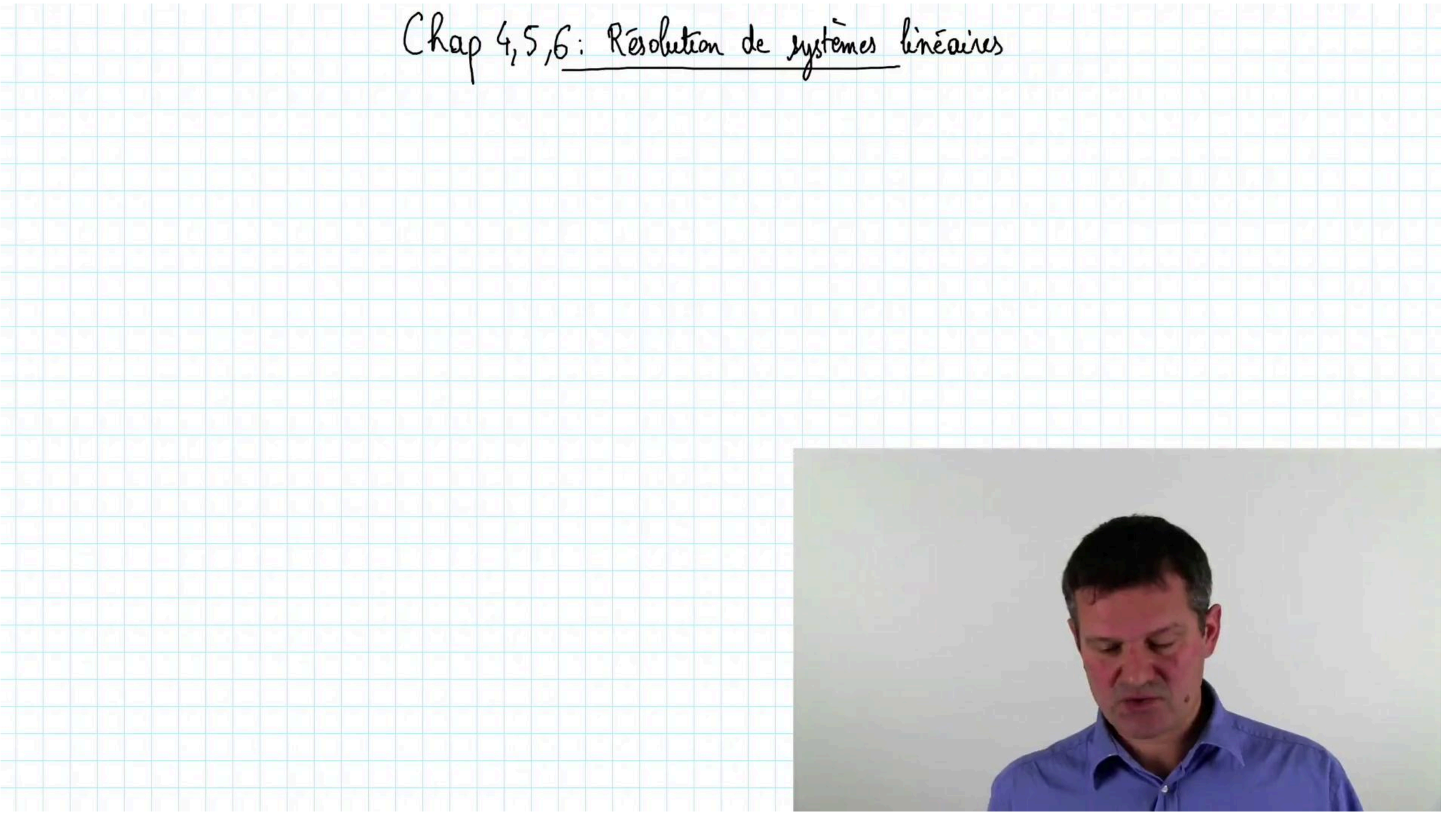
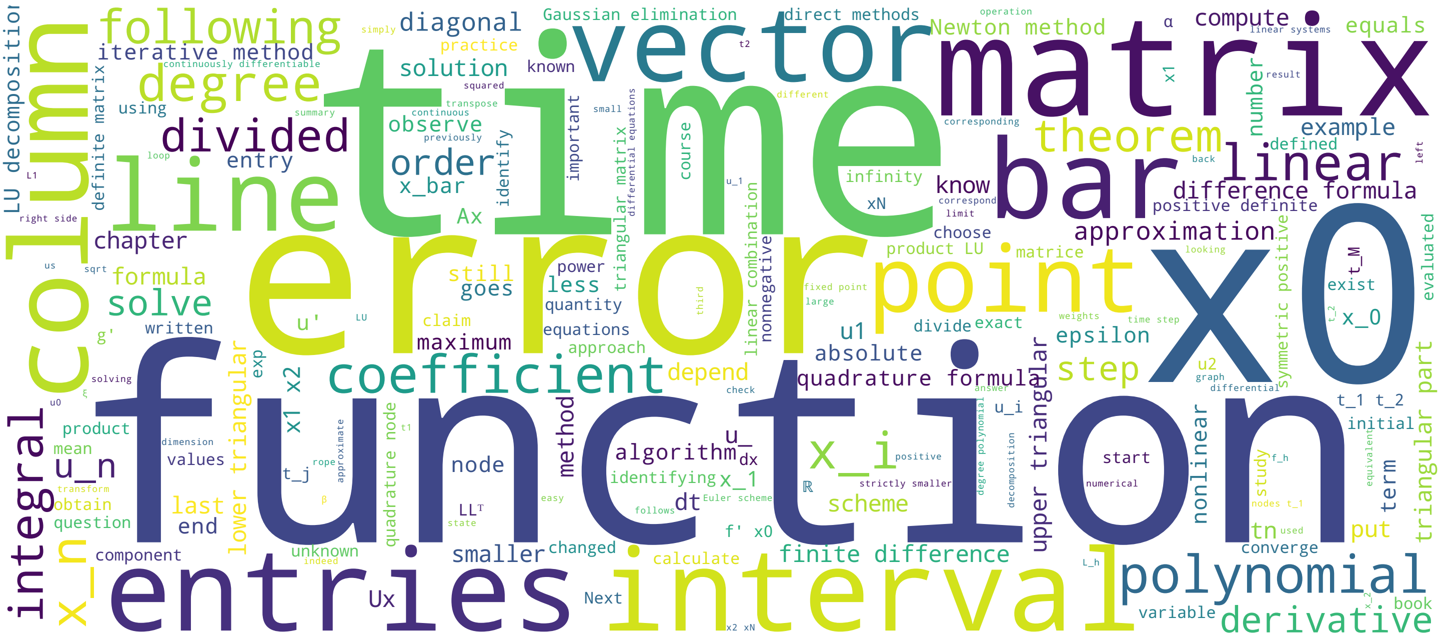

