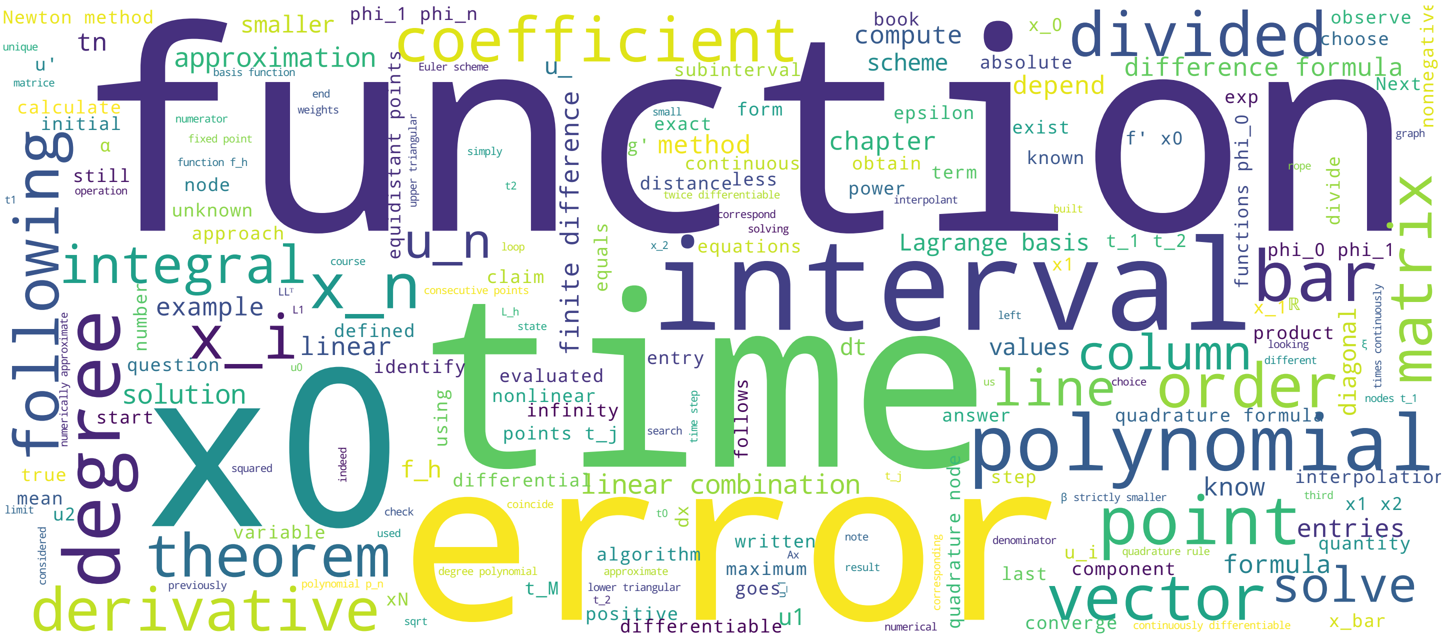
Résumé

Introduction à l'analyse numérique

Prof. Marco Picasso



Video

Résumé Chap 1 interpolation

- $p \in \mathbb{P}_n$ tq $p(t_j) = p_j$ $\quad j = 0, 1, \dots, n$

$$p(t) = \sum_{j=0}^{n} p_j \, \varphi_j(t)$$

$\varphi_0 \varphi_1 \varphi_2 \dots \varphi_n$ base de Lagr. de $\mathbb{P}_n$ ass. aux pts $t_0 t_1 t_2 \dots t_n$

$$\varphi_k(t) = \prod_{\substack{j=0 \\ j \neq k}}^{n} \frac{t - t_j}{t_k - t_j}$$

