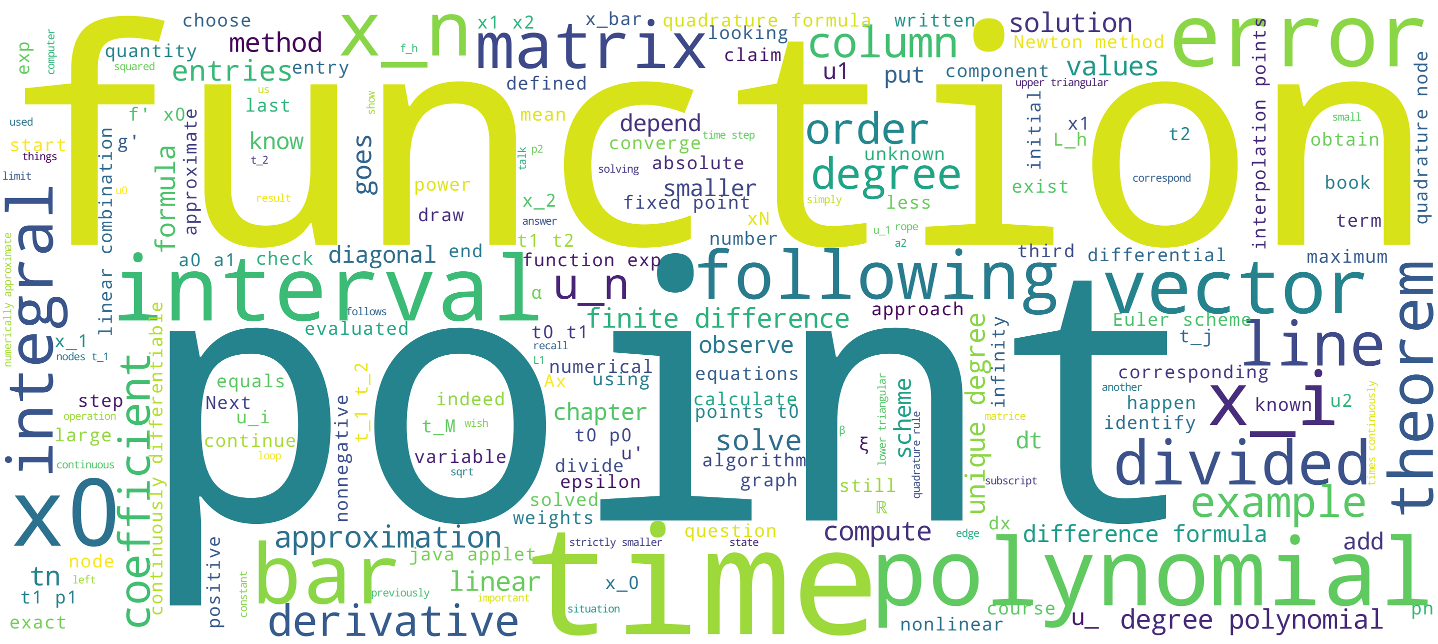
Position du problème

Introduction à l'analyse numérique

Prof. Marco Picasso



Video

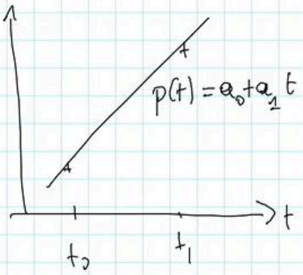
Pbm: données n entier pos.