Chap 4,5,6: Résolution de systèmes linéaires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matrix
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theorem
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LU decomposition
definite matrix
triangular matrix
linear combination
quadrature node
symmetric positive
triangular part
x_i
x_n
u_n
x_bar
infinity
know
choose
product LU
matrix
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EPFL

Video

Chap 4,5,6 : Résolution de systèmes linéaires

$A\vec{x}=\vec{b}$ N grand $N \to 10$ à 10^9

Méthodes directes : Elimin Gauss (Chap 4) Décomposition LU LL^T (Chap 5)

— itératives : important en pratique mais non abordé

Chap 4

$$\underbrace{\begin{pmatrix} & & \\ & & \\ & & \end{pmatrix}}_{A}\underbrace{\begin{pmatrix} \\ \\ \\ \end{pmatrix}}_{\vec{x}} = \underbrace{\begin{pmatrix} \\ \\ \\ \end{pmatrix}}_{\vec{b}} \longrightarrow \underbrace{\begin{pmatrix} 1 & - & - & \cdot \\ & \ddots & & \vdots \\ (0) & & \ddots & \\ & & & 1 \end{pmatrix}}_{U}\begin{pmatrix} \\ \\ \\ \end{pmatrix} = \begin{pmatrix} \\ \\ \\ \end{pmatrix} \quad \vec{x} = \vec{d}$$

Here is a summary of chapters 4,5,6 : Resolution of linear systems. Given a matrix A and a vector b, we want to solve the linear system Ax=b, so here N, the number of lines and columns of A is large, in applications, N will go from 10 to 10^9 (a billion). So we have direct methods, or iterative methods. The direct methods are : Gaussian elimination (chapter 4), or the LU decomposition for any matrix, or LL^T in the case of a symmetric positive definite matrix, that's chapter 5. Then we have iterative methods : we still didn't study this chapter which is important in practice, since if we solve partial differential equations in dimension 3 we have to use the iterative methods. It is important in practice but not studies in this course. Let's go back to chapter 4 : Gaussian elimination. Here is our linear system Ax=b, we want to transform this linear system in an equivalent one, Ux=d. Equivalent means that I have the same vector x of unknowns, x1,x2,...,xN. We changed the right side of the equation and changed the matrix. The matrix U is the upper triangular matrix with 0's on the lower triangular part, we decided to put 1's on the diagonal, and its entries in the upper triangular part.

