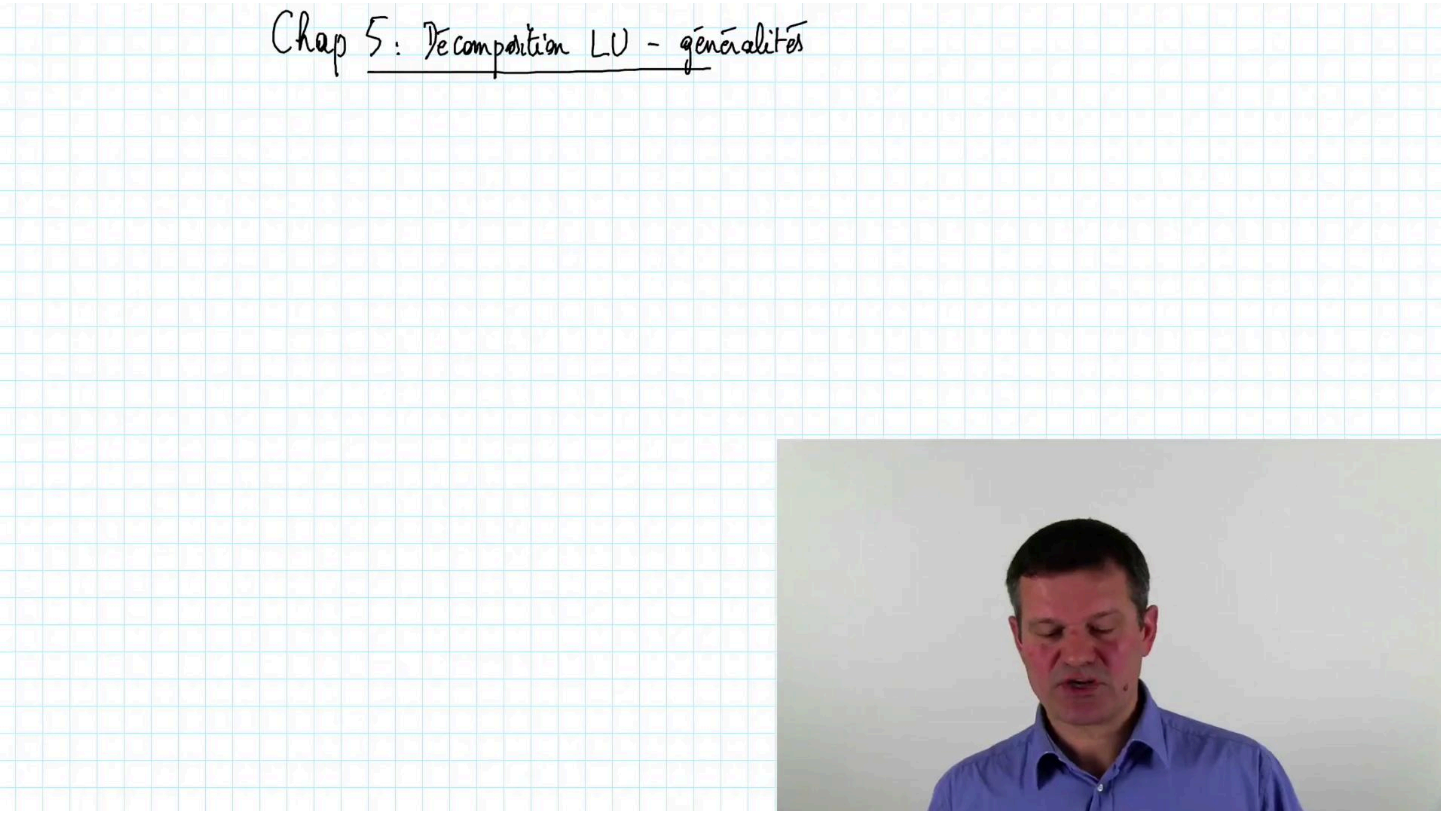
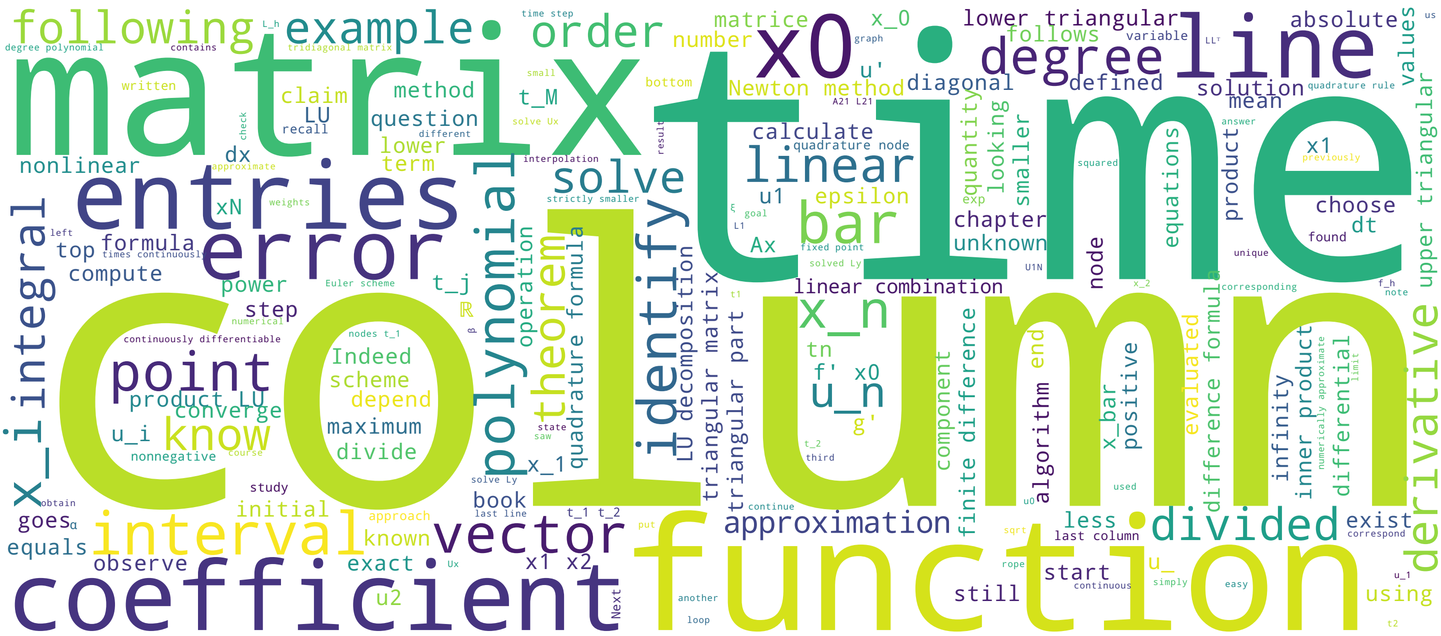

