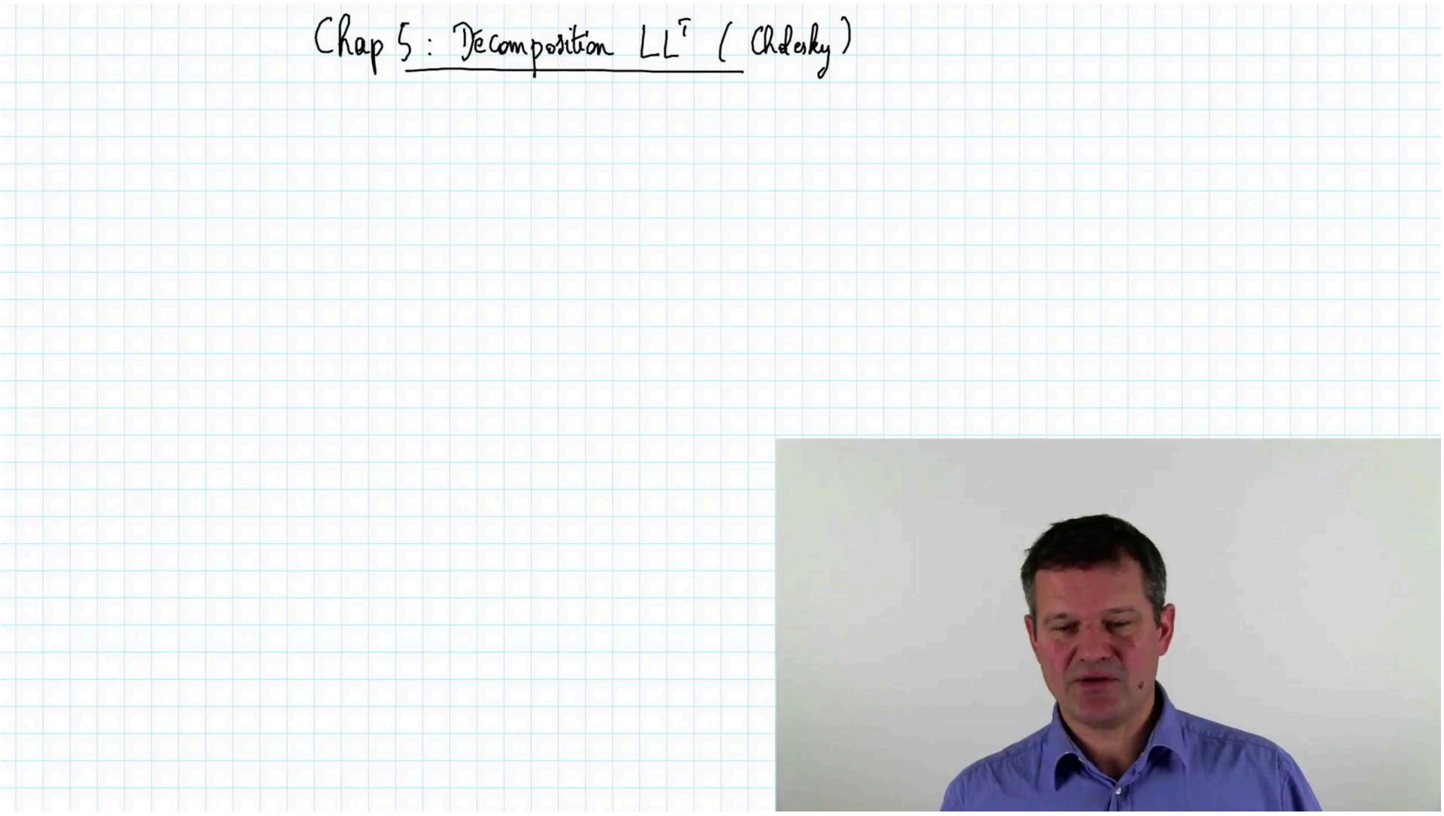
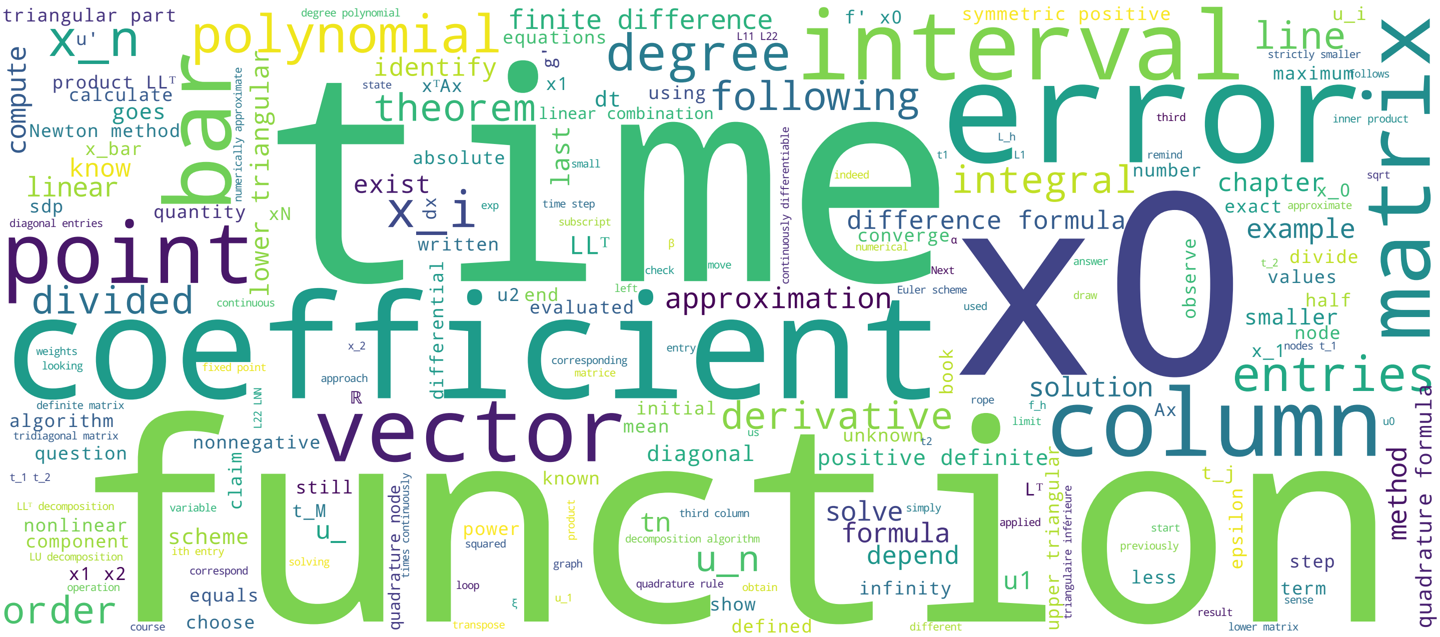

