





- Charge électrique dans un champ d'induction magnétique constant et uniforme
- Plot sur plan incliné

Mécanique | 2013 4

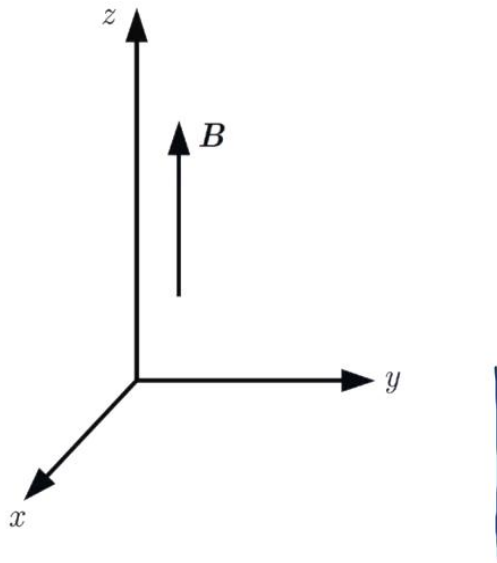
Hello. Welcome to the EPFL general physics class. In this lesson, I introduced some force models, and here, we will look at two applications. First, we'll look at the motion of an electric charge in a constant and uniform magnetic induction field, and then, we'll look at the problem of a sliding pad with friction on an inclined plane.

Notes

Summary



0m 04s



Référentiel : le laboratoire

Coordonnées cartésiennes

$$m\dot{\mathbf{v}} = q\mathbf{v} \wedge \mathbf{B}$$

$$\boldsymbol{\omega} = \frac{q\mathbf{B}}{m}$$

$$\dot{\mathbf{v}} = -\boldsymbol{\omega} \wedge \mathbf{v}$$

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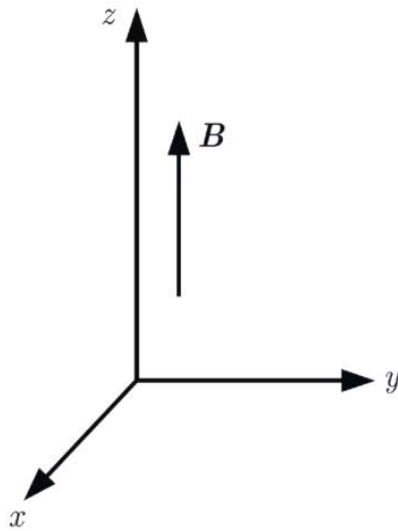
I start with the problem of a charge in a magnetic induction field \mathbf{B} . In the reference frame, I give myself an axis system of coordinates x, y, z , and I choose to take z parallel to the induction field \mathbf{B} . The reference frame is of course the laboratory in which I have reproduced the \mathbf{B} field. I will work with these cartesian coordinates. The equation of motion, Newton's second law, with the force of (the aurem) $q\mathbf{v}$ cross \mathbf{B} simply gives that result. Now, I'll write down $\boldsymbol{\omega} = \frac{q\mathbf{B}}{m}$, to get the following equation: $\dot{\mathbf{v}} = -\boldsymbol{\omega} \wedge \mathbf{v}$. So, I invite you to pause, try to remember when you've encountered an equation of this form before. We've seen an equation of this form before when we were talking about rotation, and so here we have a vector \mathbf{v} , which evolves according to this equation, this evolution, it's a rotation, and the angular vector here would be less $\boldsymbol{\omega}$. So it is a rotation. If now we want to change to a component. Well we need to calculate this vector product, which I can do here.

