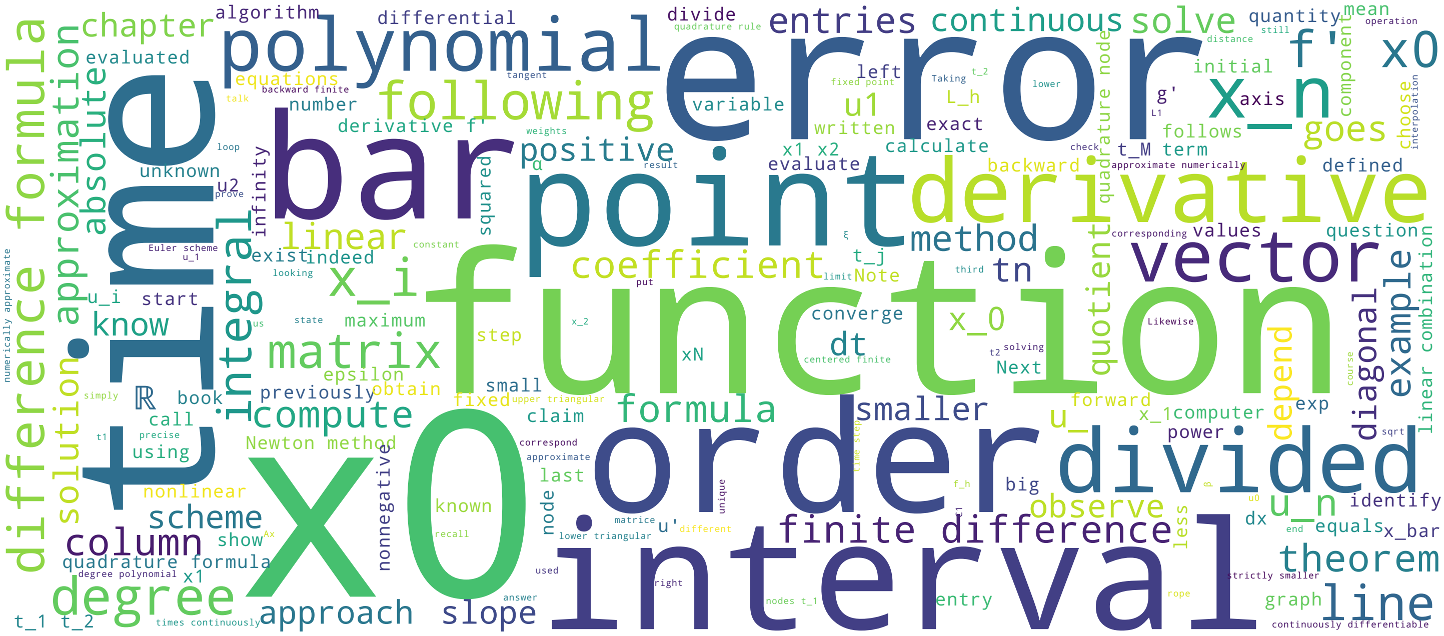
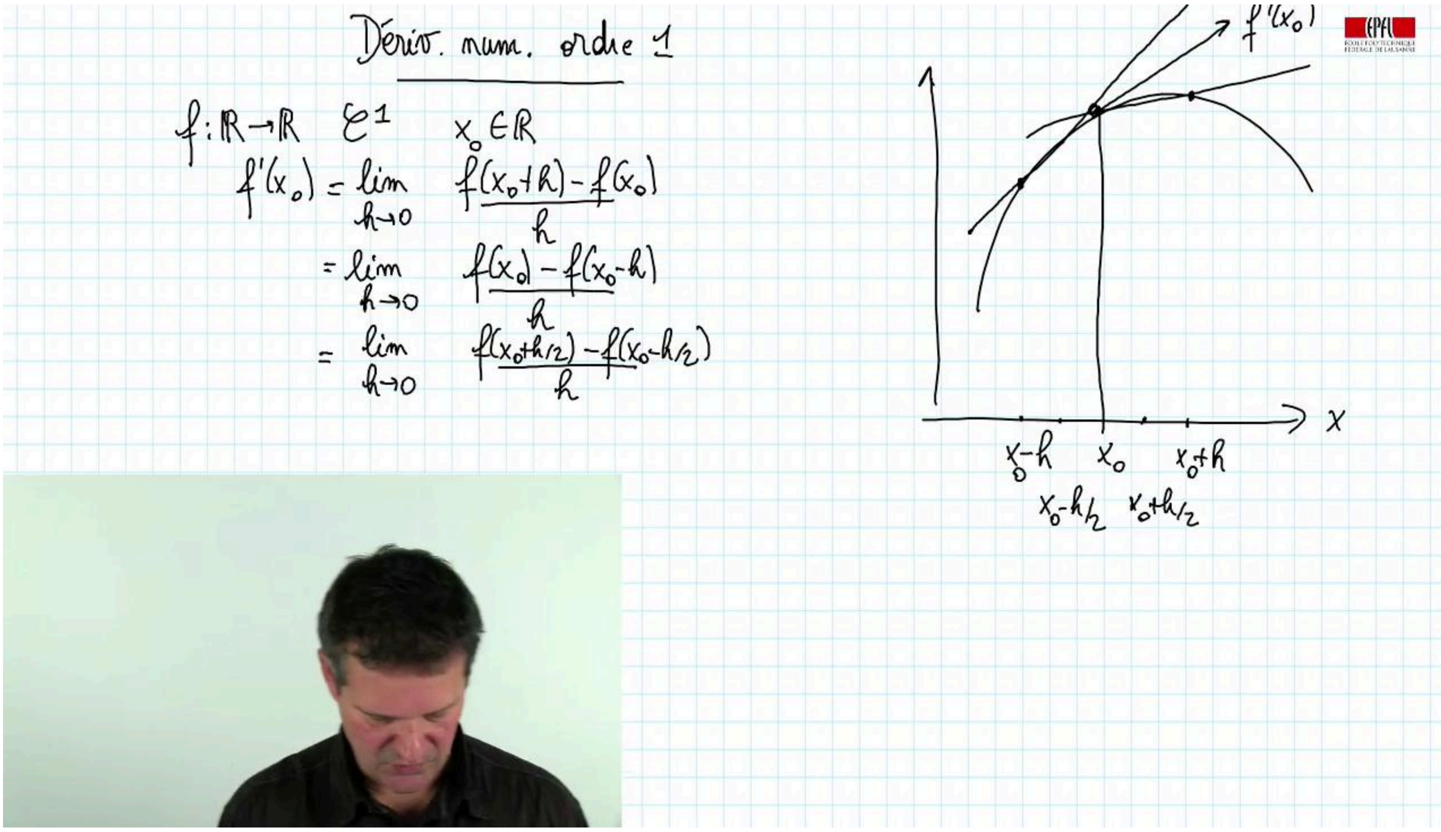
Dérivées numériques d'ordre 1 – Position du problème

