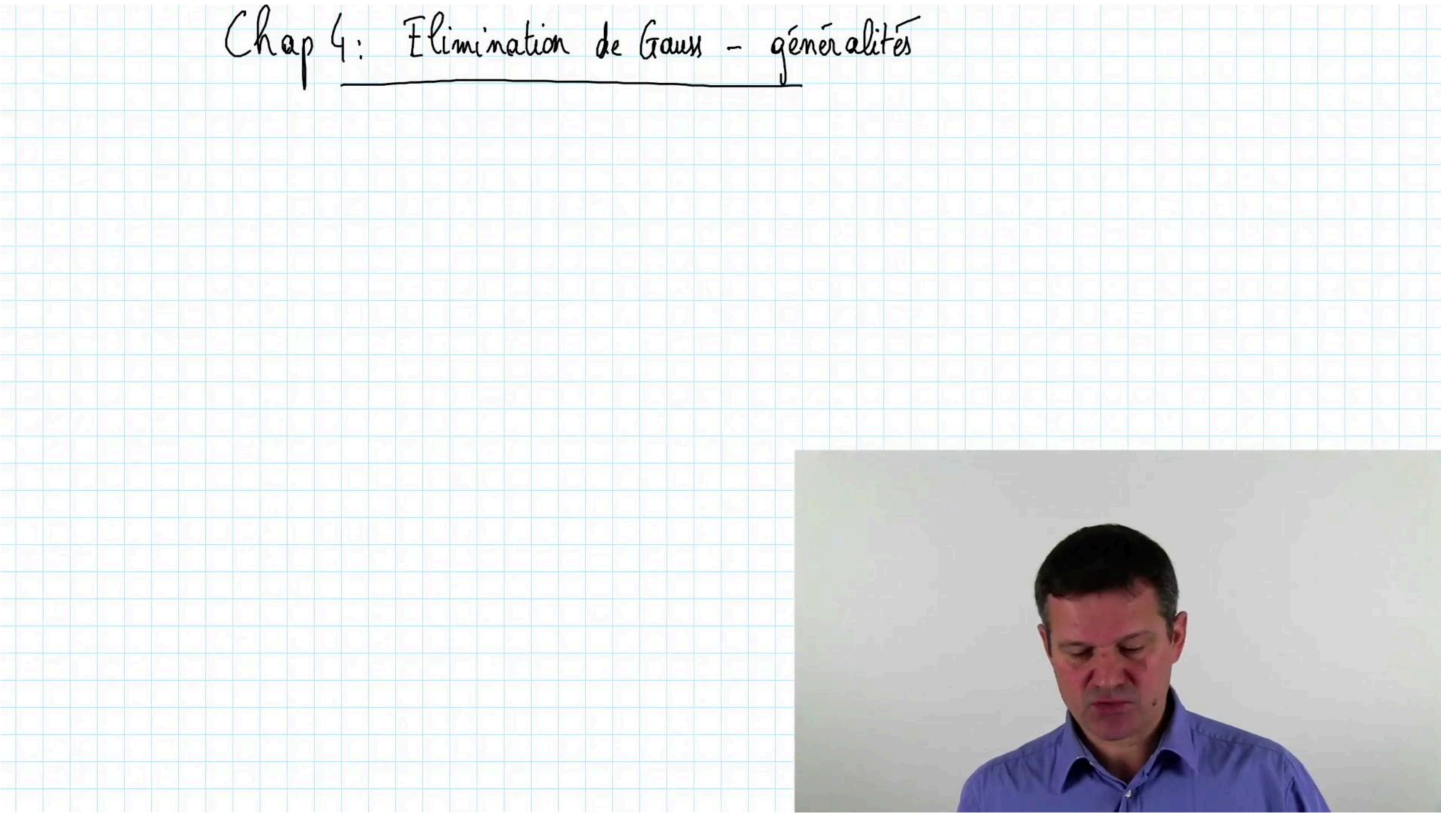
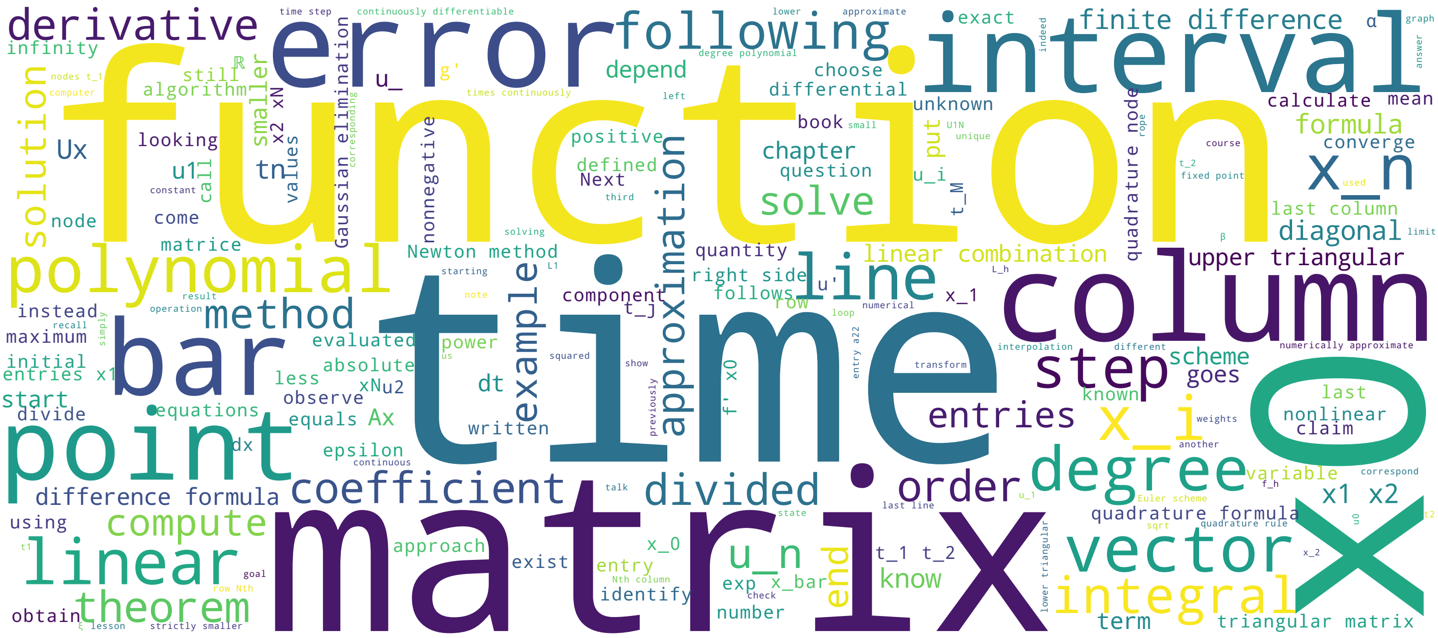