Notes

Summary



0m 30s



Référentiel : le laboratoire

Coordonnées cartésiennes

$$m\dot{\mathbf{v}} = q\mathbf{v} \wedge \mathbf{B}$$

$$\boldsymbol{\omega} = \frac{q\mathbf{B}}{m}$$

$$\dot{\mathbf{v}} = -\boldsymbol{\omega} \wedge \mathbf{v}$$

$$\begin{pmatrix} \dot{v}_x \\ \dot{v}_y \\ \dot{v}_z \end{pmatrix} = \begin{pmatrix} 0 & \omega & 0 \\ -\omega & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix}$$

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I have x, y, z, I put the omega along the z axis, so I have zero zero minus omega, and here I have vx, I'm just going to write vx, vy, vz, I have to calculate this cross product, I have omega vy, in the y direction, I have minus omega vx, and zero in the z direction. And that, now allows me to write my equations of motion, which are here.

Notes

Summary



2m 03s

Equation du mouvement, équation horaire

$$\begin{aligned}\dot{v}_x &= \omega v_y & \ddot{v}_x &= \omega \dot{v}_y = -\omega^2 v_x \\ \dot{v}_y &= -\omega v_x & \ddot{v}_y &= -\omega \dot{v}_x = -\omega^2 v_y \\ \dot{v}_z &= 0\end{aligned}$$

Conditions initiales :

$$t = 0 \quad x = x_0 \quad y = y_0 \quad z = z_0 \quad v_x = 0 \quad v_y = v_1 \quad v_z = v_{z0}$$

$$v_x = a \sin(\omega t + \phi)$$

Here they are. We do have the omega vy minus omega vx, we see that the evolution of vx depends on vy, the evolution of vy depends on vx, the equations are complete, but things get a lot simpler if you derive these two equations with respect to time. We get vx dot dot, which is worth omega foix vy dot, the vy dot, I'll look for it here, I have less omega square vx. Now the equations are decoupled, and these equations, obviously, are equations of a harmonic oscillator So I can solve them. In the direction z, we have a uniform motion. The initial conditions, well, I'll give myself (cathe) equals zero, I'm at the position x0, y0, z0. I will rotate my x axis so that there is an initial velocity zero in the x direction. I have a velocity v1 in the y direction, I assume a velocity vz0, initially, in the z direction. With that, this differential equation, that of a harmonic oscillator, is easily solved. I have a harmonic response for v, the vx function, vx with two integration constants, a and a phi.

Notes

Summary



Equation du mouvement, équation horaire

$$\begin{aligned}\dot{v}_x &= \omega v_y & \ddot{v}_x &= \omega \dot{v}_y = -\omega^2 v_x \\ \dot{v}_y &= -\omega v_x & \ddot{v}_y &= -\omega \dot{v}_x = -\omega^2 v_y \\ \dot{v}_z &= 0\end{aligned}$$

Conditions initiales :

$$\begin{aligned}t = 0 \quad x = x_0 \quad y = y_0 \quad z = z_0 \quad v_x = 0 \quad v_y = v_1 \quad v_z = v_{z0} \\ v_x = a \sin(\omega t + \phi) \quad \dot{v}_y = -\omega a \sin(\omega t + \phi) \quad v_y = a \cos(\omega t + \phi) \\ \phi = 0 \quad a = v_1\end{aligned}$$

$$x(t) = -\frac{v_1}{\omega} \cos \omega t + C \quad x(0) = x_0 = C - \frac{v_1}{\omega} \implies C = x_0 + \frac{v_1}{\omega}$$

$$x(t) = x_0 + \frac{v_1}{\omega} - \frac{v_1}{\omega} \cos \omega t \quad y(t) = y_0 + \frac{v_1}{\omega} \sin \omega t$$

If I take this v_x and put it into this equation of motion, I have v_y point that has that shape. I can integrate these equations for v_y , and here, I don't put a constant, because if I put a constant, it's going to show up in there, and it's not going to be a solution of this differential equation, so I'm forced to put a zero here. That's it. Now I have to figure out what the integration constants a and ϕ are. So, ϕ is zero because, at equals zero, I have v_x which is zero, when t equals zero, I just have a if (ϕ), so I must have ϕ which is zero, and then if I look at that equation there, I have a t equals zero, a cosine of zero, that's one, so we have v_1 which is a . Now, I can integrate the equation of v_x to find x of t . So, the sine integral, it's less omega cosine. There is a constant, the constant is given by the initial conditions. a t equals zero, we a x zero, which must be worth c minus v_1 over omega, so there you have it, I found c . And I rewrite x from t completely, for y from t , same reasoning, I just have that solution there, and now, I'm going to propose to analyze the trajectory.

