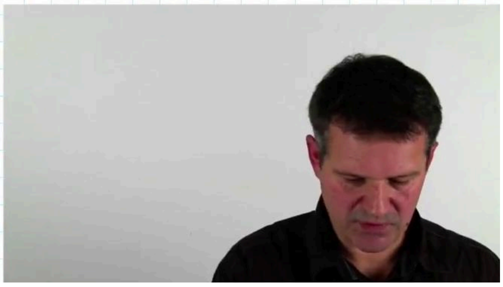
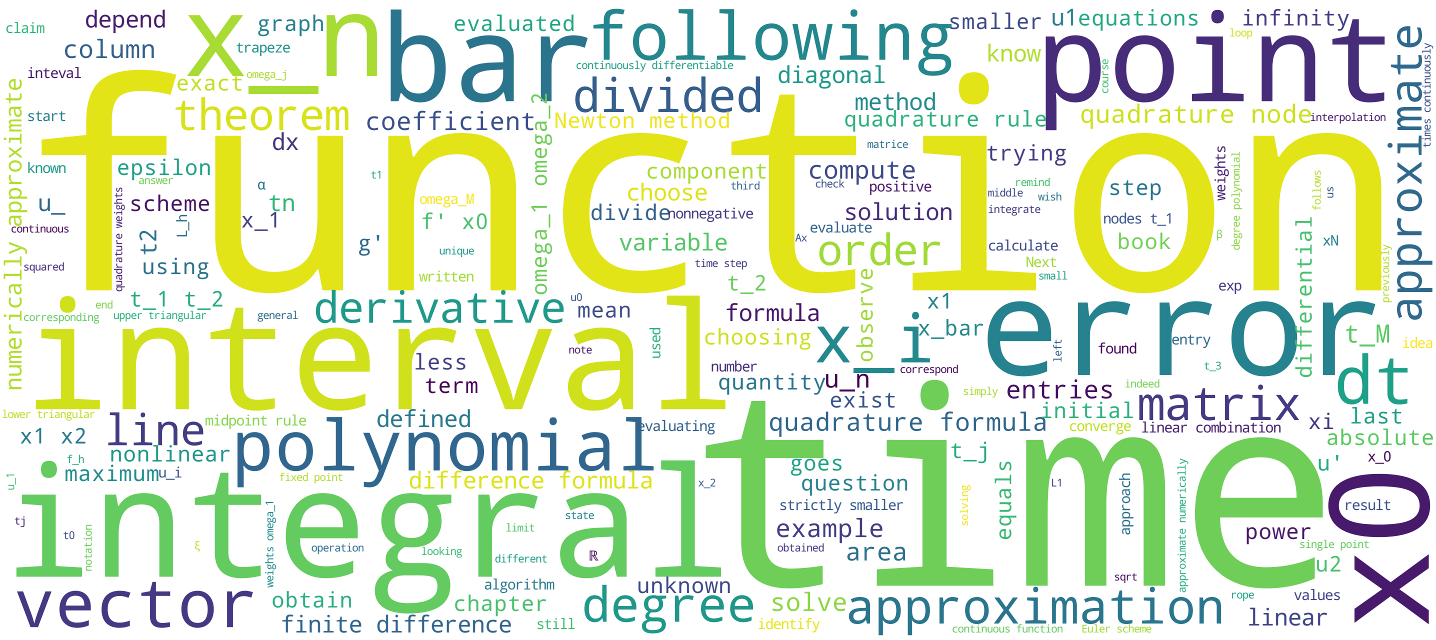
Chap. 3 : généralités (suite)





Video

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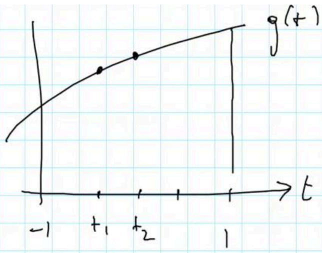

$g: [-1,1] \to \mathbb{R}$ cont. approcher num $\int_{-1}^{1} g(t)\,dt$

Formule de quadrature: M entier pos ($M = 1, 2, 3, \dots$)

$-1 \le t_1 < t_2 < \dots < t_M \le 1$ points d'intégr.

