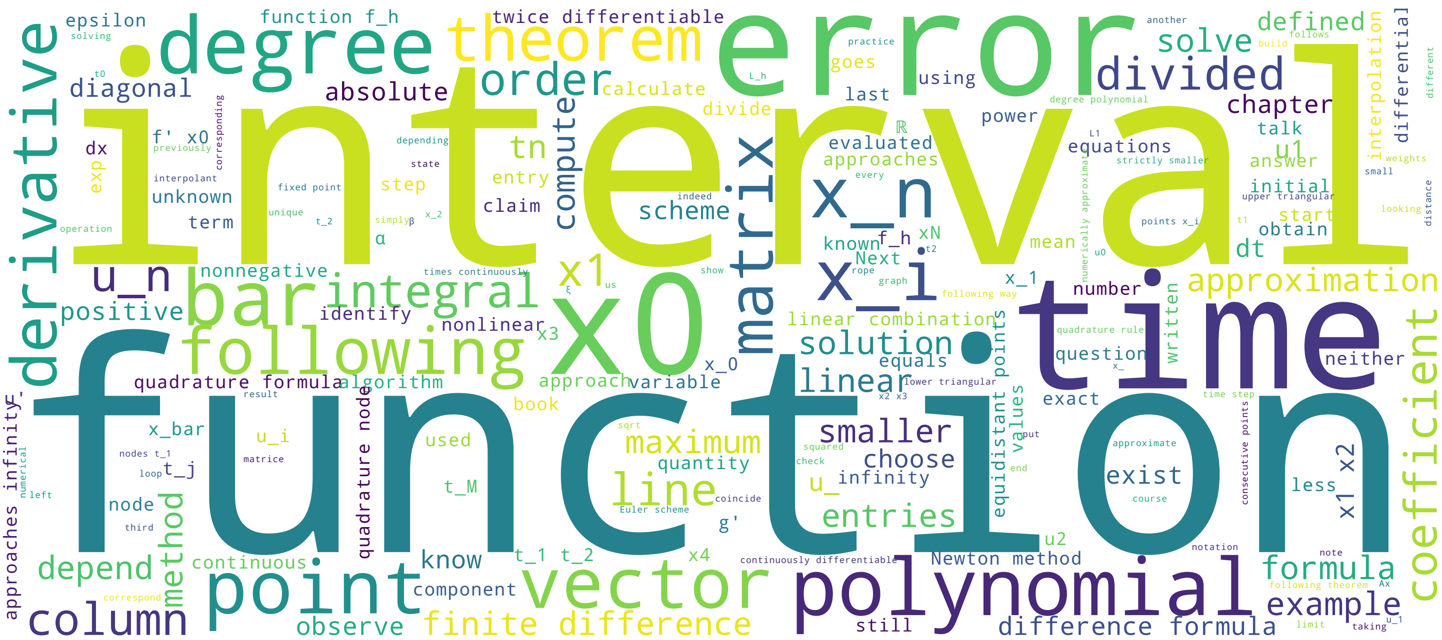
Interpolation de degré 1 par intervalles

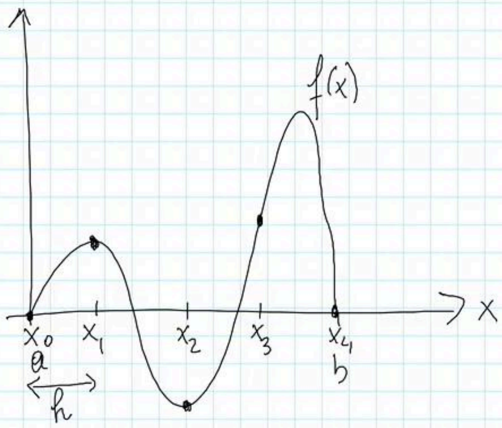
Introduction à l'analyse numérique

Prof. Marco Picasso



Video

Interpol degré 1 par intervalle :



