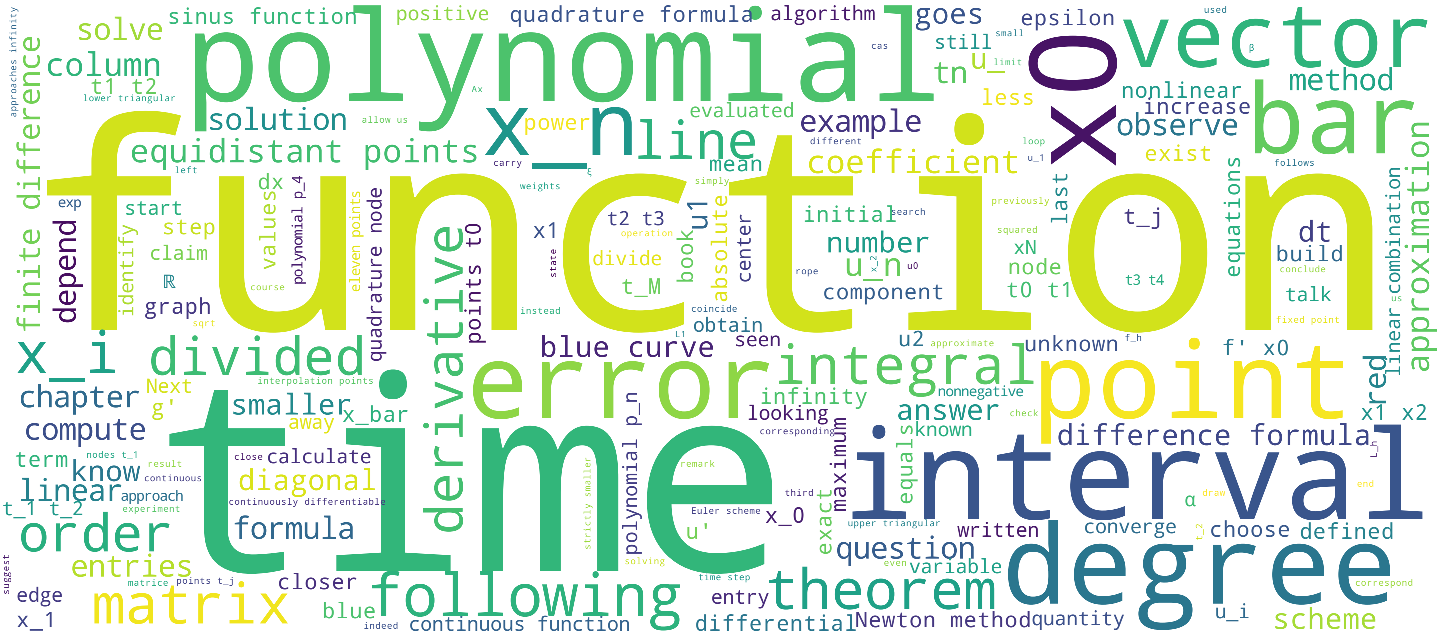
Interpolation d'une fonction continue

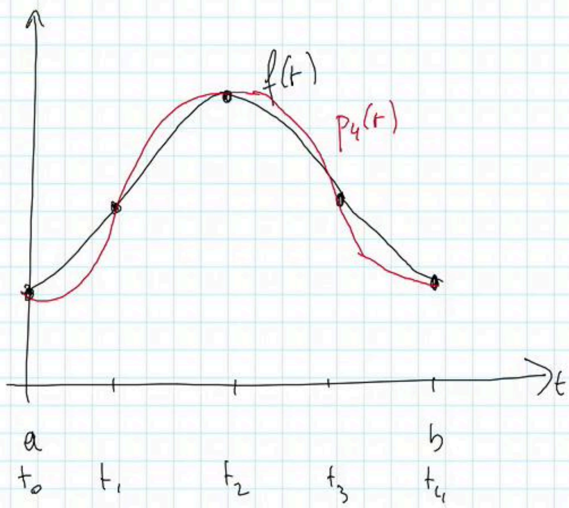
Introduction à l'analyse numérique

Prof. Marco Picasso



Video

Interpolation d'une fct continue par un polyn. (1.4 livre)



$f : [a,b] \to \mathbb{R}$ cont.

