swap the limit and the function g, and I get x_bar equal to g(x_bar).

Notes

Summary

2m 07s

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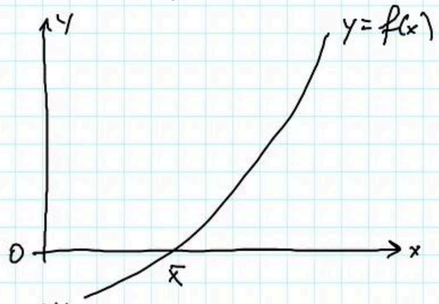

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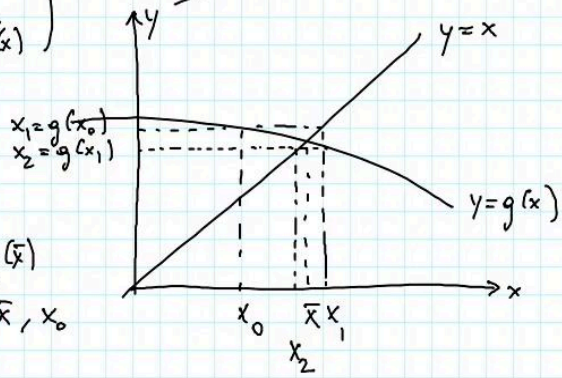

Si la suite $g(x_n)_n$ converge et si g continue, alors $\bar{x} = g(\bar{x})$

Est-ce que la suite $(x_n)_n$ converge ? Dépend de g, $\bar{x}$, x_0

Now the fundamental question is: Does the sequence converge? The sequence x_n, defined by x_n+1 equal to g(x_n), does it converge? The answer, we shall see later in theorem 8.3, depends on the function g at x_bar, x_bar being the fixed point of g, and on the initial guess x_0.

Notes

Summary

4m 11s