ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

Expériences : loi de gravitation

Mécanique, cours 13.exp

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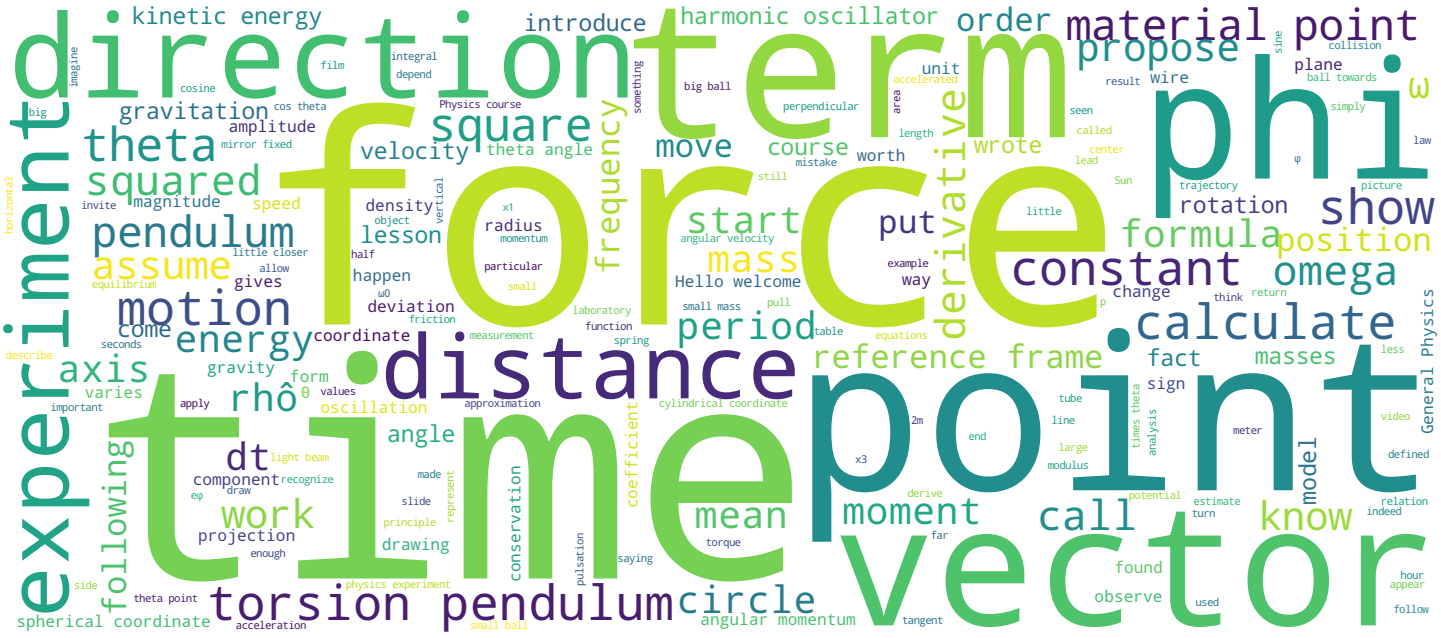
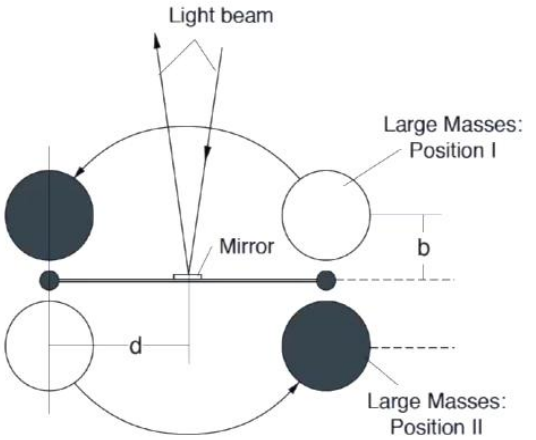


Video

EPFL

Balance de Cavendish



- Vue de haut du pendule de torsion
- Les deux petites masses sont montées sur un fil vertical
- Les deux grosses masses sont placées d'un côté ou de l'autre des petites masses, provoquant une déviation angulaire

Hello. Welcome to the ÉPFL General Physics course. In this lesson, we talked about Kepler's problem, and Newton's gravitational force. Here, I will show an experiment, the Cavendish balance, which allows to measure the force of gravitation between two masses in a laboratory. The principle of the measurement is as follows. This is, you see on this graph, the experiment seen from above. You have two masses, here, which form a torsion pendulum. There is a wire, vertically to this point which ensures, the righting torque of the torsion pendulum. There are two heavy balls drawn, here, in grey, or, in white, here, there are only two balls, big outer balls. They are either in this position, or in the other position. So, because of the gravitation, they pull, if we look at this side, the grey ball pulls the small ball towards it, or it's the white ball on the drawing that pulls the small ball towards it. And so, we are going to obtain a deviation of the pendulum, which we are going to detect thanks to a mirror fixed on the pendulum, and a set of light beams. This experiment is delicate.