Chap 5 : Décomposition LL^T (Cholesky)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algorithm
question
triangular part
product LL^T
calculate
Newton method
lower triangular
upper triangular
quadrature formula
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Video

EPFL

Chap 5 : Décomposition LL^T (Cholesky)

Déf : A $N \times N$ matrice est symétrique définie positive si (sdp)

- $A = A^T$ $(a_{ij} = a_{ji} \quad 1 \leq i,j \leq N)$
- $\forall \vec{x} \in \mathbb{R}^N \quad \vec{x}^T A \vec{x} \geq 0 \qquad \vec{x}^T A \vec{x} := (\vec{x}, A\vec{x}) = \sum_{i=1}^N x_i (A\vec{x})_i = \sum_{i,j=1}^N a_{ij} x_i x_j$
- $\vec{x}^T A \vec{x} = 0 \Leftrightarrow \vec{x} = \vec{0}$

Si A est sdp alors il existe L $(l_{ii} > 0)$ tq $A = LL^T$ L

Let's move to the LL^T decomposition, also called Cholesky decomposition, which is applied in the case where the matrix A is a symmetric positive definite matrix. First of all, let's give a definition. Let A be a NxN matrix. This matrix is symmetric positive definite, if the three following conditions are met. First, A is symmetric, $A=A^T$ (transpose of A) in the sense where the entries aij=aji for all i,j between 1 and N. Second condition: if I take any vector x in ℝ^N, and compute x^TAx, then this quantity is always positive or 0. Let me remind you that here, x^TAx is the same at the inner product between x and Ax, that is, the sum for i from 1 to N, of the ith entry of the vector xi with the ith entry of the vector Ax, which is the sum over all the subscripts i,j from 1 to N, so the sum over i, from 1 to N, sum over over j from 1 to N of aij*xi*xj. Thus the second condition is that x^TAx is nonnegative. c'est un scalaire. And the last condition, x^TAx =0 if and only if x is the 0 vector. So if A is a symmetric definite positive matrix, we will abbreviate symmetric positive definite with "sdp", if A is sdp, then we can show that there exists a unique decomposition $A=LL^T$, so there exists a lower matrix L, with diagonal entries Lii that are stricly positive, such that $A=LL^T$, again L is lower triangular.

