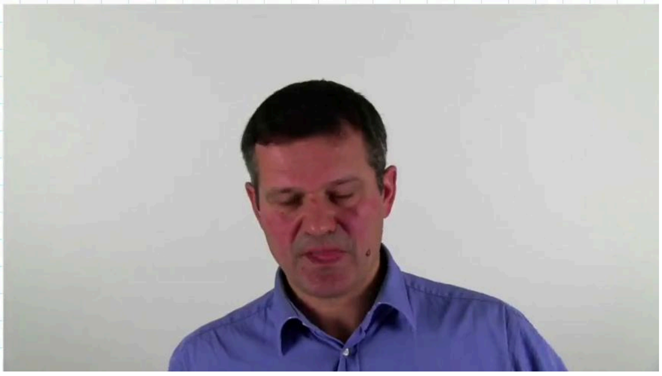
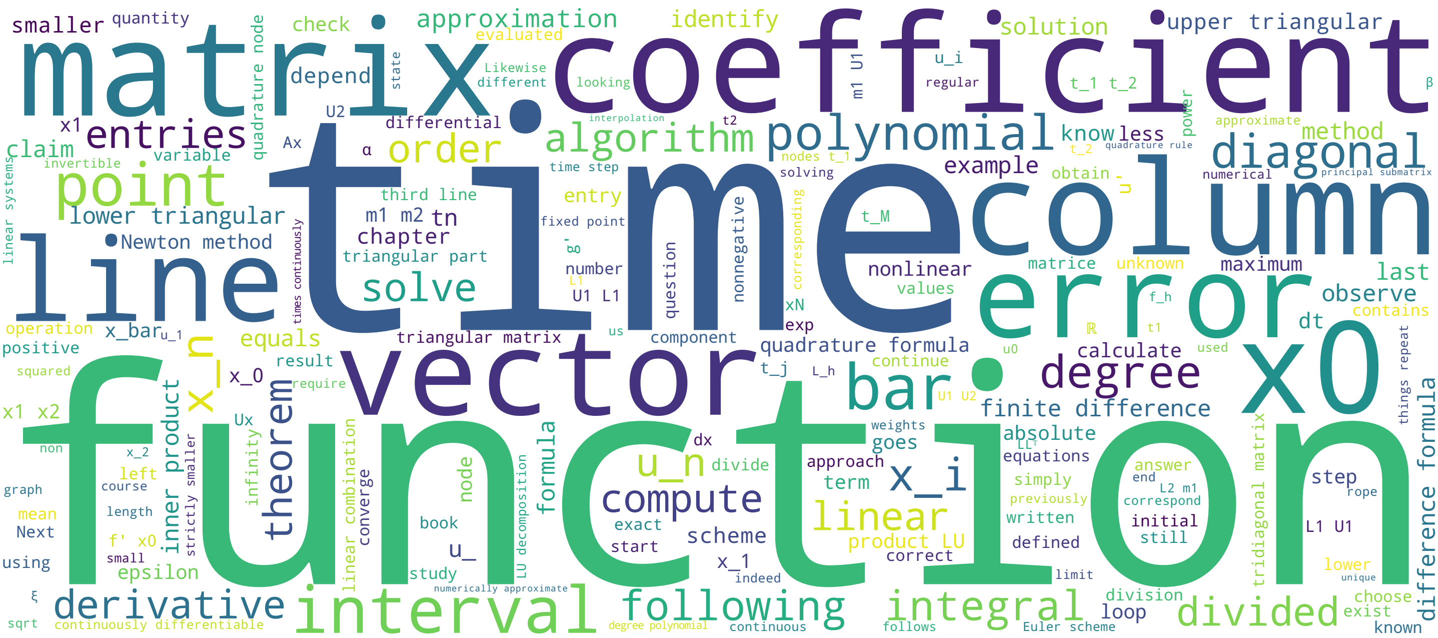
Chap 5 : Décomposition LU - Exemple