Introduction à l'analyse numérique
Prof. Marco Picasso



Video



Let's start with first-order numerical differentiation. Given a function f from ℝ to ℝ that is C1, that is that the function is continuous and its derivative f' is continuous, given x0 in ℝ, we want to approximate numerically f'(x0). The definition of f'(x0) is as follows : take f(x0+h)-f(x0), with h positive and divide by h when h goes to 0 then this quotient converges to the derivative f'(x0). The corresponding drawing is on the right handside. There is the x axis, the graph of f and a point x0. And here x0+h and f(x_0), here is the quotient. The slope of this line is the quotient (f(x0+h)-f(x0))/h. f'(x0) is the slope of the tangent so observe that when h goes to 0 the slope of this line converges to the slope of this tangent. Another possible definition is to take the quotient f(x0)-f(x0-h) divided by h and let h go to 0. so now we take a value on the left, f(x0-h) to get another approximation of the derivative. Finally, we can take the following formula : Still letting h go to 0, take f(x0+h/2) here on the graph. and take its value in x0-h/2 as well so f(x0+h/2)-f(x0-h/2) divided by h and let h go to 0. Now consider a fixed h, positive.

Notes

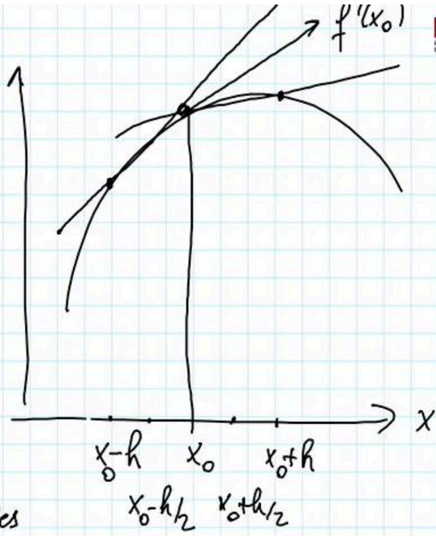
Summary

0m 03s

Dériv. num. ordre 1

$f : \mathbb{R} \to \mathbb{R}$ $\mathcal{C}^1$ $x_0 \in \mathbb{R}$

$$f'(x_0) = \lim_{h \to 0} \frac{f(x_0+h) - f(x_0)}{h}$$
$$= \lim_{h \to 0} \frac{f(x_0) - f(x_0-h)}{h}$$
$$= \lim_{h \to 0} \frac{f(x_0+h/2) - f(x_0-h/2)}{h}$$