Notes

Summary

0m 03s

Balance de Cavendish

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In our physics auditorium, we managed to show the experiment to the students during the course, thanks to the fact that the manipulative remains hung on the wall, in a corner of the auditorium.

Notes

Summary

1m 44s

I'll show you, now, the device a little closer.

Notes

Summary

2m 01s

Balance de Cavendish

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Here is the torsion pendulum: The wire is contained in this tube, the two small masses are inside this box, so they are sheltered from the air currents, and you can see, of course, the two large external masses that can be made to turn on this axis. If you look a little closer, you can see the mirror fixed on the torsion pendulum. I was saying that the experiment was tricky. If you want to make a measurement as beautiful as the one I am going to show you now, the time of two periods of 45 minutes is not enough, it took the technician four hours to record what I am going to show you now.

Notes

Summary

2m 08s

Balance de Cavendish

- Vision accélérée de l'oscillation puis la stabilisation du pendule de torsion pour les deux positions des deux grosses boules.
- Que vaut G ?

You're going to see the clock, superimposed, which is moving very fast, because the film is accelerated. You will see four hours of work in 18 seconds.

Notes

Summary

2m 53s

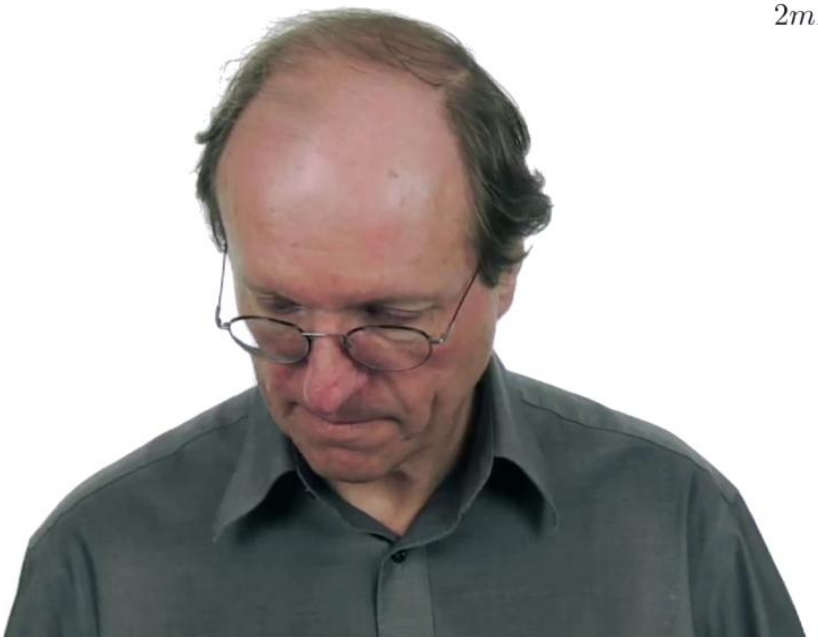
You'll see a damped harmonic oscillator, again, but accelerated, let's not forget. And now, I propose a little exercise.

Notes

Summary

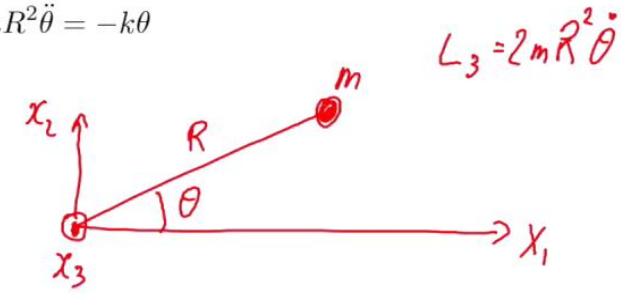
3m 14s

Balance de Cavendish : ordre de grandeur



Oscillations du pendule de torsion :

$$2mR^2\ddot{\theta} = -k\theta$$



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I think, like all my colleagues who teach physics, that it is very important to learn to work with orders of magnitude, and I propose you, here, to calculate the order of magnitude of the constant of universal gravitation g. I invite you to take a break, try to see how you will be able to do this calculation, based on the video that I just showed you. Personally, I suggest the following approach. First, we have a torsion pendulum that we have seen swinging. Then, I will model this torsion pendulum by writing its equation of motion in the small angle limit, and I wrote the following. Where did that come from? Well, if I take this point to be the center of the torsion pendulum, I have a lever arm and a small mass m at the end. Let's say that this is an axis X1 which belongs to the referential, so to the laboratory, and here is the theta angle. It's easy to convince yourself that the angular momentum in the z direction, or in the X3 direction if you like. We have a vector X3 that comes out of the table if we put X2, here. In the 3th direction, we have L3 which is 2m. I'll write large R squared, times theta point. Where R is this distance here.