EPFL

Video

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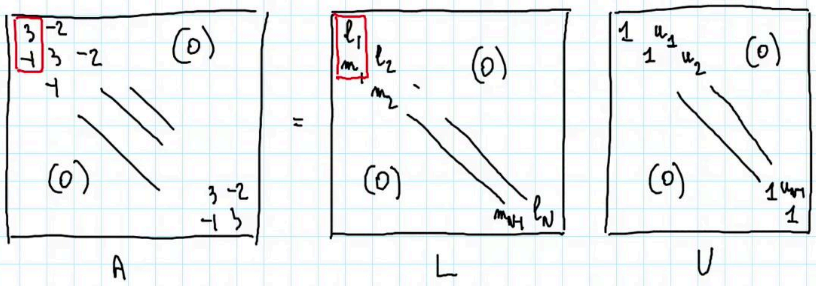

We'll now study the algorithm of LU decomposition with a tridiagonal matrix A. So we start with the tridiagonal matrix from before. It is the matrix with 3's on the diagonal, -1 just below the diagonal and -2 just above. That's the matrix A. I now want to write it as the product of matrices L and U. The matrix L is a lower triangular matrix, so I'll note L1, L2,..., LN, the diagonal coefficients. Then the subdiagonal m1, m2, until m(n-1), L is a lower triangular matrix so the upper triangular part is only contains 0's. Because of this tridiagonal structure on the matrix A, we have 0's outside those 3 diagonals, so this matrix will be lower triangular, with only a main diagonal and a diagonal below. Likewise with the matrix U. We decided that it'll have 1's on the diagonal, it is upper triangular so only has 0's on the lower triangular part, and this matrix only contains one vector above the diagonal. I wrote its entries U1, U2,..., UN-1 and the other entries are 0. As said before I must identify the coefficients of A with the coefficients of the product LU in the appropriate order. Here I identify the coefficients of the first column of A so the non-zero coefficients which are 3 and -1, I will get the coefficients of the first column of L.

Notes

Summary

0m 00s

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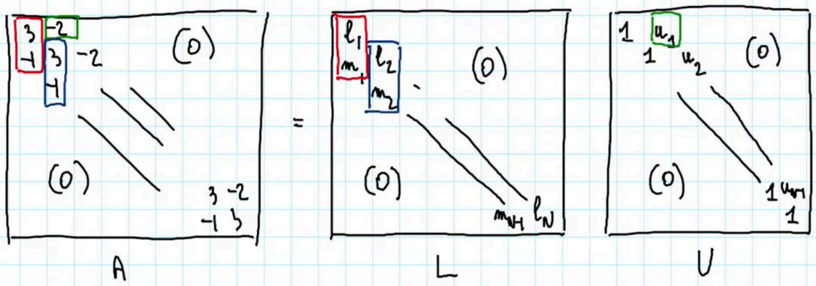

$3 = l_1$

$-1 = m_1$

$-2 = l_1 u_1 \rightarrow u_1 = -2/l_1$

$3 = m_1 u_1 + l_2 \rightarrow l_2 = 3 - m_1 u_1$

So, 3 equals the inner product between the first line of L and the first column of U. I simply get L1*1, so L1. Now I have L1. Then -1 equals the inner product of the second line of L with the first column of U, it's the coefficients 21. I obtain m1*1=-1 So I have m1 and L1. Then I must identify the coefficients of the first line of A with the first line of the product LU and I claim that I'll get the coefficients of the first line of U, so here the entry U1. I claim that identifying this coefficient I will get that entry. Here, -2 is the first line, second column. I must do the inner product between the first line of L and the second column of U and I get -2 = L1*U1, -2 = L1 * U1 and I get U1= 2/L1. Let's continue : I must identify the coefficients of the second column of A with the corresponding entries in the product LU. I claim that I'll obtain the entries of the second column of L. 3 is the result of the inner product between the second line and the first column. 3= m1*U1+L2*1. I get that L2=3*m1*U1. Let's continue, here the entry -1, third line, second column. I do the inner product : third line and second column, which will give me -1=m2.