$h > 0$ fixé (petit)

$$\left| f'(x_0) - \frac{f(x_0+h) - f(x_0)}{h} \right| = O(h)$$ Formule de Diff. finies progressive ordre 1 en h

$$\left| f'(x_0) - \frac{f(x_0) - f(x_0-h)}{h} \right| = O(h)$$ FDF rétrograde ordre 1 en h

$$\left| f'(x_0) - \frac{f(x_0+h/2) - f(x_0-h/2)}{h} \right| = O(h^2)$$ F

we compute our formulas with a computer so we want this fixed positive h to be small. We'll approach f'(x0) by the quotient (f(x0+h)-f(x0))/h and I look at the error. The error is the absolute value of the difference f'(x0)-(f(x0+h)-f(x0))/h. We'll show that this error is of order 1 in h. So we'll give a precise definition of this big O of h, O(h). This is a finite difference formula. It is a forward difference. Why do we call it forward ? Well, because we want to evaluate the derivative at x0 and I use the value at x0+h so I take a value forward. It is of order 1 so its precision is of the order 1 in h. Likewise, I can approach the derivative f'(x0) with h fixed with a backward difference formula (f(x0)-f(x0-h))/h. Taking the absolute value, we get the error. as previously this error is of order 1 in 0. So this time this is a backward finite difference formula. Why backward ? Because to approach the derivative at x0 I use the value in x0-h so we go backward. Finally, we'll now use the centered difference formula. So f(x0+h/2)-f(x0-h/2) divided by h. Note that I divided by h because the distance between x0+h/2 and x0-h/2 is indeed h. So here is the error now, and this error is not of order h anymore, but of order h^2.

Notes

Summary

2m 15s

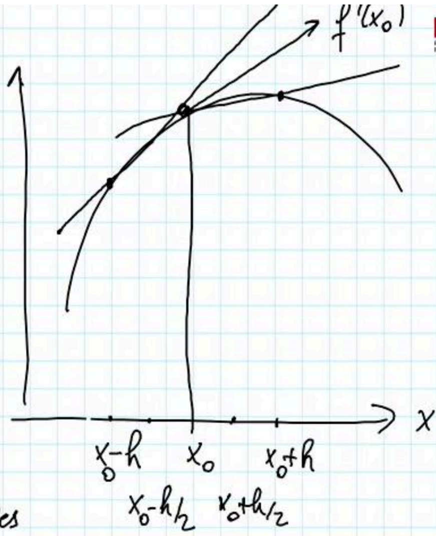
Dériv. num. ordre 1

$f:\mathbb{R}\to\mathbb{R}$ $\quad \mathcal{C}^1 \quad x_0\in\mathbb{R}$

$$f'(x_0) = \lim_{h\to 0} \frac{f(x_0+h)-f(x_0)}{h}$$

$$= \lim_{h\to 0} \frac{f(x_0)-f(x_0-h)}{h}$$

$$= \lim_{h\to 0} \frac{f(x_0+h/2)-f(x_0-h/2)}{h}$$



$h>0$ fixé (petit)

$$\left| f'(x_0) - \frac{f(x_0+h)-f(x_0)}{h} \right| = O(h)$$ Formule de Diff. finies progressive ordre 1 en h

$$\left| f'(x_0) - \frac{f(x_0)-f(x_0-h)}{h} \right| = O(h)$$ FDF rétrograde ordre 1 en h

$$\left| f'(x_0) - \frac{f(x_0+h/2)-f(x_0-h/2)}{h} \right| = O(h^2)$$ FDF centrée ordre 2 en h

This is a centered finite difference formula. Why centered? Because I want to approach the derivative at x0 and I use the values on the left and the right, x0-h/2 and x0+h/2 and this centered formula is of order 2 in h. so it is a big O of h squared, O(h^2). In the following, we'll prove and give a clear definition of those O(h) and O(h^2).

Notes

Summary

4m 38s