$\omega_1 \quad \omega_2 \quad \dots \quad \omega_M$ poids

$J(g) = \omega_1 g(t_1) + \omega_2 g(t_2) + \dots + \iota \;\, g(t_M)$

We are trying to solve the following problem: given a continuous function g on the inteval [-1,1] we wish to numerically approximate the integral between -1 and 1 of g(t) dt. The variable t varies between -1 and 1, here is the graph of g(t), this is the integral we are trying to approximate. We will now define what a quadrature formula is, quadrature formula to numerically approximate the integral between -1 and 1 of g(t) dt. We define an integer M, in general M has value 1,2 or 3, occasionally 4 or 5 but rarly more. We define quadrature nodes, values t1 strictly smaller then t2 and so on up to t_M which are in the interval [-1,1]. Here is a the quadrature node t2. The idea is to evaluate the function g at these points t_1, t_2, t_M. We shall also define the weights, named omega_1, omega_2 and so on until omega_M, which we call quadrature weights. We want to approximate the integral between -1 and 1 of g(t) dt by a quadrature rule, which is obtained by evaluating the function g in the node t1 the function g in t_2 and so on until evaluating g in t_M. Again g in t_2, g in t_3 up to g in t_M. Then you multiply these values by the weights omega_1, omega_2 until omega_M.

Notes

Summary

0m 01s

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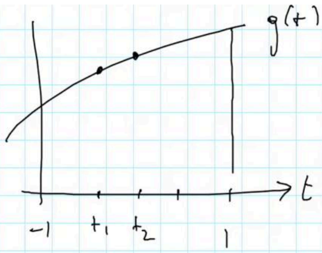
$g : [-1,1] \to \mathbb{R}$ cont. approcher num $\int_{-1}^{1} g(t)\,dt$



Formule de quadrature: M entier pos ($M = 1, 2, 3, \dots$)

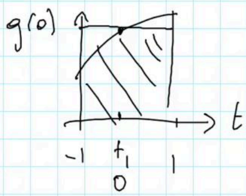
$-1 \le t_1 < t_2 < \dots < t_M \le 1$ points d'intégr.

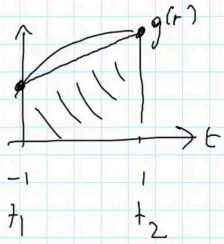
$w_1 \quad w_2 \quad \dots \quad w_M$ poids

$$J(g) = w_1 g(t_1) + w_2 g(t_2) + \dots + w_M g(t_M)$$

Ex: rectangle : $M = 1$ $t_1 = 0$ $J(g) = 2g(0)$

trapèze : $M = 2$ $t_1 = -1$ $t_2 = 1$ $J(g) = g(-1) + g(1)$





This will be the approximation of the integral between -1 and 1 of g(t) dt. For example one can use the midpoint rule (also called rectangle rule) which consists in the following: for a function g which you want to integrate over the interval -1 and 1 and you can decide to take only one single point. So, choosing one single point M = 1, then the point is t1=0, the middle of [-1,1]. Hence I will approximate the integral on [-1,1] of g(t) dt by the area of the corresponding rectangle. Since the rectangle has width 2 and height g(0), the area will be 2 times g(0). This defines J(g) for the midpoint rule, again J(g) = 2 g(0). Another example is the trapezoidal quadrature rule. In this case I will consider two quadrature nodes. By choosing two quadrature nodes, here the variable t in the interval [-1,1], I am trying to numerically approximate the integral over -1 and 1 of g(t) dt. I will choose for both quadrature nodes: t1 = -1 and t2= 1, now I can approximate the area of the trapeze which is hashed here. So for M = 2, t1 = -1 and t2 = 1. The area of the trapeze is given by g(-1) plus g(1), which is the area of this rectangle, plus the area of the triangle above the rectangle. Voilà.

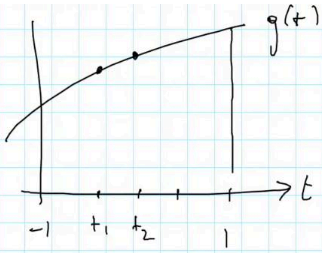
Notes

Summary

2m 19s

Chap. 3 : généralités (suite)

$g:[-1,1]\to\mathbb{R}$ cont. approcher num $\int_{-1}^{1} g(t)\,dt$

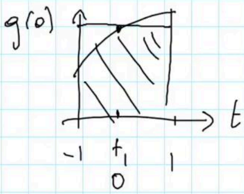


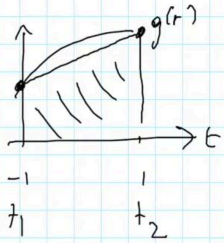
Formule de quadrature: M entier pos (M = 1, 2, 3, ...)

$-1\le t_1<t_2<\dots<t_M\le 1$ points d'intégr.

