Dérivées numériques d'ordre 1 – Formule de différences finies rétrograde

Introduction à l'analyse numérique

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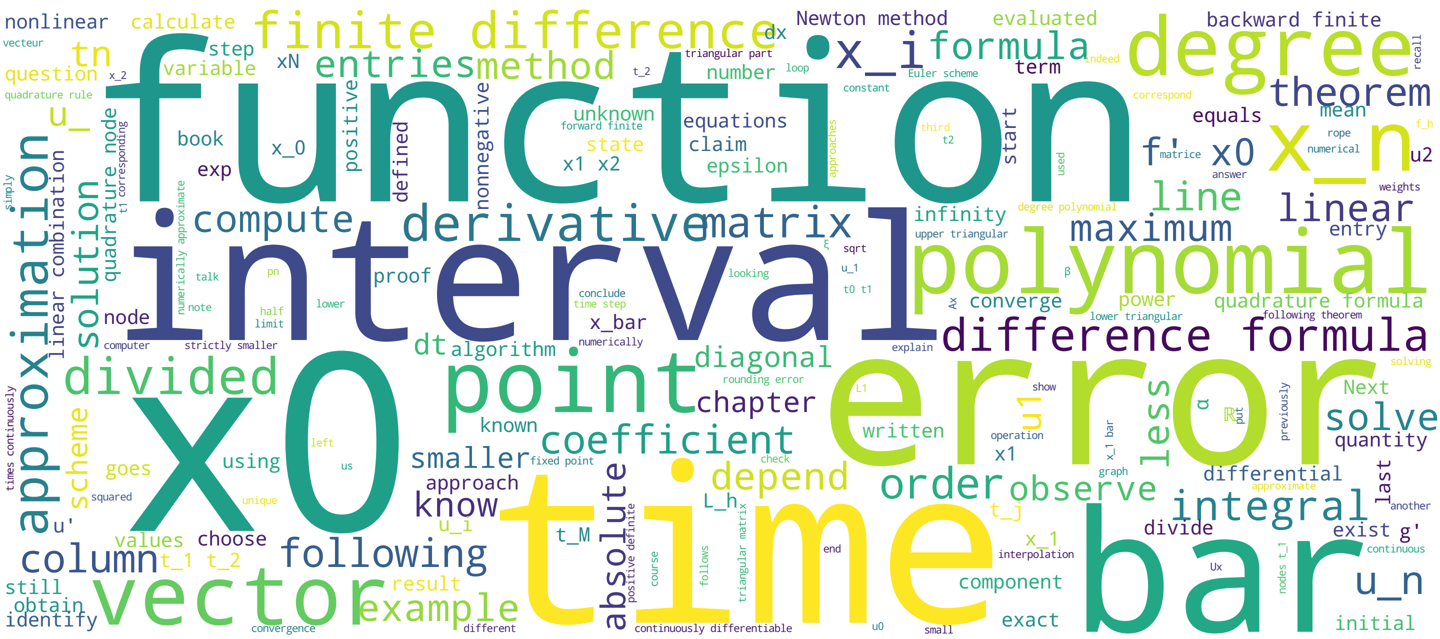


Video

Dérivée d'ordre 1 : Formule de diff. finie rétrograde

$$\left| f'(x_0) - \frac{f(x_0)-f(x_0-h)}{h} \right| = O(h)$$

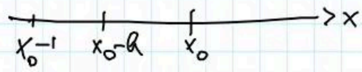
Thm 2.1 : $\forall f \in C^2 \ \forall x_0 \in \mathbb{R} \ \exists C>0 \ \forall 0<h\leq 1$ on a $\left| f'(x_0) - \frac{f(x_0)-f(x_0-h)}{h} \right| \leq Ch.$

Rem : C dép de f, x_0, mais pas de h.

Interp : fixe f, x_0 l'erreur est divisée par $\begin{smallmatrix}2\\10\end{smallmatrix}$ si h est divisé par $\begin{smallmatrix}2\\10\end{smallmatrix}$

Dem : $f(x_0-h) = f(x_0) - h f'(x_0)$ .....

$$C = \frac{1}{2} \max_{x_0-1 \leq x \leq x_0} |f''(x)|$$



We'll now explain the following result about the error between f'(x0) and its approximation by a backward finite difference formula so (f(x0) - f(x0-h))/h We now state the following theorem, Theorem 2.1 in the case of the bacward finite difference formula. For all function f and for all x0 in ℝ there is a positive c such that for all h between 0 and 1 the error f'(x0) minus its approximation by a backward finite difference formula is as before less than or equal to c*h As before because of the order of quantifiers, c can only depend on f and x0 but not h. As before we interpret the result as follows : choose a function f, and some x0 then the error, f(x0) minus its approximation with the backward finite difference formula is divided by 2 or by 10 if h is divided by 10. I leave the proof as an exercise it is similar to the previous proof we start by writing f(x0-h)=f(x0)-h*f'(x0)+ etc and to conclude we must find the c in the theorem that can depend on f and x0 but not h and this x will be the absolute value of half the maximum of the second derivative because we have x0 and x0-h and h is less than or equal to 1, so here lies x0-1 then the maximum is between x0-1 and x0 you can try numerically and observe the same phenomenon as for the forward finite difference formula.

Notes

Summary

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