Interpolation de Lagrange – cas n quelconque

Introduction à l'analyse numérique

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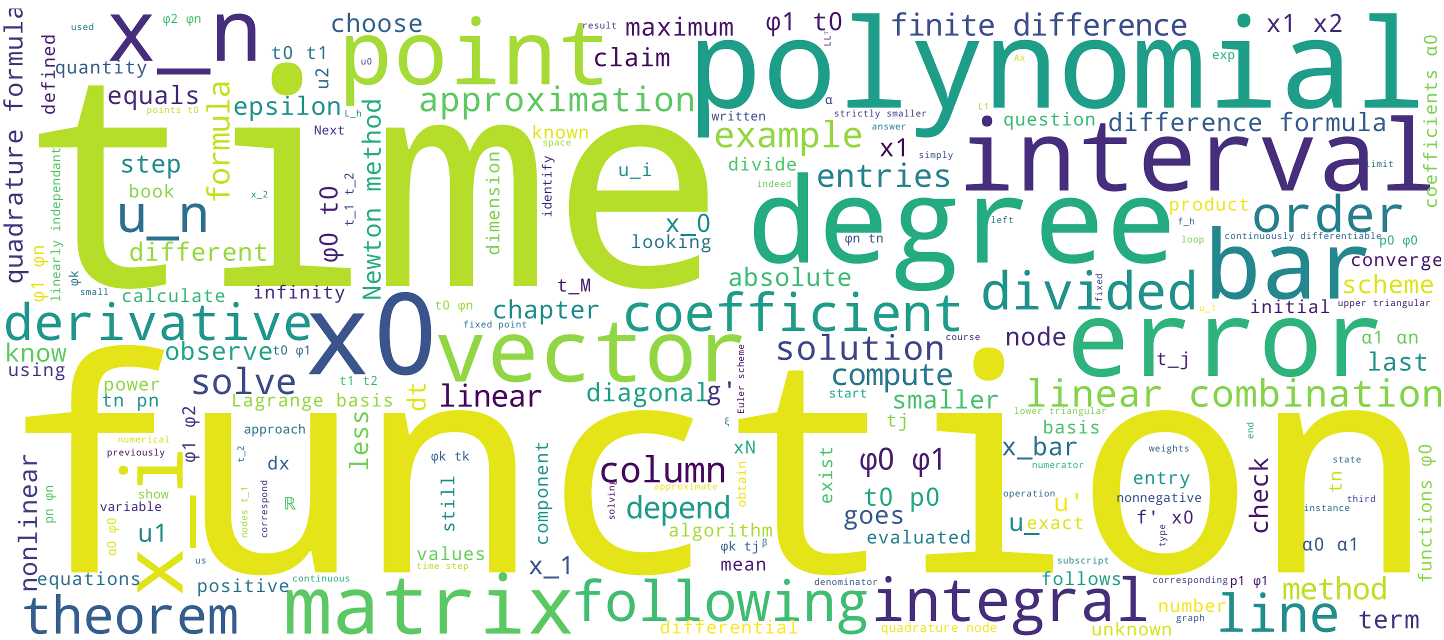

Video

EPFL

n qque : $\varphi_0\, \varphi_1\, \varphi_2 \ldots \varphi_n$ base de Lagrange de $\mathbb{P}_n$ ass. aux pts $t_0\, t_1\, t_2 \ldots t_n$

$0 \leq k \leq n$ fixé $\quad \varphi_k \in \mathbb{P}_n \quad \varphi_k(t_k) = 1 \quad \varphi_k(t_j) = 0 \quad j \neq k$

$$\varphi_k(t) = \prod_{\substack{j=0 \\ j \neq k}}^{n} \frac{t - t_j}{t_k - t_j}$$

$\varphi_0\, \varphi_1\, \varphi_2 \ldots \varphi_n$ base de $\mathbb{P}_n$: $\dim \mathbb{P}_n = n+1$

linéairement indep.

$$\alpha_0 \varphi_0(t) + \alpha_1 \varphi_1(t) + \cdots + \alpha_n \varphi_n(t) = 0 \quad \forall t \in \mathbb{R}$$

We'll now solve the problem for any n. We define φ0, φ1,..., φn the Lagrange basis for polynomials of degree n, Pn, which depends on the points t0, t1, t2,..., tn. Take an integer k, between 0 and n k is fixed. Then φk is a degree n polynomial defined as follows : we want φk(tk)=1 and φk(tj)=0 for all tj with the subscript j different from k, and between 0 and n. The formula for φk is the following φk(t) is the product on j from 0 to n, j different from k, the numerator is of the type (t-tj) such that φk(tj)=0 and on the denominator we have tk-tj, so that φk(tk)=1. I claim now that φ0, φ1, φ2,..., φn is a basis for polynomials of degree n. The dimension of the space of degree n polynomials is n+1 because any polynomial of degree n is a linear combination of the functions 1, t, t^2,..., t^n so there are n+1 functions in the basis, so the dimension of Pn is n+1 So we just need to check that φ0, φ1, φ2,..., φn are linearly independant Take any coefficients α0, α1,..., αn and a linear combination α0*φ0 +α1*φ1 +...+αn*φn Suppose this linear combination is 0 That is to say 0 for all t in R. We must show that this implies that all the coefficients α0, α1,..., αn are 0.

Notes

Summary

0m 00s

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$$\alpha_0 \varphi_0(t) + \alpha_1 \varphi_1(t) + \dots + \alpha_n \varphi_n(t) = 0 \quad \forall t \in \mathbb{R}$$

$$\alpha_0 \cdot 1 + \alpha_1 \cdot 0 + \dots + \alpha_n \cdot 0 = 0 \quad t = t_0$$

Solution du pbm : $p(t) = p_0 \varphi_0(t) + p_1 \varphi_1(t) + \dots + p_n \varphi_n(t) \in \mathbb{P}_n$

$$p(t_0) = p_0 \cdot 1 + p_1 \cdot 0 + \dots + p_n \cdot 0$$

$\vdots$

I choose for example t=t0 and I get φ0(t0)=1, φ1(t0)=0,... φn(tn)=0 so α0*φ0(t0)+α1*φ(t1)+...+αn*φ(tn) = a0*1+α1*0+...+αn*0=α0 So I get α0=0. Likewise if I choose t=t1 I get α1=0 and so on. So I indeed showed that those functions are linearly independant. Recall that I am looking for a polynomial of degree n going through (t0,p0), (t1,p1),.... (tn,pn) and I claim the solution of the problem is given by p(t). p(t) is a degree n polynomial so we can write it as a linear combination of the basis functions φ0, φ1,..., φn and the coefficients of the linear combinations are the values p0, p1,..., pn so p(t) = p0*φ0(t)+p1*φ1(t)+... +pn*φn(t). We can indeed check that it is the solution to our problem. Firsly, this polynomial is a linear combination of the functions φ0, φ1,..., φn the Lagrange basis, so it is a polynomial of degree n. And I must check that p(tj)=pj. For instance, let us show that p(t0)=p0 p(t0)=p0*φ0(t0)+p1*φ1(t0)+...+pn*φn(tn) but φ0(t0)=1, φ1(t0)=0,..., φn(t0)=0 so we have p(t0)=p0. and we continue likewise until we get p(tn)=pn

Notes

Summary

3m 03s