$t_j = a + \frac{b-a}{n} j \quad j = 0,1,\dots,n$

$p_n \in \mathbb{P}_n$ tq $p_n(t_j) = f(t_j)$ $j=0,1,\dots,n$

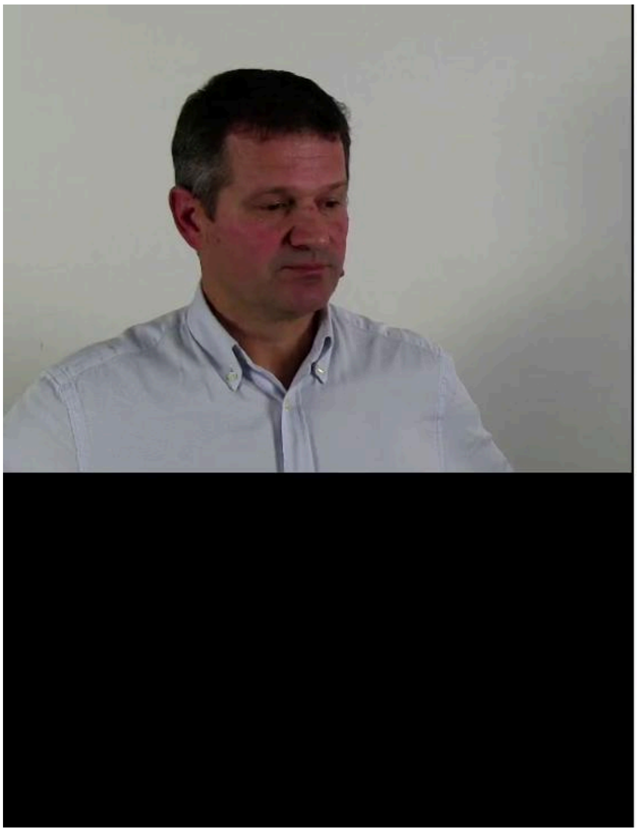
Question : $p_n \xrightarrow{n\to\infty} f$?

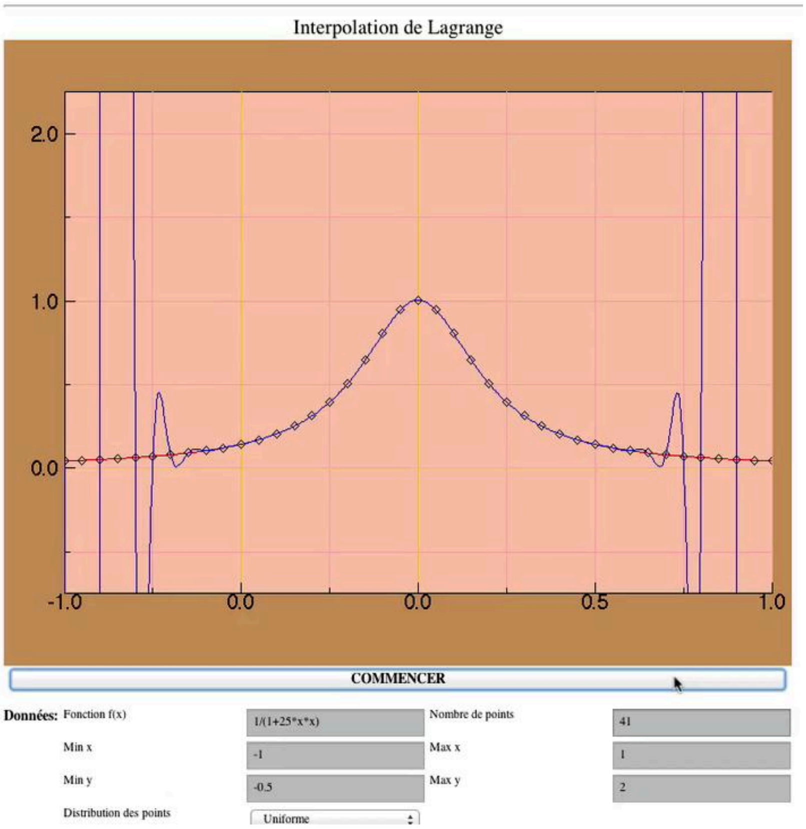
Now I will talk to you about polynomial interpolation of continuous functions. This topic is covered in the book by chapter 1.4 Given a continuous function f defined on the interval [a,b] in R (real numbers). I choose regularly spaced interpolation points, such that t_j = a + (b-a)*j/n for all j in 0,1,...,n. Here is the interval [a,b], I choose five equidistant points, t0,t1,t2,t3,t4. cinq points équidistants. Je vais prendre ici le graphe de la fonction f et je and search for the polynomial p_n of degree n which matches the function f in the equidistant points t_j; thus p_n equals the function f in t_j for all j in 0,1,...n. Here are the values f(t0), f(t1),f(t2),f(t3) and f(t4) and I can draw the polynomial p_4 of degree 4 through these five points. Given the polynomial p_4 of degree 4 passing through the equidistant points t0,t1,t2,t3,t4 which coincides with f in these points. The question we are looking to answer is the following: when I increase the number of points n, say n goes to infinity, does the polynomial get closer and closer to the function f ? I suggest a short numerical experiment with a java applet.