Chap 4: Elimination de Gauss - généralités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solve
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example
x_n
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Gaussian elimination
Newton method
linear combination
upper triangular
finite difference
difference formula
quadrature formula
triangular matrix
diagonal
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t_1 t_2
x1 x2

Video

Chap 4: Elimination de Gauss – généralités

$$\underbrace{\begin{pmatrix} a_{11} & \cdots & a_{1N} \\ \vdots & \ddots & \vdots \\ a_{N1} & \cdots & a_{NN} \end{pmatrix}}_{A} \underbrace{\begin{pmatrix} x_1 \\ \vdots \\ x_N \end{pmatrix}}_{\vec{x}} = \underbrace{\begin{pmatrix} b_1 \\ \vdots \\ b_N \end{pmatrix}}_{\vec{b}} \longrightarrow \underbrace{\begin{pmatrix} 1 & & \\ & 1 & \\ (0) & & \ddots & \\ & & & 1 \end{pmatrix}}_{U} \begin{pmatrix} x_1 \\ \vdots \\ x_N \end{pmatrix} = \begin{pmatrix} d_1 \\ \vdots \\ d \end{pmatrix}$$

Upper matrix

We'll start this lesson by recalling Gaussian elimination, you have already introduced this method in your linear algebra course. We'll see this method again in a more algorithmic