$n+1$ valeurs $t_0\ t_1\ t_2 \dots t_n$ distinctes

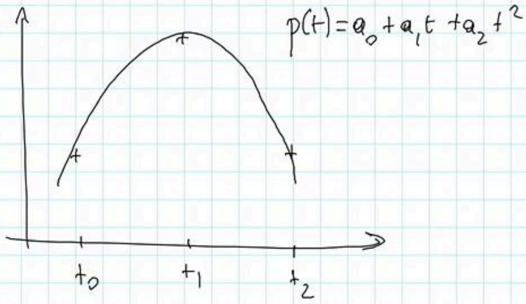
$n+1$ valeurs $p_0\ p_1\ p_2 \dots p_n$

cherché $p \in \mathbb{P}_n$ tq $p(t_j) = p_j \quad j = 0, 1, 2, \dots, n$

$n = 1$



$n = 2$

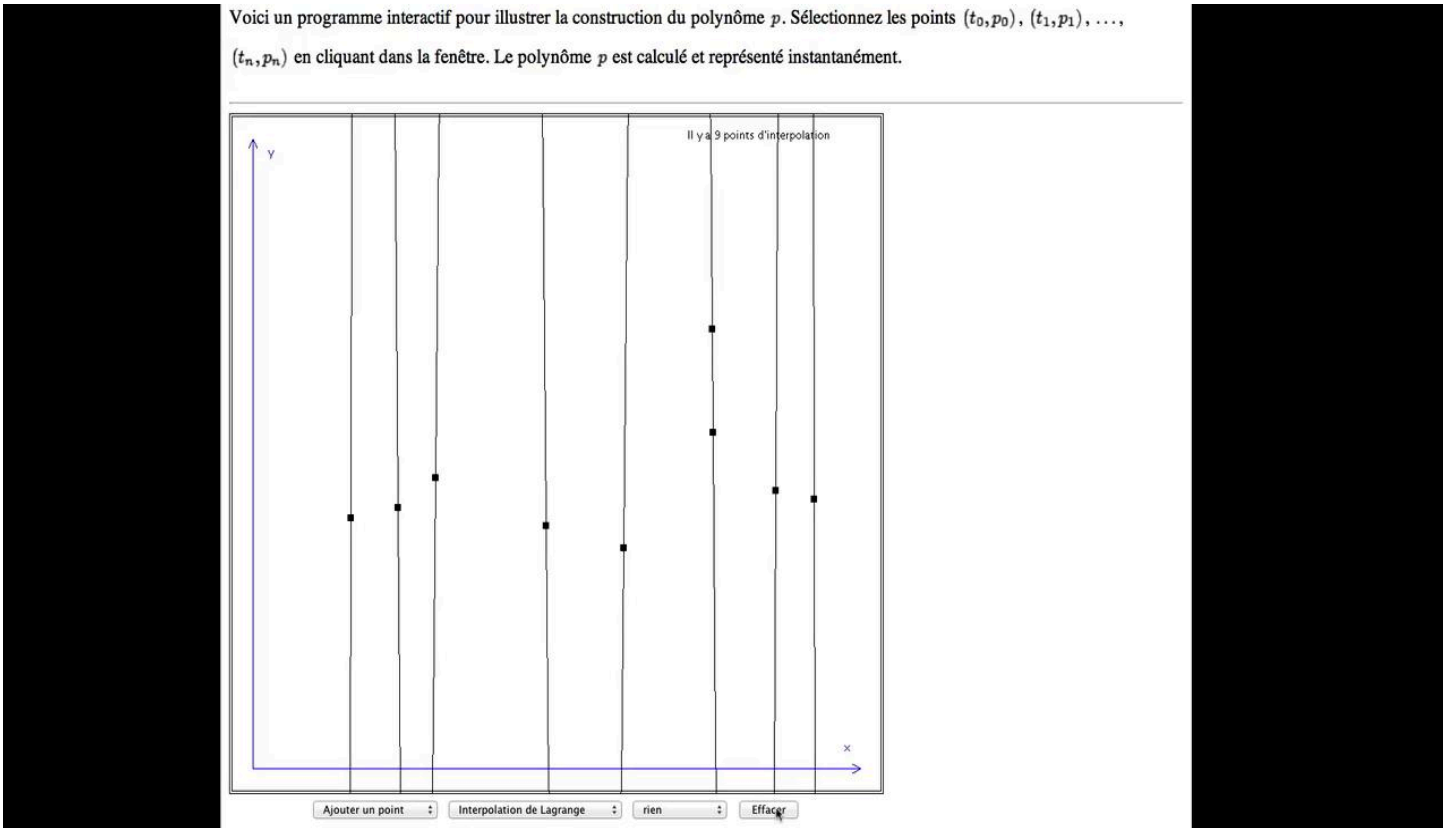


Hello, welcome to chapter 1 interpolation, the problem to be solved is the following : given n a nonnegative integer supposed to be large, and n+1 values, denoted t0,t1,t2 up to tn. Is is important for these values to be pairwise distinct, so t0 different from t1, different from t2, and so on... we take now n+1 values that we note p0,p1,p2 up to pn. These values can all be equal. We are looking for a polynomial p of degree n that goes through the points (t0,p0), (t1,p1)... up to (tn,pn). So I am looking for p in Pn. Here Pn is the set of polynomials of degee less or equal to n such that p evaluated at tj gives pj for the subscripts j from 0 to n. Then I can draw the situation with n=1, In this case I have 2 points, the corresponding value, and I look for the unique degree 1 polynomial that goes through those two points, I write it a0+a1*t. Now if n=2 I have three points so here are t0, the point (t0,p0), the points (t1,p1), and (t2,p2) and I look for the unique degree 2 polynomial going through those three points. I can write it p2(t) = a0+a1*t+a2*t^2. There is a java applet that allows us to continue with n=3,4,5.

