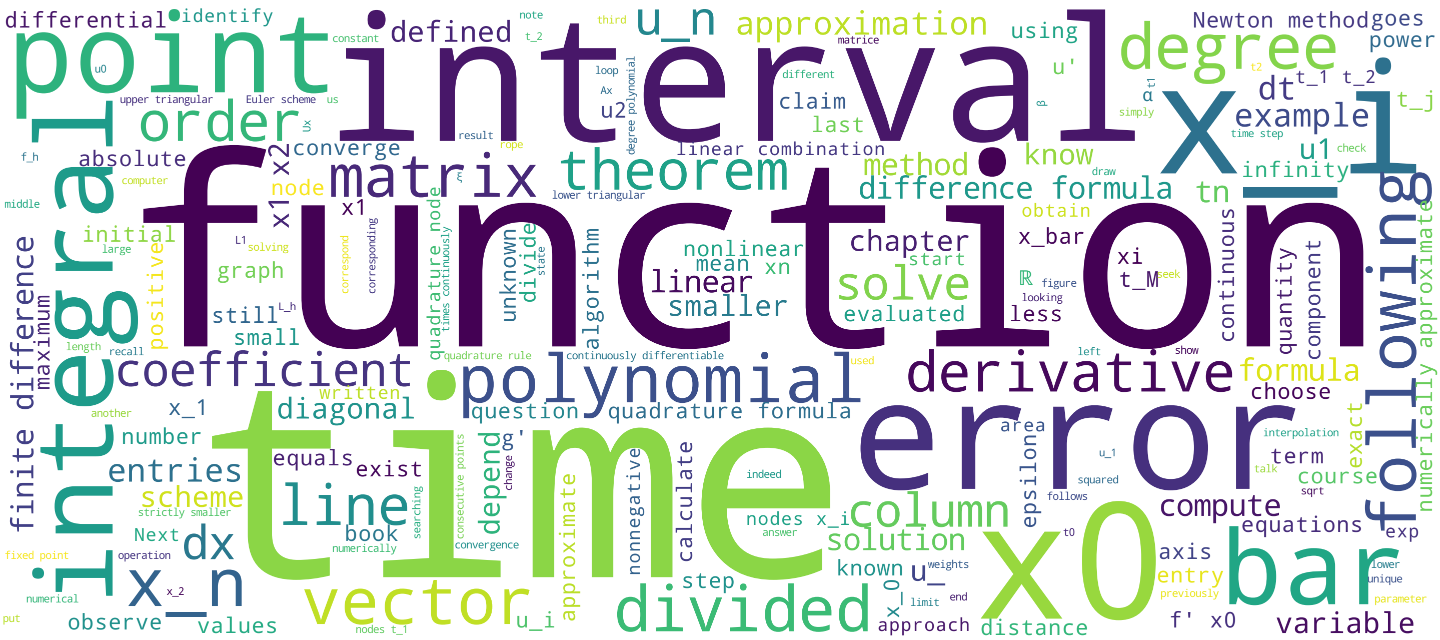

Chap. 3 : généralités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Video

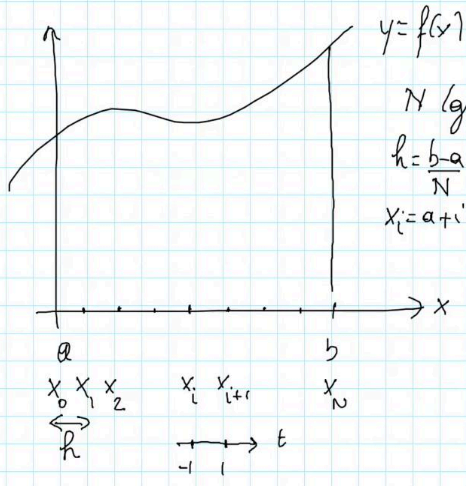
EPFL

Chap. 3 : généralités

$f : [a,b] \to \mathbb{R}$ continue.