$\omega_1 \quad \omega_2 \quad \dots \quad \omega_M$ poids

$$J(g)=\omega_1 g(t_1)+\omega_2 g(t_2)+\dots+\omega_M g(t_M)$$





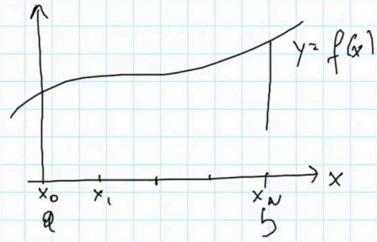
Ex: rectangle : M = 1 $\quad t_1=0 \quad J(g)=2g(0)$

trapèze : M = 2 $\quad t_1=-1 \quad t_2=1 \quad J(g)=g(-1)+g(1)$

Retour à f :

$$\int_a^b f(x)\,dx = \frac{h}{2}\sum_{i=0}^{N-1}\int_{-1}^{1} f\left(x_i+h\,\frac{t+1}{2}\right)dt$$

$$\simeq \frac{h}{2}\sum_{i=0}^{N-1}\sum_{j=1}^{M} \omega_j\, f\left(x_i+h\,\frac{t_j+1}{2}\right)$$



Returning to the initial problem: I remind you that we we want to compute the integral over [a,b] of f(x) dx, the notations are: the variable x is in the interval [a,b] and we are trying to approximate numerically the integral of the function f between a and b. Here is the graph of the function f and we divide the interval [a,b] into sub-intevals. Therefore the integral is equal to h/2 times the sum for 0 to M-1 of the integrals between -1 and 1 of f(x_i +h (t+1) / 2) dt. This was the result we reached previously, and here are the nodes x_0,x_1 up to x_N. The idea is to use this quadrature formula to approximate the function g, defined on each interval, by g(t)= f(xi + h(t+1) / 2) and we then have the following numerical approximation: for h divided by 2, we have the sum over all the intervals xi, all of these intervals here, first, second,third, fourth, and now you have the sum omega_1 times g(t_1), omega_2 times g(t_2) and so on until omega_M times g(t_M), which we can write as the sum for j from 1 to M of omega_j, which can be found here, omega_1, omega_2, etc, times the function f here, evaluated at t_j. Therefore you now have f(xi + h (tj+1)/2). So here is the approximation of this integral.

Notes

Summary

4m 27s

Chap. 3 : généralités (suite)

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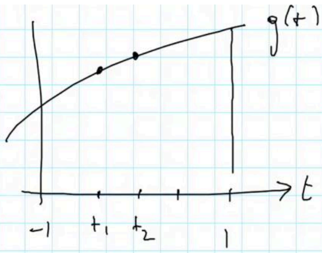

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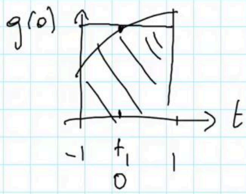
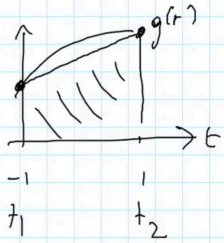
$$J(g) = w_1 g(t_1) + w_2 g(t_2) + \dots + w_M g(t_M)$$

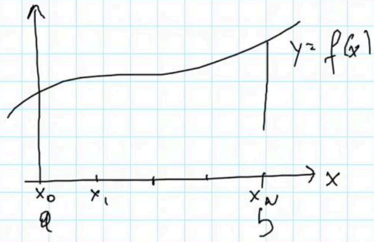
Ex : rectangle : $M = 1 \quad t_1 = 0 \qquad J(g) = 2g(0)$

trapèze : $M = 2 \quad t_1 = -1 \quad t_2 = 1 \qquad J(g) = g(-1) + g(1)$





Retour à f :
$$\int_a^b f(x)\,dx = \frac{h}{2} \sum_{i=0}^{N-1} \int_{-1}^{1} f\left(x_i + h\,\frac{t+1}{2}\right) dt$$
$$\simeq \frac{h}{2} \sum_{i=0}^{N-1} \sum_{j=1}^{M} w_j\, f\left(x_i + h\,\frac{t_j+1}{2}\right) = L_h(f)$$



Questions : • choix de $t_1 t_2 \dots t_M$? $w_1 w_2 \dots w_M$?

• erreur entre $\int_a^b f(x)\,dx$ et $L_h(f)$?

I obtain this approximation here I have used this quadrature formula defined here. This quantity here, which I will denote L_h(f) is the approximation of the integral between a and b of f(x) dx, by using this quadrature rule, defined by a certain number of nodes and weights. Now the obvious questions that arise are the following, there are at least two: how do I choose the quadrature nodes t_1, t_2 and so on until t_M with their respective weights, and the second question is what error am I making when I approximate the integral between a and b of f(x) dx, using this new quantity that I have defined to approximate this integral, namely L_h (f).

Notes

Summary

6m 44s