Chap 5: Décomposition LU - généralités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chapter
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LU decomposition
triangular matrix
quadrature formula
inner product
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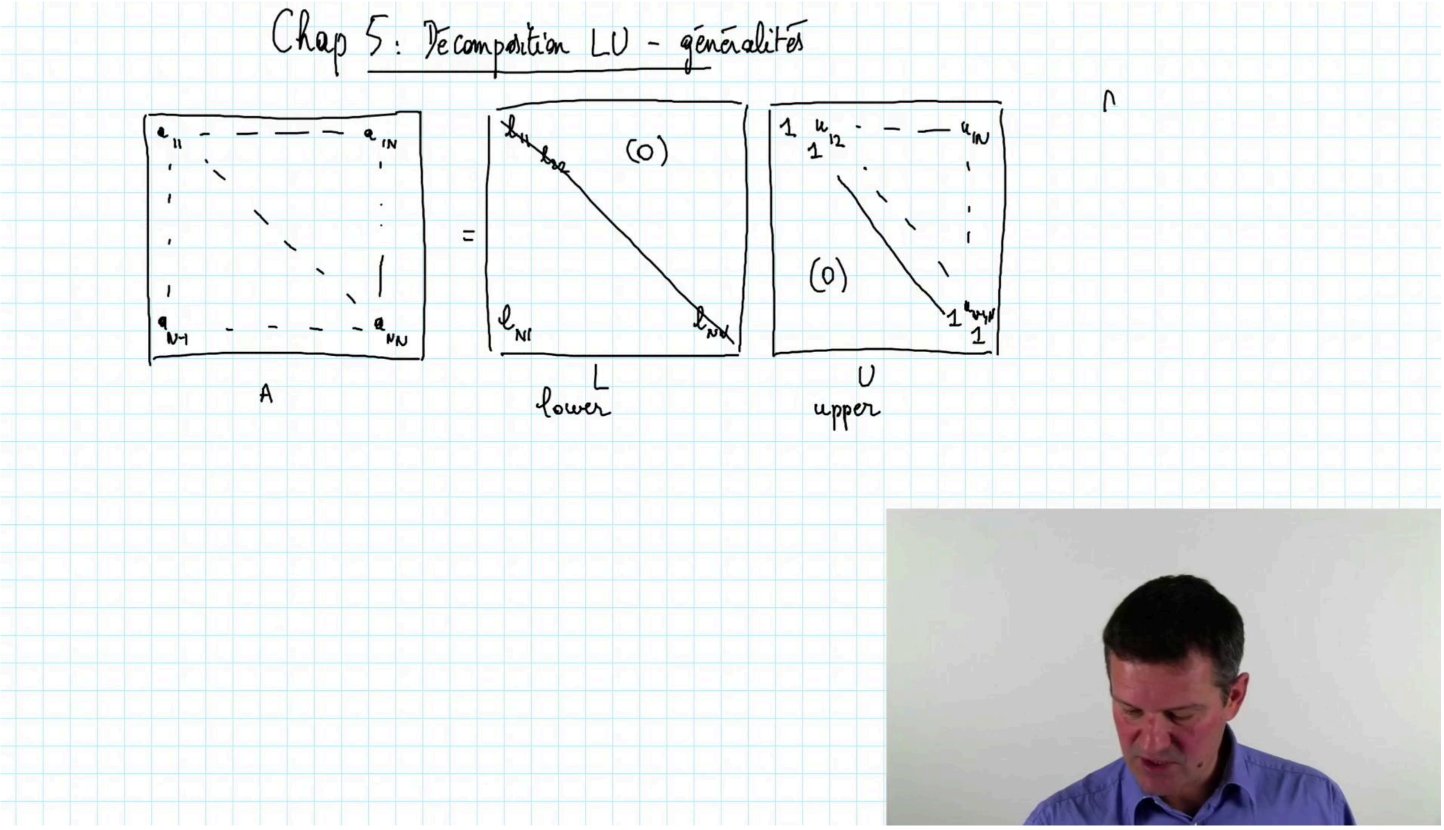