Notes

Summary

2m 31s

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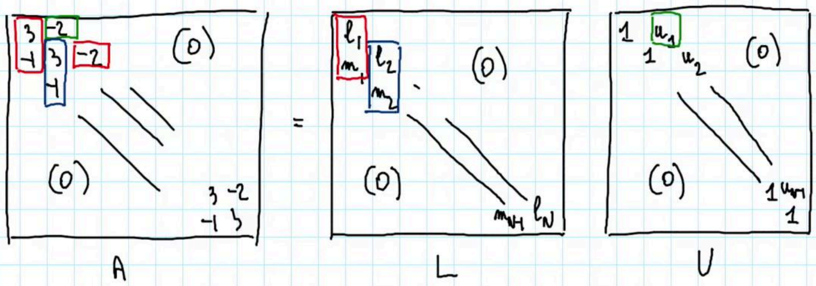

$3 = l_1$

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$3 = m_1 u_1 + l_2 \rightarrow l_2 = 3 - m_1 u_1$

$-1 = m_2$

$-2 = l_2 u_2$

Algorithme : $\vec{l}$ N vecteur l_j

$\vec{m}$ N-1 vect m_j

$\vec{u}$ N-1 vect u_j

$l_1 = 3$

Faire $i = 1, N-1$

$m_i = -1$

$u_i = -2/l_i$

$l_{i+1} = 3 - m_i u_i$

So, observe that things repeat since -1=m1, -1=m2, I can compute m2 if we so desire to check that things repeat. If I calculate the coefficient, here the entry -2, it's the third line, second column, so here the second line, third column, I'll get -2 = L2*U2, and before I had -2=L1*U1. Now I can write an algorithm. The algorithm will give me the vectors L, N-vector of coefficients Lj, the vector n, the diagonal of the matrix L, m the lower diagonal of the matrix L, it is a vector of length N-1, with entries mj and then u is the upper diagonal of the matrix U, is a (N-1)-vector with entries Uj. Let us start the algorithm. L1=3, that is the initialisation. Then I do a loop, for i going from 1 to N-1. So I calculated L1, I have to compute m1, so in the loop, mi=-1, m1=-1, m2=-1, then once I have m1 I can compute U1. Ui=-2/Li, here U1=-2/L1, U2=-2/L2. And then once I have m1 and U2, I can compute L2. When I have mi and Ui I can compute L(i+1), L(i+61)=3-mi*Ui. I check the subscripts are correct: L2 = 3-m1*U1, that's correct, and for the last subscript, (always check the first and the last), when i=N-1, I will compute LN=3-m(N-1)*U(N-1), u indice N moins 1 so the algorithm is correct.

Notes

Summary

5m 40s

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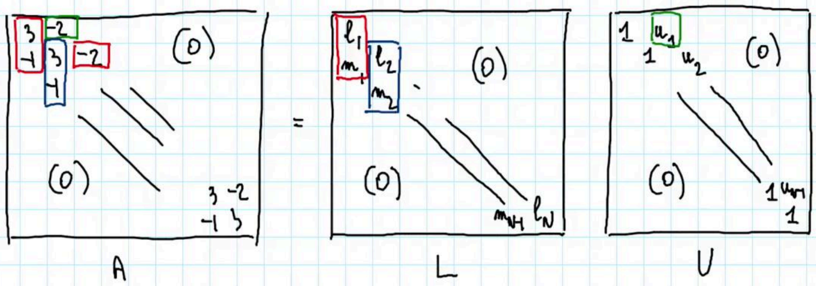

$3 = \ell_1$
$-1 = m_1$
$-2 = \ell_1 u_1 \rightarrow u_1 = -2/\ell_1$
$3 = m_1 u_1 + \ell_2 \rightarrow \ell_2 = 3 - m_1 u_1$
$-1 = m_2$
$-2 = \ell_2 u_2$

Algorithme : $\vec{\ell}$ N vecteur ℓ_j
$\vec{m}$ N-1 vect m_j
$\vec{u}$ N-1 vect u_j

$\ell_1 = 3$
Faire $i = 1, N-1$
$m_i = -1$
$u_i = -2/\ell_i$
$\ell_{i+1} = 3 - m_i u_i$

O(N) opérations
Si toutes les ss-matrices principales de A sont régulières, alors pas division par zéro.

Again, two remarks concerning this algorithm: it requires O(N) operations, since it must go through a loop from 1 to N-1, and again, can I have divisions by 0 ? The answer is the same as before: if all the principal submatrix of A are regular, that is to say invertible, then I do not have a division by 0. To solve a linear system Ax=B all is left to do is solve the two linear systems Ly=b and Ux=y but those are easy operations since we are dealing with lower and upper triangular matrices.

Notes

Summary

9m 13s