Notes

Summary

3m 27s

Balance de Cavendish : ordre de grandeur

Oscillations du pendule de torsion :

$$2mR^2\ddot{\theta} = -k\theta \qquad \omega^2 = \frac{k}{2mR^2}$$

Déviation statique du pendule de torsion :

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And you apply the angular momentum theorem, which implies dL3 over dT, equals M3, and for M3, we'll assume a couple of recalls, a moment of force proportional to theta, minus k times theta. This is the way to represent, the action of the torsion pendulum wire on the horizontal bar, like this. So, you recognize, here, the equation of a harmonic oscillator, and you know the relation between the pulsation and the coefficient which intervenes, here. We have omega square. If omega is the pulsation, omega squared is k over 2m R squared, 2m because we have, if you like, 2 material points, but that we can describe with the same theta angle. On the picture, we have seen the oscillation. So, we have an idea of the frequency of this pendulum. We don't know very well what the mass is; we can guess it. We don't have a scale, so we don't know very well what R is, but I propose to continue the analysis, first in principle, and see what we can deduce from it. So, the experiment leads to a state of equilibrium with a deviation of the light beam. And the equilibrium position is characterized by the balance between two forces, the force of gravity to the force of return, or if you want, between the torque due to the force of gravity, and the torque, or the moment of force due, to the return of the pendulum.

Notes

Summary

5m 09s

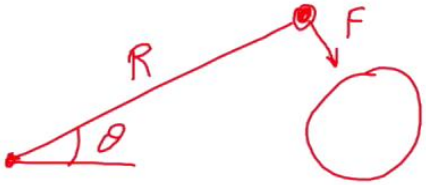
Balance de Cavendish : ordre de grandeur

Oscillations du pendule de torsion :

$$2mR^2\ddot{\theta} = -k\theta \qquad \omega^2 = \frac{k}{2mR^2}$$

Déviation statique du pendule de torsion :

$$k\Delta\theta \approx \frac{GMm}{b^2}R$$



And so I have a relation of this kind. If you want, I can, once again, draw an arm of my pendulum, and here, now, we have a big ball. So we have, a force of attraction that is exerted, like this, which is the force of gravitation. We have a distance R, so we have a moment of force which is R times F and which must counterbalance the torque, which is defined by the constant K and the theta angle. What I wrote, K times del theta, is the deviation to equilibrium, must be equal to, this moment of the force of gravitation.

Notes

Summary

7m 03s

Balance de Cavendish : ordre de grandeur

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$$2mR^2\ddot{\theta} = -k\theta \qquad \omega^2 = \frac{k}{2mR^2}$$

Déviation statique du pendule de torsion :

$$k\Delta\theta \approx \frac{GMm}{b^2}R$$

Estimations :

$b \approx R$ $\quad R \approx$ rayon de la grosse boule

$\Delta\theta \approx 1/20$

$$G = \frac{(2\pi)^2}{T^2}\frac{2Rb^2}{(4/3)\pi R^3\rho}\Delta\theta \approx 15\ 10^{-11} kg^{-1}m^3s^{-2}$$

$6.6\ 10^{-11}\ m^3\ kg^{-1}\ s^{-2}$

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So, now we have to go to the estimates, and I propose you to do the following thing. Again, of course, I can go down, look at the experiment with a meter and measure everything, but, what I would like to teach you, here, is how far we can go in a quantitative analysis of a physics experiment with just, orders of magnitude. I will take b, I had designated by b the distance between the masses. I'll assume that it's about equal to that R, that I had used to designate the distance between the center of the small mass and the axis of the torsion pendulum. I will also assume that the big balls have a radius R. So, basically, all the distances that appear in the problem, I will replace them by R. Now, the angular deviation, I estimate it to be, about one twentieth of a radiant, and when I make all the substitutions, the small mass disappears from the problem. The big R stays with me, but what I'm going to do is I'm going to introduce for the big M mass, I'm going to introduce the volume times the density. Here, rhô, it's the density of the matter. I assumed I had lead. I went to see the values of the density of lead. And I also estimated T, the period of the pendulum, watching the film carefully.

Notes

Summary

7m 57s

Balance de Cavendish : ordre de grandeur

Oscillations du pendule de torsion :

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Déviation statique du pendule de torsion :

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$6.6\ 10^{-11}\ m^3\ kg^{-1}\ s^{-2}$

Mécanique | 2013 17

And I came up with this value: 15 times 10 minus 11 kg minus one cubic meter second minus two. First of all, I'm glad my units are correct, that means I didn't make any mistake in my formulas. Well, there is a good chance that I didn't make a mistake in the formulas. Then, I find a remarkably close result. This does not mean that my approximations are better than I thought. It's a bit of a fluke, but the fact is that all these distances are close, and there may be compensating effects in my errors. That's something that happens a lot. If I had found a value 10 times too big or 10 times smaller, I would still be happy. With a factor of 100, maybe it would have been worth looking again at these approximations on distances. Finally, this is how, an order of magnitude allows to analyze quantitatively a physics experiment where we don't know all the parameters.

Notes

Summary

9m 41s