Let's now go to chapter 5 of the book : LU decomposition, Choleski decomposition. I want to solve the linear system Ax=b, I will decompose the matrix A (NxN matrix), as a product of a matrix L and a matrix U. All those matrices are NxN matrices. The matrix A has as entries a11 on the first row, ...a1N first row, last column, here the first column ends at aN1 and here is the last entry aNN, that's our matrix A. As for matrices L and U: we already talked about the matrix U, it is an upper triangular matrix and we decided before that we'll want 1's on the diagonal, 0 in the lower triangular part. Here we have the entries u12 and u1N until u(N-1,N) The matrix L is a lower triangular matrix (or just lower matrix). So the upper triangular part only contains 0, and here I have the coefficients L11, L22 until LNN here with here the entry LN1, last line first column. Thus we know A and the goal is to find the matrices L and U, upper and lower triangular matrices, such that A=LU. Again, we're given the matrix A and what we're looking for the matrices L and U such that A=LU. If we do such an operation, then it is easy to solve a linear system with A as a matrix. Why?

Notes

Summary

0m 00s

Chap 5 : Décomposition LU - généralités

$$\underbrace{\begin{pmatrix} a_{11} & \cdots & \cdots & a_{1N}\\ \vdots & \ddots & & \vdots \\ \vdots & & \ddots & \vdots \\ a_{N1} & \cdots & \cdots & a_{NN}\end{pmatrix}}_{A} = \underbrace{\begin{pmatrix} l_{11} & & & (0)\\ & l_{22} & & \\ & & \ddots & \\ l_{N1} & & & l_{NN}\end{pmatrix}}_{\substack{L \\ \text{lower}}} \underbrace{\begin{pmatrix}1 & u_{12} & \cdots & u_{1N}\\ & 1 & & \vdots \\ (0) & & \ddots & u_{N-1,N}\\ & & & 1\end{pmatrix}}_{\substack{U \\ \text{upper}}}$$

$A\vec{x}=\vec{b}$

$L\underbrace{U\vec{x}}_{\vec{y}}=\vec{b}$ $\quad \begin{cases} L\vec{y}=\vec{b} \\ U\vec{x}=\vec{y} \end{cases}$