form. So we want to solve the linear system Ax=b where A is a NxN matrix. Here is A, with N rows and N columns, its first entry is a11 and goes to a1N, first row, Nth column Then we have the first column here, so the entries a11 to aN1, and finally, at the last row, last column we have the coefficient aNN. Here is our matrix A. I am looking for the unknown vector x such that Ax=b with entries x1, x2,..., xN, and b the right side of the equation with entries b1, b2,..., bN. Here are x and b. The principle of Gaussian elimination is to transform the linear system making linear combinations of rows and columns, in a linear system that I'll write, a linear system with N lines and N columns, I'll call the matrix of the system, U U stands for upper triangular matrix, this upper triangular matrix has 1's on its diagonal and 0 below. The solution of the linear system, is still the vector x with entries x1, x2,..., xN, and the right side of the equation changed, I'll call it d, with entries d1, d2,..., dN.

Notes

Summary

0m 00s

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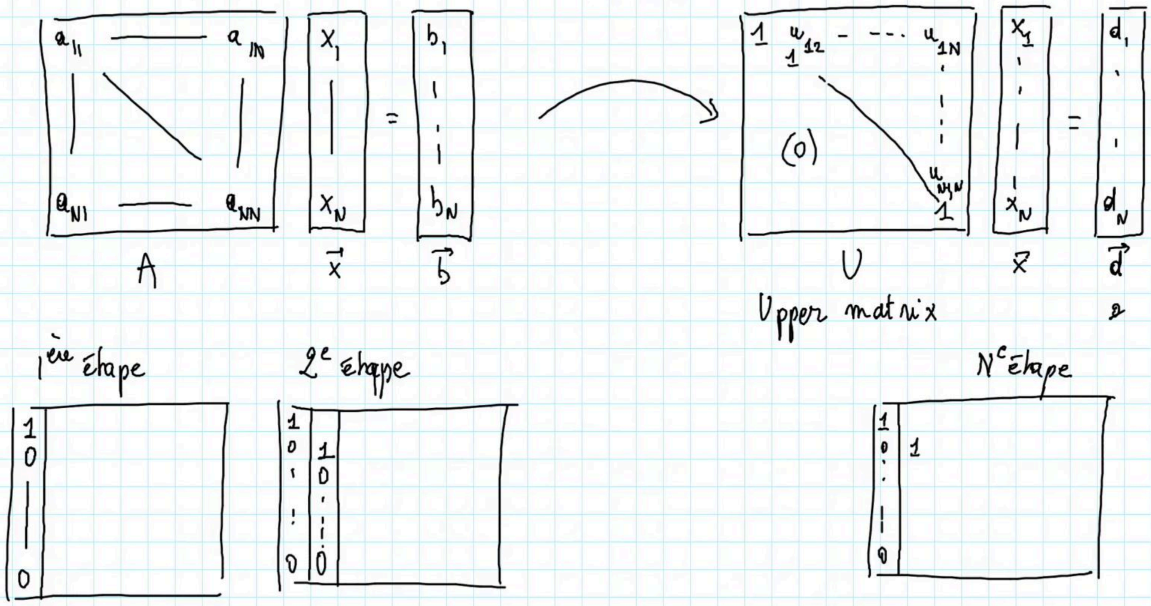
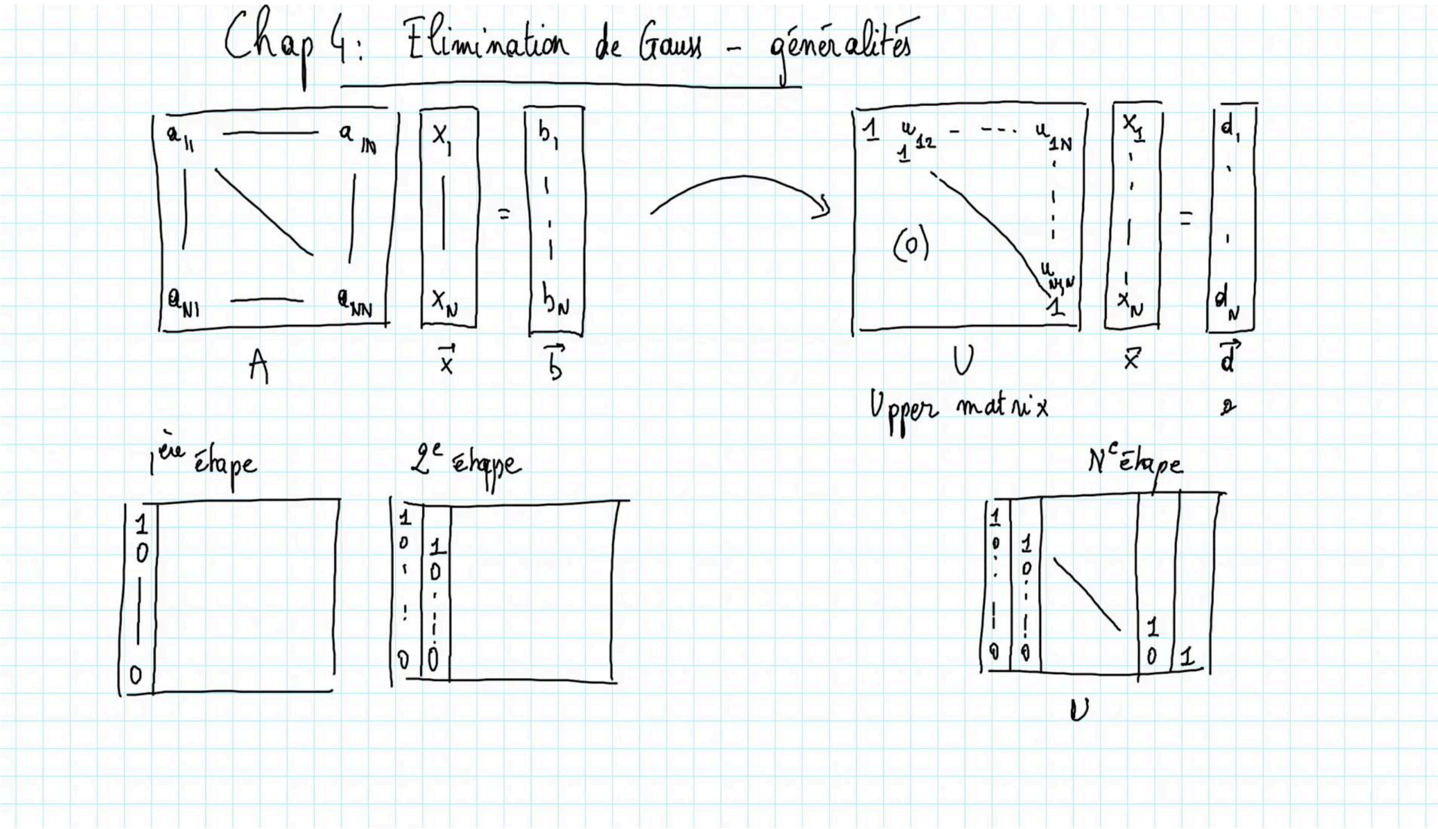


