Interpolation de degré 2 par intervalles

Introduction à l'analyse numérique

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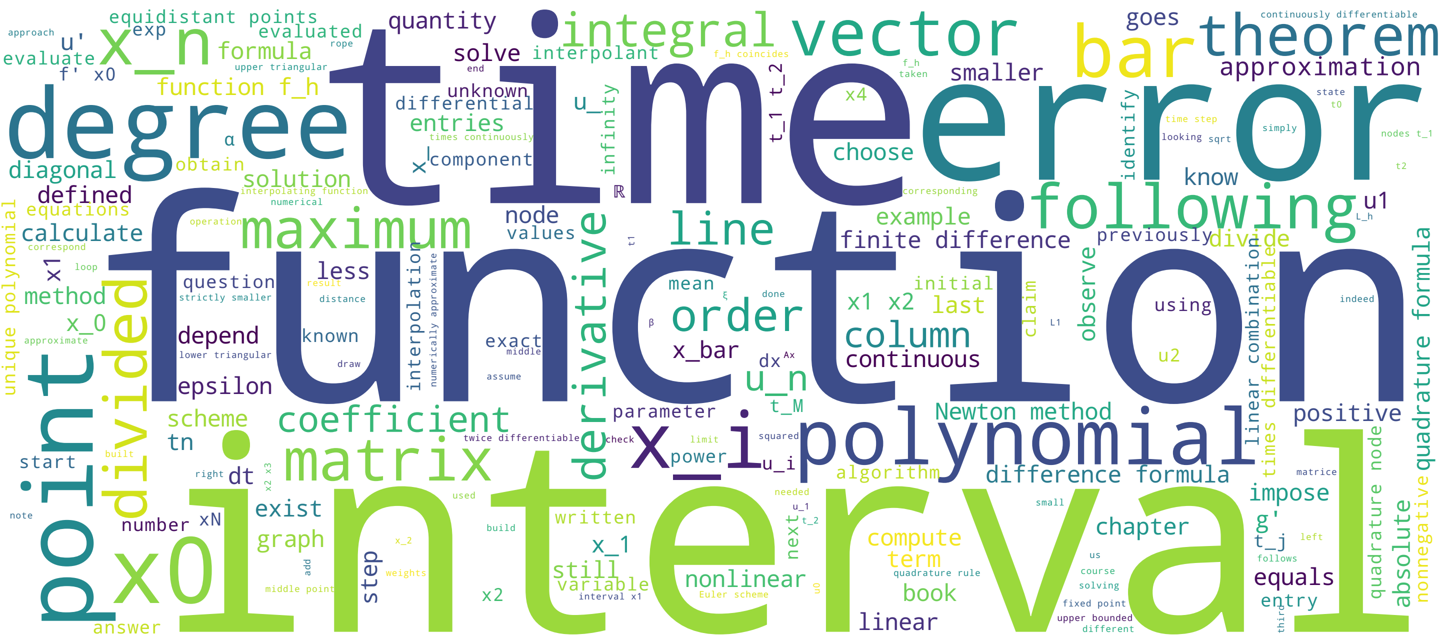
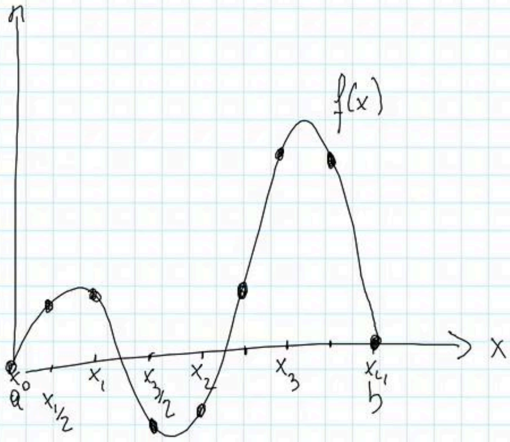


Video

Interpol degré 2 par intervalle :



$f : [a,b] \to \mathbb{R}$

$x_i = a + \underbrace{\frac{b-a}{N}}_{h} i \qquad i = 0, 1, \dots, N$

$f_h \in \mathcal{C}^0[a,b]$

$f_h(x_i) = f(x_i) \qquad i = 0, 1, \dots, N$

$f_h(x_{i+1/2}) = f(x_{i+1/2}) \qquad i = 0, 1, \dots, N-1$

$f_h|[x_i, x_{i+1}]$

This time we will consider the interpolation by interval of degree 2. As previously, we take the function f defined on the interval [a,b] in R and consider the equidistant points x_i on the interval [a,b] such that x_i = a + (b-a)/N * i for i = 0, 1,..., N. The parameter h is still the (b-a)/N. Again the graph of the function f on the interval [a,b]. I take five equidistant points x0, x1, x2, x3 and x4. The graph of the function f. I will build the function f_h which is continuous on the interval [a,b]. We must impose that this function f_h coincides with the function f at the points xi: f_h(x_i) = f(x_i) for i = 0,1,..., N. I will add one extra constraint, I will impose that the function f_h coincides with the function f in the middle of the sub-intervals: x_(i+1/2) for i = 0, 1,... N-1. These x_(i+1/2) points are the middle points of the sub-intervals x_i to x_(i+1). So x_(1/2) is the middle point of the interval x_0 to x_1. So the functions f_h and f must be equal on these intermediate points. Finally, I impose that the function f_h is a polynomial of degree 2 on each sub-interval x_i to x_(i+1) passing through the three points of each sub-interval.