L $\vec{y}$ = $\vec{b}$

U $\vec{x}$ = $\vec{y}$

Pour obtenir les coef des matrices L et U, on identifie les coeff de A et LU dans l'ordre suivant :

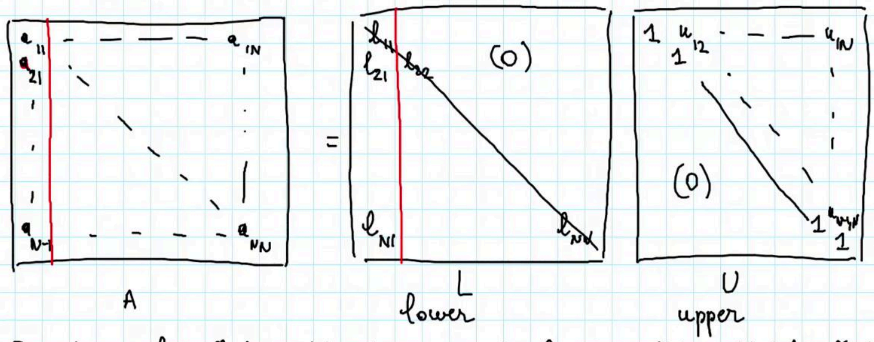
Recall that we want to solve the linear system Ax=b. We have A = LU, so we must solve LUx=b, and we will split this linear system into 2. Let y=Ux, I must solve Ly=b which is easy : recall that L is a lower triangular matrix so when I solve Ly=b, I will start from the top. I have L11*y1=b1 then I will compute y2 and so on until the bottom, so solving Ly=b when L is lower triangular is an easy task. Once I found y, I must solve Ux=y. I now know y since I solved Ly=b, so solving Ux=y is also a simple task, we saw that earlier; We have U an upper triangular matrix, here we have 0's and here 1's on the diagonal, and so we know the vector y, since we solved Ly=b and now we must solve Ux=y, and now we'll start from the bottom, we have xN=yN and so on up to the top. The question we have now is how to find the coefficients of L and U? It is quite simple, to find the coefficients of L and U. We identify the coefficients of the matrix A, with the coefficients of the product LU in the appropriate order. To know the coefficients of matrices L and U, we identify the entries of the matrix A and the matrix LU in the following order. What we need to remember, is the order in which we identify these coefficients.

Notes

Summary

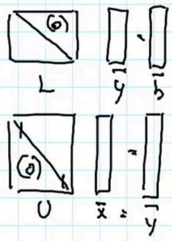
2m 25s

Chap 5 : Décomposition LU - généralités



$A\vec{x}=\vec{b}$

$\underbrace{LU\vec{x}}_{\vec{y}}=\vec{b}$ $\begin{cases} L\vec{y}=\vec{b} \\ U\vec{x}=\vec{y}\end{cases}$



Pour obtenir les coeff des matrices L et U, on identifie les coeff de A et LU dans l'ordre suivant :

1ère étape : on identifie les coeff de la 1ère col. de A et LU : on obtient les coeff de la 1ère col. de L $a_{11}=l_{11}\cdot 1$; $a_{21}=l_{21}\cdot 1$