Notes

Summary

0m 01s

Chap 4,5,6 : Résolution de systèmes linéaires

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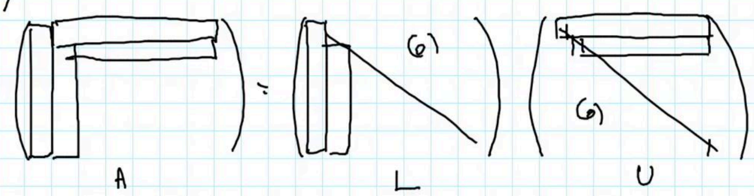
Chap 4

$$\underset{A}{\begin{pmatrix} & & \\ & & \\ & & \end{pmatrix}} \underset{\vec{x}}{\begin{pmatrix} \\ \\ \\ \end{pmatrix}} = \underset{\vec{b}}{\begin{pmatrix} \\ \\ \\ \end{pmatrix}} \longrightarrow \underset{U}{\begin{pmatrix} 1 & - & - \\ & \ddots & \vdots \\ (0) & & 1 \end{pmatrix}} \underset{\vec{x}}{\begin{pmatrix} \\ \\ \\ \end{pmatrix}} = \underset{\vec{d}}{\begin{pmatrix} \\ \\ \\ \end{pmatrix}}$$

$$\begin{pmatrix} 1 & & & & \\ 0 & 1 & & & \\ | & 0 & & & \\ | & | & & 1 & \\ 0 & 0 & & 0 & 1 \end{pmatrix}$$

Chap 5



Then it is easy to solve the linear system Ux=d. To end up with this linear system Ux=d, we proceed in N steps. First step, we put a 1 on the diagonal instead of the coefficient, and 0 on the other entries of the first column. On the second step we put a 1 on the entry 2,2 and then 0's, and so on until the last but one step where we have 1 0, or the last one where we have a 1. Then we saw chapter 5 the LU decomposition, here we wanted to write A as the product of a lower triangular matrix L with 0's in the upper triangular part and a matrix U, which is upper triangular with 0's on the lower triangular part, where we chose to have 1's on the diagonal. We get the coefficients of the matrices L and U by identifying the entries of the matrix A with the entries of the product LU and identifying the entries in the first column of A with the coefficients of the first column of LU we get all those entries. (first column of L) Then, identifying entries of the first line of A with the first line of the product LU we'll get the coefficients of the first line of the matrix U and so on. We get those entries, we identify those entries and get the second column of U, and so on until the end.

Notes

Summary

2m 12s

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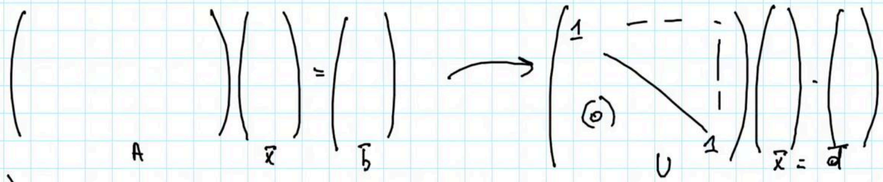
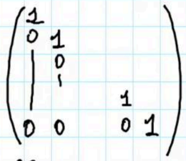
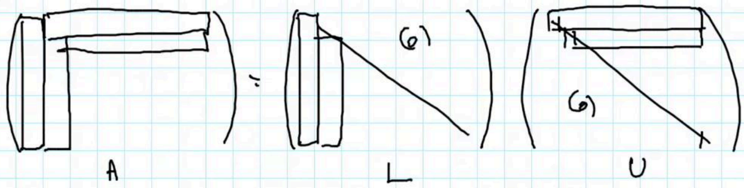
— itératives : important en pratique mais non abordé

Chap 4





Chap 5



In the case of a symmetric positive definite matrix, the A=LU decomposition will be a decomposition A=LL^T (L^T the transpose of L) the coefficients Lij on the diagonal of L are positive which simplifies the algorithm. This algorithm requires roughly twice as less operations.

Notes

Summary

4m 04s