- $f : [a,b] \to \mathbb{R}$

$$p_n(t) = \sum_{j=0}^{n} f(t_j) \varphi_j(t)$$

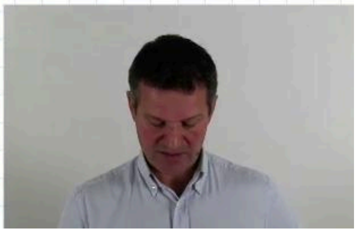
t_j équidist $\quad t_j = a + \frac{b-a}{n} j \quad j = 0, 1, \dots, n$

$p_n \xrightarrow{n \to \infty} f$? dépend de f

- interpol. intervalles $\quad f : [a,b] \to \mathbb{R} \quad f_h \in \mathcal{C}^0[a,b]$

degré 1 $\quad |f - f_h| = O(h^2)$

— 2 $\quad |f - f_h| = O(h^3)$.



A short summary of this first chapter on interpolation. The problem we tried to solve was the following: we search for a polynomial of degree n such that p(tj)= pj. The values tj and pj are given for all indexes j = 0, 1,..., n. The solution to the problem can be written as a linear combination of the functions phi_0, phi_1,..., phi_n which form the Lagrange basis of polynomials of degree n and depend on the choice of points t_0, t_1,..., t_n. Hence the solution is a linear combination of these functions phi_0, phi_1,... phi_n, and the coefficients of the linear combination are the values given by the p_j. So we have p(t) equal to the sum over j = 0,... n of the product of p_j times phi_j(t). The k-th basis function of the Lagrange basis is a polynomial of degree n which must cancel out for all points, except for j equal to k. The numerator is monomials of the form (t - t_j), and the denominator is composed of monomials (t_k - t_j) such that phi_k(t_k) is equal to 1. That must be understood for the exam. Next we considered a function f defined on a continuous interval [a,b]. We built the polynomial of degree n which coincides with the function f in a given number of points t_j equidistant on the interval [a,b].

Notes

Summary

0m 00s

Résumé Chap 1 interpolation

- $p \in \mathbb{P}_n$ tq $p(t_j) = p_j \quad j = 0, 1, \dots, n$

$$p(t) = \sum_{j=0}^{n} p_j \, \varphi_j(t) \qquad \varphi_0 \varphi_1 \varphi_2 \dots \varphi_n \text{ base de Lagr. de } \mathbb{P}_n \text{ ass. aux pts } t_0 t_1 t_2 \dots t_n$$

$$\varphi_k(t) = \prod_{\substack{j=0 \\ j \neq k}}^{n} \frac{t - t_j}{t_k - t_j}$$

- $f : [a,b] \to \mathbb{R}$

$$p_n(t) = \sum_{j=0}^{n} f(t_j)\, \varphi_j(t) \qquad t_j \text{ équidist} \quad t_j = a + \frac{b-a}{n} j \quad j = 0, 1, \dots, n$$

$p_n \xrightarrow{n \to \infty} f$? dépend de f

- interpol. intervalles $f : [a,b] \to \mathbb{R}$ $\quad f_h \in \mathcal{C}^0[a,b]$

degré 1 $\quad |f - f_h| = O(h^2)$

— 2 $\quad |f - f_h| = O(h^3)$.

Again, we have p_n a linear combination of these fixed functions, the functions forming the Lagrange basis. The coefficients of the linear combination are the values of the function f evaluated at points t_j. As n approaches infinty, we want to know if the polynomial p_n approaches the function f ? The answer is given by a theorem which depends on the derivative of order n+1 of the function f. We have considered another problem : interpolation by intervals. We built f_h, the interpolant by interval of the function f, with equidistant points. h is the distance between two consecutive points which will approach zero. So f_h is a function which is continuous, but not differentiable. This function coincides with the function f at these equidistant points and the function f_h is a polynomial of degree 1 on each subinterval. In this case the error between f and f_h is an error which follows h^2, this is true only if f is twice differentiable. Now if you consider an interpolant f_h of degree 2 on each subinterval, well you get that the distance between f and f_h is of order h^3, so the error is divided by 2^3 = 8 every time h is divided by two.

Notes

Summary

1m 33s