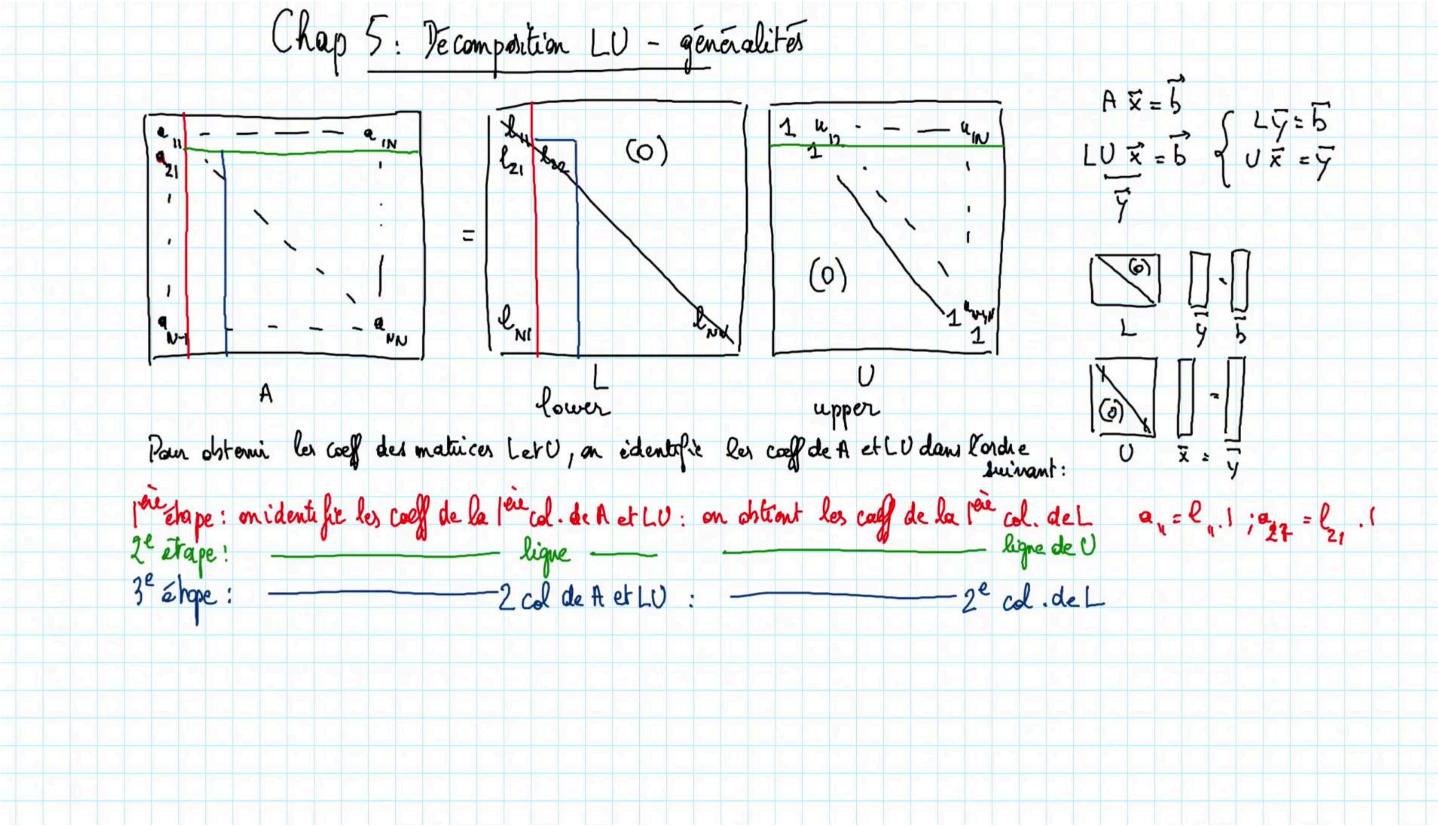
2e étape : ——— ligne ——— ——— ligne.

The first step is as follows : We identify the entries of the first column of A and LU. Here is the first line of A. If I identify coefficients of the first column of A with the coefficients of the first column of the product LU, I claim that we get the entries of the first column of the matrix L here. Indeed, let's do an example with the first coefficient, we have the coefficient A11, which is the product of the first line of L with the first column of U. Observe that when I do the product of the first line of L with the first column of U I must do the inner product of this line with that column, I simply obtain L11 * 1 so I can write A11=L11 * 1, so I found L11. I can now find L12. Here I have A12. I must do the inner product between the second line of L and still the first column of U. So here I have the coefficients A21, here is L21, so when I calculate I get A21 = L21 * 1. So A21 = L21 * 1 and so on. So if I identify the coefficients of the first column of A and the first column of LU, I get all the coefficients of the first column of L. Now I continue, step 2. This time we identify the coefficients of the first line of A with the coefficients of the first line of the product LU, and now I want to get the entries of the first line of U.

Notes

Summary

5m 14s



I take all the entries here. And I'll identify them with the product LU, and I will get all the entries that are here. For example, for the first one : we have the entry A12, I must do the inner product of the first line of L and the second column of U. I will find the entry U12 and so on, on all the entries. When I identify A1N, the inner product between the first line of L and the last column of U, I will get the coefficient U1N. Then we continue with the same method. Step 3: identify the entries of the second column of A and the second column of LU, and get the coefficients of the second column of L. Here are all the entries that I'll identify. It's those in the second column of A and I'll find the entries of the second column of L so all the entries from L22 to LN2, and I continue likewise until the end. I identify these entries and will obtain those entries and then these ones to get those, and so on until the end, and I'll have all the entries of L and U. We'll now study this LU decomposition on a tridiagonal matrix A.

Notes

Summary

8m 29s