Approcher num. $\int_a^b f(x)\,dx$

$$\int_a^b f(x)\,dx = \sum_{i=0}^{N-1} \int_{x_i}^{x_{i+1}} f(x)\,dx$$



$y = f(x)$

N (grand)

$h = \frac{b-a}{N}$ (petit)

$x_i = a + i h$

$i : 0, 1, \ldots, N$

The problem we want to solve is the following: for a given function f defined on the interval [a,b], we want to numerically approximate the integral between a and b of f(x) dx. If a draw a figure, given the x axis and the interval [a,b], the graph of the function f, and I am searching for the area between the x axis and the curve f. Firstly I divide the interval [a,b] into sub-intervals of equal length. Here I have the first point, which I name x0, the last point is xn and between them you have the points x1, x2 and so on and in the middle you have the nodes x_i followed by x_i+1. All these points are separated by a parameter, a distance which I will name h. So we have N the number of sub-intervals which should be large, h the distance between two consecutive points equal to (b-a)/N which should be small and the nodes x_i equal to a plus i times h for i between 0 up to N. Hence the integral between a and b of f(x) dx is nothing but the sum over all the sub-intervals from 0 to N-1, of the integrals between x_i and x_i+1 of f(x) dx. Now I will do a change of variable which will take me from [x_i,x_i+1] to [-1,1] and the variable is now t.

Notes

Summary

0m 00s

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$f : [a,b] \to \mathbb{R}$ continue.

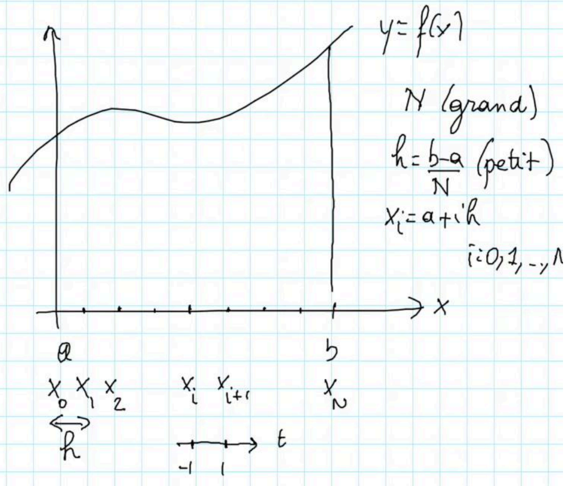
Approcher num. $\int_a^b f(x)\,dx$

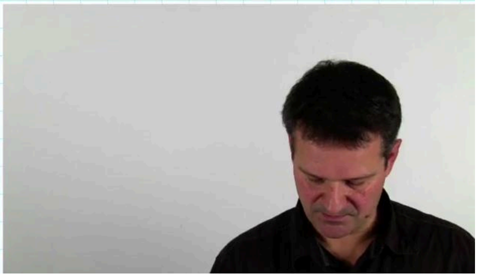
$$\int_a^b f(x)\,dx = \sum_{i=0}^{N-1} \int_{x_i}^{x_{i+1}} f(x)\,dx = \sum_{i=0}^{N-1} \frac{h}{2} \int_{-1}^{1} f\left(x_i + h\,\frac{t+1}{2}\right) dt.$$

$$x = x_i + h\,\frac{t+1}{2} \qquad dx = \frac{h}{2}\,dt$$

Pbm : $g : [-1,1] \to \mathbb{R}$ cont.

Approcher num. $\int_{-1}^{1} g(t)\,dt.$





Hence I decide that x is equal to xi + h times t+1 divided by 2, such that when t is equal to -1, well x = x_i and when t is equal to 1 well x = xi + h, also equal to x_i+1. Of course we have that dx is equal to h over 2 times dt therefore the integral between a and of f(x) dx is equal to the sum for i between 0 and N-1 of h divided by 2, which can be moved outside of the sum and we now have the integral between -1 and 1 evaluated in [xi+h (t+1)/2] dt. So now we have the following problem which involves integrals on [-1,1]. The problem to be addressed is now the following. Given a function g defined now on [-1,1] which is continuous, I seek to numerically approximate the integral between [-1,1] of g(t) dt.

Notes

Summary

2m 09s