ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

Moment cinétique et moment de force

Mécanique, cours 13.1

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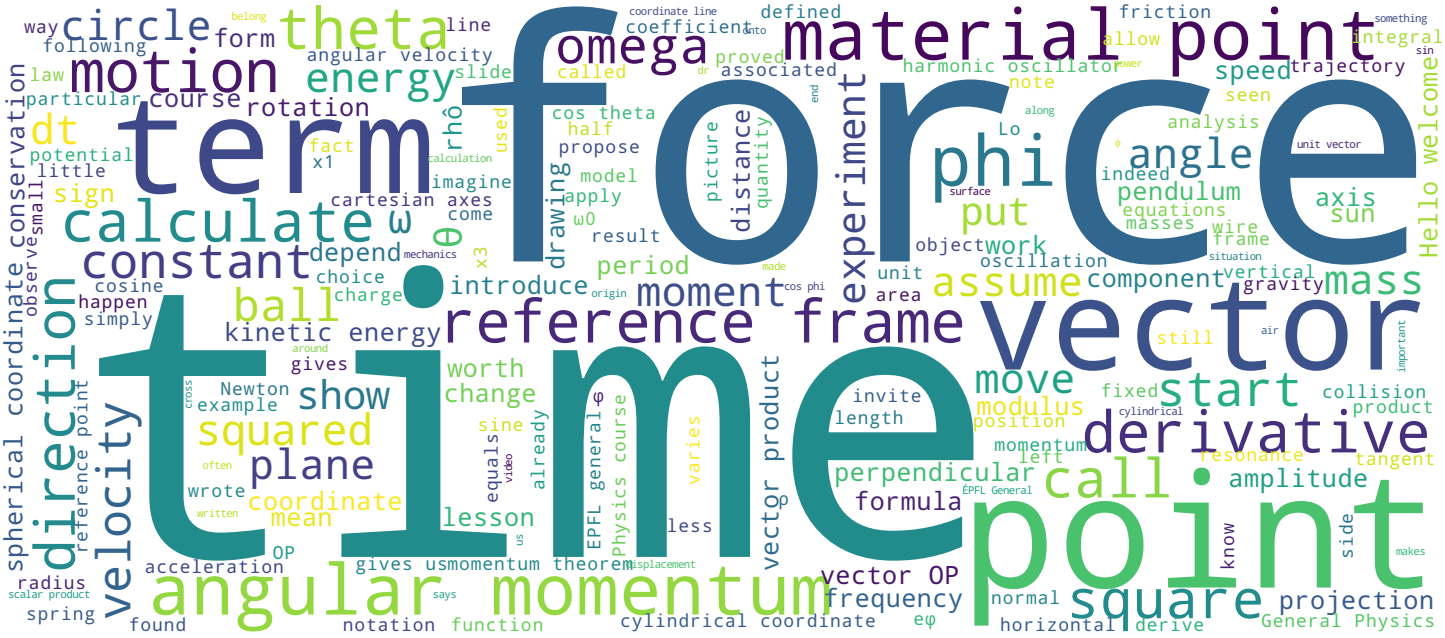
http://go.epfl.ch/traite-meca-1-13



Video

EPFL

Moment cinétique et moment de force

- Définitions
- Théorème du moment cinétique pour un point matériel

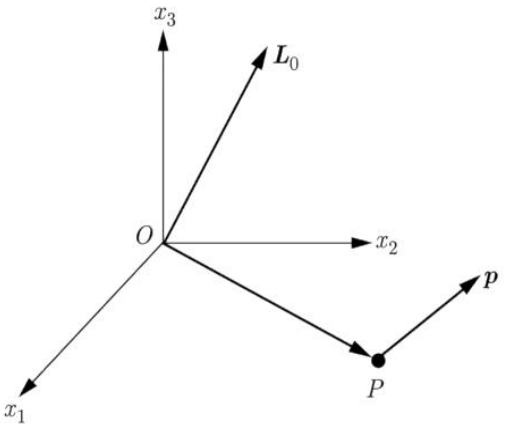
Hello, welcome to the ÉPFL General Physics course. In this lesson, I'll introduce two new physical quantities, and, in the next module, we'll immediately apply them to the analysis of the motion of the planets around the sun. So I'm going to define the angular momentum, and the momentum of a force, and we'll see that there is a close link between the two, which I call the angular momentum theorem of the sun.

