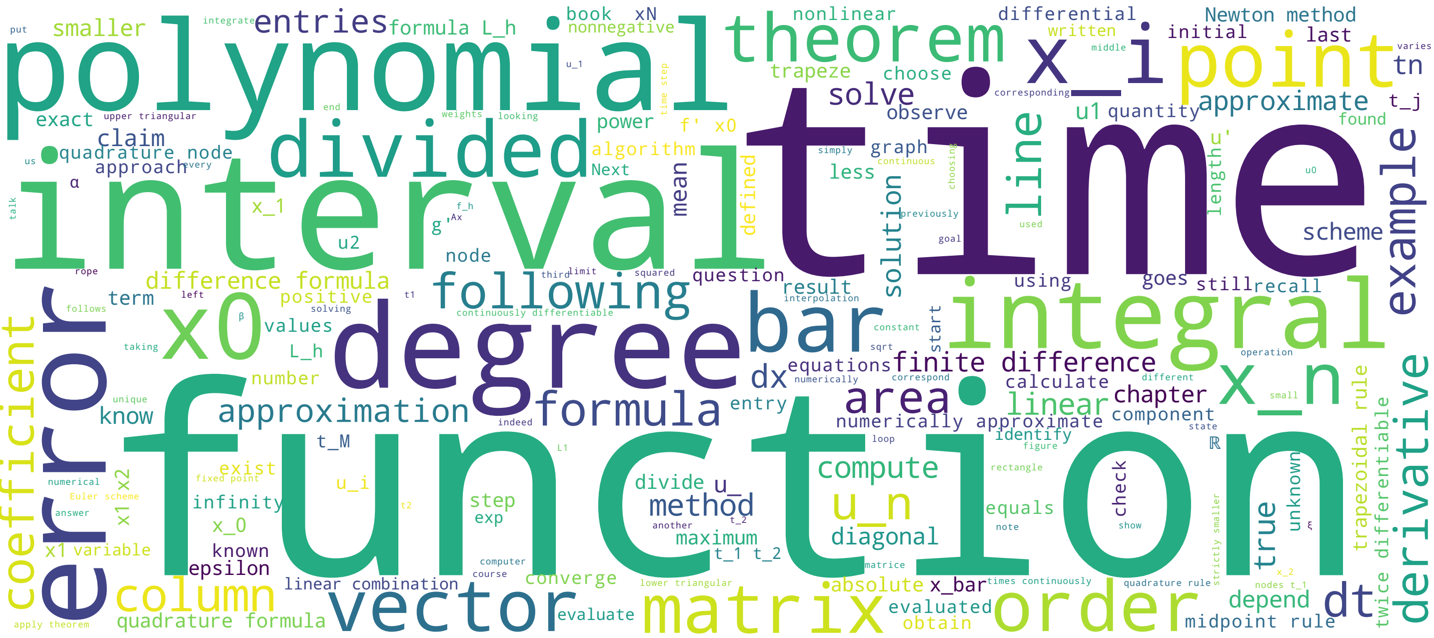

Chap 3: Formule du rectangle, du trapèze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Video

EPFL

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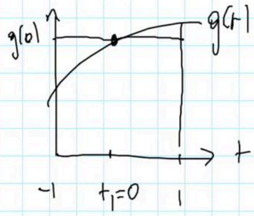
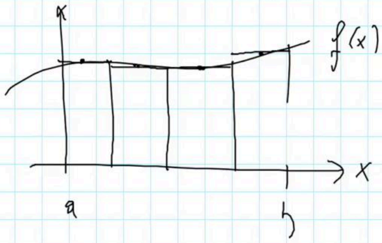
Rectangle :



$J(g) = 2g(0)$ $\quad L_h(f) = h \sum_{i=0}^{N-1} f(x_i + \frac{h}{2})$



$\int_{-1}^{1} p(t)\,dt = J(p)$ $\quad p \in \mathbb{P}_0$

Lets apply the results of theorem 3.1 to the midpoint and trapezoidal rules. et du trapèze. For the midpoint rule, I recall that the goal is to numerically approximate the integral between -1 and 1 of the function g(t) dt. The midpoint rule corresponds to choosing only one quadrature node t_1 which is 0 thus we evaluate the function g at 0 and approach the integral between -1 and 1 by the area of the rectangle of height g(0), equal to 2 times g(0), since the length of the interval is 2. Therefore the formula L_h(f) which numerically approximates the integral between a and b of f(x) dx is defined by the sum over all sub-intervals, for i from 0 to N-1, of f(x_i + h / 2). Let us illustrate the formula L_h(f). x varies from a to b, I want to integrate the function f between a and b. I divide the interval into sub-intervals of equal length, I consider the value in the middle of each sub-interval and I approximate the integral by the sum of the area of each rectangle. Clearly the formula is exact for polynomials of degree 0, that is, let p(t) be a polynomial of degree 0, the integral between -1 and 1 of p(t) dt is equal to J(p). This is true by construction.