Notes

Summary

0m 02s



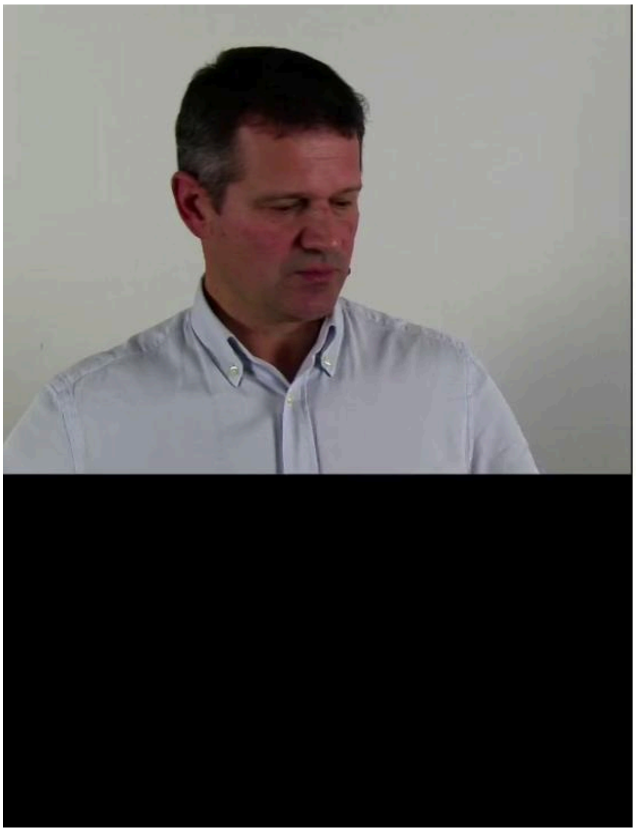


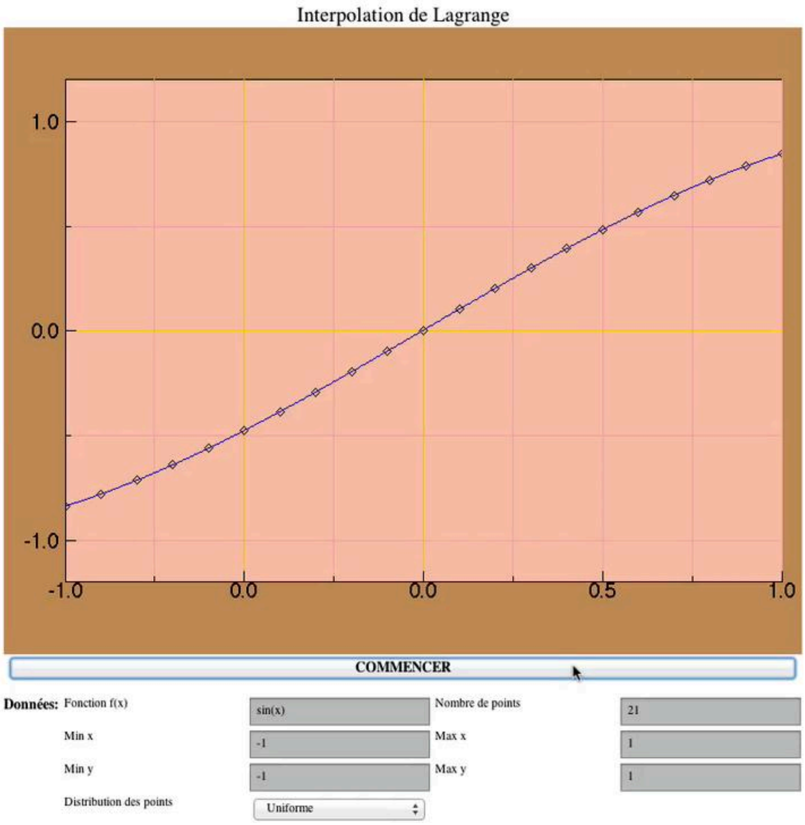
In this java app you see the graph of the function f in red; this function is the following f(t) = 1/(1+25*t^2) in the interval [-1,1]. In blue you can observe the polynomial of degree 4 through the five equidistant points. You can see that the polynomial and the function coincide. So the question to answer is: when we increase the number of points does the blue curve get closer to the red one ? Let's try ! This time I set eleven points. We can observe that the blue curve gets closer to the red one in the center, but moves further away in the edges. So we can carry on and instead of taking eleven points I take 21 points and build the polynomial of degree 20 through these 21 points; same remark, in the center the blue and red curves are nearly overlaid, but on the edges of the interval [-1,1] the blue curve is further and further away from the red one, hence p_n moves away from f. If I do the same thing except with 41 points, I build the polynomial of degree 40 through the 41 points I observe that the trend is even more pronounced. The polynomial p_40 and the function f are close to each other in the vicinity of 0. Unlike close to -1 and 1 where they are far away from each other.