Notes

Summary

0m 04s

Définition : moment cinétique



O : un point du référentiel
P : point matériel, quantité de mouvement $\boldsymbol{p}$

$$\boldsymbol{L}_0 = \boldsymbol{OP} \wedge \boldsymbol{p}$$

Attention : normal au plan défini par les deux vecteurs $\boldsymbol{OP}$ et $\boldsymbol{p}$.

I start with the definition of angular momentum. I assume I have given myself a reference frame, I consider a point O of this reference frame, I have a material point in P, given quantities of motion, and I define angular momentum which I note L index *o*, *o* being this chosen point of the reference frame, I define angular momentum Lo as the vector product of the radius vector OP and the momentum of the material point. If I make a drawing, here is as usual a system of cartesian axes which materializes the reference frame, I choose my point O at the origin of this system of cartesian axes, naturally enough. Here is my material point P. I have the vector OP and the vector P, the momentum which forms a plane, and Lo must be normal to the plane because of the geometric properties of the vector product. So, in this drawing, Lo must be understood as resulting from the right hand rule and perpendicular to the plane formed by these two vectors, here. Of course if I now take another point than O, but a point further over there, the vector OP will become larger, the modulus of Lo will change, and so it is quite clear that this vector Lo, the angular momentum, depends on the choice of this point O. That's why I propose a notation where explicitly, the reference point is given, and we speak of the angular momentum at O.

Notes

Summary

0m 31s

Définition : moment d'une force

O : un point du référentiel
P : point matériel, subit une force $\boldsymbol{F}$

$$\boldsymbol{M}_0 = \boldsymbol{OP} \wedge \boldsymbol{F}$$

Le point de référence est important !

I now move to the definition of the moment of a force. Again, I give myself a reference frame, a particular point that belongs to the reference frame, which I note O, I assume that I have a material point at P, which is undergoing a given force, F. The moment of the force at O, I define it as the vector product of OP and the force F. Like this. Again, this definition depends on the choice of O, so I give an explicit notation, where we see what reference point is used to calculate the moment of force.

Notes

Summary

2m 37s

Propriété : th. moment cinétique (pt. mat.)

$$\frac{d\boldsymbol{L}_0}{dt} = \boldsymbol{M}_0$$

Démonstration :

$$\frac{d}{dt}\boldsymbol{L}_0 = \frac{d}{dt}(\boldsymbol{OP} \wedge \boldsymbol{p}) = (\dot{\boldsymbol{OP}} \wedge \boldsymbol{p}) + (\boldsymbol{OP} \wedge \dot{\boldsymbol{p}}) = \boldsymbol{OP} \wedge \boldsymbol{F}$$

$\dot{OP} \wedge p = 0$; $\dot{p} = \vec{F}$; $OP \wedge F = \vec{\Gamma}_0$

Now I have the following result, which I call the angular momentum theorem, which says that the derivative with respect to time of the angular momentum is equal to the moment of the force, both measured with respect to the same point, of course. Demonstration: So, I start with the term on the left, I apply the definition of Lo, Lo is OP crosses the momentum, I have to, I want to calculate the derivative with respect to time of a product, I have two terms, I'll first carry the derivative over to OP, and then, to the momentum. This term gives us zero, because this is the velocity, the velocity is collinear with the quantity of motion, so I have zero. Here, I have P point, with Newton, P point, it gives F, so I find this term, which by definition is what I called Mo. So I proved this term, so I proved the angular momentum theorem.

Notes

Summary

3m 23s