Notes

Summary

0m 00s

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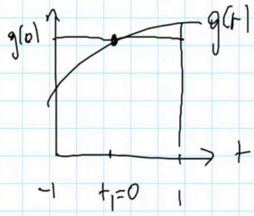
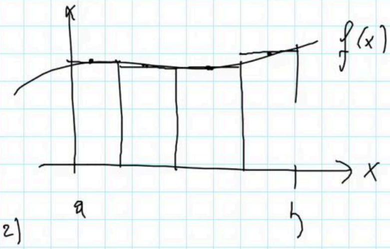
Rectangle :



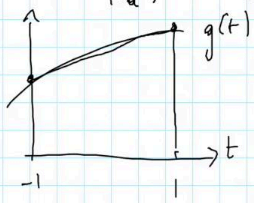
$J(g) = 2g(0)$ $\quad L_h(f) = h\sum_{i=0}^{N-1} f\left(x_i+\frac{h}{2}\right)$



$\int_{-1}^{1} p(t)\,dt = J(p) \quad \forall\, p \in \cancel{\mathbb{P}_0}\ \mathbb{P}_1$

thm 3.1 : $f \in \mathcal{C}^2[a,b]$ $\quad \left|\int_a^b f(x)\,dx - L_h(f)\right| = O(h^2)$

Trapèze :



$J(g) = g(-1)+g(1)$ $\quad L_h(f) = \frac{h}{2}\sum_{i=0}^{N-1}\left(f(x_i)+f(x_{i+1})\right)$

$\int_{-1}^{1} p(t)\,dt = J(p) \quad \forall\, p \in \mathbb{P}_1$

This is true for all polynomials of degree 0, but this is also true for all polynomials of degree 1. We should check this, but I claim this is true. Let us apply theorem 3.1. If the function f is twice differentiable on the interval [a,b], the error between the integral over [a,b] of f(x) dx and its approximation L_h(f) is of order h^2. This means that each time h is divided by 2 the error is divided by 4. For the trapezoidal rule we have a similar result. I recall the trapezoidal rule: for t varying between -1 and 1, here is the graph of the function g, and we approximate the integral by the area of the trapeze. This area is given by J(g), the area of the trapeze is g(-1) plus g(1). The formula for L_h(f) to approximate the integral between a and b of f(x) dx is this time given by h over 2 times the sum over every sub-interval, for i from 1 to N-1 of f(x_i) plus f(x_i+1). Let p be a polynomial p of degree 1 as in the figure here, I claim that the integral between -1 and 1 of p(t) dt is equal to J(p). This is false for polynomials of degree 2. For example by taking a polynomial of degree two as this one, you see immediately that the area of the trapeze is not equal to the integral between -1 and 1 of p(t) dt.

Notes

Summary

2m 15s

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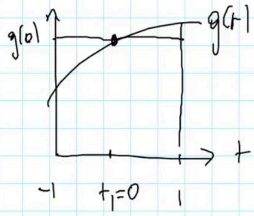
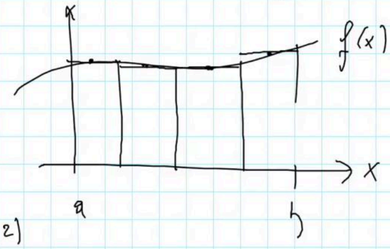
Rectangle :



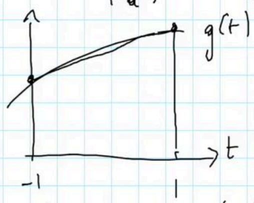
$J(g) = 2g(0)$ $\quad L_h(f) = h \sum_{i=0}^{N-1} f(x_i + \frac{h}{2})$



$\int_{-1}^{1} p(t)dt = J(p) \quad \forall\ p \in \cancel{\mathbb{P}_0}\ \mathbb{P}_1$

thm 3.1 : $f \in \mathcal{C}^2[a,b]$ $\quad \left| \int_a^b f(x)dx - L_h(f) \right| = O(h^2)$

Trapèze :



$J(g) = g(-1) + g(1)$ $\quad L_h(f) = \frac{h}{2} \sum_{i=0}^{N-1} \left(f(x_i) + f(x_{i+1})\right)$

$\int_{-1}^{1} p(t)dt = J(p) \quad \forall\ p \in \mathbb{P}_1$

thm 3.1 : $f \in \mathcal{C}^2[a,b]$ $\quad \left| \int_a^b f(x)dx - L_h(f) \right| = O(h^2).$

So this property is true for a polynomial of degree 1. By applying theorem 3.1 we obtain the same result as for the midpoint rule. If f is twice differentiable on the interval [a,b], the error between the integral over a and b of f(x) dx with the formula L_h(f) found here, is also of order h to the power 2. This means that each time h is divided by 2 then the error is divided by 4.

Notes

Summary

4m 35s