Notes

Summary

2m 26s





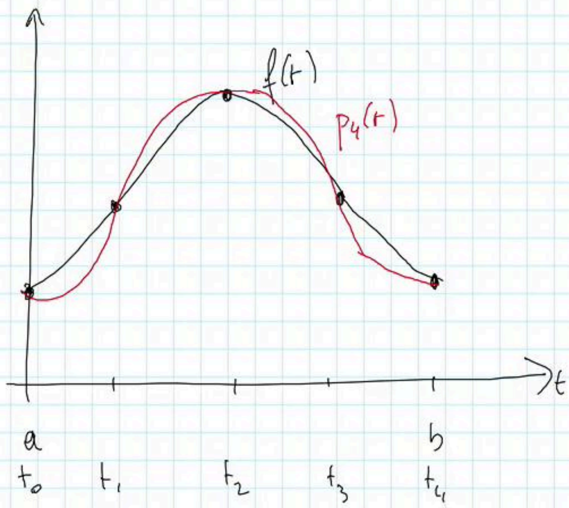
Let's do the same experiment with the sinus function on the interval [-1,1], in red the sinus function and in blue the polynomial of degree 2 through three equidistant points. You can see that there is a little difference between the polynomial and the sinus function. Using 5 equidistant points, the red and blue curves are now overlaping. With 11 points no difference can be seen. Again with 21 points the difference can't be seen with one's nacked eye.

Notes

Summary

4m 08s

Interpolation d'une fct continue par un polyn. (1.4 livre)



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Question : $p_n \xrightarrow{n \to \infty} f$?

Réponse : ça dépend de f ....

So the question: can a polynomial p_n converge to f when n approaches infinity ? i.e. when I take more and more equidistant points ? The answer from numerical observtion is: it depends ! It depends mainly on the function f. So I suggets to you a theorem to allow us to conclude in a certain number of cases. nombre de cas.

Notes

Summary

4m 47s