Notes

Summary



$$x(t) = x_0 + \frac{v_1}{\omega} - \frac{v_1}{\omega} \cos \omega t \quad y(t) = y_0 + \frac{v_1}{\omega} \sin \omega t$$

$$\left(x - x_0 - \frac{v_1}{\omega}\right)^2 + (y - y_0)^2 = \frac{v_1^2}{\omega^2}$$

$$\text{Cercle de rayon : } r = \frac{v_1}{\omega} = \frac{mv_1}{qB}$$

$$\text{Mouvement circulaire uniforme de vitesse angulaire } \omega = \frac{qB}{m},$$

indépendante du rayon (principe du cyclotron : plus la vitesse augmente plus le rayon est grand, mais la fréquence de la tension accélératrice reste la même.)

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So here is my x of t , here is my y of t , so my equations (overts), and now, I want to calculate the trajectory. So there we have a $\cos \omega t$, we have a $\sin \omega t$. I can calculate the trajectory by grouping the terms, I take x minus that term, and square it. I take y minus that term, square it. This gives me v_1 squared over ω squared times \cos squared plus \sin squared, which makes one. So, I have that term there. And this is the cartesian equation of a circle. The radius of the circle is v_1 over ω , and if I take my definition of ω , I have mv_1 over qB , so the radius of the circle is smaller the larger B is, or the radius of the circle is larger the larger v_1 is. I recall the important result, that the angular velocity cubed over m is independent of the radius. Therefore, whatever the radius, the particle takes the same time to go around the circle, whatever the radius of this circle is. This is important in the technique because it is the basic principle of operation of cycle (otron), an accelerator. As the speed of the particle increases, so does its radius, but the time to make a turn remains the same.

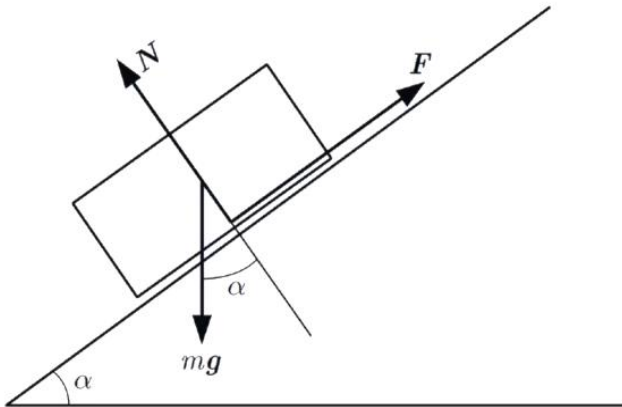
Notes

Summary



5m 55s

Frottement sec sur plan incliné



Hypothèses :

- Glissement
- Vitesse constante

$$\begin{array}{l} N = mg \cos \alpha \\ \mu_c N = mg \sin \alpha \end{array} \longrightarrow \mu_c = \tan(\alpha)$$

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I finish now with a small problem of dry friction. I imagine the following situation: a block which goes down on a plane inclined by an angle α , with respect to the horizontal, that's gravity. I assume that there is sliding, and I assume that the speed is constant. So the sum of all these three forces, there is the gravity, the reaction force of the inclined plane, the friction force, the vector sum of these three forces must give zero. If I work in projection, if I project in that direction, I have N which must be equal to $mg \cos \alpha$. $mg \cos \alpha$ is the projection of mg on that axis, and then, in that direction, I have F which must be worth $mg \sin \alpha$. That's what I wrote here. We can divide this equation by that one and get μ_c , which is worth $\tan \alpha$. So here's a geometric interpretation of μ_c , it's the tangent of the angle of the inclined plane, which is that the material point is descending at constant speed.

Notes

Summary



7m 32s