$$f:[a,b]\to\mathbb{R}$$

$$x_i = a + \overset{h}{\frac{b-a}{N}}\, i \qquad i=0,1,\dots,N$$

$$f_h \in \mathcal{C}^0[a,b]$$

$$f_h(x_i)=f(x_i) \qquad i=0,1,\dots,N$$

$$f_h|_{[x_i,x_{i+1}]} \in \mathbb{P}_1 \qquad i=0,1,\dots,N-1$$

In this video I will talk to you about interpolation of degree 1 by interval. I will consider a function f defined in the interval [a,b]. Let x_i be equidistant points in the interval [a,b] such that x_i = a+((b-a)/N)*i for i=0,1,... N. Here the interval [a,b], the variable x. The function f defined on the interval [a,b]. I choose a certain number of equidistant points on this interval [a,b] called x0, x1, x2, x3 and x4. So here is the function f(x) on the interval [a,b] and I will build the function f_h the following way This function f_h will be the interpolant by intervals of degree 1 of the function f. This function is continuous on the interval [a,b] and should coincide with the function f at points x_i for all i = 0,1,... N. In this notation we used the letter h, but what is it ? It is the following: h = (b-a)/N, the distance between two consecutive points. This h approaches 0, or equivalently N approaches infinity. So the functions f and f_h must be equal at nodes x_i. Furthermore f_h restrained such that on every sub-interval x_i to x_(i+1) f_h is a polynomial of degree 1 for i = 0,1,... N-1.

Notes

Summary

0m 00s

Interpol degré 1 par intervalle :

$f:[a,b]\to\mathbb{R}$

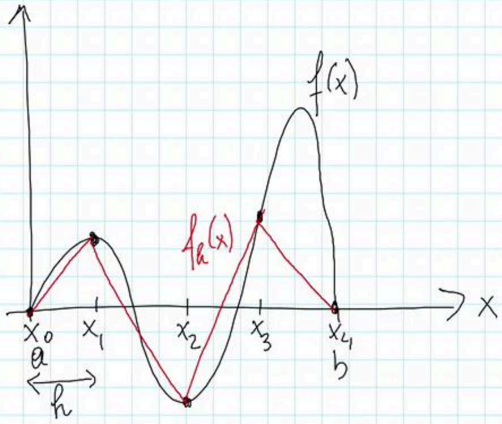
$x_i = a + \overbrace{\frac{b-a}{N}}^{h}\, i \qquad i=0,1,\dots,N$

$f_h \in \mathcal{C}^0[a,b]$

$f_h(x_i)=f(x_i) \qquad i=0,1,\dots,N$

$f_h\big|_{[x_i,x_{i+1}]} \in \mathbb{P}_1 \qquad i=0,1,\dots,N-1$

$f_h \xrightarrow[N\to\infty]{h\to 0} f$?



Thm 1.2 : $\exists C>0\ \forall f\in\mathcal{C}^2[a,b]\ \forall h>0 \quad \max_{a\leq x\leq b}|f_h(x)-f(x)| \leq C h^2 \max_{a\leq x\leq b}|f''(x)|$

Interprét : $f\in\mathcal{C}^2[a,b]$ l'erreur est au moins divisée par 2^2 chaque fois que h est divisé par 2.

So f_h on the interval x0 to x1 is a polynomial of degree 1 passing through (x0,f(x0)) and (x1,f(x1)) and so on for the sub-intervals x1 to x2, x2 to x3 and x3 to x4. Each time it is a different polynomial of degree 1. Here is the function f_h of x; the question I want to answer is: does f_h converge, somehow, to the function f as h approaches 0, or as N approaches +infinity. The error is here. I will consider the maximum of this error on the interval [a,b]. The theoretical result is the following: theorem 1.2 in the book: a positive C exists such that the function f is twice differentiable on the interval [a,b]. This is the case for my drawing. For all h positive we have the error (f_h(x)-f(x)), by taking the absolute value of this error and considering the maximum on the interval [a,b]; we get the error visible here. This error is smaller then C* (h^2) times the maximum of the second derivative of f on the interval [a,b]. According to this theorem, C does not depend on neither h nor f, hence there exists for all h and all f a value C depending on neither h nor f. The meaning of this theorem is the following in practice: we choose a function twice differentiable on the interval [a,b]. For a given h we measure the error and must observe that this error is at least divide by 2^2, so 4, each time h is divide by 2. 2

Notes

Summary

2m 11s