Here is d, and here is x. And the coefficients of the matrix U are U12 in first row, second column, until U1N at the first row, Nth column and so on we have the entry U(N-1)N. So the goal is starting with Ax=b to end up with this system Ux=d in N steps. The first step is as follows. Here is the matrix A, what we do is put a 1 here, instead of coefficient a11, by simply dividing the line by the first entry a11, provided it's nonzero, then we put a 0 here at the coefficient a21, and so on on the first column, we do several operations to get zeroes on the first column. That's the first step. Then comes the second step, The matrix is basically the same except that we have 1, 0, 0,... on the first column so in the second step we'll put a 1 instead of the entry a22, we'll do so by dividing the line by this entry a22, then we'll make a linear combination of the second and third lines to get a zero here, and so on until the end of the second column, so that's the second step. And we can imagine that in the last step, the Nth one I will end up with a linear system Ux=d as this one there, Ux=d. Thus, the matrix A was transformed in a matrix U with here 1, 0, 0,....

Notes

Summary

1m 56s



here 1,0,..., and 0 down here, until the end, at the last but one column we have 1, 0, and on the last column we have 1. So that's our matrix U, la matrice que vous obtenez du système linéaire $U\mathbf{x} = \mathbf{d}$, that we now have to solve. Solving the linear system Ux=d is easy simply because we start with the last line, and get xN=dN, then on the last but one line we get x(N-1)= d(N-1) minus this term here, which is u(N-1)N times xN, which we know and so on. So that solves easily our upper triangular linear system.

Notes

Summary

4m 11s