Notes

Summary

0m 02s

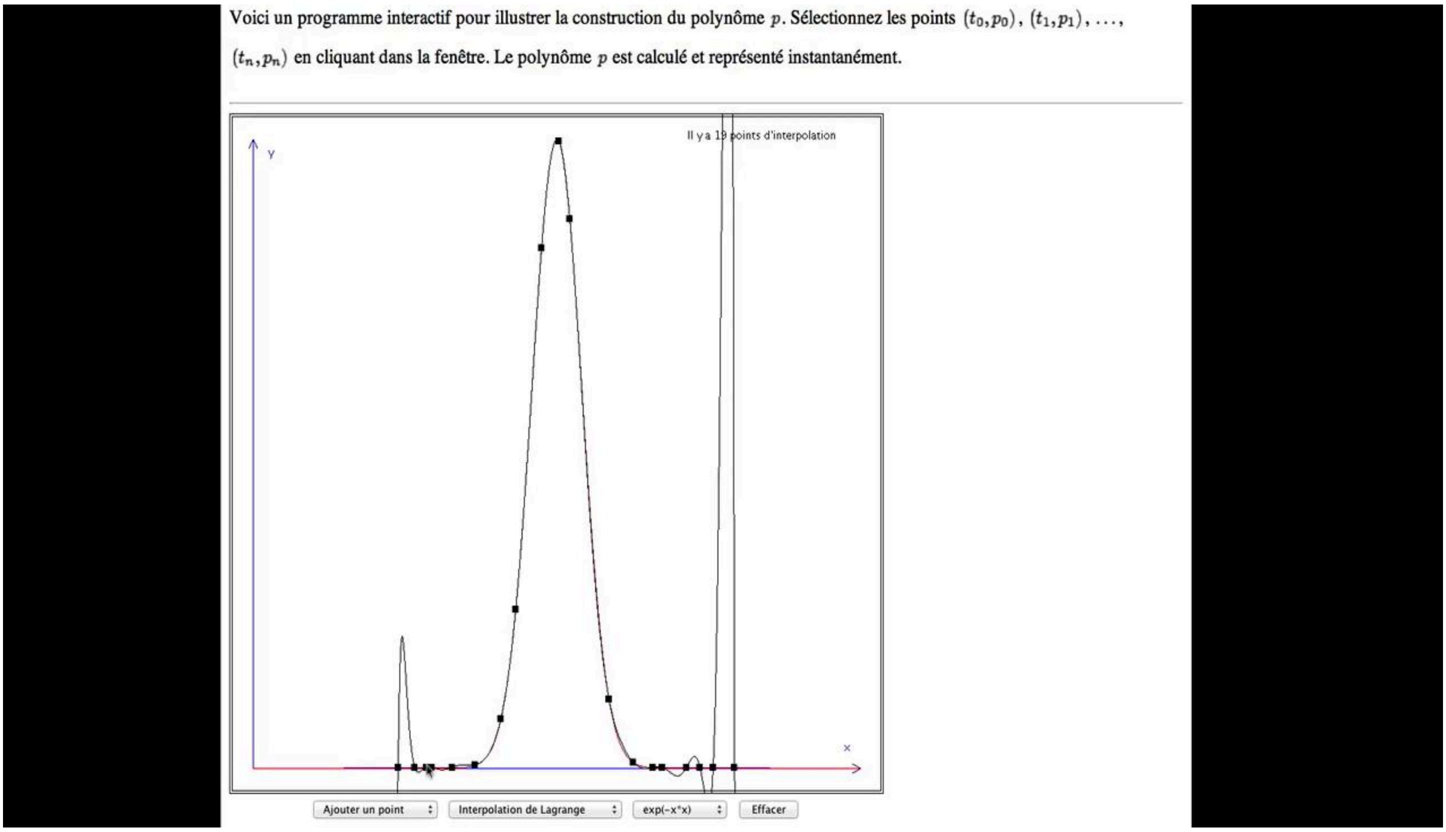


I can use this java applet to do a few experiences. I place 2 points here, and here is the unique degree 1 polynomial going through those 2 points. I add a third point, here we get the unique degree 2 polynomial going through those three points. I add a 4th point and see that the concavity of the function changes once. And so on I can keep adding as many points as I want and get here for instance 6 interpolation points and the unique degree 5 polynomial going through those points I can add even more points if I wish so. Now we wonder what would happen if I put a point right below an existing one ? Well, what will happen is that we indeed get the polynomial, of degree 8 here, going through those 9 points. But the price to pay is that this polynomial oscillates a lot more.

Notes

Summary

2m 08s



we try now a new experiment, take the function exp(-x^2). Now when I put a point, it belongs to the graph of this function, so I try to approach this function exp(-x^2) by a polynomial. So here I have 3 points here now 4,5 points. And the question I ask is : if I add a lot of points, will my polynomial approach correctly the function ? The answer depends on the placement of these points.. For example, here I have a few points. Observe that if I put one here, then things go well in the center, but quite bad on the edge of our interval. So I add some points on the side of the interval, and now I get a polynomial of high degree since I have 15 interpolation points. My polynomial has degree 14 that approaches reasonably well this function exp(-x^2)

Notes

Summary

3m 11s