Notes

Summary

0m 00s

Interpol degré 2 par intervalle :

$f : [a,b] \to \mathbb{R}$

$x_i = a + \frac{b-a}{N} i \quad i = 0, 1, \ldots, N$, with $h = \frac{b-a}{N}$

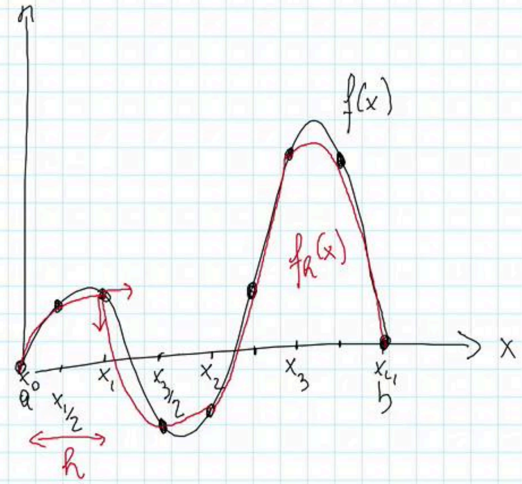
$f_h \in \mathcal{C}^0[a,b]$

$f_h(x_i) = f(x_i) \quad i = 0, 1, \ldots, N$

$f_h(x_{i+1/2}) = f(x_{i+1/2}) \quad i = 0, 1, \ldots, N-1$

$f_h|[x_i, x_{i+1}] \in \mathbb{P}_2 \quad i = 0, 1, \ldots, N-1$

$f_h \xrightarrow[N \to +\infty]{h \to 0} f$?



Thm 1.'

Previously we imposed that the function f_h was a polynomial of degree 1. Here is the interpolant of degree 2, this for all intervals for i = 0, 1,...,N-1. Now I can draw this interpolant. On the first interval x1 to x2, we have three points. This is the unique polynomial of degree 2 through these three points. On the interval x1 to x2 there are also three points. Again this is the only polynomial of degree 2 through these points. You can observe that the left derivative isn't equal to the derivative to the right at point x1. So the function is continuous, but not once differentiable. By moving on to the next interval x2 to x3, we get the unique polynomial of degree 2. Finally, on the interval x3 to x4 we also have the unique polynomial of degree 2 passing through these three points. I have now built my interpolating function of degree 2 by intervals. Again I must evaluate the error. Here is the maximum of the error. Will this error approach 0 as the parameter h, still the distance between two points, approaches 0 ? So does f_h converge, somehow, towards f as h approaches 0, equivalent to, as N approach +infinity. The answer is yes, given by theorem 1.2 of the book.

Notes

Summary

2m 22s

Interpol degré 2 par intervalle :

$f:[a,b] \to \mathbb{R}$

$x_i = a + \frac{b-a}{N} i \quad i=0,1,\dots,N$, $h = \frac{b-a}{N}$

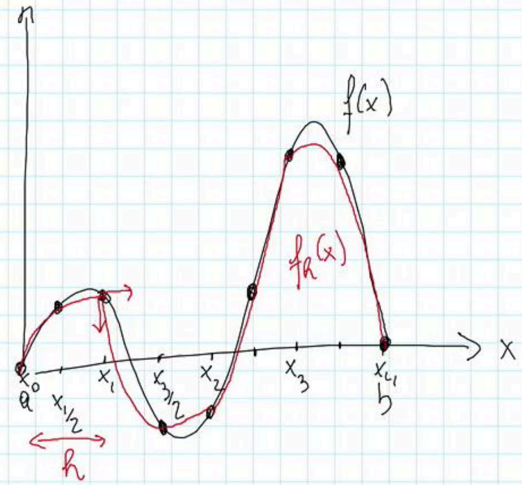
$f_h \in \mathcal{C}^0[a,b]$

$f_h(x_i) = f(x_i) \quad i=0,1,\dots,N$

$f_h(x_{i+1/2}) = f(x_{i+1/2}) \quad i=0,1,\dots,N-1$

$f_h|_{[x_i,x_{i+1}]} \in \mathbb{P}_2 \quad i=0,1,\dots,N-1$

$f_h \xrightarrow[N\to+\infty]{h\to 0} f$?



Thm 1.2 $\exists C>0 \ \forall f \in \mathcal{C}^3[a,b] \ \forall h>0 \quad \max_{a\leq x\leq b} |f_h(x)-f(x)| \leq C h^3 \max_{a\leq x\leq b} |f'''(x)|$

Interprét : $f \in \mathcal{C}^3[a,b]$ l'erreur divisée par 2^3 chaque fois que h divisé par 2

Let C be a positive constant such that for all f three times differentiable on the interval [a,b]. I must here assume f three times differentiable. Previously for the interpolating function of degree 1, f needed to be twice differentiable. So there exists C such that for all h positive, again h does not depend on f nor h, the maximum of the error on the interval [a,b], f_h(x)-f(x) taken in absolute value. I consider the maximum of this error on the interval [a,b]. Well this error is upper bounded by C * h^3 times the maximum of the third derivative, taken in absolute value. Again the signification, or the numerical experiment which can be done, is the following. We choose a function three times diferentiable, we then calculate the error between f and f_h. We can observe that this quantity, the maximum of the error on the interval [a,b] is divided by 2^3 = 8 each time h is divide by 2. chaque fois que H est divisé par deux.

Notes

Summary

3m 53s