Notes

Summary

0m 00s

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- $A = A^T$ $\quad (a_{ij} = a_{ji} \quad 1 \leq i,j \leq N)$
- $\forall \vec{x} \in \mathbb{R}^N \quad \vec{x}^T A \vec{x} \geq 0$ $\qquad \vec{x}^T A \vec{x} := (\vec{x}, A\vec{x}) = \sum_{i=1}^{N} x_i (A\vec{x})_i = \sum_{i,j=1}^{N} a_{ij} x_i x_j$
- $\vec{x}^T A \vec{x} = 0 \iff \vec{x} = \vec{0}$

Si A est sdp alors il existe L $(l_{ii} > 0)$ tq $A = LL^T$ $\quad$ L triang. inf.

$$\underbrace{\begin{pmatrix} a_{11} & \cdots & a_{1N} \\ \vdots & \ddots & \vdots \\ a_{N1} & \cdots & a_{NN} \end{pmatrix}}_{A} = \underbrace{\begin{pmatrix} l_{11} & & (0) \\ \vdots & l_{22} & \\ \vdots & \ddots & \\ l_{N1} & \cdots & l_{NN} \end{pmatrix}}_{L} \underbrace{\begin{pmatrix} l_{11} & \cdots & l_{N1} \\ & l_{22} & \vdots \\ (0) & \ddots & \vdots \\ & & l_{NN} \end{pmatrix}}_{L^T}$$

On identifie les coef de A et LL^T dans l'ordre suivant

Donc triangulaire inférieure, *lower metrics* en anglais. I can draw the following. Here is the matrix A, with entries a11,..., a1N on the first line, a11,..., aN1 on the first column, until aNN. If A is sdp, this matrix, can be written as the product of L which is a lower matrix, since it is lower triangular, it has coefficients L11, L22,..., LNN, that are strictly positive. The upper triangular part has only zeroes, and here we have the coefficient LN1, and so on. And then L^T is the transposed of L. Donc transposé. The diagonal entries are the same, L11, L22,...,LNN. La partie triangulaire supérieure qui avait des coefficients nuls se retrouve dans la partie, cette fois-ci, triangulaire inférieure, So the lower part of L^T has only 0's, And here are the coefficients LN1, and so on. To get the Cholesky (or LL^T) decomposition algorithm we need to do half the operations needed for the LU decomposition algorithm. So we identify the coefficients as before. We identify the entries of A and LL^T in the following order : As before, when I identify all the entries of the first column of A with the entries of the first column of the product LL^T, I will get all the coefficients of the first column of L.

Notes

Summary

2m 46s

Chap 5 : Décomposition LL^T (Cholesky)

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- $A = A^T$ $\quad (a_{ij} = a_{ji} \quad 1 \leq i,j \leq N)$
- $\forall \vec{x} \in \mathbb{R}^N \quad \vec{x}^T A \vec{x} \geq 0 \qquad \vec{x}^T A \vec{x} := (\vec{x}, A\vec{x}) = \sum_{i=1}^{N} x_i (A\vec{x})_i = \sum_{i,j=1}^{N} a_{ij} x_i x_j$
- $\vec{x}^T A \vec{x} = 0 \iff \vec{x} = \vec{0}$

Si A est sdp alors il existe L ($l_{ii} > 0$) tq $A = LL^T$ L triang. inf.

$$\underbrace{\begin{pmatrix} a_{11} & \cdots & a_{1N} \\ \vdots & \ddots & \vdots \\ a_{N1} & \cdots & a_{NN} \end{pmatrix}}_{A} = \underbrace{\begin{pmatrix} l_{11} & & (0) \\ \vdots & l_{22} & \\ l_{N1} & \cdots & l_{NN} \end{pmatrix}}_{L} \underbrace{\begin{pmatrix} l_{11} & \cdots & l_{N1} \\ & l_{22} & \vdots \\ (0) & & l_{NN} \end{pmatrix}}_{L^T}$$

On identifie les coef de A et LL^T dans l'ordre suivant

This time I can directly identify the coefficients of the second column of A with the coefficients of the second column of the product LL^T to get the coefficients of the second column of L. We do half the work, and so on If I take the third column, I will get the coefficients of the third column of LL^T and in the end, I will have identified all the entries, say, half of the coefficients of A, the lower triangular part with the coefficients of the product LL^T, and I will have all the coefficients of L, and so, since L^T is the transpose of L, we have all the entries of L and L^T. We'll now consider the algorithm in the case of a tridiagonal matrix.

Notes

Summary

5m 13s